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# Soft Sequences and Their Application to NC-Backgammon

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**Abstract**— Soft set theory was defined by Molodtsov in 1999 to model problems involving uncertainty. In this study, soft sequences are defined as a special case of soft sets. It is defined as a function from the set of positive integers to the power set of a universe. As a new concept, connected and disconnected soft sequences, chained soft sequences, centered soft sequences, increasing soft sequences, decreasing soft sequences, and ordered soft sequences are defined. Finally, soft sequences are applied to game theory. Using chained soft sequences, no chance (NC) backgammon —a zero-sum, strategic, and intelligence game — is played.

***Keywords*** — *Soft set, soft sequence, connected soft sequence, NC-backgammon, game theory*

## 1. Introduction

The concept of soft sets was first introduced by Molodtsov in 1999 as a mathematical framework to model uncertainties [1]. Soft sets classify a given group of objects based on parameters that characterize these objects, with the classification being determined by decision-makers. The softness in soft sets arises from the fact that the classification depends on the discretion of the decision-makers. Due to this softness, the soft set defined by each decision-maker, even when using the same set of parameters and objects, will differ.

Aktaş and Çağman [2] laid the groundwork for exploring the algebraic structure of soft set theory by introducing the concept of soft groups. Following their work, several studies have contributed to the algebraic aspects of soft sets [3–5]. The application of soft sets to decision-making began with Maji and Roy [6], and since then, numerous studies have been conducted on soft decision-making methods [7–9]. Soft sets have since found applications in nearly all areas of mathematics, including topology [10–12] and analysis [13–15].

Recently, Reddy et al. [16] have proposed soft sequences. Afterward, subsequent studies on soft sequences have been conducted [17–21]. The soft sequences therein are based on soft numbers on real numbers. In this study, we define soft sequences as a special case of soft sets. A soft sequence is described as a mapping from the set of positive integers to the power set of a universe. As a new contribution, several types of soft sequences are introduced and defined, including connected and disconnected soft sequences, chained soft sequences, centered soft sequences, increasing soft sequences, decreasing soft sequences, and ordered soft sequences. Soft sequences are then applied to game theory,

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a branch of mathematics widely used in economics and decision-making processes to analyze strategic interactions. Game theory examines scenarios where individuals, groups, or organizations, referred to as players, attempt to determine the best strategy by considering the decisions of others. It has applications in various fields, including economics, political science, biology, psychology, and computer science. For an overview of game theory, its types, and application areas, [22] is recommended.

The dependence of soft sets on human discretion makes them particularly suitable for adaptation to game theory. Deli and Çağman [23, 24] were the first to adapt soft sets to game theory. They defined soft two-person games applicable to problems involving uncertainty, extended these games to n-person scenarios, and generalized them to fuzzy soft games. They introduced solution methods for these games, such as fixed-point, elimination, lower and upper value, dominant strategy, Nash equilibrium, and probabilistic solution methods based on soft sets.

Çağman defined no-chance (NC) backgammon [25] as a two-person, zero-sum, strategic game with perfect information and intelligence. For those interested in learning more about NC-backgammon and its rules, [25–27] are recommended for consultation.

Section 2 of the present study provides some notions to be required in the following section. Section 3 presents the definitions and properties of soft sequences. Section 4 introduces the definitions and properties of connected soft sequences. Section 5 defines the application of soft sequences to game theory through the utilization of chained soft sequences for the modeling of the NC-backgammon game. The final section discusses potential directions for future studies.

## 2. Soft Sets

Molodtsov first defined soft sets [1]. Considering the symbols and notations in the Çağman [28] source, this section has been prepared. Refer to [1, 28–30] for basic information about soft sets.

**Definition 2.1.** Let $U$ be a universal set, $P(U)$ be the power set of $U$, and $E$ be the parameters characterizing the elements of the universal set. $F$ is called a soft set over $U$ if and only if $F$ is a mapping of $E$ into the $P(U)$ as follows:

$$f : E \to P(U), \quad F = \{(e, f(e)) : e \in E\}$$

If $f(e) = \emptyset$, $(e, f(e))$ is not written as an element in the soft set $F$.

Here, the function $f$ is called the approximation function of the soft set $F$. The set $f(e)$ for each $e \in E$ is called the $e$-approximation value set or $e$-approximation set. The softness of the soft set comes from the fact that $e$-approximation sets vary from person to person, that is, they depend on the decision makers.

