Mathematical Sciences and Applications E-Notes
4 (2) 52-57 (2016) ©MSAEN

New symmetric identities involving generalized Carlitz's twisted q-Bernoulli polynomials under S_5

Ugur Duran* and Mehmet Acikgoz

(Communicated by Hari Mohan SRIVASTAVA)

Abstract
In this paper, motivated by S. Araci, et al. [*Symmetric identities involving q-Frobenius-Euler polynomials under Sym (5)*, Turkish Journal of Analysis and Number Theory, Vol. 3, No. 3, pp. 90-93 (**2015**)], we obtain not only new but also some interesting identities for generalized Carlitz's twisted q-Bernoulli polynomials attached to χ using the bosonic p-adic q-integral on $\mathbb{Z}_p$ under symmetric group of degree five.

Keywords: *Symmetric identities; Generalized Carlitz's twisted q-Bernoulli polynomials; p-adic q-integral on $\mathbb{Z}_p$; Invariant under S_5.*

AMS Subject Classification (2010): *Primary: 05A19; 05A30 ; Secondary: 11B68; 11S80.*

****Corresponding author***

1. Introduction

As well known that the ordinary Bernoulli polynomials, $B_n(x)$, are defined by the following Taylor series expansion about $t = 0$:

$$\sum_{n=0}^{\infty} B_n(x)\frac{t^n}{n!} = \frac{t}{e^t - 1}e^{xt} \qquad (|t| < 2\pi). \tag{1.1}$$

Upon setting $x = 0$ in the Eq. (1.1), we have $B_n(0) := B_n$ popularly known as n-th Bernoulli number (see, e.g., [*2, 4, 7, 9-14*]).

Let $\mathbb{N}$ be the set of natural numbers, $\mathbb{N}^* = \mathbb{N} \cup \{0\}$ and p be a fixed odd prime number. Throughout the present paper, the symbols $\mathbb{Z}_p$, $\mathbb{Q}$, $\mathbb{Q}_p$ and $\mathbb{C}_p$ shall denote topological closure of $\mathbb{Z}$, the field of rational numbers, topological closure of $\mathbb{Q}$ and the field of p-adic completion of an algebraic closure of $\mathbb{Q}_p$, respectively. The normalized absolute value according to the theory of p-adic analysis is given by $|p|_p = p^{-1}$. The notation "q" can be considered as an indeterminate a complex number $q \in \mathbb{C}$ with $|q| < 1$, or a p-adic number $q \in \mathbb{C}_p$ with $|q-1|_p < p^{-\frac{1}{p-1}}$ and $q^x = \exp(x \log q)$ for $|x|_p \leq 1$. The q number of x (or q analog of x) is described as $[x]_q = \frac{1-q^x}{1-q}$. Clearly, $\lim_{q\to 1}[x]_q = x$ cf. [*2-14*].

For

$$g \in UD(\mathbb{Z}_p) = \{g \,|\, g : \mathbb{Z}_p \to \mathbb{C}_p \text{ is uniformly differentiable function}\},$$

the bosonic p-adic q-integral on $\mathbb{Z}_p$ of a function $g \in UD(\mathbb{Z}_p)$ is defined by Kim [7]:

$$I_q(g) = \int_{\mathbb{Z}_p} g(x)\, d\mu_q(x) = \lim_{N\to\infty} \frac{1}{[p^N]_q} \sum_{x=0}^{p^N-1} g(x)\, q^x. \tag{1.2}$$

Received : *15–March–2016,* ***Accepted :*** *26–May–2016*

Thus, in view of the Eq. (1.2), we have

$$qI_q(g_1) - I_q(g) = (q-1)\, f\,(0) + \frac{q-1}{\log q} f^{'}\,(0)$$

where $g_1(x) = g(x+1)$. For more details, one can take a close look at the references [3, 7, 12].

For $d \in \mathbb{N}$ with $(p, d) = 1$, let

$$X := X_d = \lim_{\overleftarrow{n}} \mathbb{Z}/dp^n\mathbb{Z} \text{ and } X_1 = \mathbb{Z}_p,$$

$$X^* = \bigcup_{\substack{0<a<dp \\ (a,p)=1}} (a + dp\mathbb{Z}_p)$$

and

$$a + dp^n\mathbb{Z}_p = \{x \in X \mid x \equiv a \pmod{dp^n}\}$$

where $a \in \mathbb{Z}$ lies in $0 \leq a < dp^n$.

