Journal for the Mathematics Education and Teaching Practices, 6(2), 77-91, Dec 2025
e-ISSN: 2717-8587
dergipark.org.tr/jmetp

Genc Bilge Publishing Ltd.
(Young Wise) © 2025
gencbilgeyayincilik.com

Research Article

Analyzing students' fraction strategies: a case study of high-achieving middle school learners

Aygül Kartal[1]* and Aygül Kartal[2]

Ministry of National Education, Çağlayan Osman Hacıalioğlu Secondary School, Rize, Türkiye

Article Info

Received: 15 October 2024
Accepted: 1 July 2025
Available online: 30 Dec 2025

Keywords:
Fractions
Mathematics Education
Middle School Students
Number Sense
Problem-Solving Strategies

Abstract

The aim of this study is to examine the cognitive strategies that academically successful middle school students employ when solving problems involving fractions, and to explore how these strategies influence their mathematical thinking and problem-solving processes. Adopting a qualitative case study design, the research involved students from 5th to 8th grades who demonstrated high achievement in mathematics. Data were gathered through a semi-structured interview form that focused on key components associated with reasoning about fractions—such as estimation, operational understanding, reference benchmarks, magnitude comparison, and equivalence. Content analysis and thematic coding techniques were used to analyze the data. The findings suggest that students' strategy use is closely related to their conceptual and procedural understanding of fractions. Students who employed intuitive, flexible strategies demonstrated deeper mathematical reasoning, while those who relied on rule-based methods produced more procedural and limited responses. These cognitive patterns were directly reflected in the nature and effectiveness of their mathematical thinking. The study also suggests that reasoning about fractions develops through instruction and can be observed at varying levels—basic, intermediate, and advanced. Accordingly, number sense should be approached as a foundational competence in mathematics education. Instructional practices should integrate visual models, estimation activities, conceptual tools, and strategy-based tasks to better support students' mathematical development.

To cite this article

Kartal, A. and Demirci, N. (2025). Analyzing students' fraction strategies: a case study of high-achieving middle school learners. *Journal for the Mathematics Education and Teaching Practices, 6*(2), 77-91. DOI: https://doi.org/10.5281/zenodo.1

Introduction

Mathematics is more than a collection of procedures and calculations; it is a discipline that fosters logical reasoning, abstract thinking, and adaptive problem-solving abilities in learners. In modern educational discourse, mathematics is recognized as a key driver of higher-order thinking skills that enable individuals to interpret and navigate complex, data-rich environments (OECD, 2021). The ability to analyze patterns, generalize relationships, and apply flexible strategies across various contexts is essential not only for academic success but also for informed decision-making in everyday life (NCTM, 2020; Boaler, 2016). Mathematical proficiency involves far more than procedural fluency; it requires a deep conceptual understanding that connects mathematical ideas and enables meaningful transfer between situations (Kilpatrick, Swafford, & Findell, 2001). Consequently, there has been a pedagogical shift from teaching isolated

[1] Corresponding Author: Dr., Ministry of National Education, Çağlayan Osman Hacıalioğlu Secondary School, Rize, Türkiye. E-mail: aygulkartal53@gmail.com ORCID: 0000-0001-5120-6816
[2] Ministry of National Education, Torbalı Science and Art Center, İzmir, Türkiye. E-mail: niymetdemirci@gmail.com ORCID: 0000-0002-2763-1879

techniques to cultivating students' reasoning, sense-making, and flexibility in approach. Within this vision, foundational constructs such as number sense play a critical role in supporting students' cognitive development and long-term mathematical achievement.

Number sense is a foundational cognitive construct that supports the development of essential mathematical competencies such as estimation, numerical reasoning, and mental computation (Sowder, 1992; McIntosh et al., 1992). It is broadly understood as an individual's intuitive grasp of numbers and their relationships, enabling adaptive strategy use and flexible problem solving. Recent research continues to refine our understanding of number sense by identifying core components—such as number recognition, magnitude comparison, and estimation—and examining their trajectories across development (Jordan, Devlin, & Botello, 2022). In particular, numerosity perception—the ability to estimate the quantity of objects without counting—has emerged as a key early indicator of number sense. A recent study by Morín, Depaepe, and Reynvoet (2025) demonstrates that children's accuracy in perceiving numerical magnitudes improves significantly with age, supporting the sharpening hypothesis, which posits that internal numerical representations become increasingly precise over time. These findings highlight the dynamic nature of number sense and its critical role not only in early mathematics but also in the transition to more complex domains such as fractions, ratios, and proportional reasoning. The developmental strengthening of numerical acuity suggests that educational interventions should aim to nurture number sense not only in early childhood but also throughout primary and middle school.

While number sense is often discussed in the context of whole numbers and early arithmetic, its importance becomes even more pronounced in domains that require abstract and relational thinking—such as fractions. Fractions are conceptually complex because they challenge students to work with quantities that are neither discrete nor whole, requiring part-whole reasoning, multiplicative thinking, and flexibility in representation (Siegler & Lortie-Forgues, 2015). Many students struggle with these ideas because they lack a robust number sense that enables them to make sense of non-unit quantities, estimate magnitudes, or predict the effects of operations. Researchers have emphasized that the same cognitive foundations supporting number sense—such as magnitude comparison, estimation, and strategy flexibility—are essential when working with fractions (Clarke & Roche, 2009; McNamara & Shaughnessy, 2015). Students who demonstrate strong number sense tend to approach fractional problems with multiple strategies, question the reasonableness of their answers, and shift flexibly between representations. In contrast, students with procedural understanding but weak number sense often rely on rote algorithms without comprehending underlying relationships, which can lead to persistent misconceptions and errors. This highlights the necessity of extending number sense frameworks into fractional contexts to better support conceptual understanding.