**Example 2.2.** Let $U = \{u_1, u_2, u_3, ..., u_{30}\}$ be the set of students in a class, and $E = \{e_1, e_2, e_3, e_4\}$ be the set of parameters characterizing the students. Here, let be the students $e_1$, tall, $e_2$, hard-working, $e_3$, athletic, $e_4$, with glasses. For a $A$ person: if tall students are $\{u_1, u_3, u_5\}$, then $f(e_1) = \{u_1, u_3, u_5\}$, if hard-working students are $\{u_1, u_4\}$, then $f(e_2) = \{u_1, u_4\}$, if the athletic students are $\{u_1, u_2, u_3\}$, then $f(e_3) = \{u_1, u_2, u_3\}$, if the students with glasses are $\{u_1, u_5\}$, then $f(e_4) = \{u_1, u_5\}$. In this case, the soft set $F_A$ formed by person $A$ is obtained as:

$$F_A = \{(e_1, \{u_1, u_3, u_5\}), (e_2, \{u_1, u_4\}), (e_3, \{u_1, u_2, u_3\}), (e_4, \{u_1, u_5\})\}$$

## 3. Soft Sequences

This section defines soft sequences and examines their basic properties. Throughout this study, $U$ is a universal set and $2^U$ is the power set of $U$.

**Definition 3.1.** A soft sequence $(a_n)$ over the universe $U$ is defined by soft sets $(a_n) = (a_1, a_2, ..., a_n, ..)$ where $a_n : \mathbb{Z}^+ \to 2^U$.

Here, the set of positive integers $\mathbb{Z}^+$ is called the index set of the soft sequence, respectively the soft sets $a_1, a_2, ..., a_n, ...$ are called the first, second,..., $n$-th term of the soft sequence and the $a_n$ term is called the general term of the soft sequence.

**Example 3.2.** Let $U = \{u_1, u_2, ..., u_n, ...\}$ be a universal set. Here, the general terms of some soft sequences over $U$ are defined:

*i.* $a_n :=$ It is a set with $n$ elements

*ii.* $b_n :=$ It is a set containing $u_2$

*iii.* $c_n :=$ It is a set with 3 elements

*iv.* $d_n :=$ It is a set containing at most two elements

*v.* $e_n :=$ It is a set with infinite elements

*vi.* $f_n := \{u_{i\times n} : i \in \mathbb{Z}^+\}$

Let the conditions of the $(a_n)$ sequence be written here according to a decision maker. According to this decision maker:

Let the term $a_1$ be $\{u_7\}$, one of the elements of $2^U$ with 1 elements, the term $a_2$ be $\{u_1, u_{100}\}$, one of the elements of $2^U$ with 2 elements, the term $a_3$ be $\{u_9, u_{117}, u_{2059}\}$, one of the elements of $2^U$ with 3 elements, and let it continue like this. In this case, the soft sequence $(a_n)$ over $U$ is obtained as:

$$(a_n) = (a_1, a_2, a_3, ...) = \Big(\{u_7\}, \{u_1, u_{100}\}, \{u_9, u_{117}, u_{2059}\}, ...\Big)$$

Other soft sequences $(b_n)$, $(c_n)$, $(d_n)$, $(e_n)$ and $(f_n)$ are obtained in a similar way.

**Definition 3.3.** Let $U$ be the universal set, $2^U$ the power set of $U$, and $A$ a finite subset of the set of positive integers. Defined from the set $A$ to the set $2^U$, where $|A| = k$

$$a_n : A \to 2^U$$

the soft set is called a finite soft sequence over $U$ and it is represented as an ordered $k$-number

$$(a_n) = (a_1, a_2, ..., a_k)$$

**Example 3.4.** Let $U = \{u_1, u_2, ..., u_9\}$ be the set of contestants who will participate in a competition and let $A = \{1, 2, 3\}$ be the set of degrees the contestants will receive. General term of soft sequence $(a_n)$ defined as:

$a_n :=$ It is the set of contestants who have the possibility of coming $n$-th in a competition

According to a juror: if the set of contestants who can come first is $\{u_3, u_4\}$, then $a_1 = \{u_3, u_4\}$, if the set of contestants who can come second is $\{u_2, u_7, u_9\}$, then $a_2 = \{u_2, u_7, u_9\}$, if the set of contestants who can comes third is $\{u_8\}$, it becomes $a_3 = \{u_8\}$. In this case, the finite soft sequence $(a_n)$ is obtained as follows:

$$(a_n) = \Big(\{u_3, u_4\}, \{u_2, u_7, u_9\}, \{u_8\}\Big)$$

**Definition 3.5.** A soft sequence $(a_n)$ over $U$ is said to be an empty soft sequence, if $a_n = \emptyset$ for all $n \in \mathbb{Z}^+$.

**Example 3.6.** $(a_n)$ is an empty soft sequence on $\mathbb{Z}$, whose general term is defined as $a_n = \{x : 2^{nx} < 0\}$.

