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Research Article / Araştırma Makalesi

Performance Evaluation of PSO Variants for Selective Harmonic Elimination in Multi-Level Inverters

Çok Seviyeli Eviricilerde Seçici Harmonik Eliminasyon İçin PSO Varyantlarının Performans Değerlendirmesi

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Abstract

In this study, the performance of different Particle Swarm Optimization (PSO) variants in solving the Selective Harmonic Elimination (SHE) equations of a Cascaded H-Bridge Multilevel Inverter (CHB-MLI) was compared. The SHE method consists of nonlinear transcendental equations, which are particularly difficult to solve analytically in high-level systems. The aim of the study is to examine the overall improvements provided by the PSO variants and to specifically evaluate their effectiveness in solving such complex engineering problems. In addition to the standard PSO, several improved versions from the literature have been considered. For each variant, the optimal switching angles that solve the SHE equations were determined and applied to a CHB-MLI model in the MATLAB/Simulink environment. The convergence behaviors of the algorithms, the total harmonic distortions (THD) and the amplitudes of the fundamental components of the output voltages were analyzed statistically. As a result of these analyses, the strengths and weaknesses of the PSO variants in the optimization processes were revealed. Based on the findings, a hybrid model was also proposed, which integrates the strong features of the successful variants. It was observed that the proposed hybrid model stands out from the other variants by exhibiting a stable and competitive performance even in the worst-case scenarios. These findings indicate that effectively developed PSO variants can be a powerful alternative for solving real-world optimization problems.

Öz

Bu çalışmada, farklı Parçacık Sürü Optimizasyonu (PSO) varyantlarının Kaskat Bağlı H-Köprü Çok Seviyeli Eviricinin (CHB-MLI) Seçici Harmonik Eliminasyon (SHE) denklemlerini çözmedeki performansları karşılaştırılmıştır SHE yöntemi, özellikle yüksek seviyeli sistemlerde analitik olarak çözülmesi oldukça zor olan doğrusal olmayan transandantal denklemlerden oluşmaktadır. Çalışmanın amacı, PSO varyantlarının genel olarak sağladığı iyileştirmeleri incelemek ve özellikle bu tür karmaşık mühendislik problemlerindeki etkinliklerini değerlendirmektir. Standart PSO'nun yanı sıra, literatürdeki geliştirilmiş bazı PSO versiyonları ele alınmıştır. Her varyant için SHE denklemlerini çözen optimal anahtarlama açıları belirlenerek MATLAB/Simulink ortamında bir CHB-MLI modeline uygulanmıştır. Algoritmaların yakınsama davranışları, çıkış gerilimlerinin toplam harmonik distorsiyonları ve temel bileşen genlikleri istatistiksel olarak analiz edilmiştir. Bu analizler sonucunda, PSO varyantlarının optimizasyon süreçlerindeki güçlü ve zayıf yönleri ortaya konmuştur. Ayrıca elde edilen bulgular doğrultusunda, başarılı varyantların seçilen güçlü özelliklerinin entegre edildiği hibrit bir model de önerilmiştir. Özellikle önerilen hibrit modelin, en kötü senaryolarda dahi istikrarlı ve rekabetçi bir performans sergileyerek diğer varyantlardan ayrıştığı gözlemlenmiştir. Bu bulgular, etkin şekilde geliştirilen PSO varyantlarının gerçek dünya optimizasyon problemlerinin çözümünde güçlü bir alternatif olabileceğini göstermektedir.

Keywords: *PSO, SHE, CHB-MLI, Optimization, PSO Variant, Multilevel Inverter*

Anahtar Kelimeler: *PSO, SHE, CHB-MLI, Optimizasyon, PSO Varyantı, Çok Seviyeli Evirici*

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1. Introduction

Particle Swarm Optimization (PSO) has a significant place in nature-inspired metaheuristic optimization algorithms. Initially proposed by Eberhart and Kennedy in 1995 [1], PSO was developed by modelling the collective behaviour of birds and fish moving in flocks or schools [2]. At its core, the algorithm consists of a swarm of particles randomly initialized in a multi-dimensional solution space. The way swarm individuals influence each other while searching for food has served as the inspiration for PSO's fundamental steps in solving optimization problems. Each particle moves within the search space by updating its position and velocity. These movements are guided by both the best position found by the individual particle ($Pbest$) and the best position discovered by the entire swarm ($Gbest$). Due to its simple structure, low computational cost, and broad applicability, PSO has been widely adopted in various engineering problems [3].

However, over time, several limitations of the classical PSO algorithm have been identified in practical applications. In particular, PSO tends to exhibit premature convergence and can become trapped in local minima, which restricts its effectiveness, especially in multimodal and complex search spaces [4]. These issues arise due to the algorithm's inability to properly balance exploration and exploitation during the search process [5]. To overcome these challenges, numerous PSO variants have been developed in the literature[6–9]. The primary goal of these variants is to adapt particle dynamics to the problem structure, ensuring a more effective exploration of the search space while also achieving faster and higher-quality solutions.

In recent years, with the increasing number of metaheuristic optimization algorithms, PSO has been perceived as a less competitive approach and is now primarily used as a benchmark tool to demonstrate the superiority of newly developed optimization techniques. However, the PSO algorithms employed for this purpose are typically either the standard version or variants with only limited enhancements. This situation has led to the neglect of the potential offered by advanced PSO variants. Therefore, evaluating the effectiveness of different PSO variants in real-world optimization problems is of great importance.

Optimization processes in real-world engineering problems often involve nonlinear and complex mathematical models. One such problem is the solution of Selective Harmonic Elimination (SHE) equations for Cascaded H-Bridge Multilevel Inverters (CHB-MLIs). The SHE method consists of nonlinear and transcendental equations [10] designed to eliminate specific harmonic components while maintaining the fundamental component amplitude at a desired level [11]. While these equations can be solved analytically for low-level systems, they are generally highly challenging for high-level inverters [12], necessitating the use of numerical or heuristic methods. In this context, due to its simple implementation and ability to efficiently explore large search spaces, advanced PSO variants are considered a strong alternative for solving SHE equations.

In this study, the performance of different PSO variants in solving SHE equations was compared, and the contributions of various improvements to the optimization process were examined. The analysis revealed that while some variants were successful in certain aspects, they fell short in others. Based on these findings, a new hybrid PSO variant that integrates the most effective strategies of successful PSO variants was developed and included in the evaluation.

With the inclusion of the hybrid model, a total of eight different PSO variants were evaluated through a series of independent runs, where optimal switching angles minimizing the objective fitness function were determined. First, the fitness values produced by each variant were visualized using box plots for comparative analysis, allowing for an overall performance evaluation of the algorithms. Then, to validate the results, the switching angles corresponding to these fitness values were applied to a three-phase star-connected CHB-MLI model in the MATLAB/Simulink environment, and the output voltage waveforms and their frequency spectrums were examined.

Accordingly, the obtained results reveal the strengths and weaknesses of the algorithms, providing significant insights into which strategies are more effective in solving SHE equations. In particular, the proposed hybrid model has distinguished itself from other variants by producing not only consistent overall performance but also demonstrating stable and competitive results even in the worst-case scenarios. Additionally, this study shows that a well-designed PSO variant can be effectively applied not only in theoretical test scenarios but also in real-world problems such as solving SHE equations, as exemplified by the newly developed hybrid variant.

2. Cascaded H-Bridge MLI and Selective Harmonic Elimination Method

CHB-MLI consists of multiple identical and discrete H-bridge inverter modules connected in series [13]. Each module has its own independent power supply, and all modules together generate a staircase-shaped output voltage. Each module forms a single step of this staircase waveform. As the number of modules increases, the output voltage waveform contains more steps, thereby achieving a closer approximation to the desired sinusoidal waveform. This modular design offers various advantages, particularly in high-voltage power systems [14,15]. For instance, when voltage levels need to be increased or decreased, additional modules can be added or removed as needed without requiring a fundamental modification to the entire system. Moreover, since each module operates independently, in case of maintenance or failure, the faulty module(s) can be isolated and replaced without disrupting the entire system. Due to these advantages, despite the increased complexity of control algorithms and the higher number of power switches required, the CHB-MLI topology is preferred over alternative multilevel inverter configurations, particularly in medium- and high-power applications.

In a CHB-MLI, the number of voltage levels depends on the number of independent H-bridge inverter modules used. The relationship between them can be expressed as shown in Equation (1), where k represents the number of H-bridge modules, and n denotes the output voltage levels:

$$n = 2k + 1 \tag{1}$$

In this study, the aim is to compare different PSO variants in solving the SHE equations of a three-phase, star-connected 11-level CHB-MLI. In this context, five modules, each with a 50 V DC source per phase, were utilized. Figure 1 illustrates the circuit diagram of each module along with the block diagram representing the module arrangement for each phase, while Figure 2 presents the phase voltage (V_{phase}) of the CHB-MLI, which exhibits a staircase waveform. As observed, each module (Vo_x) constitutes a step of this staircase waveform. To minimize switching losses, each power switch undergoes commutation twice per half-cycle (θ_X, $\pi - \theta_X$).

