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On Nano $\pi g\alpha$-Closed Sets

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Abstaract − In this paper, we define and study the properties of a nano $\pi g\alpha$-closed set which is a weaker form of a nano πg-closed set but strong than a nano πgp-closed sets and we define a new class of sets called nano $\pi g\alpha$-closed sets and some of their properties.

Keywords − *Nano π-closed set, nano πg-closed set, nano αg-closed set, nano πgp-closed set, nano gpr-closed set and nano $\pi g\alpha$-closed set*

1 Introduction

Thivagar et al. [4] introduced a nano topological space with respect to a subset X of an universe which is defined in terms of lower approximation and upper approximation and boundary region. The classical nano topological space is based on an equivalence relation on a set, but in some situation, equivalence relations are nor suitable for coping with granularity, instead the classical nano topology is extend to general binary relation based covering nano topological space

Bhuvaneswari et al. [3] introduced and investigated nano g-closed sets in nano topological spaces. Recently, Parvathy and Bhuvaneswari the notions of nano gpr-closed sets which are implied both that of nano rg-closed sets. In 2017, Rajasekaran et al. [7] introduced the notion of nano πgp-closed sets in nano topological spaces. In this paper, we define and study the properties of a nano $\pi g\alpha$-closed set which is a weaker form of a nano πg-closed set but strong than a nano πgp-closed sets and we define a new class of sets called nano $\pi g\alpha$-closed sets and some of their properties.

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2 Preliminaries

Throughout this paper $(U, \tau_R(X))$ (or X) represent nano topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset H of a space $(U, \tau_R(X))$, *n-cl*(H) and *n-int*(H) denote the nano closure of H and the nano interior of H respectively. We recall the following definitions which are useful in the sequel.

Definition 2.1. *[6] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.*

1. *The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \bigcup_{x \in U}\{R(x) : R(x) \subseteq X\}$, where R(x) denotes the equivalence class determined by x.*

2. *The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = \bigcup_{x \in U}\{R(x) : R(x) \cap X \neq \phi\}$.*

3. *The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not - X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$.*

Property 2.2. *[4] If (U, R) is an approximation space and $X, Y \subseteq U$; then*

1. $L_R(X) \subseteq X \subseteq U_R(X)$;
2. $L_R(\phi) = U_R(\phi) = \phi$ *and* $L_R(U) = U_R(U) = U$;
3. $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$;
4. $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$;
5. $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$;
6. $L_R(X \cap Y) \subseteq L_R(X) \cap L_R(Y)$;
7. $L_R(X) \subseteq L_R(Y)$ *and* $U_R(X) \subseteq U_R(Y)$ *whenever* $X \subseteq Y$;
8. $U_R(X^c) = [L_R(X)]^c$ *and* $L_R(X^c) = [U_R(X)]^c$;
9. $U_R U_R(X) = L_R U_R(X) = U_R(X)$;
10. $L_R L_R(X) = U_R L_R(X) = L_R(X)$.

Definition 2.3. *[4] Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then by the Property 2.2, R(X) satisfies the following axioms:*

1. *U and* $\phi \in \tau_R(X)$,

2. *The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$,*

3. *The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.*

This means that $\tau_R(X)$ is a topology on U called the nano topology on U with respect to X and $(U, \tau_R(X))$ as a nano topological space. The elements of $\tau_R(X)$ are called nano open sets (briefly n-open sets).

In the rest of the paper, we denote a nano topological space by $(U, \mathcal{N})$, where $\mathcal{N} = \tau_R(X)$. The nano-interior and nano-closure of a subset A of U are denoted by n--int(A) and n--cl(A), respectively.

Remark 2.4. *[4] If $[\tau_R(X)]$ is the nano topology on U with respect to X, then the set $B = \{U, \phi, L_R(X), B_R(X)\}$ is the basis for $\tau_R(X)$.*

Definition 2.5. *A subset H of a space $(U, \mathcal{N})$ is called*

1. *nano regular-open [4] if $H = n\text{-}int(n\text{-}cl(H))$.*

2. *nano pre-open [4] if $H \subseteq n\text{-}int(n\text{-}cl(H))$.*

3. *nano α-open [4] if $H \subseteq n\text{-}int(n\text{-}cl(n\text{-}int(H)))$.*

4. *nano π-open [1] if the finite union of nano regular-open sets.*

The complements of the above mentioned sets is called their respective closed sets.