Although the term "fraction number sense" is not defined through a universally accepted framework, various researchers have identified key dimensions that reflect how learners reason about and make sense of fractional quantities. According to Reeder and Utley (2007), fraction number sense involves intuitive thinking about part-whole relationships, flexible use of benchmark values, and the ability to make reasonable judgments about magnitude and equivalence. These ideas align with broader number sense principles—such as estimation, comparison, and representation flexibility—applied specifically within the context of fractions (Clarke & Roche, 2009; McNamara & Shaughnessy, 2015). In their study of prospective elementary teachers, Utley and Reeder (2012) found that even high-performing teacher candidates struggled to apply conceptual reasoning when confronted with fraction tasks that required more than procedural execution. This underscores the need to foster fraction number sense as a distinct construct that enables learners to approach problems flexibly, compare values meaningfully, and explain their reasoning through multiple representations. Recent international research further supports this perspective. For example, Sukma, Somakim, and Indaryanti (2021) demonstrated that students' ability to solve fraction problems was closely linked to their number sense skills—particularly estimation, proportional reasoning, and strategy flexibility. These findings reinforce the relevance of fraction number sense in classroom instruction and suggest that teaching practices should prioritize the development of these skills. Based on this literature, the present study adopts a framework incorporating estimation, operational reasoning, benchmark use, and equivalence construction as key components in examining students' cognitive strategies.

In the context of Turkish mathematics education, instruction on fractions has traditionally relied on the use of concrete materials and procedural algorithms, particularly in earlier grades. Tools such as fraction circles, number lines, and area models are frequently used to introduce part-whole relationships and basic operations (Güler & Uçar, 2022). However, several studies suggest that these approaches are often implemented in a limited way, focusing on mechanical application rather than fostering conceptual depth or strategic reasoning (Cetin & Yapıcı, 2023; Altun & Kara, 2021). Classroom practices in Turkey tend to emphasize correct procedures—such as converting between forms or finding common denominators—over multiple-solution strategies or estimation-based thinking. As a result, many students develop procedural fluency without a corresponding conceptual understanding of fractional magnitudes or the effects of operations. This disconnect can hinder their ability to apply flexible strategies or transfer knowledge to unfamiliar contexts (Yılmaz & Yeşildere-İmre, 2018). These findings reveal a critical gap in instructional design: while concrete representations are valuable, they must be complemented by opportunities for students to engage in comparison, justification, and strategy selection. Without these, the development of fraction number sense remains limited, even among high-achieving learners. Accordingly, this study aims to investigate how academically successful students in Turkey employ number sense components when reasoning about fractions.

Although number sense has been widely studied in relation to whole numbers and basic arithmetic, recent research highlights a significant gap in understanding how number sense operates within the domain of fractions—particularly among middle school students. While some studies have examined strategy use and flexibility in fraction problem-solving, there remains limited insight into how academically high-achieving learners employ number sense components in cognitively demanding tasks (Utley & Reeder, 2012; Sukma et al., 2021). Furthermore, few studies have investigated how these students navigate between procedural and conceptual approaches or how their strategies reflect deeper mathematical reasoning. This study seeks to contribute to the field by exploring how high-achieving middle school students in Turkey demonstrate number sense in fraction-related problem-solving contexts. By examining strategy use across multiple dimensions—such as estimation, benchmark reasoning, and equivalence construction—the study aims to offer a more nuanced understanding of students' cognitive processes and the instructional practices that support them. Addressing this need can help inform future curriculum design and teacher education by highlighting the importance of fostering conceptual flexibility alongside procedural competence.

To achieve this goal, the study addresses the following research questions:

- How do academically successful middle school students apply number sense components when solving fraction problems?
- In what ways do intuitive and flexible strategies differ from rule-based approaches in terms of conceptual understanding and solution efficiency?

These questions are investigated through a qualitative case study approach, which allows for an in-depth exploration of students' reasoning processes and strategic patterns across different grade levels. The following section describes the research design, including participants, data collection tools, and analytical procedures.

Method

Research Design

This study was designed using a qualitative approach and is based on a case study method. Case studies aim to examine a specific phenomenon or situation within its natural context, allowing for a deeper understanding of cause-and-effect relationships (Creswell, 2013). Ahmed and Williams (1997) define case study as a method in which multiple data collection tools are used to understand the complexity of an event and evaluate it in its natural setting. In this study, the components of fraction number sense are addressed in relation to students' solution strategies.

Study Group

The study group consisted of eight middle school students selected through criterion sampling. The participants were identified based on two key criteria: high academic achievement in mathematics and demonstrated interest in the subject.

Two students were selected from each grade level (5th, 6th, 7th, and 8th), and all participants volunteered to take part in the study. Information regarding the participants is presented in the table below.

Table 1: Grade levels and mathematics achievement of participants

Participant Code	Grade Level	Academic Note out of 100
S 5.1	5^{th} grade	92
S 5.2	5^{th} grade	90
S 6.1	6^{th} grade	86
S 6.2	6^{th} grade	90
S 7.1	7^{th} grade	90
S 7.2	7^{th} grade	92
S 8.1	8^{th} grade	90
S 8.2	8^{th} grade	93

Data Collection Instrument

The data collection tool used in this study was the “Fraction Number Sense Interview Form,” originally developed by Kartal (2016) based on the number sense standards proposed by McIntosh (1992). The form has undergone validity and reliability procedures. It consists of 15 open-ended questions designed to assess students' understanding of five key components of fraction number sense: equivalent representations, numerical estimation, operational effects, magnitude understanding, and use of reference points. The distribution of the questions according to these components is presented in the following table.

