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Space Time Fractional Telegraph Equation and its Application by Using Adomian Decomposition Method

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Abstract – The telegraph equations are a pair of linear differential equations which describe the voltage and current on an electrical transmission line with distance and time. In this paper the authors give a brief overview of fractional calculus and extend its application to space-time fractional telegraph equation by using Adomian decomposition method. The time- space derivates are considered as Caputo fractional derivate. The solutions are obtained in the series form.

Keywords – *Adomian decomposition method, time-space fractional telegraph equation.*

1 Introduction

Fractional Calculus is a field of mathematical study that grows out of the traditional definition of the calculus of integral and derivative operators in much the same way as fractional exponents grew from exponents with integer value. It was originated from the L-hospital and Leibnitz's inquisition about considering the result if n was taken as half in the n^{th} derivative of a function. Fractional calculus is of great importance in the field of Science and Technology as it is the generalization of ordinary differentiation and integration to arbitrary order [1]. Telegraph equations are a pair of linear differential equations that are very important due to their vast applications in high frequency transmission lines, optimization of guided communication system, propagation of electrical signals and many other physical and chemical phenomena. The theoretical background on transmission and transmission lines including open wire lines was given by Tomasi [2]. The fractional Telegraph equation has been studied extensively in literature. Cascaval [3] studied the time fractional Telegraph equation with applications to suspension flows using the Riemann-Liouville approach and presented asymptotic concepts. Orsingher and Beghin [4] obtained the fundamental solutions of time-fractional Telegraph equations of order 2α. Chen [5]

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discussed and derived the solution of the time-fractional telegraph equation with three kinds of non-homogenous boundary conditions making use of the separation of variables method. Momani [6] discussed the analytic and approximate solutions of the space and time fractional Telegraph differential equations by means of the Adomian decomposition method.

The Adomian decomposition method is a semi-analytical method for solving ordinary and partial non-linear differential equations. This method has been introduced and developed by Adomian [7,8]. This method has been used to obtain approximate solutions of a large class of linear and non-linear differential equations [8,9]. This method provides solutions in the form of power series with easily computed terms. It has many advantages over some classical techniques. After Adomian, this method has been further modified by Wazwaz [10] and more recently by Luo [11] and zhang and Luo [12]. Recently a lot of work has been done to apply this method to a large number of linear and non-linear ordinary differential equations, partial differential equations and integro-differential equations.

2 Mathematical Preliminaries

The Caputo fractional derivative of order $\alpha > 0$ is defined as [13]

$$C\,D_t^{\alpha} f(t) = \begin{cases} J^{n-\alpha} D^n f(t) = \frac{1}{\Gamma(n-\alpha)} \int_0^t (t-\tau)^{n-\alpha-1} f^{(n)}(\tau)d\tau, & n-1 < \alpha \leq n, n \in N \\ = \frac{\partial^n f}{dt^n}, & if\ \alpha = n \in N \end{cases} \tag{1}$$

Here $D^n = \frac{d^m}{dx^m}$ and J_x^{α} is called the Riemann-Liouville integral operator of order $\alpha > 0$. According to this definition,

$$CD_t^{\alpha} A = 0, \qquad f(t) = A$$
$$\Rightarrow f(t) = \text{Constant}$$

That is Caputo's fractional derivative of a constant is zero.

Furthermore the relation between Riemann-Liouville fractional integral and Caputo fractional derivative is given by the following relation,

$$J^{\alpha} CD_t^{\alpha} f(t) = J^{\alpha} J^{n-\alpha} CD^n f(t) = J^n CD^n f(t) = f(t) - \sum_{k=0}^{n-1} f^{(k)}(0) \frac{t^k}{k!}$$
$$= CD_t^{\alpha} f(t) - \sum_{k=0}^{n-1} \frac{t^{k-\alpha}}{\Gamma k - \alpha + 1} f^{(k)}(0) \tag{2}$$

The Laplace transform of Caputo's fractional derivative gives an interesting formula

$$\mathcal{L}\{CD_t^{\alpha} f(t)\} = s^{\alpha} \bar{f}(s) - \sum_{k=0}^{n-1} f^{(k)}(0) s^{\alpha-k-1} \tag{3}$$

