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Smarandache Curves of Mannheim Curve Couple According to Frenet Frame

Süleyman Şenyurt* and Abdussamet Çalışkan

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Abstract
In this paper, when the Frenet vectors of the partner curve of Mannheim curve are taken as the position vectors, the curvature and the torsion of Smarandache curves are calculated. These values are expressed depending upon the Mannheim curve. Besides, we illustrate example of our main results.

Keywords: *Mannheim curve, Mannheim partner curve, Smarandache Curves, Frenet invariants*

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****Corresponding author***

1. Introduction

A regular curve in Minkowski space-time, whose position vector is composed by Frenet frame vectors on another regular curve, is called a Smarandache curve [10]. Special Smarandache curves have been studied by some authors .
Melih Turgut and Süha Yılmaz studied a special case of such curves and called it Smarandache TB_2 curves in the space E_1^4 [10]. Ahmad T.Ali studied some special Smarandache curves in the Euclidean space. He studied Frenet-Serret invariants of a special case [1]. Muhammed Çetin , Yılmaz Tunçer and Kemal Karacan investigated special Smarandache curves according to Bishop frame in Euclidean 3-Space and they gave some differential goematric properties of Smarandache curves, also they found the centers of the osculating spheres and curvature spheres of Smarandache curves [5]. Şenyurt and Çalışkan investigated special Smarandache curves in terms of Sabban frame of spherical indicatrix curves and they gave some characterization of Smarandache curves.[4] Özcan Bektaş and Salim Yüce studied some special Smarandache curves according to Darboux Frame in E^3 [2]. Nurten Bayrak, Özcan Bektaş and Salim Yüce studied some special Smarandache curves in E_1^3 [3].

In this paper, special Smarandache curves belonging to α^* Mannheim partner curve such as T^*N^*, N^*B^*, T^*B^* and $T^*N^*B^*$ drawn by Frenet frame are defined and some related results are given.

2. Preliminaries

The Euclidean 3-space E^3 be inner product given by

$$\langle , \rangle = x_1^2 + x_2^3 + x_3^2$$

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where $(x_1, x_2, x_3) \in E^3$. Let $\alpha : I \to E^3$ be a unit speed curve denote by $\{T, N, B\}$ the moving Frenet frame . For an arbitrary curve $\alpha \in E^3$, with first and second curvature, κ and τ respectively, the Frenet formulae is given by [6], [9]

$$\begin{cases} T' = \kappa N \\ N' = -\kappa T + \tau B \\ B' = -\tau N. \end{cases} \tag{2.1}$$

Let $\alpha : I \to \mathbb{E}^3$ and $\alpha^* : I \to \mathbb{E}^3$ be the C^2- class differentiable unit speed two curves and let $\{T(s), N(s), B(s)\}$ and $\{T^*(s), N^*(s), B^*(s)\}$ be the Frenet frames of the curves α and α^*, respectively. If the principal normal vector N of the curve α is linearly dependent on the binormal vector B of the curve α^*, then (α) is called a Mannheim curve and (α^*) a Mannheim partner curve of (α). The pair (α, α^*) is said to be Mannheim pair [7], [8]. The relations between the Frenet frames $\{T(s), N(s), B(s)\}$ and $\{T^*(s), N^*(s), B^*(s)\}$ are as follows:

$$\begin{cases} T^* = \cos\theta T - \sin\theta B \\ N^* = \sin\theta T + \cos\theta B \\ B^* = N \end{cases} \tag{2.2}$$

$$\begin{cases} \cos\theta = \dfrac{ds*}{ds} \\ \sin\theta = \lambda\tau^* \dfrac{ds*}{ds} \end{cases} . \tag{2.3}$$

where $\angle(T, T^*) = \theta$, [8].

Theorem 2.1. *The distance between corresponding points of the Mannheim partner curves in $\mathbb{E}^3$ is constant, [7].*

Theorem 2.2. *Let (α, α^*) be a Mannheim pair curves in $\mathbb{E}^3$. For the curvatures and the torsions of the Mannheim curve pair (α, α^*) we have,*

$$\begin{cases} \kappa = \tau^* \sin\theta \dfrac{ds*}{ds} \\ \\ \tau = -\tau^* \cos\theta \dfrac{ds*}{ds} \end{cases} \tag{2.4}$$

and

$$\begin{cases} \kappa^* = \dfrac{d\theta}{ds*} = \theta' \dfrac{\kappa}{\lambda\tau\sqrt{\kappa^2 + \tau^2}} \\ \\ \tau^* = (\kappa \sin\theta - \tau\cos\theta) \dfrac{ds*}{ds} \end{cases} \tag{2.5}$$

Theorem 2.3. *Let (α, α^*) be a Mannheim pair curves in $\mathbb{E}^3$. For the torsions τ^* of the Mannheim partner curve α^* we have*

$$\tau^* = \frac{\kappa}{\lambda\tau}$$

3. Smarandache Curves of Mannheim Curve Couple According to Frenet Frame

Let (α, α^*) be a Mannheim pair curves in E^3 and $\{T^* N^* B^*\}$ be the Frenet frame of the Mannheim partner curve α^* at $\alpha^*(s)$. In this case, T^*N^* - Smarandache curve can be defined by

$$\beta_1(s) = \frac{1}{\sqrt{2}}(T^* + N^*). \tag{3.1}$$

Solving the above equation by substitution of T^* and N^* from (2.2), we obtain

$$\beta_1(s) = \frac{(\cos\theta + \sin\theta)T + (\cos\theta - \sin\theta)B}{\sqrt{2}}. \tag{3.2}$$

The derivative of this equation with respect to s is as follows,

$$\beta_1' = T_{\beta_1}\frac{ds_{\beta_1}}{ds} = \frac{\theta'\kappa(\sin\theta-\cos\theta)T+\kappa\sqrt{\kappa^2+\tau^2}N+\kappa(\theta'\sin\theta+\theta'\cos\theta)B}{\sqrt{2\tau^2\lambda^2\|W\|}} \tag{3.3}$$

and by substitution, we get

$$T_{\beta_1}(s) = \frac{\theta'\kappa(\sin\theta-\cos\theta)T+\kappa\sqrt{\kappa^2+\tau^2}N+\kappa(\theta'\sin\theta+\theta'\cos\theta)B}{\sqrt{\kappa^2(2\theta'^2+\kappa^2+\tau^2)}} \tag{3.4}$$

where

$$\frac{ds_{\beta_1}}{ds} = \sqrt{\frac{\kappa^2(2\theta'^2+\kappa^2+\tau^2)}{2\tau^2\lambda^2\|W\|}}. \tag{3.5}$$

In order to determine the first curvature and the principal normal of the curve $\beta_1(s)$, we formalize

$$T'_{\beta_1}(s) = \sqrt{2}\frac{\lambda^4\tau^4(\kappa^2+\tau^2)^2}{\kappa^4(2\theta'^2+\kappa^2+\tau^2)^2}\Big[(\bar{\ell}_1\cos\theta+\bar{\ell}_2\sin\theta)T+\bar{\ell}_3N+(\bar{\ell}_2\cos\theta-\bar{\ell}_1\sin\theta)B\Big], \tag{3.6}$$

where

$$\begin{cases} \bar{\ell}_1 = -\dfrac{\theta'^2\kappa^4(2\theta'^2+\kappa^2+\tau^2)}{\lambda^4\tau^4(\kappa^2+\tau^2)^2} - \dfrac{\kappa^3(\theta''\kappa^2+\theta''\tau^2-\theta'\kappa\kappa'-\theta'\tau\tau')}{\lambda^3\tau^3(\kappa^2+\tau^2)^{\frac{3}{2}}} \\ \bar{\ell}_2 = -\dfrac{\kappa^3(\kappa^3\sqrt{\kappa^2+\tau^2}+\kappa\tau^2\sqrt{\kappa^2+\tau^2}-\theta''\lambda\tau\kappa^2-\theta''\lambda\tau^3+\theta'\lambda\tau\kappa\kappa'+\theta'\lambda\tau^2\tau')}{\lambda^4\tau^4(\kappa^2+\tau^2)^{\frac{3}{2}}} \\ \quad -\dfrac{\theta'^2\kappa^4(2\theta'^2+3\kappa^2+3\tau^2)}{\lambda^4\tau^4(\kappa^2+\tau^2)^2} \\ \bar{\ell}_3 = \dfrac{\theta'\kappa^4(2\theta'^2+\kappa^2+\tau^2)}{\lambda^4\tau^4(\kappa^2+\tau^2)^{\frac{3}{2}}} - \dfrac{2\theta'\kappa^3(\theta''\kappa^2+\theta''\tau^2-\theta'\kappa\kappa'-\theta'\tau\tau')}{\lambda^3\tau^3(\kappa^2+\tau^2)^2}. \end{cases}$$

The first curvature is

$$\kappa_{\beta_1} = \|T'_{\beta_1}\|,$$

$$\kappa_{\beta_1} = \sqrt{2}\frac{\lambda^4\tau^4(\kappa^2+\tau^2)^2}{\kappa^4(2\theta'^2+\kappa^2+\tau^2)^2}\sqrt{\bar{\ell}_1^2+\bar{\ell}_2^2+\bar{\ell}_3^2}.$$

The principal normal vector field and the binormal vector field are respectively given by

$$N_{\beta_1} = \frac{(\bar{\ell}_1\cos\theta+\bar{\ell}_2\sin\theta)T+\bar{\ell}_3N+(\bar{\ell}_2\cos\theta-\bar{\ell}_1\sin\theta)B}{\sqrt{\bar{\ell}_1^2+\bar{\ell}_2^2+\bar{\ell}_3^2}}, \tag{3.7}$$

$$\begin{aligned} B_{\beta_1} = {} & \frac{(\bar{\ell}_2\cos\theta-\bar{\ell}_1\sin\theta)\kappa(\|W\|+\theta'\sin\theta)-\bar{\ell}_3\kappa\theta'\cos\theta}{\sqrt{\kappa^2(\kappa^2+\tau^2+2\theta'^2)(\bar{\ell}_1^2+\bar{\ell}_2^2+\bar{\ell}_3^2)}}\mathbf{T} \\ & +\frac{(\bar{\ell}_1\cos\theta+\bar{\ell}_2\sin\theta)\kappa\theta'\cos\theta-(\bar{\ell}_2\cos\theta-\bar{\ell}_1\sin\theta)\theta'\kappa(\sin\theta-\cos\theta)}{\sqrt{\kappa^2(\kappa^2+\tau^2+2\theta'^2)(\bar{\ell}_1^2+\bar{\ell}_2^2+\bar{\ell}_3^2)}}\mathbf{N} \\ & +\frac{\bar{\ell}_3\theta'\kappa(\sin\theta-\cos\theta)-(\bar{\ell}_1\cos\theta+\bar{\ell}_2\sin\theta)\kappa(\|W\|+\theta'\sin\theta)}{\sqrt{\kappa^2(\kappa^2+\tau^2+2\theta'^2)(\bar{\ell}_1^2+\bar{\ell}_2^2+\bar{\ell}_3^2)}}\mathbf{B}. \end{aligned} \tag{3.8}$$