Table 2. Distribution of items in the assessment tool according to number sense components

Component	Sample Question Text
Equivalent Representations	“Explain if 4/6 and 2/3 are equal. How do you know?” (Q1)
Numerical Estimation	“Estimate whether 3/4 is closer to 1 or 1/2. Explain your reasoning.” (Q3)
Operations Effects	“What happens when you multiply 1/2 by 3? How is it different from 1/2 + 3?” (Q4)
Numerical Magnitudes	“Arrange the fractions 1/3, 2/5, and 3/7 from smallest to largest.” (Q6)
Reference Use	“A store is 1/8 km and a school is 1/4 km from the midpoint of a road. Show their positions on a number line.” (Q10)

Data Collection Process

The data collection process of this study was carefully planned and implemented in accordance with the research objectives. The process consisted of the following stages:

In the initial phase, the “Fraction Number Sense Interview Form” was thoroughly reviewed to evaluate the extent to which it covered the components of fraction number sense. Expert opinions were obtained to support this evaluation, and a pilot study was conducted to ensure that the questions were clear and comprehensible. Participants were selected using criterion sampling. The selection criteria included students’ high academic achievement in mathematics and a demonstrated interest in the subject. Two students from each grade level (5th, 6th, 7th, and 8th) were voluntarily recruited, resulting in a total of eight participants. The interviews were conducted in settings where students could feel comfortable and were free from external distractions. A separate room was prepared for each interview to minimize any potential interruptions. All participants received the same instructions to ensure standardization across interviews. Each interview was conducted individually and lasted approximately 30 to 40 minutes. Written responses obtained during the interviews were compiled along with the researcher’s notes and observations. All data were stored in accordance with ethical research principles, ensuring the confidentiality of the participants. Both physical and digital data were securely stored and accessible only to the researcher. During the pilot study, linguistic issues and ambiguous expressions that students found difficult to understand were revised before moving on to the actual data collection phase. This contributed to a more efficient and fluent interview process. After the interviews were completed, student responses were reviewed to check for missing or unclear information before beginning the analysis process. In cases where clarification was needed, students were contacted again for follow-up explanations. This detailed and systematic process, aligned with the principles of qualitative research, ensured transparency, rigor, and consistency during data collection. Closely monitoring how students approached and responded to the interview questions allowed for an in-depth analysis of their strategies and thinking processes.

Data Analysis

The collected data were analyzed using a descriptive analysis method. The responses were categorized under five components of fraction number sense (equivalent representations, numerical estimation, operational effects, magnitude understanding, and use of reference points). For each component, the strategies employed by students were examined and classified as number-sense-based, rule-based, or incorrect. These strategies were coded and analyzed according to a predefined evaluation rubric.

To increase the transparency of the analysis, Table 3 presents one representative example for each number sense component. Each example includes the task, a sample student response, the coded strategy type, and an interpretive comment from the researcher. This structure was used consistently across the full dataset during coding and thematic analysis.

Table 3. Sample analysis for each fraction number sense component

Component	Example Question	Sample Student Response	Strategy Type	Researcher's Analysis
Equivalent Representation	Are 4/6 and 2/3 equal? Explain.	"I drew two circles. Both looked the same, so yes."	Number-sense-based	The student used a visual model and proportional reasoning to determine equivalence. This reflects conceptual understanding beyond procedural rule.
Numerical Estimation	Is 3/4 closer to 1 or 1/2?	"It's more than half. It's close to 1 because 2/4 is half and 3/4 is more."	Number-sense-based	The student used benchmark comparison with 1/2 and 1, indicating estimation skills and flexible magnitude reasoning.
Operational Effects	Multiply 1/2 by 3. How is it different from 1/2 + 3?	"1/2 × 3 is 3/2. That's like having 1/2 three times. The other one adds whole numbers."	Number-sense-based	The student correctly differentiated multiplication and addition through context and meaning, showing operational awareness.
Magnitude Understanding	Which is greater: 2/5 or 3/7?	"2/5 is more. Because 2/5 is 0.4 and 3/7 is less than 0.5."	Rule-based	Although the student arrived at the correct answer, the strategy was procedural (decimal conversion), not conceptually grounded.
Reference Use	A shop is 1/8 km and a school is 1/4 km from midpoint. Show it on a number line.	"1/4 is farther than 1/8. So there is 1/8 between them."	Number-sense-based	The student applied knowledge of reference equivalency (1/4 = 2/8) and visualized spatial relationships, indicating strong relational reasoning

In qualitative research, validity is related to the credibility of the findings and the extent to which the study addresses its intended purpose. To ensure validity and reliability, the following steps were taken:

Validity and Reliability

The "Fraction Number Sense Interview Form" used in this study was originally developed by the researcher in a previous master's thesis study (Kartal and Pırasa, 2022) and underwent a comprehensive validity and reliability process. Content validity was ensured through a two-tier expert review process. The initial version of the form was evaluated by a total of 12 experts, including three PhD-holding mathematics education scholars, one educational measurement and evaluation expert, and eight in-service middle school mathematics teachers (one of whom was also a graduate student). Experts were asked to rate how well each question aligned with its intended number sense component using a 5-point Likert scale and to provide written feedback on clarity, representational accuracy, and appropriateness. Based on their input, the wording, focus, and alignment of several questions were revised. The pilot implementation of the form was conducted in two phases. First, five 8th grade students participated in a preliminary trial to identify issues of comprehension and task appropriateness. A second pilot with two additional students was used to confirm adjustments. Based on these pilot results, the initial item pool of 30 questions was refined to 15 final items to ensure clarity, focus, and alignment with number sense components. Inter-coder reliability was tested by having two researchers independently code 20% of the dataset. The resulting Cohen's Kappa coefficient was $\kappa = 0.82$, indicating a high level of agreement. After resolving discrepancies through discussion, the remaining data were coded accordingly using the agreed-upon rubric. Finally, thematic saturation was reached during the analysis process. No new strategies or cognitive patterns emerged in the later stages of coding, which indicates that the identified themes were both analytically sufficient and consistently observed across different grade levels and item types.