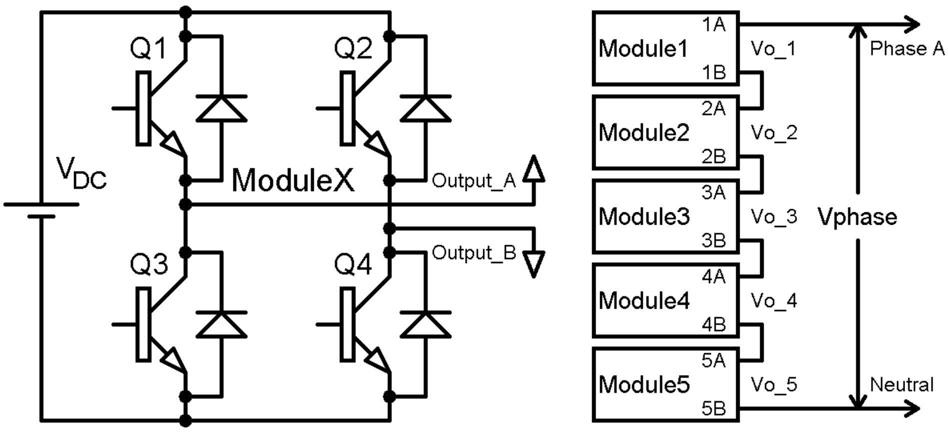


Figure 1. Circuit diagram of each module and arrangement for the 3-phase 11-level CHB-MLI

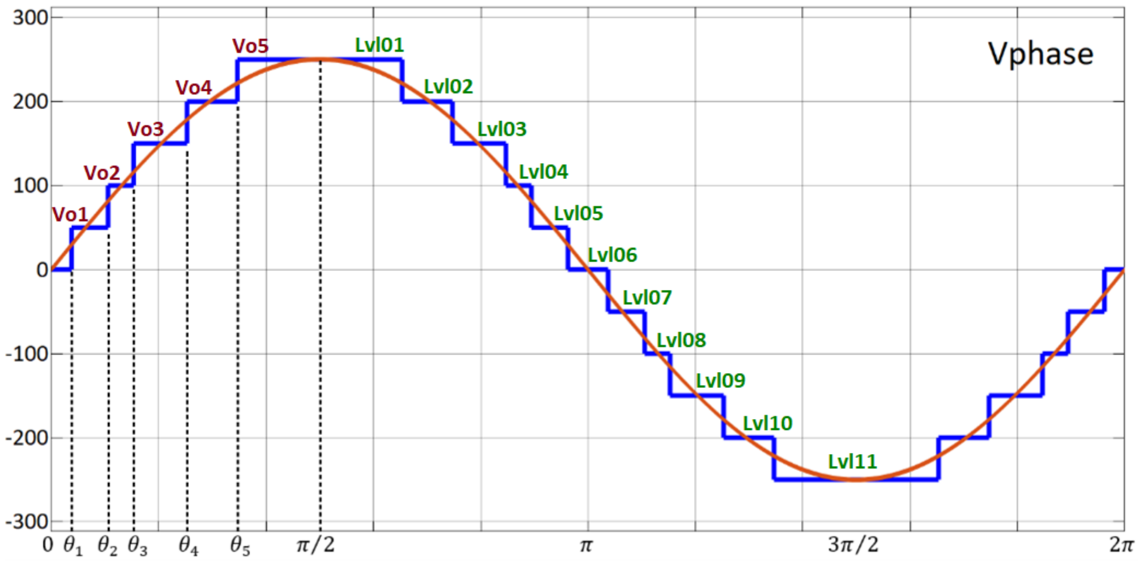


Figure 2. The output voltage waveform for one phase of MLI

Various switching strategies have been proposed in the literature for CHB-MLIs [16]. However, among these approaches, Selective Harmonic Elimination (SHE) has been the most preferred method, particularly in high-power critical applications where power quality is a limiting factor. The primary objective of the SHE method is to determine the optimal switching angles that eliminate specific low-order dominant harmonics in the inverter output voltage while maintaining the fundamental harmonic component at the desired level.

As shown in Figure 2, the Fourier series expansion of a CHB-MLI output voltage waveform can be expressed by Equation (2). Here, V_h represents the amplitude of the h^{th} harmonic. Due to quarter-wave symmetry, even harmonics do not appear in the output voltage. Assuming that all DC sources are equal and considering that five switching angles are required to generate the output voltage of an 11-level inverter, each odd harmonic can be calculated using Equation (3). The switching angles must be within the range $0 < \theta_j < \pi/2$.

$$V(\omega t) = \sum_{h=1}^{\infty} V_h \sin(h\omega t) \tag{2}$$

$$V_h = \frac{4V_{DC}}{h\pi} \sum_{j=1}^{5} \cos(h\theta_j) \quad h = 1,3,5,7,\ldots \quad j = 1,2,3,4,5 \tag{3}$$

When the SHE method is applied to an 11-level MLI, five harmonic equations are derived from Equation (3) to determine the optimal five switching angles. One of these equations represents the fundamental component, while the remaining four are selected based on which harmonics need to be eliminated. As is well known, low-order harmonics have a more dominant effect on power quality. Additionally, due to three-phase symmetry, the third and its multiples are not considered. Therefore, in an 11-level MLI, the elimination of the 5th, 7th, 11th and 13th harmonics is typically targeted. The corresponding harmonic equations are given in Equations (4)–(8).

The most significant challenge of the SHE method is solving these five nonlinear equations with five unknowns. For such nonlinear and complex equations, numerical and algebraic solution methods are often insufficient. Determining the switching angles that satisfy the required conditions can be directly considered an optimization problem. Therefore, metaheuristic optimization techniques are frequently preferred to overcome this challenge in the SHE method [17]. In this study, the performance of different PSO variants, one of the most fundamental metaheuristic optimization algorithms, is compared in solving this optimization problem. The objective is to determine the optimal five switching angles $(\theta_1, \theta_2, \theta_3, \theta_4, \theta_5)$ that maintain V_1 at the desired reference value while eliminating V_5, V_7, V_{11} and V_{13} in the given equations.

$$V_1 = \frac{4V_{DC}}{\pi}[\cos(\theta_1) + \cos(\theta_2) + \cos(\theta_3) + \cos(\theta_4) + \cos(\theta_5)] \quad (4)$$

$$V_5 = \frac{4V_{DC}}{5\pi}[\cos(5\theta_1) + \cos(5\theta_2) + \cos(5\theta_3) + \cos(5\theta_4) + \cos(5\theta_5)] \quad (5)$$

$$V_7 = \frac{4V_{DC}}{7\pi}[\cos(7\theta_1) + \cos(7\theta_2) + \cos(7\theta_3) + \cos(7\theta_4) + \cos(7\theta_5)] \quad (6)$$

$$V_{11} = \frac{4V_{DC}}{11\pi}[\cos(11\theta_1) + \cos(11\theta_2) + \cos(11\theta_3) + \cos(11\theta_4) + \cos(11\theta_5)] \quad (7)$$

$$V_{13} = \frac{4V_{DC}}{13\pi}[\cos(13\theta_1) + \cos(13\theta_2) + \cos(13\theta_3) + \cos(13\theta_4) + \cos(13\theta_5)] \quad (8)$$

3. Particle Swarm Optimization (PSO) Variants

The objective of this study is to compare the performance of different PSO variants in solving SHE equations. Among the PSO algorithm variants to be compared, it is essential to include the original version first proposed by Kennedy and Eberhart [1], which will hereafter be referred to as Standard PSO (SPSO). At this stage, the fundamental equations and operational steps of SPSO in the context of solving SHE equations will be briefly summarized. This will provide a clearer understanding of PSO's general approach to optimization problems and highlight the specific areas where the examined variants introduce improvements.

PSO is a population-based heuristic optimization algorithm that operates in a multi-dimensional search space. Each particle moves based on both its personal best position ($Pbest$) and the global best position ($Gbest$) within the swarm[18]. The position and velocity updates of the particles form the core of the algorithm, incorporating randomness to balance social and individual knowledge interactions.

The algorithm begins by generating a random initial population consisting of N particles. Each particle represents a solution set containing five angular variables (θ_i), as shown in Equation (9), specifically tailored for solving the SHE equations. This solution must satisfy the constraints given in Equation (10). At the initialization stage, since no predefined solution exists, each particle's current position is considered its personal best solution ($Pbest_i$). Additionally, the velocity parameters (V_i) are set to zero, as it is undesirable for the particles to have any predefined direction at the start of the optimization process. In this way, the candidate initial solutions for solving SHE equations are established

$$\theta_i = (\theta_{i,1}, \theta_{i,2}, \theta_{i,3}, \theta_{i,4}, \theta_{i,5}) \;\; i \in \{1,2,\dots,N\} \quad (9)$$

$$0 \le \theta_1 < \theta_2 < \theta_3 < \theta_4 < \theta_5 \le \pi/2 \quad (10)$$

$$Pbest_i = \theta_i \qquad V_i = (0,0,0,0,0) \quad (11)$$

After generating the random initial population, the first fitness scores must be computed to evaluate how close each particle is to an optimal solution. For this purpose, the fitness function derived from the SHE equations, as given in Equation (12), is used. In this equation, the first term controls the amplitude of the fundamental harmonic, while the second term aims to minimize the dominant low-order harmonics [19]. Since solving the nonlinear SHE equations is formulated as a minimization problem, a particle with a lower fitness score represents a better candidate solution. This implies achieving successful suppression of low-order harmonics while maintaining the output voltage at the desired level.