The Weyl fractional integral and the Mellin transform

The Weyl fractional integral ${}_tW_\infty^{-\alpha} f(t)$ can be regarded as the convolution of $\varphi_\alpha(-t)$ with $f(t)$ [14] so that,

$$ {}_tW_\infty^{-\alpha} f(t) = \frac{1}{\Gamma\alpha}\int_t^\infty (x-t)^{\alpha-1} f(x)dx \tag{4}$$

$$ = \varphi_\alpha(-t) * f(t) \tag{5}$$

We next calculate the Mellin transform of the Riemann-Liouville fractional integrals and derivatives.

$$ {}_0J_x^\alpha f(x) = \frac{1}{\Gamma\alpha}\int_0^x (x-t)^{\alpha-1} f(t)dt \tag{6}$$

$$ = \frac{1}{\Gamma\alpha}\int_0^x x^{\alpha-1}\left(1-\frac{t}{x}\right)^{\alpha-1} f(t)dt \tag{7}$$

Let

$$ \frac{t}{x} = \eta \Rightarrow t = x\eta \Rightarrow dt = xd\eta $$

$$ \Rightarrow \qquad {}_0J_x^\alpha f(x) = \frac{x^\alpha}{\Gamma\alpha}\int_0^\infty \varphi(\eta) f(\eta t)\, d\eta \tag{8}$$

Where

$$ \varphi(\eta) = (1-\eta)^{\alpha-1}.H(1-\eta),\ H(t) $$

is the Heaviside unit step function.

$$ H(t) = \begin{cases} 0, & t < 0 \\ 1, & t \geq 0 \end{cases} $$

Using the properties of the Mellin Transform of $f(t)$, we obtain

$$ M\{\varphi(t)\} = \hat{\varphi}(p) = \int_0^\infty t^{p-1}\varphi(t)dt = \frac{\Gamma(\alpha)\Gamma(p)}{\Gamma(\alpha+p)} \tag{9}$$

Now Mellin transform of ${}_0J_t^\alpha f(t)$ is given by

$$ M\{{}_0J_t^\alpha f(t)\} = \frac{\Gamma 1-\alpha-p}{\Gamma 1-p}.\hat{f}(p+\alpha) \tag{10}$$

The Mellin transform of ${}_0D_t^\alpha f(t)$ is given by

$$ M[{}_0D_t^\alpha f(t)] = \frac{\Gamma 1-p+\alpha}{\Gamma 1-p}.\hat{f}(p-\alpha) \tag{11}$$

We next find the Mellin transform of the Weyl fractional integral

$$ {}_xW_\infty^{-\alpha} f(x) = \frac{1}{\Gamma\alpha}\int_x^\infty (x-t)^{\alpha-1} f(t)dt \tag{12}$$

$$M\{{}_xW_\infty^{-\alpha}\, f(x)\} = \frac{\Gamma p}{\Gamma p+\alpha}.\hat{f}(p+\alpha) \quad (13)$$

or

$$\{{}_xW_\infty^{-\alpha}\, f(x)\} = M^{-1}\left\{\frac{\Gamma p}{\Gamma p+\alpha}.\hat{f}(p+\alpha)\right\} \quad (14)$$

Similarly the Mellin transform of Weyl fractional derivative is given by

$$M\{W^v\, f(x)\} = \frac{\Gamma p}{\Gamma p-v}.\hat{f}(p-v)$$

$$W^v\, f(x) = M^{-1}\left[\frac{\Gamma p}{\Gamma p-v}.\hat{f}(p-v)\right] \quad (15)$$

Adomian decomposition method

To illustrate the method consider the following differential equation of the form

$$y = f(x,y),\ y(a) = y_0 \quad (16)$$

In order to solve the problem, we put the highest degree differential operator $\mathcal{L}$ on the left side in the following way,