Findings

Exploratory Factor Analysis (EFA)

In this section, the findings are systematically presented in line with the research questions. The first sub-question addressed is: "How do the solution strategies related to the components of fraction number sense shape the mathematical thinking and problem-solving processes of academically successful middle school students?"

In accordance with this sub-question, students' strategies were analyzed across the five core components of fraction number sense: numerical estimation, operational effects, magnitude understanding, use of reference points, and equivalent representations. The data were evaluated under three categories—number-sense-based strategies, rule-based strategies, and incorrect responses—and supported with qualitative examples.

Table 4. Frequency of middle school students' fraction number sense

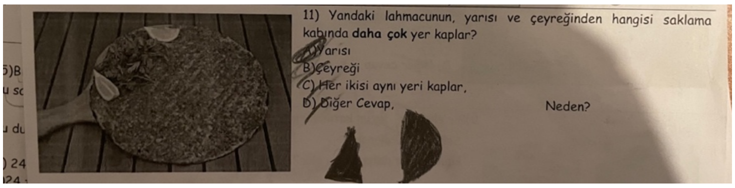
		5th Grade		6th Grade		7th Grade		8th Grade		
		5S1	5S2	6S1	6S2	7S1	7S2	8S1	8S2	NS (%)
Numerical Estimation	Q3	W	W	W	W	RB	W	RB	W	%12
	Q7	RB	RB	RB	RB	RB	RB	W	RB	
	Q12	W	NS	W	NS	NS	B	RB	W	
Operations Effects	Q4	W	RB	W	W	W	W	W	W	%4
	Q8	W	NS	W	RB	B	RB	RB	W	
	Q15	B	B	B	B	Y	B	KT	W	
Numerical Magnitudes	Q6	W	RB	B	W	RB	RB	W	W	%12
	Q9	B	RB	B	RB	NS	RB	W	RB	
	Q13	W	RB	W	NS	NS	B	RB	W	
Reference Use	Q10	B	NS	B	B	NS	B	B	B	%58
	Q11	W	NS	NS	NS	NS	NS	NS	NS	
	Q14	NS	NS	NS	NS	NS	NS	NS	RB	
Equivalent Representations	Q1	NS	NS	NS	NS	NS	NS	NS	NS	%44
	Q2	W	RB	RB	RB	NS	RB	RB	RB	
	Q5	B	B	RB	B	NS	RB	RB	RB	
Genel	NS Count	2 NS	7NS	3NS	5NS	9NS	2 NS	3 NS	2 NS	%33
	RB Count	1RB	5RB	3RB	4 RB	3 RB	7RB	7 RB	5 RB	%30
NS (%)		%13	%46	%33	%33	%60	%13	%33	%13	
RB(%)		%1	%33	%33	%26	%20	%46	%46	%33	

NS: Number-Sense-Based True Response**, RB:** Rule-Based True Response, **W**: Wrong Answer or Explanation, **B:** Blank

Numerical Estimation Component

An examination of Table 3 reveals that the percentage of correct responses to items measuring students' numerical estimation skills was calculated as 12%. It was observed that students had difficulty particularly in placing fractions accurately on a number line and in intuitively estimating their magnitude. Although most students employed rule-based strategies for this component, these approaches often led to incorrect results. For example, student 7S2 stated that “one-quarter is half of one-half,” effectively using reference points on the number line and highlighting visualization skills. This illustrates an intuitive, number-sense-based strategy.

Question 11 on the interview form asked students to compare the area taken up by one-half and one-quarter of a large lahmacun using visual reasoning. This item was designed to measure numerical estimation by prompting students to make intuitive comparisons based on part-whole relationships without relying on symbolic operations. In the case of student 7S2, the response was categorized as a number-sense-based strategy due to the use of direct visual comparison and reasoning based on physical size rather than formal calculation. The student’s drawing showed an accurate conceptualization of the fractional sizes, clearly representing that one-quarter occupies less area than one-half. In the written response, the student stated: “Because you can see in my drawing that one-quarter takes up less space. The other one is larger.” This demonstrates effective use of visual benchmarks and supports the classification of this strategy as estimation-based and conceptually grounded.



Q11. Between half and a quarter of the lahmacun shown, which one takes up more space in a storage container?

A) Half

B) Quarter

C) Both take up the same amount of space

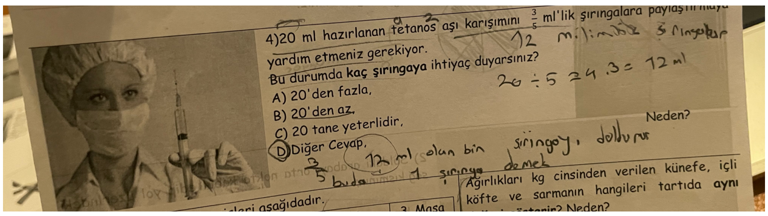
D) Other answer

Figure 1. 7S2's Response Demonstrating Fraction Number Sense

7S2: Teacher, because you can see in my drawing that one-quarter takes up less space. The other one is larger. This drawing effectively demonstrates the student’s visualization skills and their success in conceptualizing the relative magnitudes of fractions.

Operations Effects Component

Question 4 in the interview form was designed to assess students’ conceptual understanding of operational effects. In this task, students were asked how many 3/5 ml syringes are needed to divide a 20 ml tetanus vaccine dose equally. The purpose of the question is to evaluate whether students can correctly choose the operation (division vs. multiplication) and understand the logical effect of the operation within the problem context. Student 7S1 misapplied the operation. Although the correct approach is to perform 20 ÷ 3/5, the student instead attempted a procedural manipulation without interpreting the meaning of the operation. For example, the use of equations like 3/5 ÷ 20 or partial attempts to invert and multiply fractions indicates confusion. While the student appears to recall the algorithm (e.g., invert and multiply), they failed to link it meaningfully to the context of the problem. This response was therefore coded as a rule-based strategy. It reflects reliance on mechanical procedures without conceptual understanding of how and why the operation applies in this situation. This type of misconception also explains why this component had the lowest success rate in the study (only 4% correct).