Note that

$$\int_X g\,(x)\, d\mu_q\,(x) = \int_{\mathbb{Z}_p} g\,(x)\, d\mu_q\,(x)\,, \text{for } g \in UD\,(\mathbb{Z}_p)\,.$$

Let χ be a primitive Dirichlet's character with conductor $d \in \mathbb{N}$. One can find the wholly detailed study of Dirichlet's character χ in [1].

Let $T_p = \bigcup_{N\geq 1} C_{p^N} = \lim_{N\to\infty} C_{p^N}$, where $C_{p^N} = \left\{\zeta : \zeta^{p^N} = 1\right\}$ is the cyclic group of order p^N. For $\zeta \in T_p$, we indicate by $\phi_\zeta : \mathbb{Z}_p \to C_p$ the locally constant function $x \to \zeta^x$. For $q \in C_p$ with $|1-q|_p < 1$ and $\zeta \in T_p$, the Carlitz's generalized twisted q-Bernoulli polynomials attached to χ with Witt's formula are defined by the following bosonic p-adic q-integral on $\mathbb{Z}_p$, with respect to μ_q, in [12]:

$$\int_{\mathbb{Z}_p} \chi\,(y)\, \zeta^y\, [x+y]_q^n\, d\mu_q\,(y) = \beta_{n,\chi,q,\zeta}(x) \quad (n \geq 0)\,. \tag{1.3}$$

Substituting $x = 0$ into the Eq. (1.3) gives $\beta_{n,\chi,q,\zeta(0)} := \beta_{n,\chi,q,\zeta}$ that are called n-th Carlitz's generalized twisted q-Bernoulli number attached to χ.

By using the p-adic q-integral, we have

$$\beta_{n,\chi,q,\zeta}(x) = [d]_q^{n-1} \sum_{i=0}^{d-1} \chi\,(i)\, q^i \zeta^i \beta_{n,\chi,q,\zeta} \left(\frac{x+i}{d}\right).$$

In recent years, some investigations including Bernoulli numbers and polynomials and thier types have been studied and investigated by many mathematicians. For instance, Acikgoz *et al.* [2] introduced a new generalization of Bernoulli numbers based on the theory of the multiple q-calculus and they developed some theirs properties. Kim *et al.* [7] constructed q-Bernoulli numbers and polynomials by using p-adic q-integral equations on $\mathbb{Z}_p$. Also Kim [8] studied on the weighted q-Bernoulli numbers and polynomials. Kupershmidt [9] introduced reflection symmetries for the q-Bernoulli numbers and the Bernoulli polynomials. Luo *et al.* [10] gave q-extension of several well-known formulas in ordinary calculus. Additionaly, Mahmudov [11] introduced and investigated a class of Bernoulli polynomials based on q-integer. Furthermore, he derived q-analogues of some well-known formulas including Srivastava-Pinter addition theorem.

In the present paper, we consider the Carlitz's generalized twisted q-Bernoulli polynomials attached to χ and present some not only new but also interesting symmetric identities for these polynomials arising from the fermionic p-adic q-integral on $\mathbb{Z}_p$ under symmetric group of degree five shown by S_5.

2. Symmetric identities involving $\beta_{n,\chi,q,\zeta}(x)$ under S_5

Let $\zeta \in T_p$, $q \in \mathbb{C}_p$ with $|q-1|_p < 1$, $w_i \in \mathbb{N}$ be natural numbers where $i \in \mathbb{Z}$ lies in $1 \leq i \leq 5$ and χ be the trivial character. From Eqs. (1.2) and (1.3), we discover