**Definition 3.7.** A soft sequence $(a_n)$ over $U$ is said to be a constant soft sequence, if all terms of $(a_n)$ are equal to each other.

**Example 3.8.** $(a_n)$ is a constant soft sequence over $U = \{u_1, u_2, ...\}$, whose general term is defined as $a_n = \{u_1, u_2\}$.

**Definition 3.9.** A soft sequence $(a_n)$ over $U$ is said to be universal soft sequence, if $a_n = U$ for all $n \in \mathbb{Z}^+$.

**Definition 3.10.** Soft sequences $(a_n)$ and $(b_n)$ over $U$ are said to be equal soft sequences, denoted by $(a_n) = (b_n)$, if $a_n = b_n$, for all $n \in \mathbb{Z}^+$.

**Example 3.11.** Let $U = \{-1, 1\}$ is the set of objects, $a_n = \{(-1)^n\}$ and $b_n = \{\cos n\pi\}$ defined over $U$, then $(a_n) = (b_n)$ since $(a_n) = (\{-1\}, \{1\}, \{-1\}, \{1\}, ...)$ and $(b_n) = (\{-1\}, \{1\}, \{-1\}, \{1\}, ...)$.

**Definition 3.12.** Let $(a_n)$ and $(b_n)$ soft sequences over $U$. Then, intersection of $(a_n)$ and $(b_n)$, denoted by $(c_n) = (a_n) \cap (b_n)$, is a soft sequence over $U$ whose terms are defined by $c_n = a_n \cap b_n$.

**Definition 3.13.** Let $(a_n)$ and $(b_n)$ soft sequences over $U$. Then, union of $(a_n)$ and $(b_n)$, denoted by $(d_n) = (a_n) \cup (b_n)$, is a soft sequence over $U$ whose terms are defined by $d_n = a_n \cup b_n$.

**Definition 3.14.** Let $(a_n)$ be a soft sequence over $U$. Then, complement of the soft sequence $(a_n)$ denoted by $(a_n)^\circ$, is a soft sequence over $U$ whose terms are defined by $a_n^{'} = U - a_n$.

Here, $(\circ)$ is used for soft sequence complement, while $(')$ is used for set complement.

**Example 3.15.** Let the set of objects $U = \{u_1, u_2, u_3, u_4, u_5, u_6\}$, $A = \{1, 2, 3, 4\}$, and the soft sequences $(a_n)$ and $(b_n)$ defined over $U$. For $A = \{1, 2, 3, 4\}$, $(a_n)$ and $(b_n)$ are written as:

$$(a_n) = \Big(\{u_2, u_4, u_6\}, \{u_1, u_2, u_3\}, \{u_1, u_3, u_5\}, \emptyset\Big)$$

and

$$(b_n) = \Big(\{u_1, u_2\}, \{u_1, u_3, u_4, u_5, u_6\}, \{u_2, u_6\}, \{u_1, u_4\}\Big)$$

Terms of the soft sequence $(c_n) = (a_n) \cap (b_n)$ the intersection of the $(a_n)$ and $(b_n)$ is found as:

$$c_1 = a_1 \cap b_1 = \{u_2\}, \quad c_2 = a_2 \cap b_2 = \{u_1, u_3\}, \quad c_3 = a_3 \cap b_3 = \emptyset, \quad c_4 = a_4 \cap b_4 = \emptyset$$

And it is written as:

$$(c_n) = (a_n) \cap (b_n) = \Big(\{u_2\}, \{u_1, u_3\}, \emptyset, \emptyset\Big)$$

Terms of the soft sequence $(a_n) \cup (b_n)$ the union of the $(a_n)$ and $(b_n)$ is found as:

$d_1 = a_1 \cup b_1 = \{u_1, u_2, u_4, u_6\}$, $d_2 = a_2 \cup b_2 = U$, $d_3 = a_3 \cup b_3 = \{u_1, u_2, u_3, u_5, u_6\}$, $d_4 = a_4 \cup b_4 = \{u_1, u_4\}$

And it is written as:

$$(d_n) = (a_n) \cup (b_n) = \Big(\{u_1, u_2, u_4, u_6\}, U, \{u_1, u_2, u_3, u_5, u_6\}, \{u_1, u_4\}\Big)$$

Terms of the complement of the soft sequence $(a_n)$ denoted by $(a_n)^\circ$ is found as:

$a_1^{'} = U - a_1 = \{u_1, u_3, u_5\}$, $a_2^{'} = U - a_2 = \{u_4, u_5, u_6\}$, $a_3^{'} = U - a_3 = \{u_2, u_4, u_6\}$, $a_4^{'} = U - a_4 = U$

And it is written as:

$$(a_n)^\circ = \Big(\{u_1, u_3, u_5\}, \{u_4, u_5, u_6\}, \{u_2, u_4, u_6\}, U\Big)$$

**Definition 3.16.** Let $(a_n)$ and $(b_n)$ soft sequences over $U$. Then, a soft subsequence of $(b_n)$, denoted by $(a_n) \subseteq (b_n)$, if $a_n \subseteq b_n$ for all $n \in \mathbb{Z}^+$.