Definition 2.6. *A subset H of a space $(U, \mathcal{N})$ is called;*

1. *nano g-closed [2] if n-cl$(H) \subseteq G$, whenever $H \subseteq G$ and G is n-open.*

2. *nano gα-closed [9] if n-αcl$(H) \subseteq G$ whenever $H \subseteq G$ and G is nano α-open.*

3. *nano αg-closed set [9] if n-αcl$(H) \subseteq G$ whenever $H \subseteq G$ and G is n-open.*

4. *nano πg-closed [7] if n-cl$(H) \subseteq G$, whenever $H \subseteq G$ and G is nano π-open.*

5. *nano gp-closed [3] if n-pcl$(H) \subseteq G$, whenever $H \subseteq G$ and G is n-open.*

6. *nano gpr-closed [5] if n-pcl$(H) \subseteq G$, whenever $H \subseteq G$ and G is nano regular open.*

7. *nano πgp-closed [8] if n-pcl$(H) \subseteq G$, whenever $H \subseteq G$ and G is nano π-open.*

3 On Nano $\pi g\alpha$-Closed Sets

Definition 3.1. *A subset H of a space $(U, \mathcal{N})$ is nano $\pi g\alpha$-closed if n-αcl$(H) \subseteq G$ whenever $H \subseteq G$ and G is nano π-open.*

The complement of nano $\pi g\alpha$-open if $H^c = U - H$ is nano $\pi g\alpha$-closed.

Example 3.2. *Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{c\}, \{b, d\}\}$ and $X = \{c, d\}$. Then the nano topology $\mathcal{N} = \{\phi, \{c\}, \{b, d\}, \{b, c, d\}, U\}$.*

1. *then $\{a\}$ is nano $\pi g\alpha$-closed set.*

2. *then $\{b\}$ is not nano $\pi g\alpha$-closed set.*

Remark 3.3. *For a subset of a space $(U, \mathcal{N})$, we have the following implications:*

$$\begin{array}{ccc} \textbf{\textit{n-closed}} & \Rightarrow & \textbf{\textit{nano g-closed}} \\ \Downarrow & & \Downarrow \\ \textbf{\textit{nano }}\pi\textbf{\textit{-closed}} & \Rightarrow & \textbf{\textit{nano }}\pi g\textbf{\textit{-closed}} \\ \Downarrow & & \\ \textbf{\textit{nano regular-closed}} & & \end{array}$$

None of the above implications are reversible.

Theorem 3.4. *In a space $(U, \mathcal{N})$, every n-closed, every nano g-closed, every nano πg-closed, every nano αg-closed and every nano $g\alpha$-closed is nano $\pi g\alpha$-closed.*

Proof. Let $H \subseteq G$ where G is nano π-open. By hypothesis. $n\text{-}cl(H) = H \subseteq G$. Since every n-closed set is nano α-closed, $n\text{-}\alpha cl(H) \subseteq n\text{-}cl(H) \subseteq G$. Therefore H is nano $\pi g\alpha$-closed.

Let H be nano g-closed and $H \subseteq G$ where G is nano π-open. Since every nano π-open set is n-open and H is nano g-closed, $n\text{-}cl(H) \subseteq G$. Hence $n\text{-}\alpha cl(H) \subseteq n\text{-}cl(H) \subseteq G$ implies H is nano $\pi g\alpha$-closed.

Let H be a nano πg-closed set and $H \subseteq G$ where G is nano π-open. By assumption, $n\text{-}cl(H) \subseteq G$. Hence $n\text{-}\alpha cl(H) \subseteq n\text{-}cl(H) \subseteq G$ implies H is nano $\pi g\alpha$-closed.

Let H be a nano αg-closed set and $H \subseteq G$ where G is nano π-open. By Remark 3.3 and by assumption, it follows that $n\text{-}\alpha cl(H) \subseteq G$ and hence H is nano $\pi g\alpha$-closed.

Obvious every nano π-open is nano α-open.

Remark 3.5. *The converses of statements in Theorem 3.4 are not necessarily true as seen from the following Examples.*

Example 3.6. *In Example 3.3, then $\{a, b\}$ is nano $\pi g\alpha$-closed set but not n-closed.*

Example 3.7.

Let $U = \{a, b, c\}$ with $U/R = \{\{c\}, \{a, b\}\}$ and $X = \{c\}$. Then the nano topology $\mathcal{N} = \{\phi, \{c\}, U\}$.

1. *then $\{c\}$ is nano $\pi g\alpha$-closed set but not nano g-closed.*

2. *then $\{c\}$ is nano $\pi g\alpha$-closed set but not nano αg-closed.*

3. *then $\{a, c\}$ is nano $\pi g\alpha$-closed set but not nano $g\alpha$-closed.*

Theorem 3.8. *In a space $(U, \mathcal{N})$, every nano $\pi g\alpha$-closed is nano gpr-closed and nano πgp-closed.*

Proof. Let H be a nano $\pi g\alpha$-closed set and $H \subseteq G$ where G is nano regular open. By Remark 3.3 and since H is nano $\pi g\alpha$-closed set, we have $n\text{-}\alpha cl(H) \subseteq G$. Every nano α-closed set is nano pre-closed implies $n\text{-}pcl(H) \subseteq G$ and hence H is nano gpr-closed.