$$\begin{aligned}
&\int_{\mathbb{Z}_p} \chi(y)\,\zeta^{w_1w_2w_3w_4y} \\
&\times e^{[w_1w_2w_3w_4y+w_1w_2w_3w_4w_5x+w_5w_4w_2w_3i+w_5w_4w_1w_3j+w_5w_4w_1w_2k+w_5w_3w_1w_2s]_q t} d\mu_{q^{w_1w_2w_3w_4}}(y) \\
&= \lim_{N\to\infty} \frac{1}{[p^N]_{q^{w_1w_2w_3w_4}}} \sum_{y=0}^{dp^N-1} \chi(y)\,\zeta^{w_1w_2w_3w_4y} q^{w_1w_2w_3w_4y} \\
&\times e^{[w_1w_2w_3w_4y+w_1w_2w_3w_4w_5x+w_5w_4w_2w_3i+w_5w_4w_1w_3j+w_5w_4w_1w_2k+w_5w_3w_1w_2s]_q t} \\
&= \lim_{N\to\infty} \frac{1}{[w_5dp^N]_{q^{w_1w_2w_3w_4}}} \sum_{l=0}^{dw_5-1} \sum_{y=0}^{p^N-1} \chi(l)\,\zeta^{w_1w_2w_3w_4(l+dw_5y)} q^{w_1w_2w_3w_4(l+dw_5y)} \\
&\times e^{[w_1w_2w_3w_4(l+dw_5y)+w_1w_2w_3w_4w_5x+w_5w_4w_2w_3i+w_5w_4w_1w_3j+w_5w_4w_1w_2k+w_5w_3w_1w_2s]_q t},
\end{aligned} \tag{2.1}$$

which gives

$$\begin{aligned}
&\frac{1}{[w_1w_2w_3w_4]_q} \sum_{i=0}^{dw_1-1} \sum_{j=0}^{dw_2-1} \sum_{k=0}^{dw_3-1} \sum_{s=0}^{dw_4-1} \zeta^{w_5w_4w_2w_3i+w_5w_4w_1w_3j+w_5w_4w_1w_2k+w_5w_3w_1w_2s} \\
&\times \chi(i)\,\chi(j)\,\chi(k)\,\chi(s)\,\chi(l)\, q^{w_5w_4w_2w_3i+w_5w_4w_1w_3j+w_5w_4w_1w_2k+w_5w_3w_1w_2s} \times \int_{\mathbb{Z}_p} \zeta^{w_1w_2w_3w_4y} \\
&\times e^{[w_1w_2w_3w_4y+w_1w_2w_3w_4w_5x+w_5w_4w_2w_3i+w_5w_4w_1w_3j+w_5w_4w_1w_2k+w_5w_3w_1w_2s]_q t} d\mu_{q^{w_1w_2w_3w_4}}(y) \\
&= \lim_{N\to\infty} \frac{1}{[w_1w_2w_3w_4w_5dp^N]_q} \sum_{i=0}^{dw_1-1} \sum_{j=0}^{dw_2-1} \sum_{k=0}^{dw_3-1} \sum_{s=0}^{dw_4-1} \sum_{l=0}^{dw_5-1} \sum_{y=0}^{p^N-1} \\
&\times \zeta^{w_1w_2w_3w_4(l+dw_5y)+w_5w_4w_2w_3i+w_5w_4w_1w_3j+w_5w_4w_1w_2k+w_5w_3w_1w_2s} \chi(i)\,\chi(j)\,\chi(k) \\
&\times \chi(s)\,\chi(l)\, q^{w_1w_2w_3w_4(l+dw_5y)+w_5w_4w_2w_3i+w_5w_4w_1w_3j+w_5w_4w_1w_2k+w_5w_3w_1w_2s} \\
&\times e^{[w_1w_2w_3w_4(l+dw_5y)+w_1w_2w_3w_4w_5x+w_5w_4w_2w_3i+w_5w_4w_1w_3j+w_5w_4w_1w_2k+w_5w_3w_1w_2s]_q t}.
\end{aligned} \tag{2.2}$$

Note that the equation (2.2) is invariant for any permutation $\sigma \in S_5$. Thus, we get the following theorem.