**Example 3.17.** Let the set of objects $U = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9\}$ and $A = \{1, 2, 3, 4, 5\}$, the soft sequences $(a_n)$ and $(b_n)$ defined over $U$. For $A = \{1, 2, 3, 4, 5\}$, $(a_n)$ and $(b_n)$ are written as:

$$(a_n) = \Big(\{u_2, u_4, u_6\}, \{u_1, u_3, u_5\}, \{u_8, u_9\}, \{u_5, u_7, u_8, u_9\}, \emptyset\Big)$$

$$(b_n) = \Big(\{u_2, u_4, u_6, u_8\}, \{u_1, u_3, u_5, u_7, u_9\}, \{u_7, u_8, u_9\}, U, \{u_9\}\Big)$$

For all $n \in A$, $a_1 \subseteq b_1,\ a_2 \subseteq b_2,\ a_3 \subseteq b_3,\ a_4 \subseteq b_4,\ a_5 \subseteq b_5$ since the $(a_n)$ is a soft subsequence of the $(b_n)$ and written as; $(a_n)\ \subseteq (b_n)$. If $a_5 = \{u_3\}$, then $(b_n) \nsubseteq (a_n)$ since $b_4 \nsubseteq a_4$.

**Proposition 3.18.** Let $(a_n)$, $(b_n)$, and $(c_n)$ be soft sequences over $U$. Then,

*i.* $(a_n) \cap (a_n) = (a_n)$

*ii.* $(a_n) \cap (a_\emptyset) = (a_\emptyset)$

*iii.* $(a_n) \cap (a_U) = (a_n)$

*iv.* $(a_n) \cap (a_n)^\circ = (a_\emptyset)$

*v.* $(a_n) \cap (b_n) = (b_n) \cap (a_n)$

*vi.* $(a_n) \cap \Big((b_n) \cap (c_n)\Big) = \Big((a_n) \cap (b_n)\Big) \cap (c_n)$

**Proposition 3.19.** Let $(a_n)$, $(b_n)$, and $(c_n)$ be soft sequences over $U$. Then,

*i.* $(a_n) \cup (a_n) = (a_n)$

*ii.* $(a_n) \cup (a_\emptyset) = (a_n)$

*iii.* $(a_n) \cup (a_U) = (a_U)$

*iv.* $(a_n) \cup (a_n)^\circ = (a_U)$

*v.* $(a_n) \cup (b_n) = (b_n) \cup (a_n)$

*vi.* $\Big((a_n) \cup (b_n)\Big) \cup (c_n) = (a_n) \cup \Big((b_n) \cup (c_n)\Big)$

**Proposition 3.20.** Let $(a_n)$ be a soft sequence over $U$. Then,

*i.* $\Big((a_n)^\circ\Big)^\circ = (a_n)$

*ii.* $(a_\emptyset)^\circ = (a_U)$

**Proposition 3.21.** Let $(a_n)$, $(b_n)$, and $(c_n)$ be soft sequences over $U$. Then,

*i.* $\Big((a_n) \cap (b_n)\Big)^\circ = (a_n)^\circ \cup (b_n)^\circ$

*ii.* $\Big((a_n) \cup (b_n)\Big)^\circ = (a_n)^\circ \cap (b_n)^\circ$

**Proposition 3.22.** Let $(a_n)$, $(b_n)$, and $(c_n)$ be soft sequences over $U$. Then,

*i.* $(a_n) \cup \Big((b_n) \cap (c_n)\Big) = \Big((a_n) \cup (b_n)\Big) \cap \Big((a_n \cup (c_n)\Big)$

*ii.* $(a_n) \cap \Big((b_n) \cup (c_n)\Big) = \Big((a_n) \cap (b_n)\Big) \cup \Big((a_n \cap (c_n)\Big)$

## 4. Connected Soft Sequences

This section first defines the concept of connected soft sequences based on the properties of soft sequences. Then, connected soft sequences, increasing soft sequences, decreasing soft sequences, and chained soft sequences are defined and examined, along with their basic properties.