[15] $$\mathcal{L}(y) = f(x,y) \quad (17)$$

Where the differential operator $\mathcal{L}$ is given as

$$\mathcal{L} = \frac{d}{dx} \text{ and } \mathcal{L}^{-1} = \int_0^t (\,.\,)$$

Operate $\mathcal{L}^{-1}$ on both sides of equation (17) and use the initial condition $y(a) = y_0$, we get,

$$y(x) = y_0 + \mathcal{L}^{-1}f(x,y) \quad (18)$$

The solution through Adomian decomposition method is obtained in an infinite series form as

$$y(x) = \sum_{n=0}^{\infty} y_n(x) \quad (19)$$

where the components $y_n(x)$ are determined recursively. Moreover the non linear function $f(x,y)$ is defined by the infinite series of the form

$$f(x,y) = \sum_{n=0}^{\infty} A_n \quad (20)$$

by using equations (19), (20) in (18) we get

$$\sum_{n=0}^{\infty} y_n(x) = y_0 + \mathcal{L}^{-1}\sum_{n=0}^{\infty} A_n \quad (21)$$

To determine the component $y_n(x)$, the zeroth component $y_0(x)$ is identified by the term that arises from the initial condition. The remaining components are obtained by using the preceding component.

3 Solution of the Space Time Fractional Telegraph Equation by Using Adomian Decomposition Method

In this section, we have obtained a solution of following time and space fractional telegraph equation using Adomian decomposition method. The space time telegraph equation is given by

$$D_t^{2\alpha}u(x,t) + \lambda\, D_t^{\alpha}u(x,t) = \mu\, D_x^{\beta}u(x,t), \quad t \geq 0, \quad 0 < \alpha \leq 1 \tag{22}$$

subject to the boundary and initial conditions

$$\begin{cases} u(x,0) = h_1(x) \\ u_t(x,0) = h_2(x) \\ u(0,t) = s(t) \end{cases} \tag{23}$$

we write (22) in an operator form as,

$$D_t^{2\alpha}\, u = \mu\, L_x^{\beta}\, u - \lambda\, D_t^{\alpha}\, u \tag{24}$$

where $L_x^{\beta} = \frac{\partial^{\beta}}{\partial t^{\beta}}$ and the fractional differential operational $D_t^{\alpha} = \frac{\partial^{\alpha}}{\partial t^{\alpha}}$ is defined in the Caputo's sense as follows,

$$u(x,t) = \frac{\partial^{\alpha} u(x,t)}{\partial t^{\alpha}} = \begin{cases} \frac{1}{\Gamma(n-\alpha)} \int_0^t (t-\tau)^{n-\alpha-1} f^{(n)}(\tau) d\tau, & n-1 < \alpha \leq n, n \in N \\ = \frac{\partial^n f}{dt^n}, & if\ \alpha = n \in N \end{cases} \tag{25}$$

Operating with $J_t^{2\alpha} = J_0^{2\alpha}$ on both sides of equation (24) and using initial conditions, equation (23) yields

$$u(x,t) = u(x,0) + tu_t(x,0) + J_t^{2\alpha}\left[\mu\, L_x^{\beta}\, u - \lambda\, D_t^{\alpha}\, u\right] \tag{26}$$

The Adomian's decomposition method assumes a series solution for $u(x,t)$ given by

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t) \tag{27}$$

Following Adomian method analysis, equation (26) is transformed into a set of recursive relations given by

$$u_0 = h_1(x) + th_2(x) \tag{28}$$

$$u_{n+1} = J_t^{2\alpha}\left[\mu\, L_x^{\beta}\, u_n - \lambda\, D_t^{\alpha}\, u_n\right] \tag{29}$$

Using the above recursive relationship and mathematics the first three terms of the decomposition series are given by

$$u_0 = u(x,0) + t u_t(x,0) \tag{30}$$

$$u_0 = h_1(x) + t h_2(x) \tag{31}$$

$$u_1 = J_t^{2\alpha}\left[\mu\, L_x^{\beta}\, u_0 - \lambda\, D_t^{\alpha}\, u_0\right] \tag{32}$$

$$u_1 = J_t^{2\alpha}\left[\mu \frac{\partial^\beta}{\partial x^\beta}\, u_0 - \lambda \frac{\partial^\alpha}{\partial t^\alpha}\, u_0\right] \tag{33}$$

$$u_1 = J_t^{2\alpha}\left[\mu\{{h_1}^{\beta}(x) + t{h_2}^{\beta}(x)\} - \lambda\, D_t^{\alpha}\, \{h_1(x)t^0 + h_2(x)t^1\}\right] \tag{34}$$