Q4. You need to help distribute 20 ml of prepared tetanus vaccine into syringes that hold 5 ml each. In this case, how many syringes do you need?

A) More than 20

B) Less than 20

C) 20 syringes are enough

D) Other answer

Figure 2. Student 7S1’s Operational Effects Representation

At Figure 2, ttudent 7S1's response on operational effects illustrates this procedural tendency. It further highlights the need for more visual, contextualized instructional activities to support the conceptual development of operational reasoning.

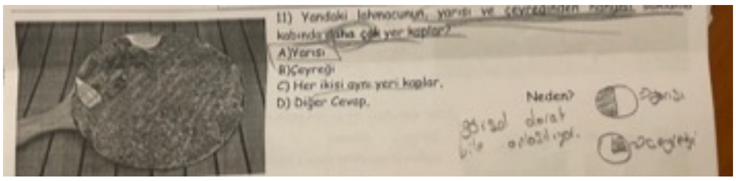
Magnitude Understanding Component

Question 6 in the interview form was designed to assess students' ability to interpret and compare the relative sizes of different fractions. The question asked: "Which is greater: 3/4 or 1/2? Explain." This item aimed to evaluate whether students could use part–whole relationships, benchmark references, or decomposition strategies to compare magnitudes intuitively.

Student 7S2 responded with: "3/4 is three one-fourths." This answer demonstrates a clear understanding of fractional composition by breaking down 3/4 into unit fractions. The student reasoned through an intuitive part-to-whole strategy, recognizing that three one-fourths is larger than a single one-half. This strategy reflects number-sense-based reasoning, grounded in a conceptual grasp of magnitude. By contrast, several students in this component relied on superficial comparisons of numerators or denominators, such as assuming that "3/4 is less than 1/2 because 3 is less than 4." These misinterpretations highlight the challenges of developing magnitude sense. However, Student 7S2's response was coded as number-sense-based because it reflected thoughtful decomposition and meaningful magnitude comparison. This finding underlines the importance of promoting flexible comparison strategies and conceptual understanding in fraction instruction.

Reference Point Use Component

Question 11 on the interview form asked students to compare which fraction—one-half or one-fourth—occupies more space on a lettuce leaf illustration. This question was designed to assess students' use of reference points by prompting them to rely on intuitive comparisons of part-whole relationships rather than symbolic calculations. Reference points such as 1/2 and 1 are foundational benchmarks in fraction sense development. Student 7S2 selected the correct answer and explained: "One-half always covers more area than one-fourth because one of two equal parts is larger than one of four equal parts." This reasoning reflects a strong conceptual understanding of how fractions relate to the whole. The student also drew a simple diagram to visually reinforce their explanation.



Q11. Between half and a quarter of the lahmacun shown, which one takes up more space in a storage container?
A) Half
B) Quarter
C) Both take up the same amount of space
D) Other answer

Figure 3. Illustration of Student 7S2's Use of Reference Points

This response was coded as a number-sense-based strategy because it relied on proportional reasoning and visual-spatial representation of benchmark fractions. The student's explanation did not depend on formal rules or calculations but instead used intuitive comparison, which is a hallmark of reference point understanding. As this item had the highest rate of correct responses (58%), it reflects students' greater familiarity with common benchmark fractions and their ability to apply them meaningfully in reasoning tasks. Student 7S2's response illustrates this competence effectively.

Equivalent Representations Component

In the equivalent representations component, the overall success rate was calculated as 44%, indicating moderate performance. This component assessed students' ability to generate, recognize, or validate equivalent fractions through simplification, expansion, or comparison. Among the participants, both rule-based and number-sense-based strategies were observed in relatively balanced proportions. Student 8S1, for example, correctly simplified the given fraction and explained: "I divided both the numerator and denominator by the same number; this always gives the correct result." This approach demonstrates accurate procedural knowledge and was coded as a rule-based strategy due to its reliance on algorithmic reasoning rather than conceptual or visual justification. However, some students attempted to justify equivalence using visual representations or real-life analogies, which were classified as number-sense-based strategies.

Although such intuitive approaches were less frequent in this component, they reflected deeper conceptual understanding when present. This diversity of strategy use suggests that equivalent representation tasks offer multiple entry points for learners and serve as a valuable bridge between procedural fluency and conceptual reasoning..

From this point, a general evaluation can be made;

Table 5. Enriched summary of students' strategy use by number sense component

Component	Success Rate	Dominant Strategy	Sample Student Statement	Interpretation
Numerical Estimation	12%	Mostly Rule-Based	*"3/4 is more than 2/4, which is half, so it's close to 1."* (7S2)	Shows benchmark use; some intuitive responses, but procedural dominance overall.
Operational Effects	4%	Rule-Based	*"I did 20 ÷ 3/5 = 20 × 5/3 = 100/3."* (7S1)	Algorithm applied, but student lacked contextual understanding of the operation.
Magnitude Understanding	12%	Mixed	*"3/4 is three one-fourths, and that's more than 1/2."* (7S2)	Demonstrates conceptual part-to-whole comparison; coded as number-sense-based.
Use of Reference Points	58%	Number-Sense-Based	*"Half is always bigger than one-fourth, because 1 of 2 is more."* (7S2)	Strong benchmark reasoning and visual explanation; this was the most successful.
Equivalent Representations	44%	Mixed	*"I divided both numbers by the same number. That always works."* (8S1)	Correct rule-based strategy, but no conceptual or visual explanation provided

Table 5 presents an enriched summary of students' strategy use across the five components of fraction number sense. Unlike the previous version, this table includes representative student statements and interpretations to illustrate how different reasoning types emerged. The data show that while rule-based strategies ensured procedural accuracy, intuitive responses often provided deeper conceptual insight. The most successful performance was observed in tasks involving reference benchmarks, where visual and verbal justifications were commonly used**.**

The analysis of the first research question revealed that students' strategy use varied significantly across the five components of fraction number sense. While reference point use was the most successful and intuitive area, components such as operational effects and estimation revealed conceptual challenges and a heavy reliance on procedural thinking. These findings suggest that academically successful students demonstrate a range of reasoning strategies depending on the nature of the task.