As shown in Equations (13) and (14), the fitness score of each particle in the swarm is evaluated using the given fitness function, and the best solution for the initialization phase is then selected.

$$f_i = \left(\left| V_{1ref} - \frac{4V_{DC}}{\pi} \sum_{j=1}^{5} \cos(\theta_{i,j}) \right| + \left(\frac{4V_{DC}}{\pi}\right)^2 \sum_{k=1}^{4} \left(\frac{1}{h_k} \sum_{j=1}^{5} \cos(h_k \theta_{i,j}) \right)^2 \right) \quad (12)$$

$$h = [5,7,11,13]$$

$$F = \begin{bmatrix} F_1 \\ \vdots \\ F_i \\ \vdots \\ F_N \end{bmatrix} = \begin{bmatrix} f(\theta_1) \\ \vdots \\ f(\theta_i) \\ \vdots \\ f(\theta_N) \end{bmatrix} \quad (13)$$

$$Fbest = min(F) \rightarrow Gbest = \theta(Fbest) \quad (14)$$

In the equations above, V_{1ref} represents the desired amplitude of the fundamental frequency component of the output voltage, h is a vector containing the orders of low-frequency harmonic components to be eliminated in a three-phase system, F denotes the fitness vector, and F_i is the fitness score of the i^{th} particle.

After the initialization phase, the iteration loop begins. In each iteration, the velocity of each particle is first updated based on its current position (θ_i), personal best position ($Pbest_i$), and the position of the best particle in the swarm ($Gbest$), as shown in Equation (15). Then, the updated velocity is added to the current position to obtain the new position, as expressed in Equation (16). The goal is to move each particle closer to the optimal solution while preserving its individual experience.

$$V_{i,j}(t+1) = V_{i,j}(t) + C_1.r1.\left(Pbest_{i,j} - \theta_{i,j}(t)\right) + C_2.r2.\left(Gbest_j - \theta_{i,j}(t)\right) \quad (15)$$

$$\theta_{i,j}(t+1) = \theta_{i,j}(t) + V_{i,j}(t+1) \quad i \in \{1,2,\dots,N\} \; j \in \{1,2,3,4,5\} \quad (16)$$

In the equations above, t represents the current iteration number, while $t+1$ denotes the next iteration. $r1$ and $r2$ are randomly generated numbers in the range [0,1]. C_1 is the cognitive coefficient that determines a particle's tendency to move toward its personal best position, whereas C_2 is the social coefficient that influences the particle's movement toward the global best position in the swarm. These two parameters together are referred to as acceleration coefficients. In SPSO, both values are set equal ($C_1 = C_2 = 2$) to ensure that particles benefit equally from their individual experiences and the swarm's collective knowledge, thereby establishing a balanced exploration and exploitation process.

At the end of each iteration, the fitness function given in Equation (12) is re-evaluated based on the updated positions of the particles. The personal best positions and the global best position are then updated according to Equations (17) and (18). The process described above is repeated until a predefined maximum number of iterations (T) is reached.

$$f(\theta_i) < f(Pbest_i) \rightarrow Pbest_i = \theta_i \quad (17)$$

$$f(Pbest_i) < f(Gbest) \rightarrow Gbest = Pbest_i \quad (18)$$

At the end of the entire optimization process, the position of the best particle in the swarm ($Gbest$) represents the optimal switching angles determined by PSO for solving the SHE equations, as shown in Equation (19). These angles will be applied to the power switches of the multilevel inverter to achieve the desired output voltage while minimizing low-order harmonics

$$Optimal\ Switching\ Angles \rightarrow Gbest = (Gbest_1, Gbest_2, Gbest_3, Gbest_4, Gbest_5) \quad (19)$$

The standard PSO algorithm exhibits certain structural weaknesses that affect the movement dynamics of particles, particularly in complex and high-dimensional problems. One of the primary weaknesses is the absence of a mechanism to retain the previous velocity component in velocity updates. In standard PSO, the velocity of each particle is updated solely based on its personal best position and the global best position, without incorporating any mechanism to preserve the particle's current velocity

The lack of dependence on previous velocity disrupts the continuity of particle movement, leading to two major issues. First, when the $Pbest$ and $Gbest$ positions are very close to the particle's current position, the velocity component reaches very small values, causing the particle to become nearly motionless. This phenomenon, commonly referred to as stagnation in the literature, results in a loss of movement capability, particularly during the convergence phase. On the other hand, as $Pbest$ and $Gbest$ move further away from the particle, it may take large and uncontrolled jumps toward these attraction points. These two extreme cases hinder an effective exploration of the search space, ultimately negatively impacting the overall performance of the algorithm.

To overcome these issues, one of the first structural modifications proposed in the literature was the introduction of an inertia weight coefficient in the velocity update equation. Therefore, the second variant in the list of PSO variants to be compared is PSOCIW (Constant Inertia Weight PSO) [20], which was the first approach in the literature to incorporate this coefficient with a fixed value. This variant enables particles to retain a portion of their current velocity, facilitating a more balanced exploration and exploitation process in the search space. In the PSOCIW variant, the velocity update equation is redefined as follows, where ω represents a constant inertia weight:

$$V_{i,j}(t+1) = \omega . V_{i,j}(t) + C_1 . r1 . \left(Pbest_{i,j} - \theta_{i,j}(t)\right) + C_2 . r2 . \left(Gbest_j - \theta_{i,j}(t)\right) \tag{20}$$

With the addition of the inertia weight, PSO gained improved velocity control and enhanced adaptability to different types of optimization problems. As a result, PSOCIW is considered a milestone in PSO literature, and the majority of modern PSO variants have been developed based on this fundamental concept. However, subsequent studies have shown that maintaining a fixed coefficient does not always ensure an effective balance between exploration and exploitation across different optimization processes. Consequently, new variants have been introduced in the literature, where the inertia weight is defined as either a time-varying or randomly selected parameter. In this context, two additional PSO variants included in the comparison are PSOTVIW (Time Varying Inertia Weight PSO) [21] and PSORIW (Random Inertia Weight PSO) [22].

PSOTVIW is one of the fundamental improvements to the fixed inertia weight approach. In this variant, the inertia weight (ω) is defined as a linearly decreasing function as iterations progress. This strategy aims to promote broader exploration in the early iterations and more focused exploitation in later iterations. The inertia weight is defined as shown in Equation (21), where ω_{max} represents the initial maximum inertia weight, and ω_{min} denotes the minimum inertia weight at the final iteration. The velocity update equation includes the inertia weight as in PSOCIW; however, in this case, the coefficient is not fixed but varies over time.

$$\omega(t) = \omega_{max} - (\omega_{max} - \omega_{min}) \cdot \frac{t}{T} \tag{21}$$

The PSORIW variant offers an alternative approach in which the inertia weight is randomly selected within a predefined range at each iteration. Instead of a deterministic change, this method adopts a dynamic and stochastic inertia management strategy, allowing different levels of exploration and exploitation to be applied in each iteration. This variability helps reduce the risk of getting trapped in local minima and enhances adaptability to different types of optimization problems. The inertia weight for each iteration is determined as follows:

$$\omega(t) = 0,5 + \frac{Rand}{2} \tag{22}$$

Both methods provide a more flexible and adaptive structure compared to PSOCIW. PSOTVIW establishes a systematic exploration-exploitation balance by modelling the natural evolution of the search process, whereas PSORIW enhances search diversity through randomness, aiming to reduce the risk of being trapped in different local minima. While the diversity introduced by PSORIW offers an advantage in highly multimodal and high-dimensional problems, the controlled approach of PSOTVIW tends to be more effective in convex and unimodal problems.

Both variants are considered significant advancements in the use of inertia weight in PSO and have influenced the development of many modern PSO derivatives. Today, hybrid approaches derived from these two strategies are widely utilized in contemporary PSO variants.

In addition to the variants discussed above, numerous studies have explored different approaches to updating the inertia weight. In this context, various PSO derivatives such as CRIWPSO (Chaotic Random Inertia Weight PSO) [23], AIWPSO (Adaptive Inertia Weight PSO) [24], GLBPSO (Global Local Best Inertia Weight PSO) [25], DAPSO (PSO with Dynamic Adaptation) [26], and IPSO (Improved PSO) [27] have been introduced in the literature. However, preliminary performance analyses indicate that these variants do not offer a significant advantage over PSORIW and PSOTVIW. Therefore, among the inertia weight-based improvements, only PSOCIW, PSORIW, and PSOTVIW have been selected as the core methods for comparison in this study.

In early PSO variants, optimization improvements primarily focused on the inertia weight component, with modifications limited to different update strategies (e.g., linear, nonlinear, or adaptive methods based on the iteration count or the relationship between $Pbest$ and $Gbest$). No changes were made to the acceleration coefficients (C_1, C_2) or the velocity-position update mechanisms, nor was an approach similar to the mutation mechanism in genetic algorithms introduced to enhance solution space diversity. These variants preserved the fundamental structure of PSO's original version, including its core velocity and position update equations.

In contrast to the previous variants, the next three variants examined in this study introduce novel approaches beyond just improvements in the inertia weight mechanism. These methods are identified as MPSOTVAC (PSO with Mutation and Time-Varying Acceleration Coefficients) [28], APSO (Adaptive PSO) [29], and IAPSO (Inertia Adaptive PSO) [30], respectively.

In MPSOTVAC, the linearly decreasing inertia weight update approach given in Equation (21) is adopted, while two major improvements are introduced to the standard PSO. The first improvement involves adapting the acceleration coefficients to change over time based on the iteration count. In this enhanced strategy, the C_1 coefficient decreases as iterations progress ($C_{1max} \rightarrow C_{1min}$), whereas the C_2 coefficient increases in contrast ($C_{2min} \rightarrow C_{2max}$). The primary objective of this approach is to enable a broader search (exploration) during the initial stages of the algorithm while ensuring faster and more stable convergence (exploitation) in later iterations.