**Definition 4.1.** Let $(a_n)$ be a soft sequence over $U$. For $i \in \mathbb{Z}^+$,

*i.* if $a_{i-1} \cap a_i \neq \emptyset$ for $1 < i$, then $a_i$ term is called left connected term and if $a_i \cap a_{i+1} \neq \emptyset$, then $a_i$ term is called right connected term,

*ii.* if $a_{i-1} \cap a_i = \emptyset$ for $1 < i$, the $a_i$ term is called the left disconnected term and if $a_i \cap a_{i+1} = \emptyset$, the $a_i$ term is called the right disconnected term,

*iii.* terms that are connected from both the left and the right are called connected terms, and terms that are disconnected from the left or the right are called disconnected terms,

*iv.* terms that are disconnected from the left and right are called disjoint terms.

**Example 4.2.** The first six terms of a soft sequence $(a_n)$ over the set $U = \{u_1, u_2, ...\}$ defined as:

$$a_1 = \{u_1\},\ a_2 = \{u_1, u_2\},\ a_3 = \{u_3\},\ a_4 = \{u_4, u_6\},\ a_5 = \{u_2, u_3, u_4, u_5\},\ a_6 = \{u_2, u_3, u_4\}, ...$$

Hence,

$$a_1 \cap a_2 = \{u_1\},\ a_2 \cap a_3 = \emptyset,\ a_3 \cap a_4 = \emptyset,\ a_4 \cap a_5 = \{u_4\},\ a_5 \cap a_6 = \{u_2, u_3, u_4\}, ...$$

since $a_1$ is connected from the right and the left connect is not defined, it is a connected term,
$a_2$ is the disconnected term because it is connected from the left and disconnected from the right,
$a_3$ is a disjoint term because it is disconnected from the right and left,
$a_4$ is the disconnected term because it is connected from the right and disconnected from the left,
$a_5$ is the connected term because it is connected from the left and the right.

**Definition 4.3.** Soft sequences in which all connected terms are called connected soft sequences. A soft sequence $(a_n)$ defined over $U$ to be connected if and only if

$$\forall\ i \in \mathbb{Z}^+, a_i \cap a_{i+1} \neq \emptyset$$

Soft sequences with at least one disconnected term are called disconnected soft sequences.
A soft sequence $(a_n)$ defined over $U$ to be disconnected if and only if

$$\exists\ i \in \mathbb{Z}^+, a_i \cap a_{i+1} = \emptyset$$

**Example 4.4.** Let the general term of a soft sequence $(a_n)$ over $U = \{u_1, u_2, ...\}$ be defined as $a_1 = 1^{U(1)}$ and $a_n = a_{n-1} \cup 1^{U(n)}$ for $1 < n$. Here $1^{U(n)}$ is a non-empty subset of the randomly selected set $U$ for the $n$-th term of the soft sequence $(a_n)$. In this case, the soft sequence $(a_n)$ is a connected soft sequence. Because, if $1^{U(1)} = \{u_2\}$ is chosen, $a_1 = \{u_2\}$, if $1^{U(2)} = \{u_2\}$ is chosen, $a_2 = \{u_2\} \cup \{u_2\} = \{u_2\}$, if $1^{U(3)} = \{u_1, u_2, u_3\}$ is chosen, $a_3 = \{u_2\} \cup \{u_1, u_2, u_3\} = \{u_1, u_2, u_3\}$, if $1^{U(4)} = \{u_5\}$ is chosen, $a_4 = \{u_1, u_2, u_3\} \cup \{u_5\} = \{u_1, u_2, u_3, u_5\}$.

As can be seen in the terms obtained while continuing indefinitely, the intersection of each successive term is different from the empty one. But the soft sequence $(b_n)$ over the same $U$, whose general term is defined as $b_n = \{u_i : \frac{n}{i} \notin \mathbb{Z}^+\}$ , is an disconnected soft sequence. Because,

$$b_1 = \emptyset,\ b_2 = \emptyset,\ b_3 = \{u_2\},\ b_4 = \{u_3\},\ b_5 = \{u_2, u_3, u_4\},\ b_6 = \{u_4, u_5\}, ...$$

As seen in the terms, the intersection of multiple consecutive terms is an empty set.

**Definition 4.5.** Let $(a_n)$ be a connected soft sequence over $U$. If $a_{i-1} \cap a_{i+1} = \emptyset$, where $i \in \mathbb{Z}^+$ and $1 < i$, then the term $a_i$ is called a chained connected term. The Venn diagram of a chained connected term $a_i$ where $x, y \in U$ is given in Figure 1.

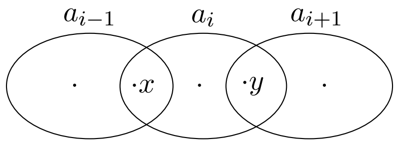


**Figure 1.** Chained connected term

Soft sequences with all terms chained connected are called chained soft sequences. The connected soft sequence $(a_n)$ defined over $U$ to be a chained soft sequence if and only if

$$\forall\ i \in \mathbb{Z}^+ \setminus \{1\}, a_{i-1} \cap a_{i+1} = \emptyset$$

where the dot, used in subsequent Venn diagrams without an element, indicates that this region may be empty or other than empty.