The second research question focuses on comparing these strategies in greater depth, particularly examining how flexible thinking based on number sense differs from rule-based approaches in terms of conceptual understanding and problem-solving effectiveness. The following section presents findings related to this comparison.

To address the second sub-question of the study — "How do flexible thinking approaches based on fraction number sense differ from rule-based strategies in terms of students' conceptual understanding and practical applications?" — the impact of these differing strategies on students' conceptual comprehension and procedural skills was examined

For this purpose, semi-structured interviews were conducted with two students: one who utilized the highest level of number-sense-based strategies (Student 7S1) and another who predominantly relied on rule-based strategies (Student 8S1). The data obtained from these interviews were analyzed thematically to evaluate how differences in strategy use were reflected in students' mathematical thinking processes.

Tablo 6. Code table for semi-structured interviews with high-performing students

Theme	Code	7S1 (Number Sense)	8S1 (Rule-Based)
Thinking Process	Instant Connection-Making	x	
	Intuitive Thinking	x	
	Visualization	x	
	Step-by-Step Progression		x
	Rule-Based Execution		x
	Systematic Thinking		x
Relating to Numbers	Breaking the Fractions	x	
	Intuitive Ratio Evaluation	x	
	Establishing Numerical Links	x	
	Rule Application		x
	Finding Common Denominators		x
	Systematic Computation		x
Approach Type	Intuitive Responses	x	
	Quick and Practical Solutions	x	
	Flexibility	x	
	Rule-Based Responses		x
	Guaranteed Accuracy		x
	Step-by-Step Process		x
Challenges and Solutions	Challenges in Problem-Solving	x	
	Alternative Thinking Strategies	x	
	Intuitive Solution Strategies	x	
	Difficulty Remembering Rules		x
	Consulting Additional Resources		x
	Learning Through Practice		x

Table 6 illustrates the thematic codes derived from semi-structured interviews with two high-performing students who demonstrated contrasting cognitive approaches. Student 7S1, who consistently employed number-sense-based strategies, provided responses that reflected intuitive and visual reasoning. For instance, when asked to compare 1/2 and 1/4, the student stated: "Half is more because one of two is more than one of four. You can see it in the drawing."

In contrast, Student 8S1 relied heavily on rule-based approaches, such as calculating least common denominators. When solving an equivalence problem, the student explained: "I found the common denominator of 4 and 8, converted the fractions, and then compared." This reflects a step-by-step, algorithm-driven problem-solving method.

These examples highlight how intuitive thinkers tend to emphasize flexibility, rapid estimation, and visual justification, whereas rule-based students favor structured methods, accuracy through formal procedures, and sequential problem-solving. This thematic divergence supports the interpretation that flexible strategies contribute to deeper conceptual understanding and adaptive thinking.

The thematic analysis presented in the table provides valuable insight into how fraction number sense strategies influence students' mathematical thinking and problem-solving processes. The responses of the two students diverged across four distinct themes:

Thinking Process

The students exhibited markedly different cognitive approaches during problem-solving. Student 7S1 utilized high-level cognitive strategies such as quick relational thinking, intuitive evaluation, and mental visualization when approaching fraction tasks. For example, the statement *"I imagine one-fourth and one-half on the number line"* illustrates the student's ability to construct conceptual and visual models mentally.

Relationship with Numbers

Significant differences were also observed in how students understood and structured fractions. Student 7S1 demonstrated part-whole reasoning by mentally partitioning the whole into fractional parts and evaluating proportions intuitively. The statement *"3/4 is three 1/4s"* reflects a holistic and conceptual relationship with numbers.

In contrast, Student 8S1 focused primarily on procedural accuracy rather than relational understanding. The explanation *"I made the denominators equal and then added the numerators"* reveals reliance on algorithmic procedures instead of conceptual reasoning. While this approach led to correct answers, it lacked conceptual depth and reduced the student's flexibility, resulting in a more time-consuming problem-solving process.

Type of Approach

The students also differed significantly in their strategic approaches. Student 7S1 tended to produce fast, practical, and flexible solutions. Through estimation, visualization, and mental trial-and-error, the student efficiently managed the problem-solving process. The statement *"In some questions, I find the solution right away using my intuition"* clearly reflects this strategy.

On the other hand, Student 8S1 adopted a more structured approach that prioritized accuracy but progressed more slowly. The student meticulously followed sequential steps to ensure procedural correctness. Although this method yielded reliable outcomes in standard problems, it limited the student's adaptability when facing novel or complex situations.

Challenges and Coping Strategies

The students' responses to challenges further highlighted their strategic differences. Student 7S1 developed alternative approaches when faced with difficulty. The statement *"I try to think of fractions in terms of whole numbers"* suggests that the student engaged in personal mental modeling to generate solutions—demonstrating a high level of independence and self-regulation.

In contrast, Student 8S1 showed a tendency to rely on external resources when encountering difficulty, particularly in recalling rules. The remark *"When I forget the rules, I check my class notes"* indicates dependency on structured support rather than internalized flexibility.

This thematic analysis reveals that flexible thinking approaches based on fraction number sense deepen students' conceptual understanding and accelerate their problem-solving processes. Such approaches strengthen students' abilities to generate alternative strategies, make estimations, and form meaningful relationships with numbers. In contrast, while rule-based strategies provide procedural accuracy, they tend to limit adaptability in novel problem contexts and often result in more superficial conceptual comprehension. These findings suggest the need for a more holistic instructional model that supports intuitive reasoning while systematically integrating rule-based understanding into teaching practices.