This strategy is expressed by the following equations:

$$C_1(t) = C_{1max} - (C_{1max} - C_{1min}) \cdot \frac{t}{T} \tag{23}$$

$$C_1(t) = C_{1max} - (C_{1max} - C_{1min}) \cdot \frac{t}{T} \tag{24}$$

The second improvement is a mutation-based velocity update, which activates a random mutation mechanism when particle velocities stagnate and no improvement occurs in the global best solution ($Gbest$). With this mechanism, if the global best value remains unchanged for a certain period, a randomly selected particle undergoes a random perturbation (increase or decrease) in one of its velocity dimensions with a certain probability. This approach aims to prevent the issue of premature convergence. The mutation process is formulated as follows:

$$f\big(Gbest(t-1)\big) - f\big(Gbest(t)\big) < 0 \rightarrow R_1 < pm \rightarrow V_{k,d} = \begin{cases} V_{k,d} + R_2 \cdot \dfrac{V_{max}}{m}, & R_2 < 0.5 \\ V_{k,d} - R_2 \cdot \dfrac{V_{max}}{m}, & R_2 \geq 0.5 \end{cases} \tag{25}$$

Here, k and d represent the index of the randomly selected particle and the randomly selected dimension, respectively. pm is the mutation probability, m is the mutation scale parameter, and R_1 and R_2 are two randomly generated numbers within the range [0,1]. In this mutation mechanism, R_1 determines whether the mutation will be applied, while R_2 determines the mutation direction (increase or decrease). V_{max} represents the velocity limit.

In IAPSO, the inertia weight of each particle is dynamically adjusted based on its distance from the global best position. In this approach, particles that move away from the global best have their inertia weight reduced, allowing them to break free from the influence of their previous velocity and be pulled more strongly toward the global best position. This mechanism preserves the exploration capability while promoting convergence toward the optimal solution. The inertia weight in IAPSO is defined as follows:

$$\omega = \omega_0 \cdot \left(1 - \frac{dist_i}{max\_dist}\right) \tag{26}$$

$$dist_i = \left(\sum_{j=1}^{D} \left(Gbest_j - \theta_{i,j}\right)^2\right)^{1/2} \tag{27}$$

$$max\_dist = \max(dist_i) \tag{28}$$

Here, ω_0 is the initial inertia weight, which is randomly selected within the range [0.5,1]. $dist_i$ represents the current Euclidean distance of the i^{th} particle and is defined as the distance between the particle and the global best position. max_dist denotes the maximum distance of any particle from the global best position in the current generation

The second major improvement of IAPSO is the addition of an adaptive momentum term to the position update equation in classical PSO. This momentum term is controlled by a randomly selected ρ parameter and takes a value in the range of 0.75 to 1.25 at each iteration, dynamically adjusting the contribution of the particle's previous position to its updated position. This mechanism allows the particle to move more freely in the search space by reducing the influence of its previous position in some cases, while in others, it reinforces the particle’s current trajectory. Thus, the risk of premature convergence is reduced, and an adaptive balance is achieved. The position update equation is defined as follows. Here, ρ is a randomly selected value in the range of $[-0.25, 0.25]$.

$$\theta_{i,j}(t+1) = (1-\rho) \cdot \theta_{i,j}(t) + V_{i,j}(t+1) \tag{29}$$

APSO is a PSO variant that stands out with its dynamic inertia weight and adaptive repositioning mechanism. In traditional PSO, the inertia weight is considered either as a fixed parameter or as one that changes based on iterations, whereas in APSO, it is dynamically determined according to each particle’s fitness value. This allows particles closer to a better solution to move more slowly, while those farther away engage in more aggressive exploration, covering larger regions of the search space. The adaptive update of the inertia weight is expressed as follows. Here, $Rank_i$ represents the ranking of the particle's fitness score within the population, and N denotes the total number of particles

$$\omega_i = \omega_{min} + (\omega_{max} - \omega_{min}) \cdot \frac{Rank_i}{N} \tag{30}$$

Another significant improvement in APSO is the random repositioning of the worst particles, as defined in Equation (31), if the global best fitness value remains unchanged for a certain period. This process aims to increase diversity within the population and prevent premature convergence

$$\begin{aligned} &f(Gbest_{t-5}) - f(Gbest_t) < thrs \rightarrow select\ \theta_{worst,1}, \theta_{worst,2}, \ldots \theta_{worst,Z} \\ &\theta_{worst,1} = \theta_{rand,1}, \theta_{worst,2} = \theta_{rand,2} \ldots \theta_{worst,z} = \theta_{rand,z} \end{aligned} \tag{31}$$

Here, $f(Gbest)$ represents the fitness value of the global best solution. If the change in this value remains below a certain threshold for five consecutive iterations, the worst Z particles in the population are selected. The positions of these selected particles are then replaced with newly generated random positions (θ_{rand}).

The conducted tests demonstrate that IAPSO achieves successful results compared to classical PSO, thanks to its dynamic inertia weight determination and momentum-based position update strategies. In particular, the Median and standard deviation (Std) metrics of the best fitness values obtained from independent runs confirm the effectiveness of the proposed improvements. However, the observation of weaker-than-expected results in some runs and the wide variation in fitness values indicate that the algorithm needs further improvements in terms of stability. This issue is believed to stem from IAPSO's fixed acceleration coefficient approach.

In this context, a new hybrid PSO variant has been proposed to address the inconsistencies in IAPSO's performance distribution while preserving its dynamic inertia weight determination and momentum-based position update strategies. This hybrid approach incorporates the adaptive acceleration coefficient adjustment strategy used in MPSOTVAC. Additionally, to further reduce the risk of premature convergence, APSO's reinitialization mechanism has also been integrated into the hybrid model. With the inclusion of this hybrid PSO variant, the total number of variants evaluated in the comparative analysis has increased to eight.

4. Results and Discussion

In this study, the effectiveness of eight different PSO variants in solving complex SHE equations was compared, and the impact of these improvements on the capability of standard PSO in real-world engineering problems was evaluated. The first of these variants is the original PSO algorithm (SPSO) without inertia weight, while six of them are existing variants in the literature that have incorporated various enhancements into the original version. The final variant is the proposed hybrid model, which selectively integrates the strong aspects of some successful variants to achieve a more balanced and robust optimization approach

The common goal of all variants is to determine the optimal switching angles by minimizing the fitness function defined in Equation (12). This function is designed to eliminate dominant harmonics up to the 14th order (excluding even harmonics and multiples of three) in the inverter output voltage while keeping the fundamental component amplitude as close as possible to the desired reference value. For all algorithms, the maximum number of iterations (T) is set to 100, the population size (N) is 50, the reference fundamental component amplitude (V_{1ref}) is 250 V (433.01 V for line voltage), and the maximum and minimum velocity limits of the particles (v_{min}, v_{max}) are set to [-1,1]. The fixed parameters specific to other variants are given in Table 1.

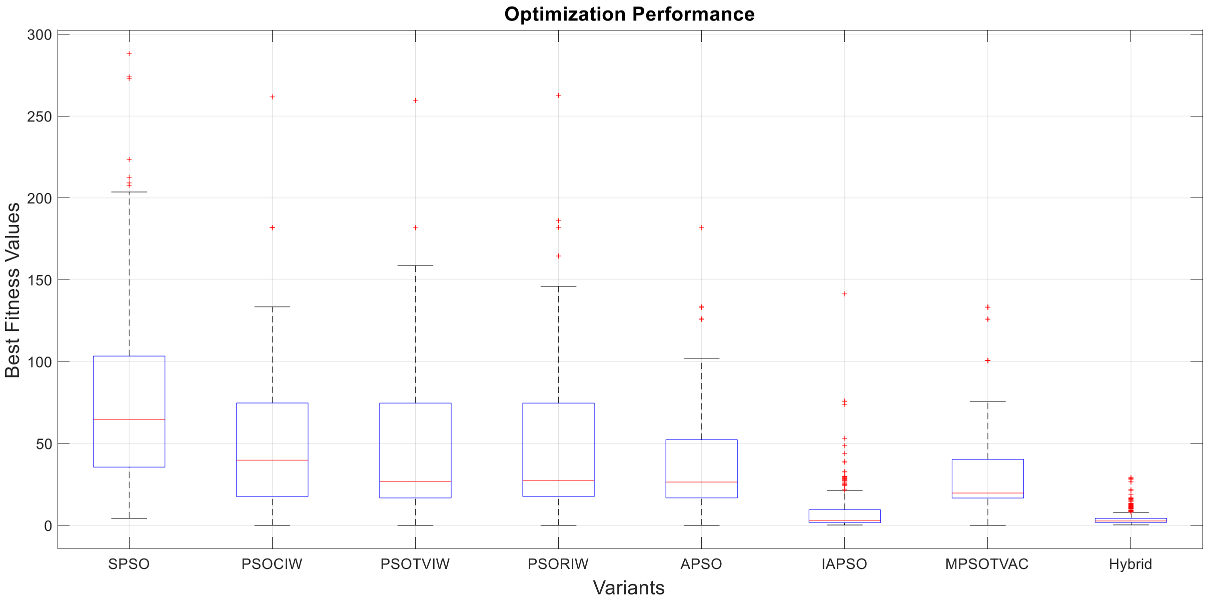
Table 1. Parameters of the PSO variants.