Theorem 2.1. *Let $\zeta \in T_p$, $q \in \mathbb{C}_p$ with $|q-1|_p < 1$, $w_i \in \mathbb{N}$ with $i \in \{1,2,3,4,5\}$ and χ be the trivial character. Then the following*

$$\begin{aligned}
&\sum_{i=0}^{dw_{\sigma(1)}-1} \sum_{j=0}^{dw_{\sigma(2)}-1} \sum_{k=0}^{dw_{\sigma(3)}-1} \sum_{s=0}^{dw_{\sigma(4)}-1} \frac{\chi(i)\,\chi(j)\,\chi(k)\,\chi(s)\,\chi(l)}{[w_{\sigma(1)}w_{\sigma(2)}w_{\sigma(3)}w_{\sigma(4)}]_q} \\
&\times \zeta^{w_{\sigma(5)}w_{\sigma(4)}w_{\sigma(2)}w_{\sigma(3)}i+w_{\sigma(5)}w_{\sigma(4)}w_{\sigma(1)}w_{\sigma(3)}j+w_{\sigma(5)}w_{\sigma(4)}w_{\sigma(1)}w_{\sigma(2)}k+w_{\sigma(5)}w_{\sigma(3)}w_{\sigma(1)}w_{\sigma(2)}s} \\
&\times q^{w_{\sigma(5)}w_{\sigma(4)}w_{\sigma(2)}w_{\sigma(3)}i+w_{\sigma(5)}w_{\sigma(4)}w_{\sigma(1)}w_{\sigma(3)}j+w_{\sigma(5)}w_{\sigma(4)}w_{\sigma(1)}w_{\sigma(2)}k+w_{\sigma(5)}w_{\sigma(3)}w_{\sigma(1)}w_{\sigma(2)}s} \\
&\times \int_{\mathbb{Z}_p} \chi(y) \exp\big([w_{\sigma(1)}w_{\sigma(2)}w_{\sigma(3)}w_{\sigma(4)}y + w_{\sigma(1)}w_{\sigma(2)}w_{\sigma(3)}w_{\sigma(4)}w_{\sigma(5)}x + w_{\sigma(5)}w_{\sigma(4)}w_{\sigma(2)}w_{\sigma(3)}i \\
&+w_{\sigma(5)}w_{\sigma(4)}w_{\sigma(1)}w_{\sigma(3)}j + w_{\sigma(5)}w_{\sigma(4)}w_{\sigma(1)}w_{\sigma(2)}k + w_{\sigma(5)}w_{\sigma(3)}w_{\sigma(1)}w_{\sigma(2)}s]_q t\big) \\
&\times \zeta^{w_{\sigma(1)}w_{\sigma(2)}w_{\sigma(3)}w_{\sigma(4)}y} d\mu_{q^{w_{\sigma(1)}w_{\sigma(2)}w_{\sigma(3)}w_{\sigma(4)}}}(y)
\end{aligned}$$

holds true for any $\sigma \in S_5$.

We observe that

$$\begin{aligned}
&[w_1w_2w_3w_4y + w_1w_2w_3w_4w_5x + w_5w_4w_2w_3i + w_5w_4w_1w_3j + w_5w_4w_1w_2k + w_5w_3w_1w_2s]_q \\
&= [w_1w_2w_3w_4]_q \left[y + w_5x + \frac{w_5}{w_1}i + \frac{w_5}{w_2}j + \frac{w_5}{w_3}k + \frac{w_5}{w_4}s\right]_{q^{w_1w_2w_3w_4}}.
\end{aligned} \tag{2.3}$$

From Eqs. (2.1) and (2.3), we obtain

$$\begin{aligned}&\int_{\mathbb{Z}_p}\chi(y)\,e^{[w_1w_2w_3w_4y+w_1w_2w_3w_4w_5x+w_5w_4w_2w_3i+w_5w_4w_1w_3j+w_5w_4w_1w_2k+w_5w_3w_1w_2s]_q t}\\&\times\zeta^{w_1w_2w_3w_4y}d\mu_{q^{w_1w_2w_3w_4}}(y)\\&=\sum_{n=0}^{\infty}[w_1w_2w_3w_4]_q^n\\&\times\left(\int_{\mathbb{Z}_p}\chi(y)\,\zeta^{w_1w_2w_3w_4y}\left[y+w_5x+\frac{w_5}{w_1}i+\frac{w_5}{w_2}j+\frac{w_5}{w_3}k+\frac{w_5}{w_4}s\right]_{q^{w_1w_2w_3ww_4}}^n d\mu_{q^{w_1w_2w_3w_4}}(y)\right)\frac{t^n}{n!}\\&=\sum_{n=0}^{\infty}[w_1w_2w_3w_4]_q^n\,\beta_{n,\chi,q^{w_1w_2w_3w_4},\zeta^{w_1w_2w_3w_4}}\left(w_5x+\frac{w_5}{w_1}i+\frac{w_5}{w_2}j+\frac{w_5}{w_3}k+\frac{w_5}{w_4}s\right)\frac{t^n}{n!}.\end{aligned}\tag{2.4}$$