In order to calculate the torsion of the curve β_1, we differentiate

$$\begin{aligned}
\beta_1'' = & \frac{1}{\sqrt{2}\lambda^3\tau^2\|W\|^2}\Big[\big(-\kappa^4\|W\|\sin\theta-\theta'^2\kappa^2\|W\|\cos\theta-\theta'^2\kappa^2\|W\|\sin\theta \\
& -\kappa^2\tau^2\|W\|\sin\theta+2\kappa\tau^2\theta'\lambda\tau'\cos\theta-2\kappa\tau^2\theta'\lambda\tau'\sin\theta-\kappa\tau^3\theta'\lambda'\sin\theta-\kappa\tau^3\lambda\theta''\cos\theta \\
& +\kappa\tau^3\theta'\lambda'\cos\theta+\tau^3\theta'\lambda\kappa'\sin\theta-\tau^3\theta'\lambda\kappa'\cos\theta-\kappa^3\theta'\lambda\tau'\sin\theta+\kappa\tau^3\lambda\theta''\sin\theta \\
& -\kappa^3\tau\lambda\theta''\cos\theta-\kappa^3\tau\theta'\lambda'\sin\theta+\kappa^3\tau\theta'\lambda'\cos\theta+\kappa^3\theta'\lambda\tau'\cos\theta+\kappa^3\tau\lambda\theta''\sin\theta\big)\mathbf{T}+ \\
& \big(\kappa^2\tau^2\theta'+\kappa^4\theta'-\|W\|\kappa\tau^3\lambda'+\|W\|\tau^3\lambda\kappa'-\|W\|\kappa^3\tau\lambda'-\|W\|\kappa^3\tau'+\|W\|\kappa^2\tau\lambda\kappa' \\
& -\|W\|\kappa\tau^2\tau'\big)\mathbf{N}+\big(-\kappa^4\|W\|\cos\theta-\theta'^2\kappa^2\|W\|\cos\theta+\theta'^2\kappa^2\|W\|\sin\theta \\
& -\kappa^2\tau^2\|W\|\cos\theta-2\kappa\tau^2\theta'\lambda\tau'\sin\theta-2\kappa\tau^2\theta'\lambda\tau'\cos\theta-\kappa\tau^3\theta'\lambda'\sin\theta+\kappa\tau^3\lambda\theta''\sin\theta \\
& +\kappa\tau^3\lambda\theta''\cos\theta+\tau^3\theta'\lambda\kappa'\sin\theta+\tau^3\theta'\lambda\kappa'\cos\theta-\kappa^3\tau\theta'\lambda'\cos\theta-\kappa^3\theta'\lambda\tau'\sin\theta \\
& +\kappa^3\tau\lambda\theta''\cos\theta-\kappa^3\theta'\lambda\tau'\cos\theta-\kappa^3\tau\theta'\lambda'\sin\theta+\kappa^3\tau\lambda\theta''\sin\theta-\kappa\tau^3\theta'\lambda'\cos\theta\big)\mathbf{B}\Big].
\end{aligned}$$

and thus

$$\beta_1''' = \frac{(\bar{j}_1\cos\theta+\bar{j}_2\sin\theta)T+\bar{j}_3N+(-\bar{j}_1\sin\theta+\bar{j}_2\cos\theta)B}{\sqrt{2}}$$

where

$$\begin{cases}
\bar{j}_1 = \dfrac{2\kappa^5\tau^2\theta'\kappa^3\tau^4\theta'+\kappa^3\tau^2\theta'^3-3\kappa^4\tau\theta'\lambda\theta''\sqrt{\kappa^2+\tau^2}-3\kappa^2\tau^3\theta'\lambda\theta''\sqrt{\kappa^2+\tau^2}}{\lambda^3\tau^3\|W\|^5} \\
\quad -\dfrac{3\kappa\tau^3\theta'^2\lambda\kappa'\sqrt{\kappa^2+\tau^2}-2\kappa^2\tau^4\theta'\lambda\kappa'\lambda'+6\kappa^3\tau^3\theta'\lambda\lambda'\tau'+4\kappa\tau^5\theta'\lambda\lambda'\tau'}{\lambda^3\tau^3\|W\|^5} \\
\quad -\dfrac{2\kappa^5\tau\theta'\lambda\lambda'\tau'-6\theta'^2\kappa^2\sqrt{\kappa^2+\tau^2}\tau^2\lambda\tau'+2\kappa^2\tau^3\theta'\lambda^2\kappa'\tau'-3\kappa^3\tau^3\theta'\lambda^2\tau''}{\lambda^3\tau^3\|W\|^5} \\
\quad +\dfrac{2\kappa\tau^5\theta'\lambda^2\tau''-\kappa^2\tau^4\theta'\lambda^2\kappa'-2\kappa^2\tau^4\lambda^2\kappa'\theta'+2\kappa^5\tau^2\lambda\lambda'\theta''+4\kappa^3\tau^4\lambda\lambda'\theta''}{\lambda^3\tau^3\|W\|^5} \\
\quad +\dfrac{2\kappa\tau^6\lambda\lambda'\theta''+2\kappa^5\tau\lambda^2\tau'\theta''+6\kappa^3\tau^3\lambda^2\tau''\theta''-2\kappa\tau^6\theta'\lambda'^2-2\kappa^5\theta'\lambda^2\tau'^2}{\lambda^3\tau^3\|W\|^5} \\
\quad +\dfrac{4\tau^5\lambda^2\tau'\theta''+\kappa^5\tau^2\theta'\lambda\lambda''+2\kappa^3\tau^4\theta'\lambda\lambda''+\kappa\tau^6\theta'\lambda\lambda''+\kappa^5\tau\theta'\lambda^2\tau''+\kappa^5\theta'^3}{\lambda^3\tau^3\|W\|^5} \\
\quad +\dfrac{3\kappa^4\theta'^2\lambda\tau'\sqrt{\kappa^2+\tau^2}+3\kappa^4\tau\theta'^2\lambda'\sqrt{\kappa^2+\tau^2}+3\kappa^2\tau^3\theta'^2\lambda'\sqrt{\kappa^2+\tau^2}}{\lambda^3\tau^3\|W\|^5} \\
\quad +\dfrac{2\tau^6\theta'\lambda\kappa'\lambda'+4\tau^5\theta'\lambda^2\kappa'\tau'-5\kappa^3\tau^2\theta'\lambda^2\tau'^2-6\kappa\tau^4\theta'\lambda^2\tau'^2+3\kappa\tau^4\theta'\lambda^2\kappa'^2}{\lambda^3\tau^3\|W\|^5} \\
\quad +\dfrac{\kappa^7\theta'-\tau^6\theta'\lambda^2\kappa''-\kappa^5\tau^2\lambda^2\theta'''-2\kappa^3\tau^4\lambda^2\theta'''-\kappa\tau^6\lambda^2\theta'''-2\tau^6\lambda^2\kappa'\theta''}{\lambda^3\tau^3\|W\|^5} \\
\quad -\dfrac{2\kappa^5\tau^2\theta'\lambda'^2+4\kappa^3\tau^4\theta'\lambda'^2}{\lambda^3\tau^3\|W\|^5}
\end{cases}$$

$$
\begin{cases}
\bar{\jmath}_2 = -\dfrac{3\kappa^5\tau\lambda\kappa'\|W\| + 6\kappa^3\tau^3\lambda\kappa'\|W\| + 3\kappa\tau^5\lambda\kappa'\|W\| - 6\kappa^4\tau^2\tau'\lambda\|W\|}{\lambda^3\tau^3\|W\|^5} \\[2ex]
\quad + \dfrac{3\kappa^2\tau^4\tau'\lambda\|W\| - \kappa\tau^6\theta'\lambda\lambda'' - \kappa^5\tau\theta'\lambda^2\tau'' + \kappa^2\tau^4\theta'\lambda^2\kappa'' + 3\kappa^3\tau^3\theta'\lambda^2\tau''}{\lambda^3\tau^3\|W\|^5} \\[2ex]
\quad - \dfrac{2\kappa^2\tau^4\lambda^2\kappa'\theta'' - 2\kappa\tau^5\theta'\lambda^2\tau'' - 6\kappa^3\tau^3\lambda^2\tau'\theta'' - 6\kappa\tau^4\theta'\lambda^2\tau'^2 + 2\kappa^2\tau^4\theta'\lambda\kappa'\lambda'}{\lambda^3\tau^3\|W\|^5} \\[2ex]
\quad - \dfrac{4\kappa\tau^5\lambda^2\tau'\theta'' + 2\kappa^5\tau^2\lambda\lambda'\theta'' + 4\kappa^3\tau^4\lambda\lambda'\theta'' + 2\kappa\tau^6\lambda\lambda'\theta'' + 2\tau^6\theta'\lambda\kappa'\lambda'}{\lambda^3\tau^3\|W\|^5} \\[2ex]
\quad - \dfrac{2\kappa^5\tau\lambda^2\tau'\theta'' + \kappa^5\tau^2\theta'\lambda\lambda'' + 2\kappa^3\tau^4\theta'\lambda\lambda'' + 3\kappa^4\theta'\tau\theta''\lambda\|W\|}{\lambda^3\tau^3\|W\|^5} \\[2ex]
\quad - \dfrac{3\kappa^2\theta'\tau^3\theta''\lambda\|W\| + 3\kappa\theta'^2\tau^3\lambda\kappa'\|W\| + \kappa^5\theta'^3 + \kappa^7\theta' + 2\kappa^5\tau^2\theta' + \kappa^3\tau^4\theta'}{\lambda^3\tau^3\|W\|^5} \\[2ex]
\quad - \dfrac{\kappa^3\tau^2\theta'^3 - 3\kappa^6\tau'\lambda\|W\| - 3\kappa^6\tau\lambda'\|W\| - 6\kappa^4\tau^3\lambda'\|W\| - 3\kappa^2\tau^5\lambda'\|W\|}{\lambda^3\tau^3\|W\|^5} \\[2ex]
\quad + \dfrac{\kappa^5\tau^2\lambda^2\theta''' + 2\kappa^3\tau^4\lambda^2\theta''' + \kappa\tau^6\lambda^2\theta''' + \tau^6\theta'\lambda^2\kappa'' + 2\tau^6\lambda^2\kappa'\theta'' + 2\kappa^5\tau^2\theta'\lambda'^2}{\lambda^3\tau^3\|W\|^5} \\[2ex]
\quad + \dfrac{4\kappa^3\tau^4\theta'\lambda'^2 + 2\kappa\tau^6\theta\lambda'^2 + 2\kappa^5\theta'\lambda^2\tau'^2 + 3\kappa^4\theta'^2\tau\lambda'\|W\| + 2\kappa^2\tau^3\theta'\lambda^2\kappa'\tau'}{\lambda^3\tau^3\|W\|^5} \\[2ex]
\quad + \dfrac{3\kappa^2\theta'^2\tau^3\lambda'\|W\| + 3\kappa^4\theta'^2\lambda\tau'\|W\| + 5\kappa^3\tau^2\theta'\lambda^2\tau'^2 - 3\kappa\tau^4\theta'\lambda^2\kappa'^2 - 4\tau^5\theta'\lambda^2\kappa'\tau'}{\lambda^3\tau^3\|W\|^5} \\[2ex]
\quad + \dfrac{6\kappa^3\tau^3\theta'\lambda\lambda'\tau' + 4\kappa\tau^5\theta'\lambda\lambda'\tau' + 2\kappa^5\tau\theta'\lambda\lambda'\tau' + 6\theta'^2\kappa^2\|W\|\tau^2\lambda\tau'}{\lambda^3\tau^3\|W\|^5}
\end{cases}
$$