Let H be a nano $\pi g\alpha$-closed set and $H \subseteq G$ where G is nano π-open. By hypothesis, $n\text{-}\alpha cl(H) \subseteq G$. Now $n\text{-}pcl(H) \subseteq n\text{-}\alpha cl(H) \subseteq G$ implies that H is nano πgp-closed.

Theorem 3.9. *In a space $(U,\mathcal{N})$, every nano gp-closed set is nano $\pi g\alpha$-closed.*

Proof. Obvious.

Remark 3.10. *The converses of statements in Theorem 3.9 are not necessarily true as seen from the following Examples.*

Example 3.11. *In Example 3.7, then $\{c\}$ is nano $\pi g\alpha$-closed but not nano gp-closed.*

Theorem 3.12. *In a space $(U,\mathcal{N})$, if H is nano regular open and nano $\pi g\alpha$-closed, then H is nano α-closed and hence n-clopen.*

Proof. If H is nano regular open and nano $\pi g\alpha$-closed, then $n\text{-}\alpha cl(H) \subseteq H$. This implies H is a nano α-closed. Since every nano α-closed and nano regular open set is n-closed, H is n-clopen .

Theorem 3.13. *In a space $(U,\mathcal{N})$, for $x \in U$, its complement $U - \{x\}$ is nano $\pi g\alpha$-closed or nano π-open.*

Proof. Suppose $U - \{x\}$ is not nano π-open. Then U is the only nano π-open set containing $U - \{x\}$. This implies $n\text{-}\alpha cl(U - \{x\}) \subseteq U$. Hence $U - \{x\}$ is nano $\pi g\alpha$-closed.

Theorem 3.14. *In a space $(U,\mathcal{N})$, if H is nano $\pi g\alpha$-closed and $H \subseteq K \subseteq n\text{-}\alpha cl(H)$, then K is nano $\pi g\alpha$-closed.*

Proof. Let $K \subseteq G$ where G is nano π-open. Then $H \subseteq K$ implies $H \subseteq G$. Since H is nano $\pi g\alpha$-closed we have $n\text{-}\alpha cl(H) \subseteq G$. Also $K \subseteq n\text{-}\alpha cl(H)$ implies $n\text{-}\alpha cl(K) \subseteq n\text{-}\alpha cl(H)$. Thus $n\text{-}\alpha cl(K) \subseteq G$ and so K is nano $\pi g\alpha$-closed.

Theorem 3.15. *In a space $(U,\mathcal{N})$, let H be a nano $\pi g\alpha$-closed set in U. Then $n\text{-}\alpha cl(H) - H$ does not contain any non-empty nano π-closed set.*

Proof. Let P be a non-empty nano π-closed set such that $P \subseteq n\text{-}\alpha cl(H) - H$. Then $P \subseteq n\text{-}\alpha cl(H) \cap (U - H) \subseteq U - H$ implies $H \subseteq U - P$. H is nano $\pi g\alpha$-closed and $U - P$ is nano π-open implies that nano $n\text{-}\alpha cl(H) \subseteq U - P$. That is $P \subseteq (n\text{-}\alpha cl(H))^c$. Now $P \subseteq n\text{-}\alpha cl(H) \cap (n\text{-}\alpha cl(H))^c$ implies P is empty.

Theorem 3.16. *In a space $(U,\mathcal{N})$, if H is a nano $\pi g\alpha$-closed set, then $n\text{-}\pi cl(x) \cap H \neq \phi$ holds for each $x \in n\text{-}\alpha cl(H)$.*

Proof. Let H be a nano $\pi g\alpha$-closed set. Suppose $n\text{-}\pi cl(x) \cap H = \phi$, for some $x \in n\text{-}\alpha cl(H)$. We have $H \subseteq U - n\text{-}\pi cl(x)$. Since H is nano $\pi g\alpha$-closed set, $n\text{-}\alpha cl(H) \subseteq U - n\text{-}\pi cl(x)$ implies $x \notin n\text{-}\alpha cl(H)$ which is a contradiction. Hence $n\text{-}\pi cl(x) \cap H \neq \phi$ holds for each $x \in n\text{-}\alpha cl(H)$.