By Eq. (2.4), we have

$$\begin{aligned}&\int_{\mathbb{Z}_p}[w_1w_2w_3w_4y+w_1w_2w_3w_4w_5x+w_5w_4w_2w_3i+w_5w_4w_1w_3j+w_5w_4w_1w_2k+w_5w_3w_1w_2s]_q^n\\&\times\chi(y)\,\zeta^{w_1w_2w_3w_4y}d\mu_{q^{w_1w_2w_3w_4}}(y)\\&=[w_1w_2w_3w_4]_q^n\,\beta_{n,\chi,q^{w_1w_2w_3w_4},\zeta^{w_1w_2w_3w_4}}\left(w_5x+\frac{w_5}{w_1}i+\frac{w_5}{w_2}j+\frac{w_5}{w_3}k+\frac{w_5}{w_4}s\right),\ (n\geq 0).\end{aligned}\tag{2.5}$$

Therefore, by Theorem 2.1 and Eq. (2.5), we have the following theorem.

Theorem 2.2. *Let $\zeta\in T_p$, $q\in\mathbb{C}_p$ with $|q-1|_p<1$, $w_i\in\mathbb{N}$ with $i\in\{1,2,3,4,5\}$ and χ be the trivial character. Then the following*

$$\begin{aligned}&\left[w_{\sigma(1)}w_{\sigma(2)}w_{\sigma(3)}w_{\sigma(4)}\right]_q^{n-1}\sum_{i=0}^{dw_{\sigma(1)}-1}\sum_{j=0}^{dw_{\sigma(2)}-1}\sum_{k=0}^{dw_{\sigma(3)}-1}\sum_{s=0}^{dw_{\sigma(4)}-1}\chi(i)\,\chi(j)\,\chi(k)\,\chi(s)\\&\times\zeta^{w_{\sigma(5)}w_{\sigma(4)}w_{\sigma(2)}w_{\sigma(3)}i+w_{\sigma(5)}w_{\sigma(4)}w_{\sigma(1)}w_{\sigma(3)}j+w_{\sigma(5)}w_{\sigma(4)}w_{\sigma(1)}w_{\sigma(2)}k+w_{\sigma(5)}w_{\sigma(3)}w_{\sigma(1)}w_{\sigma(2)}s}\\&\times q^{w_{\sigma(5)}w_{\sigma(4)}w_{\sigma(2)}w_{\sigma(3)}i+w_{\sigma(5)}w_{\sigma(4)}w_{\sigma(1)}w_{\sigma(3)}j+w_{\sigma(5)}w_{\sigma(4)}w_{\sigma(1)}w_{\sigma(2)}k+w_{\sigma(5)}w_{\sigma(3)}w_{\sigma(1)}w_{\sigma(2)}s}\\&\times\beta_{n,\chi,q^{w_{\sigma(1)}w_{\sigma(2)}w_{\sigma(3)}w_{\sigma(4)}},\zeta^{w_{\sigma(1)}w_{\sigma(2)}w_{\sigma(3)}w_{\sigma(4)}}}\left(w_{\sigma(5)}x+\frac{w_{\sigma(5)}}{w_{\sigma(1)}}i+\frac{w_{\sigma(5)}}{w_{\sigma(2)}}j+\frac{w_{\sigma(5)}}{w_{\sigma(3)}}k+\frac{w_{\sigma(5)}}{w_{\sigma(4)}}s\right)\end{aligned}$$

holds true for any $\sigma\in S_5$.