Variant	Parameter
SPSO	$C_{1,2} = 0.75$
PSOCIW	$C_{1,2} = 0.75$, $\omega = 0.7$
PSOTVIW	$C_{1,2} = 0.75$, $\omega_{min,max} = (0.4, 0.9)$
PSORIW	$C_{1,2} = 0.75$
APSO	$C_{1,2} = 0.75$, $\omega_{min,max} = (0.4, 0.9)$, $Z = 5$, $thrs = 1$
IAPSO	$C_{1,2} = 0.75$, $\rho = [-0.25, 0.25]$
MPSOTVAC	$C_{1,2min} = 0.25$, $C_{1,2max} = 1.25$, $\omega_{min,max} = (0.4, 0.9)$, $pm = 0.1$, $m = 5$
Proposed	$C_{1,2min} = 0.25$, $C_{1,2max} = 1.25$, $\omega_{min,max} = (0.4, 0.9)$, $\rho = [-0.25, 0.25]$, $Z = 5$, $thrs = 1$

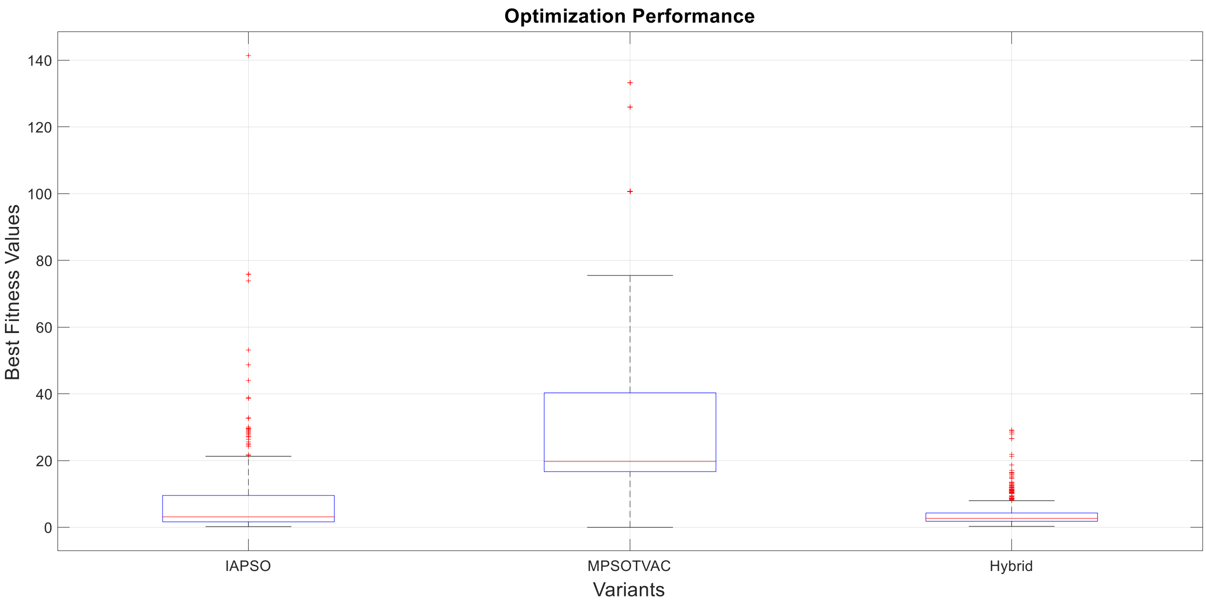
For each variant, 501 independent runs were conducted, and the best fitness values obtained from each run were recorded. Among these, the Best, Median, and Worst fitness values, along with their standard deviations for each variant, were determined and presented in Table 2. Additionally, to visualize the distribution of these values and better highlight the performance differences between the variants, a box plot was generated and shown in Figure 3a. Furthermore, to analyze the performance of the last three variants in more detail, a second box plot was created, including only these three variants, and presented in Figure 3b.

Table 2. Optimization results of PSO variants

PSO Variants	Statistical Summary of Fitness Values			
	Best	Median	Worst	Std
SPSO	4.3234	64.6005	288.1462	47.7406
PSOCIW	7.53E-06	39.8323	261.7193	43.4600
PSOTVIW	3.63E-05	26.6706	259.5298	42.3202
PSORIW	***7.01E-06***	27.2649	262.5894	42.8051
APSO	7.71E-05	26.4647	181.8058	32.6880
IAPSO	0.2290	3.1401	141.3887	11.1067
MPSOTVAC	1.44E-05	19.7548	133.2802	26.5570
Hybrid	0.2895	***2.6952***	***29.1667***	***4.4168***



(a)



(b)

Figure 3. Boxplot graphic of **(a)** all PSO variants, **(b)** last 3 variants

Upon examining the box plots and Table 2, the first noticeable point is that the inclusion of inertia weight significantly enhances optimization performance. Variants with inertia weight achieve better results across all metrics (Best, Median, Worst, and Std) indicating not only improved solution quality but also greater stability compared to standard PSO.

Another important observation from the results is that there is no significant performance difference among PSOCIW, PSOTVIW, and PSORIW. Although PSORIW achieved the best result in the Best metric among all variants, this outcome is possibly due to the stochastic nature of the algorithm and is not statistically meaningful, with only a slight improvement over the others. In the Median metric, PSOTVIW and PSORIW exhibited slightly lower values than PSOCIW. Overall, the presence of inertia weight led to better results in all three algorithms compared to standard PSO. However, considering the method of determining inertia weight, the slightly higher Median value of PSOCIW suggests that dynamically changing inertia weight approaches improve solution quality more effectively

The obtained results indicate that the additional improvements of APSO did not provide a significant advantage in the context of this optimization problem in terms of Best and Median metrics. Nevertheless, in the Worst metric, APSO achieved a lower error value compared to other methods and exhibited a relatively better performance in the worst-case scenarios. Furthermore, its lower standard deviation suggests that the algorithm reduces solution variability and operates more stably. However, this stability is associated with a narrower distribution of solutions rather than an overall improvement in solution quality

In general, when the results are examined, it is observed that the last three variants (IAPSO, MPSOTVAC, and the proposed Hybrid) have demonstrated significantly better performance compared to the first five variants, except for the Best metric. However, since the Best metric can exhibit substantial variations across different runs, it is not considered a reliable standalone criterion for evaluating the overall performance of the algorithms. In contrast, these three algorithms, which produced lower values in the Median, Worst, and Std metrics, not only generated better solutions during the optimization process but also improved the consistency of these solutions, demonstrating a competitive performance. These findings indicate that the applied enhancements have improved optimization performance and that these three methods explore the search space more efficiently.

IAPSO has demonstrated a significant improvement in the Median and Std metrics compared to the first five variants and has also produced better results than MPSOTVAC. Although the difference between IAPSO and MPSOTVAC is smaller than that observed with the other variants, it is still noteworthy. In the Worst metric, an improvement has been achieved compared to the first five variants; however, this difference is relatively small, and IAPSO remains slightly behind MPSOTVAC.

On the other hand, when examining the box plot, it is evident that IAPSO has a considerable number of outliers. This indicates that while the algorithm produces very good results in some runs, it yields unexpectedly high fitness values under certain conditions. Despite these fluctuations, IAPSO stands out as a strong alternative alongside the proposed hybrid variant for solving SHE equations, particularly due to the improvements it provides in the Median and Std metrics.

When evaluating its overall performance, MPSOTVAC demonstrates significantly better results compared to the first five variants. Although MPSOTVAC achieved the best value among the last three variants in the Best metric, this comparison is of limited significance due to the reasons discussed earlier. In the Worst metric, it also performed better than IAPSO; however, the difference is not substantial. Therefore, while MPSOTVAC delivers a relatively acceptable performance in solving SHE equations, it falls behind the proposed hybrid variant and IAPSO.

The proposed hybrid variant integrates three powerful improvement strategies under a single framework: the adaptive momentum term added to the position update equation in IAPSO, the time-varying acceleration coefficients from MPSOTVAC, and the reinitialization mechanism from APSO. Examining the results, the proposed hybrid variant has produced the most consistent and high-quality results across all performance metrics. Its lowest Median value among all variants indicates that it consistently generates higher-quality solutions. Additionally, its superior performance in the Worst metric demonstrates that it maintains acceptable optimization performance even in unfavorable scenarios. The low Std value confirms that the solution quality remains stable, indicating that the algorithm provides consistent performance. Boxplots reveal that while the proposed hybrid variant has outliers, these values are more concentrated within a narrower range compared to other algorithms, minimizing extreme deviations. This suggests that the algorithm reduces variability among solutions and minimizes performance fluctuations.

The convergence curves of the last three variants are presented in Figure 4 to further analyze their optimization behavior in addition to their overall performance. Upon examining the curves, it is observed that IAPSO, MPSOTVAC, and the Hybrid variant all successfully converged toward the global best solution. However, differences can be seen in terms of convergence speed, stability, and improvement in solution quality.