$$u_1 = J_t^{2\alpha}\left[\mu\{{h_1}^{\beta}(x) + t{h_2}^{\beta}(x)\} - \lambda\, \left\{h_1(x)\frac{\Gamma(1+0)}{\Gamma(1+0-\alpha)}t^{0-\alpha} + h_2(x)\frac{\Gamma(1+1)}{\Gamma(1+1-\alpha)}t^{1-\alpha}\right\}\right] \tag{35}$$

$$u_1 = J_t^{2\alpha}\left[\mu\{{h_1}^{\beta}(x) + t{h_2}^{\beta}(x)\} - \lambda\, \left\{h_1(x)\frac{t^{-\alpha}}{\Gamma(1-\alpha)} + h_2(x)\frac{t^{1-\alpha}}{\Gamma(2-\alpha)}\right\}\right] \tag{36}$$

$$u_1 = \left[\mu\{{h_1}^{\beta}(x)(J_t^{2\alpha}\, t^0) + {h_2}^{\beta}(x)(J_t^{2\alpha} t^1)\} - \lambda\left\{\frac{h_1(x)}{\Gamma(1-\alpha)}(J_t^{2\alpha} t^{-\alpha}) + \frac{h_2(x)}{\Gamma(2-\alpha)}(J_t^{2\alpha} t^{1-\alpha})\right\}\right] \tag{37}$$

$$u_1 = \left[\mu\{{h_1}^{\beta}(x)(J_t^{2\alpha}\, t^0) + {h_2}^{\beta}(x)(J_t^{2\alpha} t^1)\} - \lambda\left\{\frac{h_1(x)}{\Gamma(1-\alpha)}(J_t^{2\alpha} t^{-\alpha}) + \frac{h_2(x)}{\Gamma(2-\alpha)}(J_t^{2\alpha} t^{1-\alpha})\right\}\right] \tag{38}$$

$$u_1 = \left[\mu\left\{{h_1}^{\beta}(x)\frac{\Gamma(1+0)}{\Gamma(1+0+2\alpha)}t^{0+2\alpha} + {h_2}^{\beta}(x)\frac{\Gamma(1+1)}{\Gamma(1+1+2\alpha)}t^{1+2\alpha}\right\} - \lambda\left\{\frac{h_1(x)}{\Gamma(1-\alpha)}\frac{\Gamma(1-\alpha)}{\Gamma(1-\alpha+2\alpha)}t^{-\alpha+2\alpha} + \frac{h_2(x)}{\Gamma(2-\alpha)}\frac{\Gamma(1+1-\alpha)}{\Gamma(1+1-\alpha+2\alpha)}t^{1-\alpha+2\alpha}\right\}\right] \tag{39}$$

$$u_1 = \mu\left\{{h_1}^{\beta}(x)\frac{t^{2\alpha}}{\Gamma(2\alpha+1)} + {h_2}^{\beta}(x)\frac{t^{2\alpha+1}}{\Gamma(2\alpha+2)}\right\} - \lambda\left\{h_1(x)\frac{t^{\alpha}}{\Gamma(\alpha+1)} + h_2(x)\frac{t^{\alpha+1}}{\Gamma(\alpha+2)}\right\} \tag{40}$$

$$u_2 = J_t^{2\alpha}\left[\mu\, L_x^{\beta}\, u_1 - \lambda\, D_t^{\alpha}\, u_1\right] \tag{41}$$

$$u_2 = J_t^{2\alpha}\left[\mu \frac{\partial^\beta}{\partial x^\beta}\, u_1 - \lambda \frac{\partial^\alpha}{\partial t^\alpha}\, u_1\right] \tag{42}$$

$$u_2 = J_t^{2\alpha}\left[\mu \frac{\partial^\beta}{\partial x^\beta}\left\{\mu\left({h_1}^{\beta}(x)\frac{t^{2\alpha}}{\Gamma(2\alpha+1)} + {h_2}^{\beta}(x)\frac{t^{2\alpha+1}}{\Gamma(2\alpha+2)}\right) - \lambda\left(h_1(x)\frac{t^{\alpha}}{\Gamma(\alpha+1)} + h_2(x)\frac{t^{\alpha+1}}{\Gamma(\alpha+2)}\right)\right\} - \lambda\frac{\partial^\alpha}{\partial t^\alpha}\left\{\mu\left({h_1}^{\beta}(x)\frac{t^{2\alpha}}{\Gamma(2\alpha+1)} + {h_2}^{\beta}(x)\frac{t^{2\alpha+1}}{\Gamma(2\alpha+2)}\right) - \lambda\left(h_1(x)\frac{t^{\alpha}}{\Gamma(\alpha+1)} + h_2(x)\frac{t^{\alpha+1}}{\Gamma(\alpha+2)}\right)\right\}\right] \tag{43}$$