$$
\begin{cases}
\bar{\jmath}_3 = -\dfrac{5\theta'\kappa^2\lambda\tau^2\tau' - 3\theta'\kappa\lambda\tau^3\kappa' - \theta'\kappa^3\lambda\tau\kappa' + 2\lambda\tau^4\kappa\lambda'\|W\| + 2\lambda^2\tau^3\kappa'\tau'\|W\| + \kappa^5\|W\|}{\lambda^3\tau^3\|W\|^3} \\[2ex]
\quad - \dfrac{2\kappa\lambda^2\tau^2\tau'^2\|W\| - \kappa^3\tau^2\|W\| - \theta'^2\kappa^3\|W\| + 2\kappa^2\lambda^2\tau\kappa'\tau'\|W\| + 2\kappa^2\lambda\tau^2\kappa'\lambda'\|W\|}{\lambda^3\tau^3\|W\|^3} \\[2ex]
\quad + \dfrac{2\kappa^2\lambda\tau^3\theta'' + \lambda^2\tau^4\kappa''\|W\| - 3\theta'\kappa^4\lambda\tau' - 3\theta'\kappa^4\tau\lambda' - 3\theta'\kappa^2\tau^3\lambda' + 2\kappa^3\tau^2\lambda'^2\|W\|}{\lambda^3\tau^3\|W\|^3} \\[2ex]
\quad + \dfrac{2\kappa\tau^4\lambda'^2\|W\| + 2\kappa^3\lambda^2\tau'^2\|W\| + \kappa^2\lambda^2\tau^2\kappa''\|W\| + 2\kappa^3\lambda\tau\lambda'\tau'\|W\| + 2\kappa\lambda\tau^3\lambda'\tau'\|W\|}{\lambda^3\tau^3\|W\|^3} \\[2ex]
\quad - \dfrac{\kappa^3\lambda\tau^2\lambda''\|W\| + \kappa\lambda\tau^4\lambda''\|W\| + \kappa^3\lambda^2\tau\tau''\|W\| + \kappa\lambda^2\tau^3\tau''\|W\| + 2\kappa^4\lambda\tau\theta''}{\lambda^3\tau^3\|W\|^3}.
\end{cases}
$$

The torsion is then given by

$$
\tau_{\beta_1} = \frac{det(\beta_1', \beta_1'', \beta_1''')}{\|\beta_1' \wedge \beta_1''\|^2} \quad ,
$$

$$\tau_{\beta_1} = \frac{\begin{array}{l}\sqrt{2}\Big[\kappa^5\theta'\bar{j}_3 + \kappa^5\|W\|\bar{j}_1 + 2\kappa^4\theta\tau\lambda'\bar{j}_1 + 2\kappa^4\theta'\lambda\tau'\bar{j}_1 - \kappa^4\tau\lambda\theta''\bar{j}_2 - \kappa^4\tau\lambda\theta''\bar{j}_1 \\ +\bar{j}_2\kappa^3\theta'\kappa'\lambda\tau - \kappa^3\theta'\tau\lambda\kappa'\bar{j}_1 + \theta'^3\kappa^3\bar{j}_3 + \kappa^3\theta'^2\bar{j}_1\|W\| + \kappa^3\tau^2\theta'\bar{j}_3 + \kappa^3\theta'^2\|W\| \\ +\kappa^3\tau^2\|W\|\bar{j}_1 + 2\kappa^2\tau^3\theta'\lambda'\bar{j}_1 + \bar{j}_2\kappa^2\theta'\tau'\lambda\tau^2 + 3\kappa^2\theta'\tau'\lambda\tau^2\bar{j}_1 - \kappa^2\tau^3\lambda\theta''\bar{j}_2 \\ -\kappa^2\tau^3\lambda\theta''\bar{j}_1 + \theta'^2\kappa^2\lambda\tau\|W\| - 2\kappa\tau^3\theta'\lambda\kappa'\bar{j}_1\Big]\lambda^3\tau^3\|W\|^3\end{array}}{\begin{array}{l}\Big[2\kappa^8\tau^2\lambda^2\theta''^2 + 4\kappa^6\lambda^2\tau^4\theta''^2 + 2\kappa^4\lambda^2\tau^6\theta''^2 + 16\kappa^8\tau^2\theta'^2 + 8\kappa^6\tau^4\theta'^2 \\ +5\kappa^6\tau^2\theta'^4 - 4\kappa^7\tau^2\theta'\lambda^2\kappa'\theta'' - 4\kappa^6\tau^3\theta'\lambda^2\tau'\theta'' - 4\kappa^5\tau^4\theta'\lambda^2\kappa'\theta'' + \kappa^6\theta'^6 \\ +4\kappa^5\tau^3\theta'^2\lambda^2\kappa'\tau' - 4\kappa'^4\tau^5\theta'\lambda^2\tau'\theta'' - 4\kappa^9\tau\theta'\lambda\theta'' - 8\kappa^7\tau^3\theta'\lambda\theta'' + 4\kappa^{12} \\ -2\kappa^7\tau\theta'^3\lambda\theta'' - 4\kappa^5\tau^5\theta'\lambda\theta'' - 2\kappa^5\tau^3\theta'^3\lambda\theta'' + 2\kappa^6\tau^2\theta'^2\lambda^2\kappa'^2 + 5\kappa^8\theta'^4 \\ +4\kappa^8\theta'^2\kappa'\lambda\tau + 4\kappa^7\theta'^2\tau'\lambda\tau^2 + 4\kappa^6\tau^3\theta'^2\lambda\kappa' + 2\kappa^6\tau\theta'^4\lambda\kappa' + 4\kappa^5\tau^4\theta'^2\lambda\tau' \\ +2\kappa^5\tau^2\theta'^4\lambda\tau' + 12\kappa^{10}\tau^2 + 8\kappa^{10}\theta'^2 + 12\kappa^8\tau^4 + 4\kappa^6\tau^6 + 2\kappa^4\tau^4\theta'^2\lambda^2\tau'^2\Big]\end{array}}.$$

N^*B^* - Smarandache curve can be defined by

$$\beta_2(s) = \frac{1}{\sqrt{2}}(N^* + B^*). \tag{3.9}$$

Solving the above equation by substitution of N^* and B^* from (2.2), we obtain

$$\beta_2(s) = \frac{\sin\theta T + N + \cos\theta B}{\sqrt{2}}. \tag{3.10}$$

The Frenet invariants of the Smarandache curve, β_2 are given as following:

$$\mathbf{T}_{\beta_2} = \frac{-\kappa(\theta'\cos\theta + \|W\|\sin\theta)T + \kappa\|W\|N + \kappa(\theta'\sin\theta - \|W\|\cos\theta)B}{\sqrt{\kappa^2(\theta'^2 + 2\kappa^2 + 2\tau^2)}},$$

$$\mathbf{N}_{\beta_2} = \frac{(\bar{\triangle}_1\cos\theta + \bar{\triangle}_2\sin\theta)T + \bar{\triangle}_3 N + (\bar{\triangle}_2\cos\theta - \bar{\triangle}_1\sin\theta)B}{\sqrt{\bar{\triangle}_1^{\,2} + \bar{\triangle}_2^{\,2} + \bar{\triangle}_3^{\,2}}},$$

$$\begin{aligned}\mathbf{B}_{\beta_2} = {} & \frac{-\kappa(\theta'\sin\theta\bar{\triangle}_3 - \cos\theta\|W\|\bar{\triangle}_2 - \cos\theta\|W\|\bar{\triangle}_3 + \sin\theta\|W\|\bar{\triangle}_1)}{\sqrt{\kappa^2(\theta'^2 + 2\kappa^2 + 2\tau^2)(\bar{\triangle}_1^{\,2} + \bar{\triangle}_2^{\,2} + \bar{\triangle}_3^{\,2})}}\mathbf{T} \\ & + \frac{\kappa(-\|W\|\bar{\triangle}_1 + \theta'\bar{\triangle}_2)}{\sqrt{\kappa^2(\theta'^2 + 2\kappa^2 + 2\tau^2)(\bar{\triangle}_1^{\,2} + \bar{\triangle}_2^{\,2} + \bar{\triangle}_3^{\,2})}}\mathbf{N} \\ & + \frac{-\kappa(\theta'\cos\theta\bar{\triangle}_3 + \cos\theta\|W\|\bar{\triangle}_1 + \sin\theta\|W\|\bar{\triangle}_2 + \sin\theta\|W\|\bar{\triangle}_3)}{\sqrt{\kappa^2(\theta'^2 + 2\kappa^2 + 2\tau^2)(\bar{\triangle}_1^{\,2} + \bar{\triangle}_2^{\,2} + \bar{\triangle}_3^{\,2})}}\mathbf{B},\end{aligned}$$

$$\kappa_{\beta_2} = \sqrt{2}\frac{\lambda^4\tau^4\|W\|^4}{\kappa^4(\theta'^2 + 2\kappa^2 + 2\tau^2)^2}\sqrt{\bar{\triangle}_1^{\,2} + \bar{\triangle}_2^{\,2} + \bar{\triangle}_3^{\,2}},$$