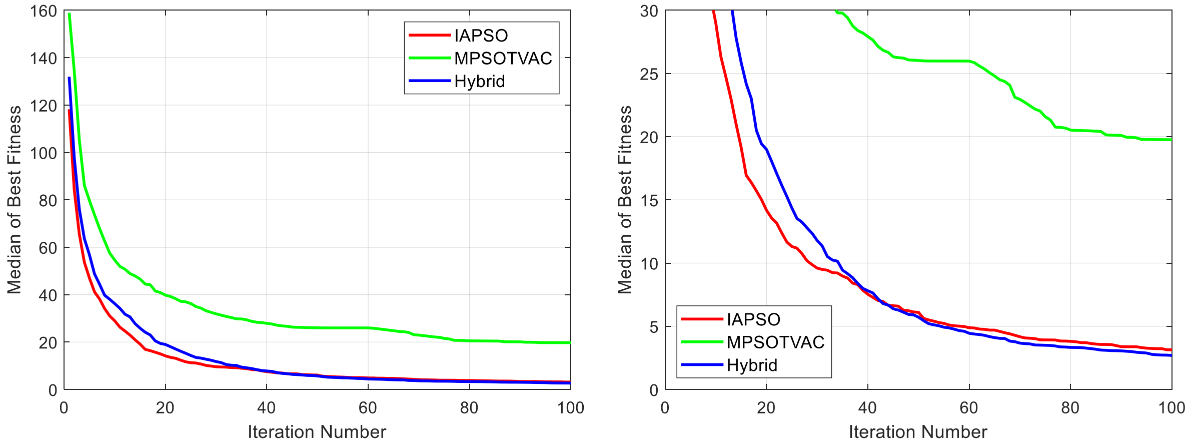


Figure 4. Convergence curves of the last 3 variants **(a)** normal, **(b)** magnified view

IAPSO exhibited a rapid decline in the early iterations due to its fixed acceleration coefficients, aggressively steering towards the best solution. However, maintaining fixed acceleration coefficients caused the algorithm to lose flexibility in later iterations, leading to a solution quality inferior to that of the Hybrid variant.

MPSOTVAC demonstrated a more stable decreasing trend thanks to its mechanism for adjusting acceleration coefficients based on iterations. However, its inertia weight update strategy was not as effective as that of IAPSO, and the algorithm lagged behind both IAPSO and the Hybrid variant in terms of solution quality.

The Hybrid variant, built upon the strong foundation of IAPSO, incorporated MPSOTVAC's controlled acceleration coefficient adjustment strategy, effectively balancing exploration and exploitation, making it the most efficient algorithm in the convergence process. Initially, it prioritized a broader exploration phase, which resulted in a slower convergence rate compared to IAPSO in the early iterations. However, in later iterations, it transitioned into a more aggressive exploitation phase, achieving the highest solution quality and surpassing the other two variants.

Overall, while all three algorithms exhibited a successful optimization process, the Hybrid variant delivered the most stable and best-performing results. IAPSO gained an early advantage in the initial iterations but fell behind the Hybrid variant due to its fixed acceleration coefficients. Meanwhile, MPSOTVAC showed a steady decline but remained the weakest variant among the three in terms of solution quality.

To validate the results obtained in the optimization processes, the optimal switching angles obtained by the last three variants for each performance metric were recorded (Table 3) and applied to a three-phase 11-level CHB-MLI model in MATLAB/Simulink. As a result of the simulation, the Total Harmonic Distortion (THD) up to the 14^{th} order and the fundamental component amplitude errors of the inverter output voltage were calculated (Table 4)

Additionally, to visually evaluate the performance of the algorithms, inverter line voltages, their fundamental components along with their desired reference counterparts, and frequency spectrums showing other dominant harmonics are presented in the following figures. Among these, Figure 5 presents the results obtained by applying the switching angles produced by PSO variants in the Best criterion, Figure 6 in the Median criterion, and Figure 7 in the Worst criterion

Table 3. Optimal switching angles found by the last 3 PSO variants

PSO Variants	Perfor. Metrics	Fitness Scores	Optimal Switching Angles (θ)				
			θ_1	θ_2	θ_3	θ_4	θ_5
IAPSO	Best	0.22896	8.0723	19.5195	29.5207	47.7187	63.1720
	Median	3.14005	8.3307	19.1536	28.9665	47.2366	63.1539
	Worst	141.389	6.9622	38.0693	88.4607	89.2185	89.4513
MPSOTVAC	Best	1.44E-05	7.8621	19.3727	29.6524	47.6809	63.2109
	Median	19.7548	5.2253	24.5971	38.9990	49.6409	72.9554
	Worst	133.280	8.1392	37.0757	87.6129	89.0261	89.9973
Hybrid	Best	0.28952	8.1324	19.4678	29.7515	47.7013	63.0676
	Median	2.69516	8.5098	18.7479	29.5546	48.2781	62.9309
	Worst	29.1667	15.9531	24.2180	41.6987	59.2115	61.7156

Table 4. Simulation results of last 3 PSO variants

	% THD (Until 14^{th})			% Voltage Error of V1		
Perfor. Metrics	IAPSO	MPSO TVAC	Hybrid	IAPSO	MPSO TVAC	Hybrid
Best	0.19	0.01	0.22	-0.09	-0.09	-0.09
Median	0.61	0.27	0.66	0.23	-7.83	-0.12
Worst	2.41	0.77	1.29	-53.44	-53.03	-8.27

Upon examining the simulation results, in the Best criterion, all three algorithms produced fundamental component amplitudes that were very close to the target value. However, in terms of THD, MPSOTVAC (0.01) achieved the lowest value by a large margin, making it the most successful approach. In contrast, the Hybrid (0.22) and IAPSO (0.19) variants produced significantly higher THD values compared to MPSOTVAC but remained very close to each other. This indicates that the low fitness value of MPSOTVAC in the Best criterion is largely due to its superior THD minimization capability.

In the Median criterion, MPSOTVAC once again achieved the lowest THD value. However, this was accomplished at the expense of fundamental component amplitude accuracy. The error rate (-7.83%) was significantly higher compared to the Hybrid (-0.12%) and IAPSO (0.23%) variants. The Hybrid and IAPSO algorithms also produced very similar results in this criterion in terms of both THD and fundamental component amplitude error. While one exhibited slightly higher THD, the other had a marginally larger fundamental component amplitude error.

In the Worst criterion, the differences between the algorithms became most pronounced. In this scenario, the clear superiority of the proposed Hybrid variant was evident. MPSOTVAC maintained its tendency to achieve the lowest THD value. However, while achieving this optimization success, it suffered a significant reduction in the fundamental component voltage, nearly halving the voltage value. IAPSO exhibited the weakest performance in this criterion. It produced the worst results in terms of both THD and fundamental component amplitude, displaying severe instability during the optimization process.

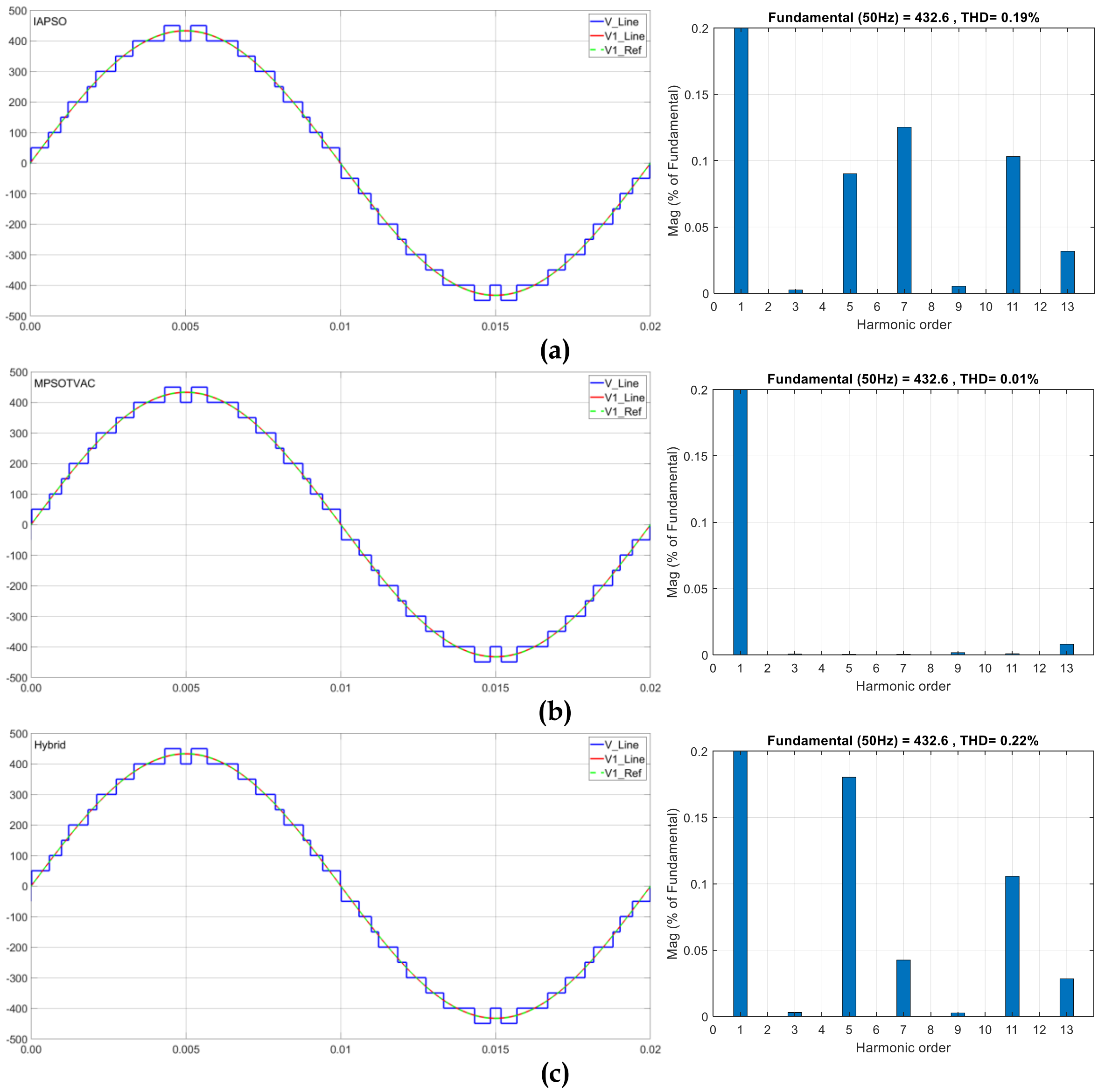
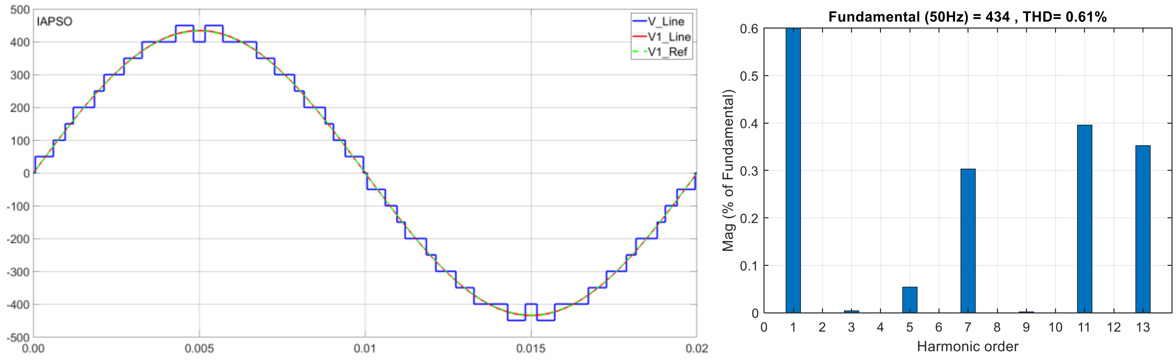