We obtain, by using the definition of $[x]_q$, that

$$\begin{aligned}&\left[y+w_5x+\frac{w_5}{w_1}i+\frac{w_5}{w_2}j+\frac{w_5}{w_3}k+\frac{w_5}{w_4}s\right]_{q^{w_1w_2w_3w_4}}^n\\&=\sum_{m=0}^{n}\binom{n}{m}\left(\frac{[w_5]_q}{[w_1w_2w_3w_4]_q}\right)^{n-m}[w_2w_3w_4i+w_1w_3w_4j+w_1w_2w_4k+w_1w_2w_3s]_{q^{w_5}}^{n-m}\\&\times q^{m(w_5w_4w_2w_3i+w_5w_4w_1w_3j+w_5w_4w_1w_2k+w_5w_3w_1w_2s)}\,[y+w_5x]_{q^{w_1w_2w_3w_4}}^m,\end{aligned}\tag{2.6}$$

which yields

$$\begin{aligned}&[w_1w_2w_3w_4]_q^{n-1}\sum_{i=0}^{dw_1-1}\sum_{j=0}^{dw_2-1}\sum_{k=0}^{dw_3-1}\sum_{s=0}^{dw_4-1}\zeta^{w_5w_4w_2w_3i+w_5w_4w_1w_3j+w_5w_4w_1w_2k+w_5w_3w_1w_2s}\\&\times q^{w_5w_4w_2w_3i+w_5w_4w_1w_3j+w_5w_4w_1w_2k+w_5w_3w_1w_2s}\chi(i)\,\chi(j)\,\chi(k)\,\chi(s)\\&\times\int_{\mathbb{Z}_p}\chi(y)\,\zeta^{w_1w_2w_3w_4y}\left[y+w_5x+\frac{w_5}{w_1}i+\frac{w_5}{w_2}j+\frac{w_5}{w_3}k+\frac{w_5}{w_4}s\right]_{q^{w_1w_2w_3w_4}}^n d\mu_{q^{w_1w_2w_3w_4}}(y)\\&=\sum_{m=0}^{n}\binom{n}{m}[w_1w_2w_3w_4]_q^{m-1}[w_5]_q^{n-m}\,\beta_{m,\chi,q^{w_1w_2w_3w_4},\zeta^{w_1w_2w_3w_4}}(w_5x)\,\overline{C}_{n,m,q^{w_5},\zeta^{w_5}}(w_1,w_2,w_3,w_4\mid\chi),\end{aligned}\tag{2.7}$$

where

$$\overline{C}_{n,m,q,\zeta}(w_1,w_2,w_3,w_4 \mid \chi) \tag{2.8}$$
$$= \sum_{i=0}^{dw_1-1}\sum_{j=0}^{dw_2-1}\sum_{k=0}^{dw_3-1}\sum_{s=0}^{dw_4-1} \zeta^{w_2w_3w_4i+w_1w_3w_4j+w_1w_2w_4k+w_1w_2w_3s}\chi(i)\chi(j)\chi(k)\chi(s)$$
$$\times\, q^{(m+1)(w_2w_3w_4i+w_1w_3w_4j+w_1w_2w_4k+w_1w_2w_3s)}\left[w_2w_3w_4i+w_1w_3w_4j+w_1w_2w_4k+w_1w_2w_3s\right]_q^{n-m}.$$

As a result, by (2.8), we arrive at the following theorem.

Theorem 2.3. *Let $\zeta \in T_p$, $q \in \mathbb{C}_p$ with $|q-1|_p < 1$, $w_i \in \mathbb{N}$ with $i \in \{1,2,3,4,5\}$ and χ be the trivial character. For $n \geq 0$, the following expression*

$$\sum_{m=0}^{n}\binom{n}{m}\left[w_{\sigma(1)}w_{\sigma(2)}w_{\sigma(3)}w_{\sigma(4)}\right]_q^{m-1}\left[w_{\sigma(5)}\right]_q^{n-m}$$
$$\times\beta_{m,\chi,q^{w_{\sigma(1)}w_{\sigma(2)}w_{\sigma(3)}w_{\sigma(4)}},\zeta^{w_{\sigma(1)}w_{\sigma(2)}w_{\sigma(3)}w_{\sigma(4)}}}\left(w_{\sigma(5)}x\right)$$
$$\times\overline{C}_{n,m,q^{w_{\sigma(5)}},\zeta^{w_{\sigma(5)}}}\left(w_{\sigma(1)},w_{\sigma(2)},w_{\sigma(3)},w_{\sigma(4)} \mid \chi\right)$$

holds true for some $\sigma \in S_5$.