**Example 4.6.** All terms of the soft sequence $(a_n)$, whose general term is $a_n = \{u_n, u_{n+1}\}$ defined over $U = \{u_1, u_2, ...\}$, are chain connected soft terms. Because,

$$a_1 = \{u_1, u_2\},\ a_2 = \{u_2, u_3\},\ a_3 = \{u_3, u_4\},\ a_4 = \{u_4, u_5\},\ ...,\ a_n = \{u_n, u_{n+1}\},\ ...$$

As seen in the terms, the intersections of all consecutive odd terms and all consecutive even terms are empty sets. Then, this soft sequence is a chained soft sequence.

**Definition 4.7.** Let $(a_n)$ be a connected soft sequence over $U$. If $a_{i-1} \cap a_i \cap a_{i+1} \neq \emptyset$, where $i \in \mathbb{Z}^+$ and $1 < i$, then the term $a_i$ is called a centered connected term. The Venn diagram of a centered connected term $a_i$ where $x \in U$ is given in Figure 2.

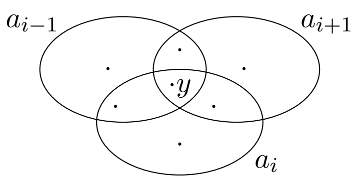


**Figure 2.** Centered connected term

Soft sequences with all terms centered connected are called centered soft sequences. The connected soft sequences $(a_n)$ defined over $U$ to be a centered soft sequences if and only if

$$\bigcap_{i \in \mathbb{Z}^+} a_i \neq \emptyset$$

The intersection of terms of centered soft sequences is called the center of this soft sequence, and it is shown as:

$$\text{cent}(a_n) = \bigcap_{i \in \mathbb{Z}^+} a_i$$

**Example 4.8.** All terms of the soft sequence $(a_n)$, whose general term is $a_n = \{u_i : i|n\}$ defined over $U = \{u_1, u_2, ...\}$, are centered connected. Because,

$$a_1 = \{u_1\},\ a_2 = \{u_1, u_2\},\ a_3 = \{u_1, u_3\}, a_4 = \{u_1, u_2, u_4\},\ a_5 = \{u_1, u_5\},\ ....$$

As the terms show, $u_1$ is the common element of all consecutive terms. Then, this soft sequence is a centered soft sequence. In this case, the center of the soft sequence $(a_n)$ is obtained as $\text{cent}(a_n) = \{u_1\}$.

**Definition 4.9.** Let $(a_n)$ be a connected soft sequence over $U$. If $a_{i-1} \subseteq a_i \subseteq a_{i+1}$, where $i \in \mathbb{Z}^+$ and $1 < i$, then the term $a_i$ is called increasingly connected term. The Venn diagram of an increasingly connected term $a_i$ where $x, y \in U$ is given in Figure 3.

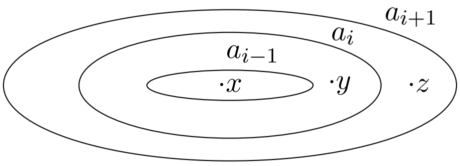


**Figure 3.** Increasingly connected term

Soft sequences with increasingly connected terms are called increasing soft sequences. The connected soft sequence $(a_n)$ defined over $U$ to be an increasing soft sequence if and only if

$$\forall\ i \in \mathbb{Z}^+, a_i \subseteq a_{i+1}$$

**Proposition 4.10.** Every increasing soft sequence is a centered soft sequence and $\mathrm{cent}(a_n) = a_1$.

*Proof.* If $(a_n)$ is an increasing soft sequence, $a_i \subseteq a_{i+1}$ for for all $i \in \mathbb{Z}^+$, in this case the terms of the soft sequence are $a_1 \subseteq a_2 \subseteq a_3 \subseteq ... \subseteq a_n \subseteq ....$ Since $a_1 \subseteq a_i$ for $1 < i$ in the increasing sequence, the term $a_1$ is a subset of all terms, that is, $\bigcap_{i\in\mathbb{Z}^+} a_i = a_1$. Then, $\mathrm{cent}(a_n) = \bigcap_{i\in\mathbb{Z}^+} a_i = a_1$. □

**Example 4.11.** All terms of the soft sequences $(a_n)$, whose general term is $a_n = \{u_1, u_2, ..., u_n\}$ defined over $U = \{u_1, u_2, ...\}$, are increasingly connected. Because,

$$a_1 = \{u_1\},\ a_2 = \{u_1, u_2\},\ a_3 = \{u_1, u_2, u_3\},\ a_4 = \{u_1, u_2, u_3, u_4\},\ a_5 = \{u_1, u_2, u_3, u_4, u_5\},\ ...$$

As seen in the terms, it inclusions each successive term as the index grows. Then, this soft sequence increases. From proposition 4.10, this increasing soft sequence is moreover, a centered soft sequence, with the center $\mathrm{cent}(a_n) = a_1 = \{u_1\}$.