$$u_2 = J_t^{2\alpha}\left[\left\{\frac{\mu^2 h_1{}^{2\beta}(x)}{\Gamma(2\alpha+1)}t^{2\alpha} + \frac{\mu^2 h_2{}^{2\beta}(x)}{\Gamma(2\alpha+2)}t^{2\alpha+1} - \frac{\mu\lambda h_1{}^{\beta}(x)}{\Gamma(\alpha+1)}t^{\alpha} - \frac{\mu\lambda h_2{}^{\beta}(x)}{\Gamma(\alpha+2)}t^{\alpha+1}\right\} - \left\{\frac{\mu\lambda h_1{}^{\beta}(x)}{\Gamma(2\alpha+1)}(D_t^{\alpha}t^{2\alpha}) + \frac{\mu\lambda h_2{}^{\beta}(x)}{\Gamma(2\alpha+2)}(D_t^{\alpha}t^{2\alpha+1}) - \frac{\lambda^2 h_1(x)}{\Gamma(\alpha+1)}(D_t^{\alpha}t^{\alpha}) - \frac{\lambda^2 h_2(x)}{\Gamma(\alpha+2)}(D_t^{\alpha}t^{\alpha+1})\right\}\right] \quad (44)$$

Which on simplification gives

$$u_2 = J_t^{2\alpha}\left[\left\{\frac{\mu^2 h_1{}^{2\beta}(x)}{\Gamma(2\alpha+1)}t^{2\alpha} + \frac{\mu^2 h_2{}^{2\beta}(x)}{\Gamma(2\alpha+2)}t^{2\alpha+1} - \frac{\mu\lambda h_1{}^{\beta}(x)}{\Gamma(\alpha+1)}t^{\alpha} - \frac{\mu\lambda h_2{}^{\beta}(x)}{\Gamma(\alpha+2)}t^{\alpha+1}\right\} - \left\{\frac{\lambda\mu h_1{}^{\beta}(x)}{\Gamma(\alpha+1)}t^{\alpha} + \frac{\lambda\mu h_2{}^{\beta}(x)}{\Gamma(\alpha+2)}t^{\alpha+1} - \lambda^2 h_1(x)t^0 - \lambda^2 h_2(x)t^1\right\}\right] \quad (45)$$

$$u_2 = \left[\frac{\mu^2 h_1{}^{2\beta}(x)}{\Gamma(2\alpha+1)}(J_t^{2\alpha}t^{2\alpha}) + \frac{\mu^2 h_2{}^{2\beta}(x)}{\Gamma(2\alpha+2)}(J_t^{2\alpha}t^{2\alpha+1}) - \frac{\mu\lambda h_1{}^{\beta}(x)}{\Gamma(\alpha+1)}(J_t^{2\alpha}t^{\alpha}) - \frac{\mu\lambda h_2{}^{\beta}(x)}{\Gamma(\alpha+2)}(J_t^{2\alpha}t^{\alpha+1}) - \frac{\lambda\mu h_1{}^{\beta}(x)}{\Gamma(\alpha+1)}(J_t^{2\alpha}t^{\alpha}) - \frac{\lambda\mu h_2{}^{\beta}(x)}{\Gamma(\alpha+2)}(J_t^{2\alpha}t^{\alpha+1}) + \lambda^2 h_1(x)(J_t^{2\alpha}t^0) + \lambda^2 h_2(x)(J_t^{2\alpha}t^1)\right] \quad (46)$$