References

[1] T. M. Apostol, *Introduction to Analytic Number Theory*, New York; Splinger-Verlag, **(1976).**

[2] M. Acikgoz, S. Araci, U.Duran, *New extensions of some known special polynomials under the theory of multiple q-calculus*, Turkish Journal of Analysis and Number Theory, Vol. 3, No. 5, (**2015**) pages 128-139.

[3] S. Araci, U. Duran, M. Acikgoz, *Symmetric identities involving q-Frobenius-Euler polynomials under Sym (5)*, Turkish Journal of Analysis and Number Theory, Vol. 3, No. 3, pp. 90-93 (**2015**).

[4] J. Choi, P. J. Anderson, H. M. Srivastava, *Carlitz's q-Bernoulli and q-Euler numbers and polynomials and a class of generalized q-Hurwitz zeta functions*, Applied Mathematics and Computation, Vol. 215, Issue 3, October (**2009)**, pp. 1185–1208.

[5] J. Choi, P. J. Anderson and H. M. Srivastava, *Some q-extensions of the Apostol-Bernoulli and the Apostol-Euler polynomials of order n, and the multiple Hurwitz Zeta function*, Applied Mathematics and Computation, 199 (**2008**), 723-737

[6] U. Duran, M. Acikgoz, S. Araci, *Symmetric identities involving weighted q-Genocchi polynomials under S_4*, Proceedings of the Jangjeon Mathematical Society,18 (**2015**), No. 4, pp 455-465.

[7] T. Kim, *q-Volkenborn integration*, Russian Journal of Mathematical Physics 9.3 (**2002**), pp. 288-299.

[8] T. Kim, *On the weighted q-Bernoulli numbers and polynomials*, Advances Studies Contemporary Mathematics, 21 (**2011**), no. 2, 231-236.

[9] B. A. Kupershmidt, *Reflection Symmetries of q-Bernoulli Polynomials*, Journal of Nonlinear Mathematical Physics, 12 (Suppl. 1), 412-422 (**2005**).

[10] Q.-M. Luo, H. M. Srivastava, *q-extension of some relationships between the Bernoulli and Euler polynomials*, Taiwanese Journal of Mathematics, Vol. 15, No. 1 (**2011**), pp. 241-257.

[11] N. I. Mahmudov, *On a class of q-Benoulli and q-Euler polynomials*, Advance in Difference Equations., (**2013)** 2013:108.

[12] C. S. Ryoo, *Symmetric identities for Carlitz's generalized twisted q-Bernoulli numbers and polynomials associated with p-adic q-integral on $\mathbb{Z}_p$*, International Mathematical Forum, Vol. 10 (**2015)**, no. 9, 435-441.

[13] H. M. Srivastava, *Some generalizations and basic (or q-) extensions of the Bernoulli, Euler and Genocchi polynomials*, Applied Mathematics & Information Sciences. 5 (**2011**), 390-444.

[14] H. M. Srivastava, T. Kim, Y. Simsek *q-Bernoulli numbers and polynomials associated with multiple q-zeta functions and basic L-series*, Russian Journal of Mathematical Physics, 12 (**2005**), 241-268.

Affiliations

UĞUR DURAN
ADDRESS: Department of Mathematics, Faculty of Arts and Science, University of Gaziantep, TR-27310 Gaziantep, Turkey.
E-MAIL: duran.ugur@yahoo.com

MEHMET AÇIKGÖZ
ADDRESS: Department of Mathematics, Faculty of Arts and Science, University of Gaziantep, TR-27310 Gaziantep, Turkey.
E-MAIL: acikgoz@gantep.edu.tr