**Proposition 4.12.** The set of terms of increasing soft sequences is an order relation according to the subset relation $\subseteq$.

**Definition 4.13.** Let $(a_n)$ be a connected soft sequence over $U$. If $a_{i+1} \subseteq a_i \subseteq a_{i-1}$, where $i \in \mathbb{Z}^+$ and $1 < i$, then the term $a_i$ is called decreasingly connected term. The Venn diagram of the connected term $a_i$ decreasing to $x, y, z \in U$ is given in Figure 4.

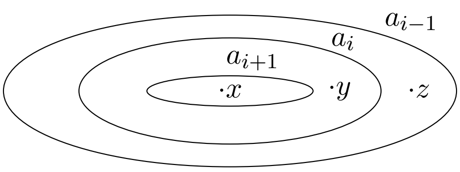


**Figure 4.** Decreasingly connected term

Soft sequences with decreasingly connected terms are called decreasing soft sequences. The connected soft sequence $(a_n)$ defined over $U$ to be a decreasing soft sequence if and only if

$$\forall\ i \in \mathbb{Z}^+, a_{i+1} \subseteq a_i$$

**Proposition 4.14.** Every decreasing soft sequence is a centered soft sequence and $\mathrm{cent}(a_n) = \lim_{n\in\mathbb{Z}^+} a_n$.

*Proof.* A decreasing soft sequence $(a_n)$ by definition satisfies the condition $a_{i+1} \subseteq a_i$ for all $i \in \mathbb{Z}^+$. In this case, the terms of the soft sequence are $a_1 \supseteq a_2 \supseteq a_3 \supseteq ... \supseteq a_n \supseteq ....$ Moreover, since it is a connected soft sequence, $a_{i-1} \cap a_i \neq \emptyset$ for for all $1 < i$. Then, $\bigcap_{i\in\mathbb{Z}^+} a_i \neq \emptyset$ since this will be a decreasing soft sequence, it is a soft sequence with a center, and its center is $\bigcap_{i\in\mathbb{Z}^+} a_i = a_i$ since the limit for the decreasing soft sequences will be the intersection of the terms, $\bigcap_{i\in\mathbb{Z}^+} a_i = \lim_{n\in\mathbb{Z}^+} a_n$ and from here $\text{cent}(a_n) = \lim_{n\in\mathbb{Z}^+} a_n$ is found. □

**Example 4.15.** All terms of the soft sequences $(b_n)$, whose general term is $b_n = \left[0, \dfrac{1}{n}\right]$ defined over $U = [0, 1]$, are decreasingly connected. Because,

$$b_1 = [0,1]\,, \quad b_2 = \left[0, \frac{1}{2}\right], \quad b_3 = \left[0, \frac{1}{3}\right], \quad b_4 = \left[0, \frac{1}{4}\right], \quad ..., \quad b_n = \left[0, \frac{1}{n}\right], \quad ...$$

As seen in the terms, each term becomes a subset of the previous one as the index grows. Then, this soft sequence is a decreasing soft sequence. From Proposition 4.14, this decreasing soft sequence is furthermore, a centered soft sequence and its center is $\text{cent}(b_n) = \lim_{n\in\mathbb{Z}^+}[0, \frac{1}{n}] = [0, 0] = \{0\}$.

**Proposition 4.16.** The set of terms of decreasing soft sequences is an order relation according to the inclusion relation $\supseteq$.

**Definition 4.17.** Increasing or decreasing soft sequences are called ordered soft sequences.

## 5. Playing NC-Backgammon with Soft Sequences

In this section, the soft sequences are applied to the game theory. Using chained soft sequences, NC-backgammon, a zero-sum, strategic and intelligence game is played. Çağman defined NC-backgammon [25] as a two-person, zero-sum, strategic, perfect information, and intelligence game. For a detailed explanation of NC-backgammon and its gameplay, the reader is directed to [26, 27].

Herein, the utilization of chained soft sequences within the framework of NC-backgammon is systematically presented.