Discussion

This section interprets the findings of the study in relation to existing literature on number sense and mathematical thinking. Each major component of fraction number sense is discussed individually to examine how different strategy types—particularly number-sense-based versus rule-based approaches—affected students' understanding and problem-solving processes. The discussion also addresses the educational implications of these findings and how they contribute to the broader understanding of strategy use in fraction learning.

Among the five components investigated, the use of reference points emerged as the most successful area, with 58% of students responding correctly. This high success rate can be attributed to the familiarity of benchmark fractions such as 1/2 and 1, which are commonly emphasized in primary and middle school curricula. Many students used intuitive reasoning supported by visual benchmarks or verbal justification. For instance, Student 7S2 explained: "One-half is more than one-fourth because one of two parts is bigger than one of four," accompanied by a proportional drawing.

This finding supports the argument made by Reys and Reys (1992), who emphasized that a strong understanding of benchmark fractions forms the foundation of number sense. It also aligns with McNamara and Shaughnessy (2015), who highlight visual reasoning and reference benchmarks as central to students' fractional thinking. The ability to mentally locate and compare fractions using reference points appears to be a gateway to more advanced estimation and comparison skills.

In stark contrast, the operational effects component demonstrated the lowest rate of correct responses—only 4%. Most students relied heavily on symbolic manipulation and rule application without fully understanding the contextual meaning of the operations. A common error involved confusing multiplication with division, especially when working with real-life contexts such as "How many 3/5 ml syringes are needed for 20 ml?" Student 7S1, for example, correctly performed the division algorithm ($20 \div 3/5 = 20 \times 5/3$), but failed to explain what the result represented in practical terms. This finding echoes concerns raised by Woodward (1998), who emphasized that overreliance on procedures may lead to superficial correctness without conceptual depth. It also reflects the challenges identified in Turkish mathematics instruction (Akgün and Yıldız, 2016), where concrete representations are used but deeper reasoning is rarely emphasized. This suggests a need for instructional design that moves beyond procedural fluency to foster students' ability to reason meaningfully about the effects of operations.

The comparative analysis between Student 7S2, who consistently used number-sense-based strategies, and Student 8S1, who relied on rule-based approaches, revealed striking cognitive differences. Student 7S2 used intuitive reasoning, benchmarks, and flexible estimation to arrive at answers, often relying on drawings or verbal explanation to justify solutions. In contrast, Student 8S1 preferred structured algorithms, such as finding common denominators or inverting fractions for division, and emphasized accuracy through formal steps. These contrasting profiles align with McNamara and Shaughnessy (2015), who argue that flexible thinkers demonstrate deeper conceptual understanding and adaptability, while rule-based thinkers often perform well in structured contexts but may struggle with non-routine problems. The analysis showed that number-sense-based strategies were associated with quicker reasoning, better visualization, and more self-explanation, whereas rule-based strategies provided step-by-step security but limited generalization. This supports the pedagogical view that combining both approaches in instruction could optimize both understanding and procedural reliability.

When examined holistically, the findings indicate that students do not rely on a single type of reasoning, but instead shift between intuitive and procedural strategies depending on the task context. Components such as reference point use and magnitude understanding naturally prompted flexible thinking, while operational effects often triggered rule-based responses. Notably, the effectiveness of strategies varied not only by task type but also by the students' ability to integrate conceptual understanding with procedural execution. This interplay between intuitive and algorithmic approaches suggests that number sense is not a fixed trait but a flexible cognitive resource that can be strengthened through targeted instructional strategies. Students who were able to link part-whole relationships, estimate relative sizes, and visualize operations demonstrated stronger mathematical reasoning. In contrast, those who applied algorithms without conceptual grounding struggled to adapt their thinking to unfamiliar problems.

These results carry important implications for mathematics education. First, they support the notion that instruction should explicitly develop number-sense-based strategies alongside procedural fluency. As emphasized by McIntosh et al. (1992), number sense is both teachable and foundational to meaningful problem-solving. Second, the findings highlight the pedagogical value of visual models, benchmark-based reasoning, and flexible thinking prompts in enhancing students' conceptual grasp of fractions. From a research perspective, this study contributes to the growing body of literature that seeks to define and operationalize fraction number sense as a multidimensional construct. By examining student responses across five key components, and comparing cognitive profiles in depth, this work provides a nuanced perspective on how strategy use reflects and shapes mathematical thinking.

In sum, the study underscores the importance of promoting flexible, number-sense-based strategies in fraction instruction to develop both conceptual depth and procedural control—two pillars essential to mathematical competence.

Conclusion

This study examined the strategy use of academically successful middle school students in solving tasks related to fraction number sense, and explored how these strategies influenced their mathematical thinking and problem-solving processes. The findings demonstrated that number sense is not merely a procedural skill, but a multidimensional cognitive competence. Students were observed to shift between different types of strategies depending on the nature of the task, indicating that number sense is not a fixed trait, but rather a flexible mental resource that can be activated and developed through experience.

A key distinction in students' responses emerged between number-sense-based (intuitive, flexible) and rule-based (algorithmic, step-by-step) strategies. Students who adopted number-sense-based strategies tended to solve problems more quickly, efficiently, and with deeper conceptual understanding. They made use of visual representations, reference points, and estimation. In contrast, students who relied solely on rule-based strategies were often successful in executing procedures correctly but struggled with interpreting the contextual meaning of problems and generating alternative solutions. These differences are shaped not only by conceptual understanding but also by individual learning experiences and preferences.

The study underscores the importance of addressing number sense as a distinct and central focus in mathematics instruction. Number sense is not something that can be taught directly; rather, it must be fostered through well-structured learning activities. Instructional tools such as visual materials, benchmark numbers, estimation tasks, and context-rich problem situations play a vital role in supporting the development of number sense. The ability to decompose numbers, mentally place them on number lines, and make intuitive comparisons before applying symbolic procedures highlights its dynamic and developable nature.