$$u_2 = \left[\frac{\mu^2 h_1{}^{2\beta}(x)}{\Gamma(2\alpha+1)}\frac{\Gamma(2\alpha+1)}{\Gamma(1+2\alpha+2\alpha)}t^{2\alpha+2\alpha} + \frac{\mu^2 h_2{}^{2\beta}(x)}{\Gamma(2\alpha+2)}\frac{\Gamma(1+2\alpha+1)}{\Gamma(1+2\alpha+1+2\alpha)}t^{2\alpha+1+2\alpha} - \frac{\mu\lambda h_1{}^{\beta}(x)}{\Gamma(\alpha+1)}\frac{\Gamma(\alpha+1)}{\Gamma(1+\alpha+2\alpha)}t^{\alpha+2\alpha} - \frac{\mu\lambda h_2{}^{\beta}(x)}{\Gamma(\alpha+2)}\frac{\Gamma(1+\alpha+1)}{\Gamma(1+\alpha+1+2\alpha)}t^{\alpha+1+2\alpha} - \frac{\lambda\mu h_1{}^{\beta}(x)}{\Gamma(\alpha+1)}\frac{\Gamma(\alpha+1)}{\Gamma(1+\alpha+2\alpha)}t^{\alpha+2\alpha} - \frac{\lambda\mu h_2{}^{\beta}(x)}{\Gamma(\alpha+2)}\frac{\Gamma(1+\alpha+1)}{\Gamma(1+\alpha+1+2\alpha)}t^{\alpha+1+2\alpha} + \lambda^2 h_1(x)\frac{\Gamma(1+0)}{\Gamma(1+0+2\alpha)}t^{0+2\alpha} + \lambda^2 h_2(x)\frac{\Gamma(1+1)}{\Gamma(1+1+2\alpha)}t^{1+2\alpha}\right] \quad (47)$$

$$u_2 = \mu^2\left(h_1{}^{2\beta}(x)\frac{t^{4\alpha}}{\Gamma(2\alpha+1)} + h_2{}^{2\beta}(x)\frac{t^{4\alpha+1}}{\Gamma(4\alpha+2)}\right) - 2\mu\lambda\left(h_1{}^{\beta}(x)\frac{t^{3\alpha}}{\Gamma(3\alpha+1)} + h_2{}^{\beta}(x)\frac{t^{3\alpha+1}}{\Gamma(3\alpha+2)}\right) + \lambda^2\left(h_1(x)\frac{t^{2\alpha}}{\Gamma(2\alpha+1)} + h_2(x)\frac{t^{2\alpha+1}}{\Gamma(2\alpha+2)}\right) \quad (48)$$

and so on. In this manner the rest of components of the decomposition series can be obtained. the solution in series form is given by

$$u(x,t) = u_0(x,t) + u_1(x,t) + u_2(x,t) + \cdots \quad (49)$$

$$u(x,t) = h_1(x) + th_2(x) + \mu\left(h_1{}^{\beta}(x)\frac{t^{2\alpha}}{\Gamma(2\alpha+1)} + h_2{}^{\beta}(x)\frac{t^{2\alpha+1}}{\Gamma(2\alpha+2)}\right) - \lambda\left(h_1(x)\frac{t^{\alpha}}{\Gamma(\alpha+1)} + h_2(x)\frac{t^{\alpha+1}}{\Gamma(\alpha+2)}\right) + \mu^2\left(h_1{}^{2\beta}(x)\frac{t^{4\alpha}}{\Gamma(2\alpha+1)} + h_2{}^{2\beta}(x)\frac{t^{4\alpha+1}}{\Gamma(4\alpha+2)}\right) - 2\mu\lambda\left(h_1{}^{\beta}(x)\frac{t^{3\alpha}}{\Gamma(3\alpha+1)} + h_2{}^{\beta}(x)\frac{t^{3\alpha+1}}{\Gamma(3\alpha+2)}\right) + \lambda^2\left(h_1(x)\frac{t^{2\alpha}}{\Gamma(2\alpha+1)} + h_2(x)\frac{t^{2\alpha+1}}{\Gamma(2\alpha+2)}\right) \quad (50)$$