Let $A$ and $B$ be two players who will play NC-backgammon. Let player $A$ be the first to start and select a number. Let $U = \{1, 2, 3, 4, 5, 6\}$ be a universe and $b_i \in U$, for all $i \in \mathbb{N}$. Then, a soft sequences $(a_n)$ can be defined as

$$a_n : \mathbb{Z}^+ \to 2^U$$

$$a_n = \begin{cases} \{b_0, b_1\}, & n = 1 \ (b_0 \text{ and } b_1 \text{ are selected by } A \text{ and } B \text{ respectively, sucht that } b_1 \neq b_0) \\ \{b_{2k}, b_{2k-1}\}, & n = 2k \ (b_{2k} \text{ is selected by } A, \text{ sucht that } b_{2k} \neq b_{2k-1},\, b_{2k} \neq b_{2k-2}) \\ \{b_{2k}, b_{2k+1}\}, & n = 2k+1 \ (b_{2k+1} \text{ is selected by } B, \text{ sucht that } b_{2k+1} \neq b_{2k},\, b_{2k+1} \neq b_{2k-1}) \end{cases}$$

As shown in Figure 5, $a_n$ is the general term of a chained soft sequence.

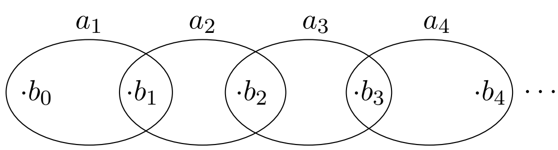


**Figure 5.** Chained soft sequence for NC-backgammon

In this case, the chained soft sequence $(a_n)$ is used to specify the rules for players $A$ and $B$ in NC-backgammon. Let player $A$ be the first to start the game.

In turn 0: If $n = 1$, then neither player moves any checkers with the $a_1 = \{b_0, b_1\}$.

In turn $A$: If $n \geq 2$, $n = 2k$, $k \in \mathbb{Z}^+$, then player $A$ moves their checkers with number pair $a_{2k} = \{b_{2k}, b_{2k-1}\}$ as if the pair is obtained by rolling two dice.

In turn $B$: If $n \geq 3$, $n = 2k+1$, $k \in \mathbb{Z}^+$, then player $B$ moves their checkers with number pair $a_{2k+1} = \{b_{2k}, b_{2k+1}\}$ as if the pair is obtained by rolling two dice.

This procedure continues until the game is finished.

**Example 5.1.** As it is above, player A starts the game first.

$n = 1$, in turn 0: If $A$ choose a number $b_0 = 5$ and $B$ choose a number $b_1 = 4$ among the six numbers. In this case, neither player moves any checkers with the $a_1 = \{5, 4\}$.

$n = 2$, in turn A: If $A$ choose the number $b_2 = 1$, then $A$ moves their checkers with the numbers $a_2 = \{1, 4\}$.

$n = 3$, in turn B: If $B$ choose the number $b_3 = 6$, then $B$ moves their checkers with the numbers $a_3 = \{1, 6\}$.

$n = 4$, in turn A: If $A$ choose the number $b_2 = 4$, then $A$ moves their checkers with the numbers $a_4 = \{4, 6\}$.

$n = 5$, in turn B: If $B$ choose the number $b_3 = 2$, then $B$ moves their checkers with the numbers $a_5 = \{4, 2\}$.

It continues until the game is over. These moves occur in Table 1.

**Table 1.** Game Moves

| $n$ | Players | $a_n$ | Numbers |
|---|---|---|---|
| 1. | $A, B$ | $a_1 = \{b_0, b_1\}$ | $\{5, 4\}$ |
| 2. | $A$ | $a_2 = \{b_2, b_1\}$ | $\{1, 4\}$ |
| 3. | $B$ | $a_3 = \{b_2, b_3\}$ | $\{1, 6\}$ |
| 4. | $A$ | $a_4 = \{b_4, b_3\}$ | $\{4, 6\}$ |
| 5. | $B$ | $a_5 = \{b_4, b_5\}$ | $\{4, 2\}$ |
| $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ |

In this example, the chained soft sequence $(a_n)$ is obtained as follows:

$$(a_n) = (\{b_0, b_1\}, \{b_1, b_2\}, \{b_2, b_3\}, \{b_3, b_4\}, \{b_4, b_5\}, \{b_5, b_6\}, ...)$$

## 6. Conclusion

The study carried out in this paper, soft sequences were defined as a specialized of soft sets. It is determined from the set of positive integers to the power set of a universe. As a new concept, we defined connected and disconnected soft sequences, chained soft sequences, centered soft sequences, increasing soft sequences, decreasing soft sequences, and ordered soft sequences. Finally, the soft sequences are applied to game theory. Furthermore, NC-backgammon, a zero-sum, strategic, and intelligence game that uses chained soft sequences, was played. In the future, researchers can study the limits of soft sequences, which are not included herein due to their non-use. Soft sequences can be applied to other areas of game theory and social problems.

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