Figure 5. Line voltage waveform, fundamental harmonic and its reference, frequency spectrum of **(a)** IAPSO; **(b)** MPSOTVAC; **(c)** Hybrid variants at Best metric

In contrast, the Hybrid variant, even in the worst-case scenario, only produced a -8.27% fundamental component amplitude error, which is at a level that MPSOTVAC could only achieve in the Median criterion. The THD value of the Hybrid variant in this criterion increased slightly compared to the previous scenarios, falling behind MPSOTVAC. However, it should be noted that MPSOTVAC achieved this low THD value at the cost of a significant drop in the fundamental component voltage.

Overall, the conducted analyses clearly highlight the strengths and weaknesses of the algorithms. IAPSO, although generally a successful algorithm that produced low fitness values, showed instability in some runs, resulting in suboptimal outcomes. Indeed, its poor performance in the Worst criterion confirmed this, as previously indicated by the broad distribution and high standard deviation (Std) values observed in the box plot.



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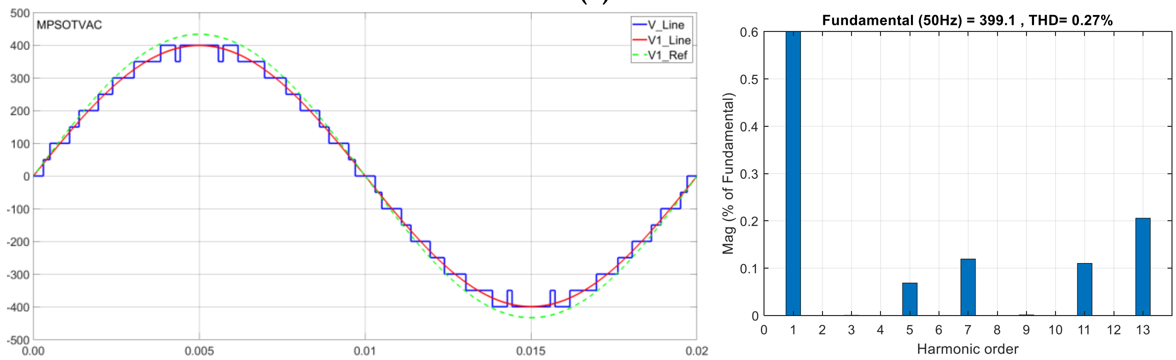

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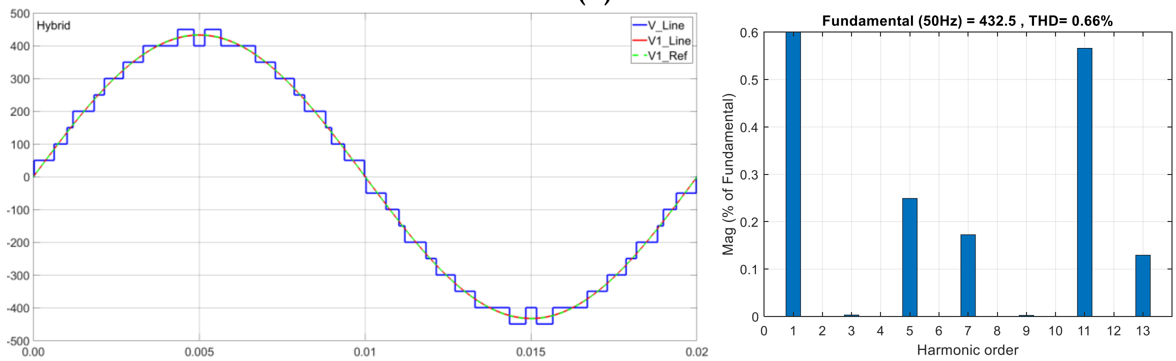

(c)

Figure 6. Line voltage waveform, fundamental harmonic and its reference, frequency spectrum of **(a)** IAPSO; **(b)** MPSOTVAC; **(c)** Hybrid variants at Median metric

MPSOTVAC produced an outstanding result in the Best criterion in terms of both THD and fundamental component amplitude. However, its success in achieving low THD in other criteria came at the expense of fundamental component amplitude errors that reached unacceptable levels in the Median and especially the Worst criteria. This suggests that while MPSOTVAC has the potential to produce excellent results, this success cannot always be achieved consistently.

The proposed Hybrid variant was developed precisely due to IAPSO's vulnerability to such instabilities, despite its generally strong performance, and the simulation results validated the correctness of this approach. While the Hybrid variant exhibited an average performance in the Best and Median criteria, it achieved the most successful results by far in the Worst criterion, demonstrating its overall stability.

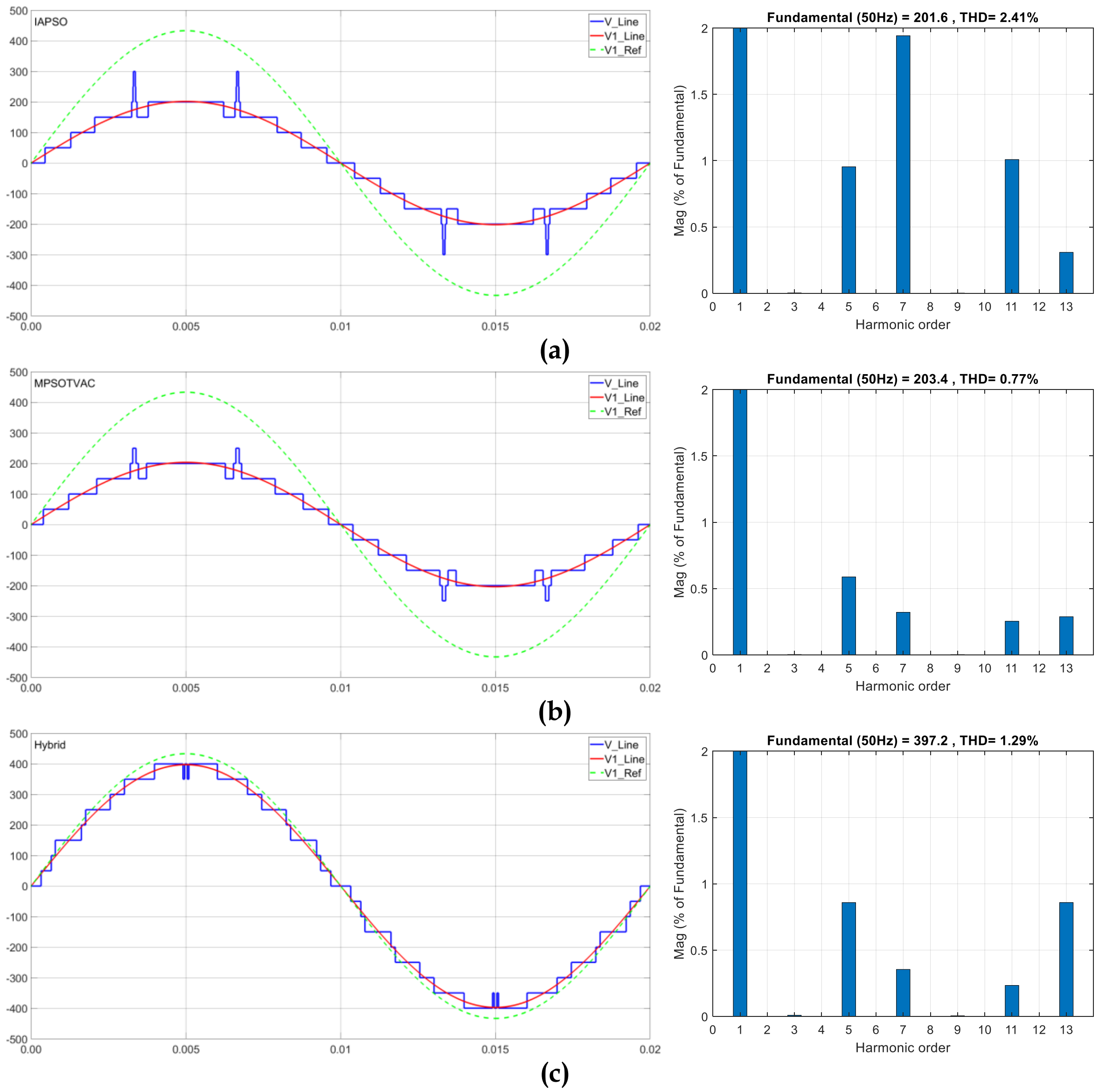

Figure 7. Line voltage waveform, fundamental harmonic and its reference, frequency spectrum of **(a)** IAPSO; **(b)** MPSOTVAC; **(c)** Hybrid variants at Worst metric

5. Conclusion

In this study, the capabilities of eight different PSO variants in solving complex SHE equations were compared, and the impact of the proposed improvements on the performance of standard PSO in real-world engineering problems was evaluated. The evaluated variants include the original version of PSO (SPSO), existing enhanced versions from the literature, and the proposed hybrid model, which integrates selected strong features from some of the most effective variants.