Special case

Setting $\alpha = 1$ and $\beta = 2$ in equation (50) we obtain the solution of classical telegraph equation by

$$u(x,t) = h_1(x) + th_2(x) + \mu\left(h_1{}^{\beta}(x)\frac{t^2}{2!} + h_2{}^{\beta}(x)\frac{t^3}{3!}\right) - \lambda\left(h_1(x)t + h_2(x)\frac{t^2}{2!}\right) + \mu^2\left(h_1{}^{2\beta}(x)\frac{t^4}{4!} + h_2{}^{2\beta}(x)\frac{t^5}{5!}\right) - 2\mu\lambda\left(h_1{}^{\beta}(x)\frac{t^3}{3!} + h_2{}^{\beta}(x)\frac{t^4}{4!}\right) + \lambda^2\left(h_1(x)\frac{t^2}{2!} + h_2(x)\frac{t^4}{4!}\right) \quad (51)$$

4 Conclusions

Clear conclusion can be drawn from the analytical results in equation (50) and equation (51) that the Adomian method provides highly accurate numerical solutions without spatial discretization for the problem. It is evident that the overall errors can be made smaller by adding new terms of the decomposition series.

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References

[1] A. A. Kilbas, H. M. Srivastava & J. J. Trujillo, (2006), Theory and Applications of Fractional Differential Equations: *Elsevier, Amsterdam, The Netherlands.*

[2] W. Tomasi, (2004), Electronic Communication Systems, *Prentice Hall, New Jersey.*

[3] R. C. Cascaval, E. C. Eckstein, L. Frota & J. A. Goldstein , (2002), Fractional Telegraph Equations: *J Math. Anal. Appl.*, 276, 145-159.

[4] E. Orsingher & L. Beghin, (2004), Time Fractional Telegraph Equation and Telegraph Process with Brownian Time: *Prob. Theory Relat. Fields*, 128, 141-160.

[5] J. Chen, F. Liu & V. Anh, (2008), Analytical Solution for the Time-Fractional Telegraph Equation by the method of Separating Variables: *J. Math. Anal. Appl.*, 338, 1364-1377.

[6] S. Momani, (2005), Analytic and Approximate solutions of the space and time fractional telegraph equations: *Appl. Math. Comput.,* 170, 1126-1134.

[7] G. Adomian, (1986), Non-linear Stochastic Operator Equations: *Academic Press, San Diego.*

[8] G. Adomian, (1994), Solving Frontier Problems of Physics: The Decomposition Method: *Kluwer Acad. Pub., Boston.*

[9] G. Adomian, (1988), A Review of the Decomposition Method in Applied Mathematics: *J. Math. Anal. Appl.* 135, 501-544.

[10] A. M. Wazwaz, (2008), A Study on Linear and Nonlinear Schrodinger equations by the Variational Iteration method: *Chaos, Solitons and Fractals* 37, 1136-1142.

[11] X. G. Luo, (2005), A two step Adomian decomposition method: *Appl. Math Comput.* 170(1), 570-583.

[12] B. Q. Zhang, X. G. Luo and Q. B. Wu, (2006), The restrictions and improvement of the Adomian decomposition method: *Appl. Math. Comput*. 177, 99-104.

[13] M. Caputo, (1969), Elasticita e Dissipazione: *Zanichelli, Bologa, Italy.*
[14] H. Weyl, (1917), Vierteljahrsschr. d. Naturf. Ges.; Zurich, 62, 296–302.
[15] E. A. Ibijola, B.J. Adegboyegun and O.Y. Halid, (2008), On Adomian decomposition method (ADM) for numerical solutions of ordinary differential equations: *Advances in Natural and Applied Sciences*, 3(3) 165-169.
[16] A. Atangana, (2015), On the stability and convergence of the time-fractional variable order telegraph equation: *Journal of Computational Physics* 293, 104-114.
[17] M. A. E. Herzallah, (2010), On abstract fractional telegraph equation: *J. Comput. Nonlinear Dyn*. 5, 5pp.
[18] P. Zjaung, F. Liv, (2006), Implicit difference approximation for the time fractional diffusion equation: *J. Appl. Math Comput.*, 22, 87-99.
[19] M. Azreg-Ainov, (2009), A developed new algorithm for evaluating Adomian polynomials: *CMES*, 42(1) 1-18.
[20] E. Babolian, A. R. Vahidi and G. H. Asadi Cordshooli, (2005), Solving differential equations by decomposition method: *Applied Mathematics and Computation*, 167, 1150-115.