$$\tau_{\beta_2} = \frac{\begin{array}{l}\sqrt{2}\Big[2\kappa^5\theta'\bar{\hbar}_3 - \kappa^4\theta''\lambda\tau\bar{\hbar}_3 - \kappa^4\theta''\lambda\tau\bar{\hbar}_2 + 2\kappa^3\theta'\tau^2\bar{\hbar}_3 + \kappa^3\theta'\kappa'\lambda\tau\bar{\hbar}_3 \\ +2\lambda^2\tau^2\|W\|\kappa^3\bar{\hbar}_1 + \kappa^3\theta'^2\|W\|\bar{\hbar}_1 + \kappa^3\theta'\kappa'\lambda\tau\bar{\hbar}_2 + \theta'^3\kappa^3\bar{\hbar}_3 + \kappa^2\theta'\tau'\lambda\tau^2\bar{\hbar}_3 \\ -\kappa^2\theta''\lambda\tau^3\bar{\hbar}_2 - \kappa^2\theta''\lambda\tau^3\bar{\hbar}_3 + \kappa^2\theta'\tau'\lambda\tau^2\bar{\hbar}_2 + 2\kappa\lambda^2\tau^4\|W\|\bar{\hbar}_1\Big]\lambda^3\tau^3\|W\|^3\end{array}}{\begin{array}{l}\Big[2\lambda^2\tau^2\theta'^2\kappa^6\kappa'^2 + 2\lambda^2\tau^4\theta'^2\tau'^2\kappa^4 + 2\theta'^4\kappa^6\lambda\tau\kappa' + 2\theta'^4\kappa^5\lambda\tau^2\tau' - 8\kappa^7\theta'\theta''\lambda\tau^3 \\ +\theta'^6\kappa^6 + 4\kappa^8\theta'^2\kappa'\lambda\tau + 4\kappa^7\theta'^2\tau'\lambda\tau^2 - 4\kappa^9\theta'\theta''\lambda\tau + 4\lambda^2\tau^3\theta'^2\kappa^5\kappa'\tau' \\ -4\kappa^5\theta'\theta''\lambda\tau^5 - 2\kappa^7\theta'^3\theta''\lambda\tau + 4\kappa^6\theta'^2\lambda\tau^3\kappa' - 2\kappa^5\theta'^3\theta''\lambda\tau^3 + 4\kappa^5\theta'^2\lambda\tau^4\tau' \\ -4\kappa^6\lambda^2\tau^3\theta''\theta'\tau' - 4\kappa^5\lambda^2\tau^4\theta''\theta'\kappa' - 4\kappa^4\lambda^2\tau^5\theta''\theta'\tau' + 5\kappa^8\theta'^4 + 8\kappa^{10}\theta'^2 \\ +8\kappa^6\theta'^2\tau^4 + 2\kappa^8\theta''^2\lambda^2\tau^2 + 12\kappa^{10}\tau^2 + 12\kappa^8\tau^4 + 4\kappa^6\tau^6 + 16\kappa^8\theta'^2\tau^2 \\ +4\kappa^{12} + +2\kappa^4\lambda^2\tau^6\theta''^2 - 4\kappa^7\lambda^2\tau^2\theta''\theta'\kappa' + 4\kappa^6\lambda^2\tau^4\theta''^2 + 5\kappa^6\theta'^4\tau^2\Big]\end{array}},$$

where

$$\begin{cases} \bar{\triangle}_1 = \dfrac{-2\lambda\theta''\kappa^5\tau + 2\lambda\theta'\kappa^4\kappa'\tau - 2\lambda\theta''\kappa^3\tau^3 + 2\lambda\theta'\kappa^3\tau^2\tau' + 2\theta'\kappa^6 + 2\theta'\kappa^4\tau^2 + \theta'''\kappa^4}{\lambda^4\tau^4(\kappa^2+\tau^2)^{\frac{3}{2}}} \\ \bar{\triangle}_2 = -\dfrac{\lambda\theta''\theta'\kappa^5\tau - \lambda\theta'^2\kappa^4\kappa'\tau + \lambda\theta''\theta'\kappa^3\tau^3 - \lambda\theta'^2\kappa^3\tau^2\tau' - 2\kappa^8 - 4\kappa^6\tau^2 - 3\theta'^2\kappa^6}{\lambda^4\tau^4(\kappa^2+\tau^2)^2} \\ \quad -\dfrac{2\kappa^4\tau^4 + 3\theta'^2\kappa^4\tau^2 + \theta'^4\kappa^4}{\lambda^4\tau^4(\kappa^2+\tau^2)^2} \\ \bar{\triangle}_3 = \dfrac{-\lambda\theta''\theta'\kappa^5\tau + \lambda\theta'^2\kappa^4\kappa'\tau - \lambda\theta''\theta'\kappa^3\tau^3 + \lambda\theta'^2\kappa^3\tau^2\tau' - 2\kappa^8 - 4\kappa^6\tau^2 - \theta'^2\kappa^6}{\lambda^4\tau^4(\kappa^2+\tau^2)^2} \\ \quad -\dfrac{2\kappa^4\tau^4 + \theta'^2\kappa^4\tau^2}{\lambda^4\tau^4(\kappa^2+\tau^2)^2}, \end{cases}$$

$$\begin{cases} \bar{\hbar}_1 = -\dfrac{\theta'\kappa''\lambda^3\tau^5\kappa^2 + 2\theta''\kappa'\lambda^3\tau^5\kappa^2 - 2\theta''\kappa^5\lambda'\lambda^2\tau^3 - 4\theta''\kappa^3\lambda'\lambda^2\tau^5 - 2\theta''\kappa\lambda'\lambda^2\tau^7}{\lambda^4\tau^4\|W\|^5} \\ \quad +\dfrac{2\theta''\kappa^5\tau'\lambda^3\tau^2 + 6\theta''\kappa^3\tau'\lambda^3\tau^4 + 4\theta''\kappa\tau'\lambda^3\tau^6 + \theta'\kappa^5\lambda''\lambda^2\tau^3 + 2\theta'\kappa^3\lambda''\lambda^2\tau^5}{\lambda^4\tau^4\|W\|^5} \\ \quad +\dfrac{\theta'\kappa\lambda''\lambda^2\tau^7 + 3\theta'\kappa^3\tau''\lambda^3\tau^4 + 2\theta'\kappa\tau''\lambda^3\tau^6 - 2\theta'''\kappa^3\lambda^3\tau^5 - \theta'''\kappa\lambda^3\tau^7 - \kappa^8\theta'}{\lambda^4\tau^4\|W\|^5} \\ \quad +\dfrac{\kappa^6\lambda^2\tau^2\theta'' + 2\kappa^4\lambda^2\tau^4\theta'' + \kappa^2\lambda^2\tau^6\theta'' - 2\theta''\kappa'\lambda^3\tau^7 + \theta'^3\kappa^5\lambda\tau + \theta'^3\kappa^3\lambda\tau^3}{\lambda^4\tau^4\|W\|^5} \\ \quad -\dfrac{6\theta'\kappa\lambda^3\tau^5\tau'^2 - 2\kappa^5\lambda^2\tau^2\theta'\kappa' - 5\kappa^3\lambda^2\tau^4\theta'\kappa' + 7\kappa^4\lambda^2\tau^3\theta'\tau' - \theta'\kappa^5\tau''\lambda^3\tau^2}{\lambda^4\tau^4\|W\|^5} \\ 6 \qquad -\dfrac{4\kappa^2\lambda^2\tau^5\theta'\tau' + 5\theta'\kappa^3\lambda^3\tau^3\tau'^2 - 3\kappa\lambda^2\tau^6\theta'\kappa + 3\kappa^6\lambda\tau^2\theta'\lambda' + 6\kappa^4\lambda\tau^4\theta'\lambda'}{\lambda^4\tau^4\|W\|^5} \\ \quad -\dfrac{3\kappa^2\lambda\tau^6\theta'\lambda' + 3\kappa^6\lambda^2\tau\theta'\tau' - 2\theta'\kappa'\lambda'\lambda^2\tau^7 - 4\theta'\kappa'\tau\lambda^3\tau^6 + 2\theta'\kappa^5\tau'^2\lambda^3\tau}{\lambda^4\tau^4\|W\|^5} \\ \quad -\dfrac{2\theta'\kappa^5\lambda'^2\lambda\tau^3 + 4\theta'\kappa^3\lambda'^2\lambda\tau^5 + 2\theta'\kappa\lambda'^2\lambda\tau^7 - 3\theta'\kappa'^2\lambda^3\tau^5\kappa + 2\kappa^6\theta'\tau^2}{\lambda^4\tau^4\|W\|^5} \end{cases}$$

$$\begin{cases} -\dfrac{\kappa^4\theta'\tau^4+\theta'\kappa''\lambda^3\tau^7+2\theta'\kappa^2\lambda^3\tau^4\kappa'\tau'-2\theta'\kappa^2\lambda'\lambda^2\tau^5\kappa'+6\theta'\kappa^3\lambda'\lambda^2\tau^4\tau'}{\lambda^4\tau^4\|W\|^5} \\ -\dfrac{4\theta'\kappa\lambda'\lambda^2\tau^6\tau'+2\theta'\kappa^5\lambda'\tau'\lambda^2\tau^2+\theta'''\kappa^5\lambda^3\tau^3}{\lambda^4\tau^4\|W\|^5} \quad , \end{cases}$$

$$\begin{cases} \bar{\hbar}_2 = \dfrac{\kappa^5\lambda''\lambda^2\tau^3+\kappa^5\tau''\lambda^3\tau^2-\kappa^4\kappa''\lambda^3\tau^3+2\kappa^3\lambda''\lambda^2\tau^5+2\kappa^3\tau''\lambda^3\tau^4-2\kappa^2\kappa''\lambda^3\tau^5}{\lambda^4\tau^4\|W\|^4} \\ +\dfrac{\kappa\tau''\lambda^3\tau^6\kappa\lambda''\lambda^2\tau^7-3\theta'\kappa^2\lambda^2\tau^4\theta''-3\theta'\kappa^4\lambda^2\tau^2\theta''-\kappa''\lambda^3\tau^7-6\kappa^5\lambda'\tau^3}{\lambda^4\tau^4\|W\|^4} \\ +\dfrac{2\kappa^5\lambda\tau^3-3\kappa^3\lambda'\tau^5+\kappa^3\lambda\tau^5-3\lambda'\kappa^7\tau-3\tau'\lambda\kappa^7+\lambda\kappa^7\tau+4\kappa^2\kappa'\lambda'\lambda^2\tau^5}{\lambda^4\tau^4\|W\|^4} \\ +\dfrac{4\kappa^2\kappa'\tau'\lambda^3\tau^4-2\kappa\lambda'\tau'\lambda^2\tau^6-2\kappa^5\lambda'\tau'\lambda^2\tau^2+2\kappa^4\kappa'\lambda'\lambda^2\tau^3+2\kappa^4\kappa'\tau'\lambda^3\tau^2}{\lambda^4\tau^4\|W\|^4} \\ -\dfrac{4\kappa^3\lambda'\tau'\lambda^2\tau^4-3\theta'^2\kappa^4\lambda^2\tau\tau'-3\theta'^2\kappa^2\lambda\tau^4\lambda'-3\theta'^2\kappa^4\lambda\tau^2\lambda'+3\theta'^2\kappa\lambda^2\tau^4\kappa'}{\lambda^4\tau^4\|W\|^4} \\ +\dfrac{6\theta'^2\kappa^2\lambda^2\tau^3\tau'+3\kappa'\lambda\tau\kappa^6-2\kappa^5\lambda'^2\lambda\tau^3-2\kappa\tau'^2\lambda^3\tau^5+2\kappa'\lambda'\lambda^2\tau^7+2\kappa'\tau'\lambda^3\tau^6}{\lambda^4\tau^4\|W\|^4} \\ -\dfrac{2\kappa^5\tau'^2\lambda^3\tau+6\kappa^5\tau'\lambda\tau^2-\kappa^5\theta^2\lambda\tau-6\kappa^4\kappa\lambda\tau^3+4\kappa^3\lambda'^2\lambda\tau^5+4\kappa^3\tau'^2\lambda^3\tau^3}{\lambda^4\tau^4\|W\|^4} \\ -\dfrac{3\kappa^3\tau'\lambda\tau^4-\kappa^3\theta'^2\lambda\tau^3-3\kappa^2\kappa\lambda\tau^5+2\kappa\lambda'^2\lambda\tau^7}{\lambda^4\tau^4\|W\|^4} \quad , \end{cases}$$