Corollary 3.17. *Let H be nano $\pi g\alpha$- closed in $(U,\mathcal{N})$. Then H is nano α-closed $\Longleftrightarrow n\text{-}\alpha cl(H) - H$ is nano π-closed.*

Lemma 3.18. *Let $(U,\mathcal{N})$ be a space and H is subset of U. Then the following properties are equivalent.*

1. *H is n-clopen.*

2. *H is nano regular open and nano $\pi g\alpha$-closed.*

3. *H is nano π-open and nano $\pi g\alpha$-closed.*

Proof. Follows from Theorem 3.12 and Remark 3.3.

Proposition 3.19. *In a space $(U, \mathcal{N})$, the union of two nano $\pi g\alpha$-closed sets is nano $\pi g\alpha$-closed.*

Proof. Let $H \cup K \subseteq G$ where G is nano π-open. Since H and K are nano $\pi g\alpha$-closed sets, $n\text{-}\alpha cl(H) \subseteq G$ and $n\text{-}\alpha cl(K) \subseteq G$. Now $n\text{-}\alpha cl(H \cup K) = n\text{-}\alpha cl(H) \cup n\text{-}\alpha cl(K) \subseteq G$. Hence $H \cup K$ is nano $\pi g\alpha$-closed.

Example 3.20. *In Example 3.7, then $H = \{a\}$ and $K = \{b\}$ is nano $\pi g\alpha$-closed sets. Clearly $H \cup K = \{a, b\}$ is nano $\pi g\alpha$-closed.*

Remark 3.21. *In sa space $(U, \mathcal{N})$,*

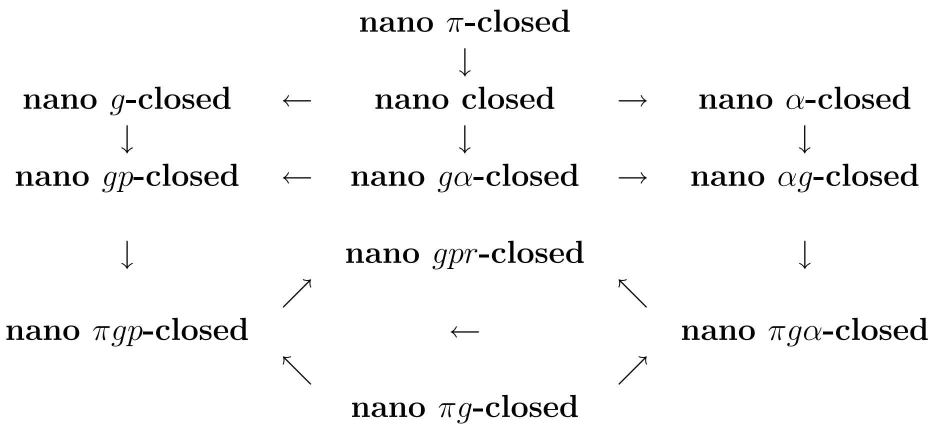
1. $n\text{-}\alpha cl(U - H) = U - n\text{-}int(H)$
2. *for any $H \subseteq U$, $n\text{-}\alpha int(n\text{-}\alpha cl(H) - H) = \phi$.*

Theorem 3.22. *A subset H of a space $(U, \mathcal{N})$ is nano $\pi g\alpha$-open $\Longleftrightarrow$ $P \subseteq n\text{-}\alpha int(H)$ whenever P is nano π-closed and $P \subseteq H$.*

Proof. Necessity. Let H be nano $\pi g\alpha$-open. Let P be a nano π-closed set such that $P \subseteq H$. Then $U - H \subseteq U - P$ where $U - P$ is nano π-open. Then $U - H$ is nano $\pi g\alpha$-closed implies $n\text{-}\alpha cl(U - H) \subseteq U - P$. By Remark 3.21. $U - n\text{-}\alpha int(H) \subseteq U - P$. That is $P \subseteq n\text{-}\alpha int(H)$.

Sufficiency. Suppose P is a nano π-closed set and $P \subseteq H$ implies $P \subseteq n\text{-}\alpha int(H)$. Let $U - H \subseteq G$ where G is nano π-open. Then $U - G \subseteq H$ and $U - G$ is nano π-closed. By hypothesis, $U - G \subseteq n\text{-}\alpha int(H)$. That is $U - n\text{-}\alpha int(H) \subseteq G$ implies $n\text{-}\alpha cl(U - H) \subseteq G$. This implies $U - H$ is nano $\pi g\alpha$-closed and H is nano $\pi g\alpha$-open.

Remark 3.23. *From the above Propositions, Examples and Remarks, we obtain the following diagram, where $A \longrightarrow B$ represents A implies B but not conversely.*



None of the above implications are reversible

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