The first and most fundamental improvement introduced to PSO is the addition of an inertia weight. This component regulates the balance between exploration and exploitation during the optimization process, allowing the algorithm to produce more stable and successful results. However, using inertia weight alone has proven insufficient, approaches that lack complementary mechanisms have failed to consistently deliver the desired performance in certain cases.

The findings of the study indicate that three variants stand out in terms of performance. IAPSO, with its dynamic inertia weight adjustment and momentum-based position update mechanism, has generally produced successful solutions. However, it has exhibited instability in some runs, leading to unexpectedly poor results. This inconsistency suggests that IAPSO may suffer from solution quality fluctuations, making it unreliable across all scenarios.

MPSOTVAC, which modifies acceleration coefficients based on iteration count, has achieved the best results in some runs but has failed to maintain this success consistently. In particular, it has shown large deviations in fundamental component amplitude, resulting in an overall unstable performance.

The proposed hybrid PSO variant builds upon the strengths of IAPSO while integrating MPSOTVAC's controlled acceleration coefficient adaptation mechanism and APSO's reinitialization strategy. The results demonstrate that the hybrid variant is not only effective in specific cases but also emerges as the most stable and reliable algorithm overall. It has consistently generated the highest-quality solutions on average while maintaining competitive performance even in worst-case scenarios.

This study provides a comprehensive evaluation of the strengths and weaknesses of different PSO variants, serving as an important reference for future research. Furthermore, the findings highlight that the optimization performance of standard PSO can be significantly improved through appropriate modifications, with the proposed hybrid approach achieving the best balance. The results also suggest that advanced PSO variants can serve as an effective alternative for solving complex engineering problems such as SHE equations.

Conflict of Interest

The author declares that he has no conflict of interest.

References

[1] Eberhart R, Kennedy J. A new optimizer using particle swarm theory. MHS95 Proc Sixth Int Symp Micro Mach Hum Sci [Internet]. 1995 [cited 2025 Mar 17]. p. 39–43. Available from: https://ieeexplore.ieee.org/document/494215.

[2] Gad AG. Particle Swarm Optimization Algorithm and Its Applications: A Systematic Review. Arch Comput Methods Eng. 2022;29(5):2531–2561.

[3] Xu H, Deng Q, Zhang Z, et al. A hybrid differential evolution particle swarm optimization algorithm based on dynamic strategies. Sci Rep. 2025;15(1):4518.

[4] Shaqarin T, Noack BR. A Fast-Converging Particle Swarm Optimization through Targeted, Position-Mutated, Elitism (PSO-TPME). Int J Comput Intell Syst. 2023;16(1):6.

[5] Jain M, Saihjpal V, Singh N, et al. An Overview of Variants and Advancements of PSO Algorithm. Appl Sci. 2022;12(17):8392.

[6] Zhan Z-H, Zhang J, Li Y, et al. Adaptive Particle Swarm Optimization. IEEE Trans Syst Man Cybern Part B Cybern. 2009;39(6):1362–1381.

[7] Chauhan P, Deep K, Pant M. Novel inertia weight strategies for particle swarm optimization. Memetic Comput. 2013;5(3):229–251.

[8] Fan S-KS, Chiu Y-Y. A decreasing inertia weight particle swarm optimizer. Eng Optim. 2007;39(2):203–228.

[9] Feng Y, Yao Y-M, Wang A-X. Comparing with Chaotic Inertia Weights in Particle Swarm Optimization. 2007 Int Conf Mach Learn Cybern [Internet]. 2007 [cited 2025 Feb 23]. p. 329–333. Available from: https://ieeexplore.ieee.org/abstract/document/4370164.

[10] Yang K, Zhang Q, Zhang J, et al. Unified Selective Harmonic Elimination for Multilevel Converters. IEEE Trans Power Electron. 2017;32(2):1579–1590.

[11] Dahidah MSA, Konstantinou G, Agelidis VG. A Review of Multilevel Selective Harmonic Elimination PWM: Formulations, Solving Algorithms, Implementation and Applications. IEEE Trans Power Electron. 2015;30(8):4091–4106.

[12] Yang K, Yuan Z, Yuan R, et al. A Groebner Bases Theory-Based Method for Selective Harmonic Elimination. IEEE Trans Power Electron. 2015;30(12):6581–6592.
[13] Bertin T, Despesse G, Thomas R. Comparison between a Cascaded H-Bridge and a Conventional H-Bridge for a 5-kW Grid-Tied Solar Inverter. Electronics. 2023;12(8):1929.
[14] Mao W, Zhang X, Hu Y, et al. A Research on Cascaded H-Bridge Module Level Photovoltaic Inverter Based on a Switching Modulation Strategy. Energies. 2019;12(10):1851.
[15] Lingom PM, Song-Manguelle J, Nyobe-Yome JM, et al. A Comprehensive Review of Compensation Control Techniques Suitable for Cascaded H-Bridge Multilevel Inverter Operation with Unequal DC Sources or Faulty Cells. Energies. 2024;17(3):722.
[16] Sharma B, Manna S, Saxena V, et al. A comprehensive review of multi-level inverters, modulation, and control for grid-interfaced solar PV systems. Sci Rep. 2025;15(1):661.
[17] Memon MA, Mekhilef S, Mubin M, et al. Selective harmonic elimination in inverters using bio-inspired intelligent algorithms for renewable energy conversion applications: A review. Renew Sustain Energy Rev. 2018;82:2235–2253.
[18] Zhang H, Li D. Applications of computer vision techniques to cotton foreign matter inspection: A review. Comput Electron Agric. 2014;109:59–70.
[19] Bektaş Y, Karaca H, Taha TA, et al. Red deer algorithm-based selective harmonic elimination technique for multilevel inverters. Bull Electr Eng Inform. 2023;12(5):2643–2650.
[20] Shi Y, Eberhart R. A modified particle swarm optimizer. 1998 IEEE Int Conf Evol Comput Proc IEEE World Congr Comput Intell Cat No98TH8360 [Internet]. 1998 [cited 2025 Feb 23]. p. 69–73. Available from: https://ieeexplore.ieee.org/document/699146.
[21] Shi Y, Eberhart RC. Empirical study of particle swarm optimization. Proc 1999 Congr Evol Comput-CEC99 Cat No 99TH8406 [Internet]. 1999 [cited 2025 Feb 23]. p. 1945-1950 Vol. 3. Available from: https://ieeexplore.ieee.org/document/785511.
[22] Eberhart RC, Shi Y. Tracking and optimizing dynamic systems with particle swarms. Proc 2001 Congr Evol Comput IEEE Cat No01TH8546 [Internet]. 2001 [cited 2025 Feb 23]. p. 94–100 vol. 1. Available from: https://ieeexplore.ieee.org/document/934376.
[23] Feng Y, Teng G-F, Wang A-X, et al. Chaotic Inertia Weight in Particle Swarm Optimization. Second Int Conf Innov Comput Informatio Control ICICIC 2007 [Internet]. 2007 [cited 2025 Feb 23]. p. 475–475. Available from: https://ieeexplore.ieee.org/document/4428117.
[24] Qin Z, Yu F, Shi Z, et al. Adaptive Inertia Weight Particle Swarm Optimization. In: Rutkowski L, Tadeusiewicz R, Zadeh LA, et al., editors. Artif Intell Soft Comput – ICAISC 2006. Berlin, Heidelberg: Springer; 2006. p. 450–459.
[25] Arumugam MS, Rao MVC. On the performance of the particle swarm optimization algorithm with various inertia weight variants for computing optimal control of a class of hybrid systems. Discrete Dyn Nat Soc. 2006;2006(1):079295.
[26] Yang X, Yuan J, Yuan J, et al. A modified particle swarm optimizer with dynamic adaptation. Appl Math Comput. 2007;189(2):1205–1213.
[27] Jiao B, Lian Z, Gu X. A dynamic inertia weight particle swarm optimization algorithm. Chaos Solitons Fractals. 2008;37(3):698–705.
[28] Ratnaweera A, Halgamuge SK, Watson HC. Self-organizing hierarchical particle swarm optimizer with time-varying acceleration coefficients. IEEE Trans Evol Comput. 2004;8(3):240–255.
[29] Panigrahi BK, Ravikumar Pandi V, Das S. Adaptive particle swarm optimization approach for static and dynamic economic load dispatch. Energy Convers Manag. 2008;49(6):1407–1415.
[30] Suresh K, Ghosh S, Kundu D, et al. Inertia-Adaptive Particle Swarm Optimizer for Improved Global Search. 2008 Eighth Int Conf Intell Syst Des Appl [Internet]. 2008 [cited 2025 Mar 17]. p. 253–258. Available from: https://ieeexplore.ieee.org/document/4696340.