$$\begin{cases} \bar{\hbar}_3 = +\dfrac{\tau''\lambda^2\kappa^2\tau^3-\kappa''\lambda^2\kappa^3\tau^2+\lambda''\lambda\kappa^4\tau^2+\lambda''\lambda\kappa^2\tau^4+\tau''\lambda^2\kappa^4\tau-\kappa\kappa''\lambda^2\tau^4}{\lambda^4\tau^4\|W\|^2} \\ +\dfrac{2\kappa'\lambda'\lambda\kappa^3\tau^2+2\kappa'\tau'\lambda^2\kappa^3\tau-2\lambda'\tau'\lambda\kappa^4\tau-2\lambda'\tau'\lambda\kappa^2\tau^3+\lambda\kappa^5\tau-2\lambda'^2\kappa^4\tau^2}{\lambda^4\tau^4\|W\|^2} \\ -\dfrac{2\lambda'^2\kappa^2\tau^4+2\tau'^2\lambda^2\kappa^4-\lambda\kappa^3\tau^3+3\kappa\kappa'\lambda^2\tau^4-2\kappa\kappa'\lambda'\lambda\tau^4-2\kappa\kappa'\tau'\lambda^2\tau^3}{\lambda^4\tau^4\|W\|^2} \\ +\dfrac{\theta'^2\kappa^3\lambda\tau-3\kappa'\lambda^2\kappa^3\tau^2+3\lambda'\lambda\kappa^4\tau^2+3\lambda'\lambda\kappa^2\tau^4+3\tau'\lambda^2\kappa^4\tau+3\tau'\lambda^2\kappa^2\tau^3}{\lambda^4\tau^4\|W\|^2} \\ +\dfrac{2\tau'^2\lambda^2\kappa^2\tau^2}{\lambda^4\tau^4\|W\|^2}. \end{cases}$$

T^*B^* - Smarandache curve can be defined by

$$\beta_3(s) = \frac{1}{\sqrt{2}}(T^*+B^*). \tag{3.11}$$

Solving the above equation by substitution of T^* and B^* from (2.2), we obtain

$$\beta_3(s) = \frac{\sin\theta T + N + \cos\theta B}{\sqrt{2}}. \tag{3.12}$$

The Frenet invariants of the Smarandache curve, β_3 are given as following:

$$\mathbf{T}_{\beta_3} = \sin\theta T + \cos\theta B \ ,$$

$$\mathbf{N}_{\beta_3} = \frac{\bar{o_1}T + \bar{o_2}N + \bar{o_3}B}{\sqrt{\bar{o_1}^2 + \bar{o_1}^2 + \bar{o_3}^2}} \ ,$$

$$\mathbf{B}_{\beta_3} = \frac{-\bar{o_2}\cos\theta T + (\bar{o_1}\cos\theta - \bar{o_3}\sin\theta)N + \bar{o_2}\sin\theta B}{\sqrt{\bar{o_1}^2 + \bar{o_1}^2 + \bar{o_3}^2}} \ ,$$

$$\kappa_{\beta_3} = \frac{\sqrt{\bar{o_1}^2 + \bar{o_1}^2 + \bar{o_3}^2}}{\theta'\lambda^2\tau^2\|W\|},$$

$$\tau_{\beta_3} = \frac{\begin{array}{l}\sqrt{2}\Big[\kappa^5\theta'\bar{p_3} - 2\kappa^5\theta'\bar{p_1} + \kappa^5\|W\|\bar{p_1} + \kappa^3\theta'^3\bar{p_3} - 2\kappa^3\bar{p_3}\theta'^2\|W\| \\ +\kappa^3\bar{p_1}\theta'^2\|W\| + \kappa^3\theta'\tau^2\bar{p_3} - 2\kappa^3\theta'\tau^2\bar{p_1} + \kappa^3\tau^2\|W\|\bar{p_1}\Big]\lambda^3\tau^3\|W\|^3\end{array}}{\begin{array}{l}\Big[-8\kappa^8\|W\|\theta'\tau^2 + \kappa^6\theta'^6 + \kappa^{12} + \kappa^6\tau^6 + 3\kappa^{10}\tau^2 + 7\kappa^{10}\theta'^2 + 3\kappa^8\tau^4 \\ +7\kappa^6\tau^4\theta'^2 + 7\kappa^8\theta'^4 + 7\kappa^6\theta'^4\tau^2 - 4\kappa^6\|W\|\theta'^5 - 8\kappa^6\|W\|\theta'^3\tau^2 \\ -4\kappa^{10}\|W\|\theta' - 4\kappa^6\|W\|\theta'\tau^4 - 8\kappa^8\|W\|\theta'^3 + 14\kappa^8\tau^2\theta'^2\Big]\end{array}},$$

where

$$\begin{cases} \bar{o_1} = -\sqrt{2}\lambda^2\tau^2\theta'\cos\theta\|W\| + \kappa^2\theta'^2\cos\theta \\ \bar{o_2} = \sqrt{2}\lambda^2\kappa^2\tau^2 + \sqrt{2}\lambda^2\tau^4 - \kappa^2\theta'\|W\| \\ \bar{o_3} = \sqrt{2}\lambda^2\tau^2\theta'\sin\theta\|W\| - \kappa^2\theta'^2\sin\theta, \end{cases}$$

$$\begin{cases} \bar{p_1} = \dfrac{3\kappa^4\tau\theta'^2\lambda' - 3\kappa^4\tau\theta'\lambda'\|W\| - 3\kappa^4\tau\theta'\lambda\theta'' + \kappa^4\tau\lambda\theta''\|W\| + 3\kappa^4\theta'^2\lambda\tau' - 3\kappa^4\theta'\lambda\tau'\|W\|}{\lambda^3\tau^3\|W\|^4} \\ \quad + \dfrac{2\kappa^3\tau\theta'\|W\|\lambda\kappa' + 3\kappa^2\tau^3\theta'^2\lambda' - 3\kappa^2\tau^3\theta'\lambda'\|W\| - 3\kappa^2\tau^3\theta'\lambda\theta'' + \kappa^2\tau^3\lambda\theta''\|W\|}{\lambda^3\tau^3\|W\|^4} \\ \quad + \dfrac{6\kappa^2\tau^2\theta'^2\lambda\tau' - 4\kappa^2\tau^2\theta'\|W\|\lambda\tau' - 3\kappa\tau^3\theta'^2\lambda\kappa' + 3\kappa\tau^3\theta'\lambda\kappa'\|W\|}{\lambda^3\tau^3\|W\|^4}, \end{cases}$$

$$\begin{cases} \bar{p_2} = \dfrac{4\lambda\tau^5\theta'\tau'\kappa\theta' + 6\lambda\tau^3\theta'\tau'\kappa^3\theta' + 2\lambda\tau\theta'\tau'\kappa^5\theta' + 2\kappa'\lambda^2\tau^3\tau'\kappa^2\theta' - 2\kappa'\lambda\tau^4\lambda'\kappa^2\theta'}{\lambda^3\tau^3\|W\|^5} \\ \quad + \dfrac{4\kappa'\lambda\tau^4\lambda'\kappa^2\|W\| + 2\kappa'\lambda\tau^2\theta'\kappa^4\|W\| + 4\kappa'\lambda^2\tau^3\tau\kappa^2\|W\| + 2\kappa'\lambda^2\tau\tau'\kappa^4\|W\|}{\lambda^3\tau^3\|W\|^5} \\ \quad - \dfrac{2\lambda\tau^5\theta'\tau'\kappa\|W\| + 4\lambda\tau^3\theta'\tau'\kappa^3\|W\| + 2\lambda\tau\theta'\tau'\kappa^5\|W\| + 2\lambda^2\tau'^2\kappa^5\|W\|}{\lambda^3\tau^3\|W\|^5} \\ \quad - \dfrac{2\tau^6\theta'^2 + 4\tau^4\theta'^2\kappa^3\|W\| + 2\tau^2\theta'^2\kappa^5\|W\| - 2\tau^6\theta'^2\kappa\theta' - 4\tau^4\theta'^2\kappa^3\theta'\kappa\|W\|}{\lambda^3\tau^3\|W\|^5} \end{cases}$$

$$
\begin{cases}
+\dfrac{2\tau^2\theta'^2\kappa^5\theta' + 2\lambda^2\tau'^2\kappa^5\theta' + \tau^2\kappa^3\theta'^2\|W\| + \lambda^2\tau^6\kappa\theta''' + 2\lambda^2\tau^4\kappa^3\theta''' + \lambda^2\tau^2\kappa^5\theta'''}{\lambda^3\tau^3\|W\|^5} \\
+\dfrac{\lambda^2\tau^6\kappa''\theta' + 2\kappa'\lambda^2\tau^6\theta'' - \lambda^2\tau^6\kappa''\|W\| + \kappa^5\theta'^2\|W\| - \tau^4\kappa^3\theta' - 2\tau^2\kappa^5\theta'}{\lambda^3\tau^3\|W\|^5} \\
+\dfrac{\tau^4\kappa^3\|W\| + 2\tau^2\kappa^5\|W\| + \tau^2\kappa^3\theta'^3 + \kappa^5\theta'^3 - \kappa^7\theta' + \kappa^7\|W\| - 4\lambda^2\tau^2\tau'^2\kappa^3\|W\|}{\lambda^3\tau^3\|W\|^5} \\
+\dfrac{\lambda^2\tau^5\kappa\tau''\|W\| + 2\lambda^2\tau^3\kappa^3\tau''\|W\| + \lambda^2\tau\kappa^5\tau''\|W\| + \lambda\tau^6\kappa\lambda''\|W\| + 2\lambda\tau^4\kappa^3\lambda''\|W\|}{\lambda^3\tau^3\|W\|^5} \\
+\dfrac{\lambda\tau^2\kappa^5\lambda''\|W\| + 2\kappa'\lambda\tau^6\theta'\|W\| + 2\kappa'\lambda^2\tau^5\tau'\|W\| - 2\lambda\tau^6\lambda'\kappa\theta'' - 4\lambda\tau^4\lambda'\kappa^3\theta''}{\lambda^3\tau^3\|W\|^5} \\
-\dfrac{2\lambda\tau^2\theta'\kappa^5\theta'' + 2\lambda^2\tau\tau'\kappa^5\theta'' + 2\kappa'\lambda\tau^6\lambda'\theta' + \lambda\tau^6\kappa\lambda''\theta' + 2\lambda\tau^4\kappa^3\lambda''\theta' + \lambda\tau^2\kappa^5\lambda''\theta'}{\lambda^3\tau^3\|W\|^5} \\
-\dfrac{\lambda^2\tau\kappa^5\tau''\theta' + 2\lambda^2\tau^4\kappa^2\kappa''\|W\| + \lambda^2\tau^2\kappa^4\kappa''\|W\| + 3\theta'\kappa\lambda^2\tau^4\kappa'^2 + 2\theta'\kappa\lambda^2\tau^5\tau''}{\lambda^3\tau^3\|W\|^5} \\
+\dfrac{6\theta'\kappa\lambda^2\tau^4\tau'^2 + \theta'\kappa^2\lambda^2\tau^4\kappa'' - 3\theta'\kappa^3\lambda^2\tau^3\tau'' + 5\theta'\kappa^3\lambda^2\tau^2\tau^2 + 2\kappa'\lambda^2\tau^4\kappa^2\theta''}{\lambda^3\tau^3\|W\|^5} \\
-\dfrac{6\lambda^2\tau^3\tau'\kappa^3\theta'' + 4\kappa'\lambda^2\tau^5\tau'\theta' + 2\lambda^2\tau^4\tau'^2\kappa\|W\| + 4\lambda^2\tau^5\tau\kappa\theta''}{\lambda^3\tau^3\|W\|^5} \quad ,
\end{cases}
$$

$$
\begin{cases}
\bar{p_3} = \dfrac{2\kappa^4\tau\theta''\lambda - 3\kappa^4\tau\theta'\lambda' + 3\kappa^4\tau\lambda'\|W\| - 3\kappa^4\lambda\theta'\tau' + 3\kappa^4\lambda\tau'\|W\| + \kappa^3\tau\lambda\theta'\kappa'}{\lambda^3\tau^3\|W\|^3} \\
\quad +\dfrac{2\kappa^2\tau^3\theta''\lambda - 3\kappa^3\tau\lambda\kappa'\|W\| - 3\kappa^2\tau^3\theta'\lambda' + 3\kappa^2\tau^3\lambda'\|W\| - 5\kappa^2\tau^2\lambda\theta'\tau'}{\lambda^3\tau^3\|W\|^3} \\
\quad +\dfrac{3\kappa^2\tau^2\lambda\tau'\|W\| + 3\kappa\tau^3\lambda\theta'\kappa' - 3\kappa\tau^3\lambda\kappa'\|W\|}{\lambda^3\tau^3\|W\|^3}.
\end{cases}
$$

$T^*N^*B^*$ - Smarandache curve can be defined by

$$\beta_4(s) = \frac{1}{\sqrt{2}}(T^* + N^* + B^*). \tag{3.13}$$

Solving the above equation by substitution of T^*, N^* and B^* from (2.2), we obtain

$$\beta_4(s) = \frac{(\sin\theta + \cos\theta)T + N + (\cos\theta - \sin\theta)B}{\sqrt{3}}. \tag{3.14}$$

The Frenet invariants of the Smarandache curve, β_4 are given as following:

$$\mathbf{T}_{\beta_4} = \left(\frac{-\kappa\|W\|\sqrt{\|W\|}\sin\theta-\kappa\theta'\sqrt{\|W\|}\big(\cos\theta-\sin\theta\big)}{\sqrt{-2\kappa^2(\kappa^2\theta'-\kappa^2\|W\|+\theta'\tau^2-\tau^2\|W\|-\theta'^2\|W\|)}}\right)T$$
$$+\left(\frac{\kappa\|W\|\sqrt{\|W\|}}{\sqrt{-2\kappa^2(\kappa^2\theta'-\kappa^2\|W\|+\theta'\tau^2-\tau^2\|W\|-\theta'^2\|W\|)}}\right)N$$
$$+\left(\frac{-\kappa\|W\|\sqrt{\|W\|}\cos\theta+\kappa\theta'\sqrt{\|W\|}\big(\cos\theta+\sin\theta\big)}{\sqrt{-2\kappa^2(\kappa^2\theta'-\kappa^2\|W\|+\theta'\tau^2-\tau^2\|W\|-\theta'^2\|W\|)}}\right)B\ ,$$

$$\mathbf{N}_{\beta_4} = \frac{(\bar{g_1}\cos\theta+\bar{g_2}\sin\theta)T+\bar{g_3}N+(g_2\cos\theta-g_1\sin\theta)B}{\sqrt{\bar{g_1}^2+\bar{g_2}^2+\bar{g_3}^2}}\ ,$$

$$\mathbf{B}_{\beta_4} = \left(\frac{-\kappa\sqrt{\|W\|}\big(\sin\theta\|W\|\bar{g_1}-\cos\theta\|W\|(\bar{g_2}+\bar{g_3})+\theta'\bar{g_3}(\sin\theta+\cos\theta)\big)}{\sqrt{-2\kappa^2(\bar{g_1}^2+\bar{g_2}^2+\bar{g_3}^2)(\kappa^2\theta'-\kappa^2\|W\|+\theta'\tau^2-\tau^2\|W\|-\theta'^2\|W\|)}}\right)\mathbf{T}$$
$$+\left(\frac{-\|W\|\bar{g_1}+\theta'\bar{g_1}+\theta'\bar{g_2}}{\sqrt{-2\kappa^2(\bar{g_1}^2+\bar{g_2}^2+\bar{g_3}^2)(\kappa^2\theta'-\kappa^2\|W\|+\theta'\tau^2-\tau^2\|W\|-\theta'^2\|W\|)}}\right)\mathbf{N}$$
$$+\left(\frac{\kappa\sqrt{\|W\|}\big(-\cos\theta\|W\|\bar{g_1}-\sin\theta\|W\|(\bar{g_2}+\bar{g_3})+\theta'\bar{g_3}(\sin\theta-\cos\theta)\big)}{\sqrt{-2\kappa^2(\bar{g_1}^2+\bar{g_2}^2+\bar{g_3}^2)(\kappa^2\theta'-\kappa^2\|W\|+\theta'\tau^2-\tau^2\|W\|-\theta'^2\|W\|)}}\right)\mathbf{B}\ ,$$

$$\kappa_{\beta_4} = \frac{\sqrt{3}\lambda^8\tau^8\|W\|^4\sqrt{\bar{g_1}^2+\bar{g_2}^2+\bar{g_3}^2}}{2\kappa^4(\kappa^2\tau^2\lambda^2+\tau^4\lambda^2+{\theta'}^2\lambda^2\tau^2-{\theta'}^2\kappa^2)^2}\ ,$$

$$\tau_{\beta_4} = \frac{\begin{array}{l}\sqrt{3}\Big[2\kappa^5\theta'\bar{f}_3-2\kappa^5\theta'\bar{f}_1+2\kappa^5\|W\|\bar{f}_1-\kappa^4\tau\lambda\theta''\bar{f}_2-\kappa^4\tau\lambda\theta''\bar{f}_3-\kappa^2\tau^3\lambda\theta''\bar{f}_2-\kappa^2\tau^3\lambda\theta''\bar{f}_3\\ -\kappa^4\tau\lambda\theta''\bar{f}_1+\bar{f}_2\kappa^3\tau\theta'\lambda\kappa'+\bar{f}_3\kappa^3\tau\theta'\lambda\kappa'+\bar{f}_1\kappa^3\tau\theta'\lambda\kappa'+2\kappa^3\tau^2\theta'\bar{f}_3-\kappa^2\tau^3\lambda\theta''\bar{f}_1\\ +\bar{f}_3\kappa^2\tau^2\theta'\lambda\tau'+\bar{f}_1\kappa^2\tau^2\theta'\lambda\tau'+\bar{f}_2\kappa^2\tau^2\theta'\lambda\tau'-2\kappa^3\tau^2\theta'\bar{f}_1+2\kappa^3\tau^2\|W\|\bar{f}_1+2\theta'^3\kappa^3\bar{f}_3\\ -2\kappa^3\theta'^2\bar{f}_3\|W\|+2\kappa^3\theta'^2\bar{f}_1\|W\|\Big]\lambda^3\tau^3\|W\|^3\end{array}}{\begin{array}{l}\Big[-16\kappa^8\theta'^3\|W\|-8\kappa^{10}\theta'\|W\|+16\kappa^6\tau^2\theta'^4+32\kappa^8\tau^2\theta'^2+16\kappa^6\tau^4\theta'^2-8\kappa^6\theta'^5\|W\|\\ +4\kappa^{12}+16\kappa^8\theta'^4+16\kappa^{10}\theta'^2+12\kappa^{10}\tau^2+12\kappa^8\tau^4+4\kappa^6\tau^6+4\kappa^6\theta'^6-4\kappa^9\tau\lambda\theta''\|W\|\\ -8\kappa^7\tau^3\lambda\theta''\|W\|-4\kappa^5\tau^5\lambda\theta''\|W\|+3\kappa^4\tau^4\theta'^2\lambda^2\tau'^2-4\kappa^7\tau\theta'^3\lambda\theta'-4\kappa^5\tau^3\theta'^3\lambda\theta''\\ +3\kappa^6\tau^2\theta'^2\lambda^2\kappa'^2+4\kappa^6\tau\theta'^4\lambda\kappa'+4\kappa^5\tau^2\theta'^4\lambda\tau'+4\kappa^8\tau\theta'\lambda\kappa'\|W\|+4\kappa^6\tau^3\theta'\lambda\kappa'\|W\|\\ +4\kappa^7\tau^2\theta'\lambda\tau'\|W\|+4\kappa^5\tau^4\theta'\lambda\tau'\|W\|-6\kappa^6\tau^3\theta'\lambda^2\tau'\theta''-6\kappa^7\tau^2\theta'\lambda^2\kappa'\theta''+3\kappa^8\tau^2\lambda^2\theta''^2\\ +6\kappa^5\tau^3\theta'^2\lambda^2\kappa'\tau'-6\kappa^4\tau^5\theta'\lambda^2\tau'\theta''-16\kappa^6\tau^2\theta'^3\|W\|-16\kappa^8\tau^2\theta'\|W\|-8\kappa^6\tau^4\theta'\|W\|\\ -6\kappa^5\tau^4\theta'\lambda^2\kappa'\theta''+6\kappa^6\tau^4\lambda^2\theta''^2+3\kappa^4\tau^6\lambda^2\theta''^2\Big]\end{array}}\ ,$$

where

$$\begin{cases}
\bar{g_1} = \dfrac{\kappa^3\tau^3\theta'\lambda\theta'' - 2\kappa^3\tau^3\lambda\theta''\|W\| - \theta'^2\kappa^3\tau'\lambda\tau^2 + 2\kappa^3\|W\| - \tau^2\theta'\lambda\tau' - \theta'^2\kappa^4\kappa'\lambda\tau}{\lambda^4\tau^4\|W\|^4} \\[2ex]
\quad + \dfrac{\kappa^5\tau\theta'\lambda\theta'' + 2\kappa^4\tau\theta'\lambda\|W\| - \kappa' - 2\kappa^5\tau\lambda\theta''\|W\| - 4\kappa^4\tau^2\theta'^2 + 2\kappa^4\tau^2\theta'\|W\|}{\lambda^4\tau^4\|W\|^4} \\[2ex]
\quad - \dfrac{2\theta'^4\kappa^4 - 4\kappa^4\theta'^3\|W\| + 4\kappa^6\theta'^2 - 2\kappa^6\theta'\|W\|}{\lambda^4\tau^4\|W\|^4} \\[2ex]
\bar{g_2} = \dfrac{\kappa^3\tau^3\theta'\lambda\theta'' + \kappa^3\tau^3\lambda\theta''\|W\| - \theta'^2\kappa^3\tau'\lambda\tau^2 - \kappa^3\|W\|\tau^2\theta'\lambda\tau' - \theta'^2\kappa^4\kappa'\lambda\tau - 2\kappa^8}{\lambda^4\tau^4\|W\|^4} \\[2ex]
\quad - \dfrac{\kappa^4\tau\theta'\lambda\|W\|\kappa' - \kappa^5\tau\lambda\theta''\|W\| + 2\kappa^4\tau^4 + 4\kappa^4\tau^2\theta'^2 - 2\kappa^4\tau^2\theta'\|W\| - \kappa^5\tau\theta'\lambda\theta''}{\lambda^4\tau^4\|W\|^4} \\[2ex]
\quad - \dfrac{4\kappa^6\tau^2 + 2\theta'^4\kappa^4 - 2\kappa^4\theta'^3\|W\| + 4\kappa^6\theta'^2 - 2\kappa^6\theta'\|W\|}{\lambda^4\tau^4\|W\|^4} \\[2ex]
\bar{g_3} = \dfrac{\kappa^3\tau^3\lambda\theta''\|W\| - 2\kappa^3\tau^3\theta'\lambda\theta'' + 2\theta'^2\kappa^3\tau'\lambda\tau^2 - \kappa^3\|W\|\tau^2\theta'\lambda\tau' + 2\theta'^2\kappa^4\kappa'\lambda\tau}{\lambda^4\tau^4\|W\|^4} \\[2ex]
\quad - \dfrac{\kappa^4\tau\theta'\lambda\|W\|\kappa' - \kappa^5\tau\lambda\theta''\|W\| + 2\kappa^4\tau^4 + 4\kappa^4\tau^2\theta'^2 - 4\kappa^4\tau^2\theta'\|W\| + 4\kappa^6\tau^2}{\lambda^4\tau^4\|W\|^4} \\[2ex]
\quad + \dfrac{2\kappa^4\theta'^3\|W\| - 4\kappa^6\theta'^2 + 4\kappa^6\theta'\|W\| - 2\kappa^8 - 2\kappa^5\tau\theta'\lambda\theta''}{\lambda^4\tau^4\|W\|^4} \qquad ,
\end{cases}$$

$$\begin{cases}
\bar{f_1} = -\dfrac{\kappa^2\tau^4\theta'\lambda^2\kappa'' + 7\kappa^4\tau^2\theta'\lambda\tau' + 4\kappa^2\tau^4\theta'\lambda\tau' + 2\kappa^2\tau^4\lambda^2\kappa'\theta'' + 5\kappa^3\tau^2\theta'\lambda^2\tau'^2}{\lambda^3\tau^3\|W\|^5} \\[2ex]
\quad + \dfrac{2\kappa^5\tau\theta'\lambda\kappa' + 5\kappa^3\tau^3\theta'\lambda\kappa' + 3\kappa\tau^5\theta'\lambda\kappa' + 6\kappa^3\tau^3\lambda^2\tau'\theta'' + 4\kappa\tau^5\lambda^2\tau'\theta'' + \kappa^5\theta'^3}{\lambda^3\tau^3\|W\|^5} \\[2ex]
\quad + \dfrac{4\tau^5\theta'\lambda^2\kappa'\tau' + 2\kappa^5\tau^2\lambda\lambda'\theta' + 4\kappa^3\tau^4\lambda\lambda'\theta'' + 2\kappa\tau^6\lambda\lambda'\theta'' + 2\kappa^5\tau\lambda^2\tau'\theta'' + \kappa^7\theta'}{\lambda^3\tau^3\|W\|^5} \\[2ex]
\quad + \dfrac{2\tau^6\theta'\lambda\lambda'\kappa' + \kappa^5\tau^2\theta'\lambda\lambda'' + 2\kappa^3\tau^4\theta'\lambda\lambda'' + \kappa\tau^6\theta'\lambda\lambda'' + \kappa^5\tau\theta'\lambda^2\tau'' + 2\kappa^2\tau^4\theta'\lambda\lambda'\kappa'}{\lambda^3\tau^3\|W\|^5} \\[2ex]
\quad + \dfrac{3\kappa^3\tau^3\theta'\lambda^2\tau'' + 2\kappa\tau^5\theta'\lambda^2\tau'' + 3\kappa\tau^4\theta'\lambda^2\kappa'^2 - 6\kappa\tau^4\theta'\lambda^2\tau'^2 + 6\theta'^2\kappa^2\|W\|\tau^2\lambda\tau'}{\lambda^3\tau^3\|W\|^5} \\[2ex]
\quad - \dfrac{2\kappa^2\tau^3\theta'\lambda^2\kappa'\tau' - \kappa^3\tau^2\theta'^3 - 2\kappa^5\tau^2\theta' - \kappa^3\tau^4\theta' + \kappa^5\tau^2\lambda^2\theta''' + 3\kappa\tau^3\theta'^2\lambda\kappa'\|W\|}{\lambda^3\tau^3\|W\|^5} \\[2ex]
\quad - \dfrac{2\kappa^3\tau^4\lambda^2\theta''' + \kappa\tau^6\lambda^2\theta''' + \tau^6\theta'\lambda^2\kappa'' + 3\kappa^6\tau\theta'\lambda' + 6\kappa^4\tau^3\theta'\lambda' + 2\kappa^5\tau\theta'\lambda\lambda'\tau'}{\lambda^3\tau^3\|W\|^5}
\end{cases}$$

$$
\left\{
\begin{aligned}
&-\frac{3\kappa^2\tau^5\theta'\lambda' + 3\kappa^6\theta'\lambda\tau' - \kappa^6\tau\lambda\theta'' - 2\kappa^4\tau^3\lambda\theta'' + 6\kappa^3\tau^3\theta'\lambda\lambda'\tau' + 3\kappa^2\tau^3\theta'\lambda\theta''\|W\|}{\lambda^3\tau^3\|W\|^5}\\
&-\frac{2\tau^6\lambda^2\kappa'\theta'' + 2\kappa^5\tau^2\theta'\lambda'^2 + 4\kappa^3\tau^4\theta'\lambda'^2 + 2\kappa\tau^6\theta'\lambda'^2 + 2\kappa^5\theta'\lambda^2\tau'^2 + 4\kappa\tau^5\theta'\lambda\lambda'\tau'}{\lambda^3\tau^3\|W\|^5}\\
&+\frac{3\kappa^4\tau\theta'^2\lambda'\|W\| + 3\kappa^2\tau^3\theta'^2\lambda'\|W\| + 3\kappa^4\theta'^2\lambda\tau'\|W\| - 3\kappa^4\tau\theta'\lambda\theta''\|W\| + \kappa^2\tau^5\lambda\theta''}{\lambda^3\tau^3\|W\|^5}
\end{aligned}
\right.
$$

$$
\left\{
\begin{aligned}
\bar{f}_2 = &\frac{3\kappa^7\tau^2 + 3\kappa^5\tau^4 + \tau^4\kappa^3\theta'^2 - \kappa''\lambda^2\tau^8 + 3\lambda\kappa^8\tau' + 3\tau^7\kappa^2\lambda' + 9\tau^5\kappa^4\lambda' + 9\tau^3\kappa^6\lambda'}{\lambda^3\tau^3\|W\|^6}\\
&+\frac{3\tau\kappa^8\lambda' - 2\tau^8\kappa\lambda'^2 - 6\tau^6\kappa^3\lambda'^2 - 6\tau^4\kappa^5\lambda'^2 - 2\tau^2\kappa^7\lambda'^2 - 2\lambda^2\kappa^7\tau'^2 + 2\tau^2\kappa^5\theta'^2}{\lambda^3\tau^3\|W\|^6}\\
&-\frac{+\kappa^7\theta'\|W\| + \kappa^5\theta'^3\|W\| - \tau^6\kappa^3 - \kappa^7\theta'^2 - 6\lambda\tau^6\kappa^2\kappa'\lambda' - 6\lambda^2\tau^5\kappa^2\kappa'\tau' + 2\lambda\tau^7\kappa\tau'\lambda'}{\lambda^3\tau^3\|W\|^6}\\
&+\frac{6\lambda^2\tau^3\kappa^4\kappa'\tau' - 6\lambda\tau^5\kappa^3\tau'\lambda' - 6\lambda\tau^3\kappa^5\tau'\lambda' - 2\lambda\tau\kappa^7\tau'\lambda' + 6\lambda\tau^4\kappa^4\kappa'\lambda' + 2\lambda\tau^2\kappa^6\kappa'\lambda'}{\lambda^3\tau^3\|W\|^6}\\
&+\frac{2\lambda^2\tau\kappa^6\kappa'\tau' - 3\theta'\kappa^6\lambda\tau\theta'' + 9\theta'^2\kappa^4\lambda\tau^2\tau' - 3\theta'^2\kappa^3\lambda\tau^3\kappa' - 6\theta'\kappa^4\lambda\tau^3\theta'' + 6\theta'^2\kappa^2\lambda\tau^4\tau'}{\lambda^3\tau^3\|W\|^6}\\
&+\frac{2\tau^2\kappa^5\theta'\lambda'^2\|W\| - 3\theta'^2\kappa\lambda\tau^5\kappa' - 3\theta'\kappa^2\lambda\tau^5\theta' + 2\tau^6\kappa\theta'\lambda'^2\|W\| + 4\tau^4\kappa^3\theta'\lambda'^2\|W\|}{\lambda^3\tau^3\|W\|^6}\\
&+\frac{2\lambda^2\kappa^5\theta'\tau'^2\|W\| + \kappa''\lambda^2\tau^6\theta'\|W\| + 2\lambda^2\tau^6\kappa\theta''\|W\| + \lambda^2\tau^6\kappa\theta'''\|W\| + 2\lambda^2\tau^4\kappa^3\theta'''\|W\|}{\lambda^3\tau^3\|W\|^6}\\
&+\frac{\lambda^2\tau^2\kappa^5\theta'''\|W\| + 2\lambda^2\tau^3\kappa^2\theta'\kappa'\tau'\|W\| + 2\lambda\tau\kappa^5\theta'\tau'\lambda'\|W\| + 4\theta'\kappa\lambda'\lambda\tau^5\|W\|\tau'}{\lambda^3\tau^3\|W\|^6}\\
&+\frac{6\theta'\kappa^3\lambda'\lambda\tau^3\|W\|\tau' - 2\theta'\kappa^2\lambda'\lambda\tau^4\|W\|\kappa' + \theta'\kappa^2\lambda^2\tau^4\|W\|\kappa'' - 2\theta'\kappa\lambda^2\tau^5\|W\|\tau''}{\lambda^3\tau^3\|W\|^6}\\
&+\frac{6\theta'\kappa\lambda^2\tau^4\|W\|\tau'^2 - 3\theta'\kappa\lambda^2\tau^4\|W\|\kappa'^2 - 3\theta'\kappa^3\lambda^2\tau^3\|W\|\tau'' + 5\theta'\kappa^3\lambda^2\tau^2\|W\|\tau'^2}{\lambda^3\tau^3\|W\|^6}\\
&-\frac{4\lambda\tau^4\kappa^3\lambda'\theta''\|W\| + 2\lambda\tau^2\kappa^5\lambda'\theta''\|W\| + 4\lambda^2\tau^5\kappa\tau'\theta''\|W\| + 6\lambda^2\tau^3\kappa^3\tau'\theta''\|W\|}{\lambda^3\tau^3\|W\|^6}\\
&-\frac{4\lambda^2\tau^5\theta'\kappa'\tau'\|W\| + \lambda\tau^6\kappa\theta'\lambda''\|W\| + 2\lambda\tau^4\kappa^3\theta'\lambda'\|W\| + \lambda\tau^2\kappa^5\theta'\lambda''\|W\|}{\lambda^3\tau^3\|W\|^6}\\
&+\frac{2\lambda^2\tau^4\kappa^2\kappa'\theta''\|W\| + 2\lambda^2\tau^7\kappa'\tau' - 3\kappa''\lambda^2\tau^6\kappa^2 - 3\kappa''\lambda^2\tau^4\kappa^4 - \kappa''\lambda^2\tau^2\kappa^6}{\lambda^3\tau^3\|W\|^6}\\
&+\frac{\lambda^2\tau\kappa^7\tau'' + \lambda\tau^8\kappa\lambda'' + 3\lambda\tau^4\kappa^5\lambda'' + \lambda\tau^2\kappa^7\lambda'' + \lambda^2\tau^7\kappa\tau'' + 3\lambda^2\tau^5\kappa^3\tau'' + 3\lambda^2\tau^3\kappa^5\tau''}{\lambda^3\tau^3\|W\|^6}\\
&+\frac{9\lambda\tau^2\kappa^6\tau' - 3\lambda\tau\kappa^7\kappa' - 2\lambda^2\tau^6\kappa\tau'^2 - 6\lambda^2\tau^4\kappa^3\tau'^2 - 6\lambda^2\tau^2\kappa^5\tau'^2 + 2\lambda\tau^8\kappa'\lambda' + 3\theta'^2\kappa^6\tau\lambda'}{\lambda^3\tau^3\|W\|^6}\\
&+\frac{3\theta'^2\kappa^6\lambda\tau' - \tau^2 + 6\theta'^2\kappa^4\tau^3\lambda' + 3\theta'^2\kappa^2\tau^5\lambda'\kappa^3\theta'^3\|W\| - \tau^4\kappa^3\theta'\|W\| - 2\tau^2\kappa^5\theta'\|W\| + \kappa^9}{\lambda^3\tau^3\|W\|^6}\\
&+\frac{3\lambda\tau^6\kappa^2\tau' - 9\lambda\tau^5\kappa^3\kappa' + 9\lambda\tau^4\kappa^4\tau' - 9\lambda\tau^3\kappa^5\kappa' + 2\lambda^2\tau\kappa^5\tau'\theta''\|W\| + 2\lambda\tau^6\theta'\kappa'\lambda'\|W\|}{\lambda^3\tau^3\|W\|^6}\\
&+\frac{3\lambda\tau^6\kappa^3\lambda'' - 3\lambda\tau^7\kappa\kappa' - 2\lambda\tau^6\kappa\lambda'\theta''\|W\| + \lambda^2\tau\kappa^5\theta'\tau''\|W\|}{\lambda^3\tau^3\|W\|^6} \quad ,
\end{aligned}
\right.
$$

$$\left\{\begin{aligned}
\bar{f}_3 = &-\frac{2\lambda\tau^4\kappa'\lambda'\|W\| + 2\lambda^2\tau^3\kappa'\tau'\|W\| + \kappa^3\lambda\tau^2\lambda''\|W\| + \kappa\lambda\tau^4\lambda''\|W\|}{\lambda^3\tau^3\|W\|^3} \\
&+\frac{2\kappa\lambda^2\tau^2\tau'^2\|W\| - \kappa^3\lambda^2\tau\tau''\|W\| - \kappa\lambda^2\tau^3\tau''\|W\| + 3\kappa^4\lambda\tau'\|W\|}{\lambda^3\tau^3\|W\|^3} \\
&+\frac{3\kappa^4\tau\lambda'\|W\| + 3\kappa^2\tau^3\lambda'\|W\| + \lambda^2\tau^4\kappa''\|W\| + 2\kappa^3\tau^2\lambda'^2\|W\|}{\lambda^3\tau^3\|W\|^3} \\
&+\frac{2\kappa\tau^4\lambda'^2\|W\| + 2\kappa^3\lambda^2\tau'^2\|W\| - \kappa^3\theta'^2\|W\| - 3\kappa\lambda\tau^3\kappa'\|W\| - \kappa^5\|W\|}{\lambda^3\tau^3\|W\|^3} \\
&+\frac{2\kappa^4\lambda\tau\theta'' - 3\kappa^4\theta'\tau\lambda' - 3\kappa^2\tau^3\theta'\lambda' + 2\kappa^2\tau^3\lambda\theta'' - 3\kappa^4\theta'\lambda\tau' - \kappa^3\tau^2\|W\|}{\lambda^3\tau^3\|W\|^3} \\
&+\frac{3\kappa\tau^3\theta'\lambda\kappa' - 5\tau^2\kappa^2\theta'\lambda\tau' + \theta'\kappa^3\lambda\tau\kappa' - 3\kappa^3\lambda\tau\kappa'\|W\| + 3\kappa^2\lambda\tau^2\tau'\|W\|}{\lambda^3\tau^3\|W\|^3}\,.
\end{aligned}\right.$$

Example

Let us consider the unit speed Mannheim curve and Mannheim partner curve:

$$\alpha(s) = \frac{1}{\sqrt{2}}\big(-\cos s, -\sin s, s\big) \quad \text{and} \quad \alpha^*(s) = \frac{1}{\sqrt{2}}(-2\cos s, -2\sin s, s)\cdot$$

The Frenet invariants of the partner curve, $\alpha^*(s)$ are given as following:

$$\begin{cases} T^*(s) = \dfrac{1}{\sqrt{5}}\big(2\sin s, -2\cos s, 1\big), \quad N^*(s) = \dfrac{1}{\sqrt{5}}\big(\sin s, \cos s, -2\big), \quad B^*(s) = \big(\cos s, \sin s, 0\big) \\ \kappa^*(s) = \dfrac{2\sqrt{2}}{5}, \quad \tau^*(s) = \dfrac{\sqrt{2}}{5}\cdot \end{cases}$$

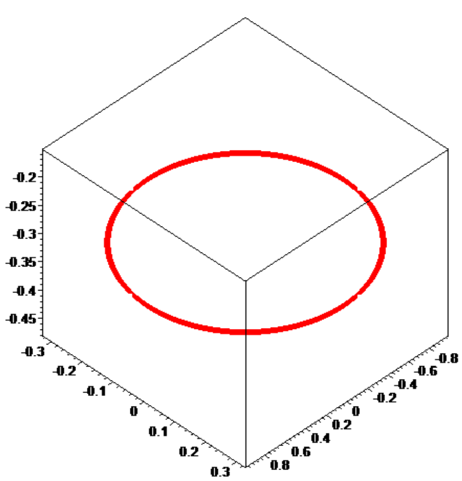
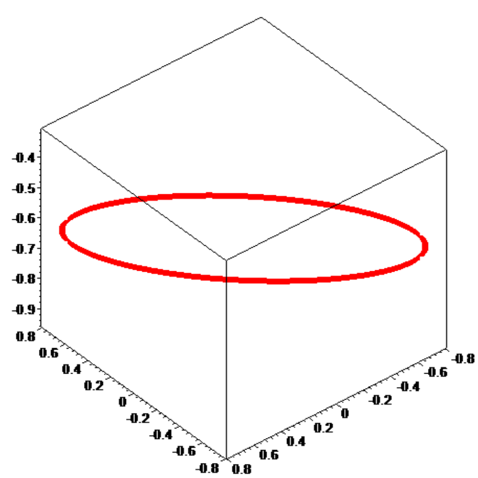


Figure 1. T^*N^*-Smarandache Curve



N^*B^*-Smarandache Curve

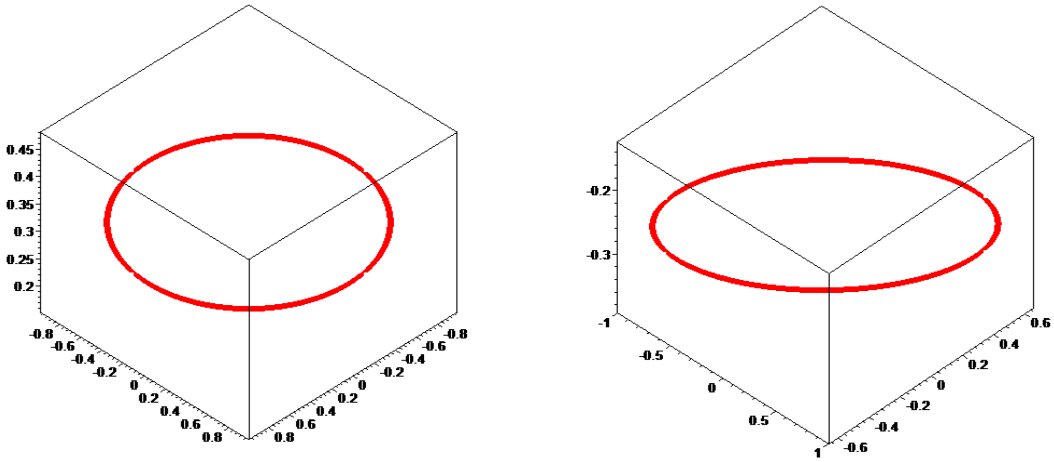


Figure 2. T^*B^*-Smarandache Curve

$T^*N^*B^*$-Smarandache Curve

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Affiliations

SÜLEYMAN ŞENYURT
ADDRESS: Ordu University,Faculty of Arts and Sciences, Department of Mathematics, Ordu-Turkey.
E-MAIL: senyurtsuleyman@hotmail.com

ABDUSSAMET ÇALIŞKAN
ADDRESS: Ordu University,Faculty of Arts and Sciences, Department of Mathematics, Ordu-Turkey.
E-MAIL: abdussamet65@gmail.com