Based on the findings, it can be suggested that number sense develops across identifiable stages—basic, intermediate, and advanced. At the basic level, students recognize whole-part relationships and can compare quantities. At the intermediate level, they begin using mental computation, estimation, and reference strategies. At the advanced level, students are capable of abstract numerical reasoning, generating multiple solution paths, and evaluating operational outcomes. This staged perspective can help educators observe students' number sense levels and adapt their instruction accordingly.

In conclusion, this study affirms that number sense is a holistic and evolving competence, closely associated with both students' academic performance and the depth of their conceptual reasoning. Mathematics education should be redesigned to promote number sense through multidimensional instructional approaches that integrate intuitive reasoning, visual representation, and strategic flexibility. The idea that number sense can be cultivated gradually through instruction, supported at different developmental levels, offers a valuable roadmap for future teaching practices and research directions.

Recommendations

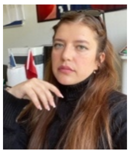
Based on the findings of this study, the following recommendations are proposed in order of priority: Address conceptual weaknesses in operational understanding. Since the "operational effects" component showed the lowest success rate (4%), instruction should focus on developing conceptual clarity around operations involving fractions. Teachers should provide contextualized problems that highlight the meaning of operations rather than only teaching procedures such as "invert and multiply. "Continue to strengthen benchmark-based reasoning.As "reference point use" was the most successful component (58%), curricula should emphasize the use of benchmarks like 1/2 and 1 across grade levels. Instructional tasks should include number lines, visual representations, and real-life estimation scenarios to deepen intuitive number sense. Encourage visual and part-whole strategies in teaching equivalent representations. In light of students' balanced use of intuitive and rule-based strategies in equivalent fraction tasks, educators should incorporate both symbolic manipulation and visual modeling. Using fraction bars, area models, and comparison diagrams can support students' understanding of equivalence beyond rules. Create learning environments that promote flexible thinking. Findings revealed that students who used number-sense-based strategies solved problems with greater efficiency and understanding. Teachers should therefore provide open-ended tasks that allow students to select and justify strategies. Promoting metacognitive reflection on multiple solution paths can enhance strategic flexibility. Design teacher education modules focused on number sense progression. Given that number sense was observed at varying cognitive levels (basic, intermediate, advanced), teacher preparation programs should train educators to recognize and support these stages. Developing teachers' ability to diagnose student thinking can improve instructional adaptation.

Develop assessment tools that measure strategic thinking, not just correctness. Evaluations should include diagnostic items that reveal students' reasoning processes. Rubrics that differentiate between intuitive, rule-based, and mixed strategies can offer richer insights into students' mathematical thinking and guide instruction more effectively. Conduct longitudinal and comparative studies on number sense. Future research should examine how number sense develops over time and across diverse student groups. Studies comparing grade levels, ability levels, and instructional settings will further clarify the theoretical structure and instructional potential of fraction number sense.

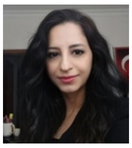
Acknowledgment

The current study is based on data that was collected by Dr. Aygül Kartal and Dr. Niymet Demirci. In the conducted research, the researchers implemented the study within the instructional process at their institution, with the consent of the families and students, and with the permission of the institution's administration; thus, all ethical requirements were fulfilled.

Biodata of Author



Dr. **Aygül Kartal** is currently serving as a vice principal at a public school in Rize, Türkiye. Her research interests focus on mathematics education, particularly Montessori-based instruction, number sense, and the development of creative thinking skills. She has published academic work and developed instructional materials in these areas. **E-mail**: aygulkartal53@gmail.com **ORCID**: 0000-0001-5120-6816



Dr. Niymet Demirci is currently working as a classroom teacher at Torbalı Science and Art Center in Türkiye. Her research interests include mathematics education, number sense, instructional strategies, and the mathematical thinking skills of gifted students. She has authored various academic publications and actively participates in educational projects related to these areas. **E-mail**: niymetdemirci@gmail.com **ORCID**: 0000-0002-2763-1879

References

Akgün, L., & Yıldız, C. (2016). Kesir kavramlarının öğretiminde öğretmen rehberliğinde yapılan iş birlikli öğrenmenin başarıya ve kalıcılığa etkisi, (The effect of teacher-guided cooperative learning on success and retention in teaching fraction concepts). *Eğitim ve Bilim, 41*(186), 1–21. https://doi.org/10.15390/EB.2016.4526

Ahmed, W., & Williams, M. (1997). Case study method in educational research: From theory to practice. *Contemporary Education, 68*(1), 27–30.

Bailey, D. H., Hoard, M. K., Nugent, L., & Geary, D. C. (2012). Competence with fractions predicts gains in mathematics achievement. *Journal of Experimental Child Psychology, 113*(3), 447–455. https://doi.org/10.1016/j.jecp.2012.06.004

Bailey, D. H., Hoard, M. K., Nugent, L., & Geary, D. C. (2014). Competence with fractions predicts gains in mathematics achievement. *Journal of Experimental Child Psychology, 118*, 110–118. https://doi.org/10.1016/j.jecp.2013.09.009

Bailey, D. H., Siegler, R. S., & Geary, D. C. (2015). Developmental change in the influence of domain-general abilities and domain-specific knowledge on mathematics achievement. *Journal of Educational Psychology, 107*(3), 880–893. https://doi.org/10.1037/edu0000024

Behr, M. J., Harel, G., Post, T. R., & Lesh, R. (1992). *Rational number, ratio, and proportion*. In D. A. Grouws (Ed.), Handbook of research on mathematics teaching and learning (pp. 296–333). Macmillan Publishing.

Cetin, H., & Yapıcı, M. (2023). 8. sınıf öğrencilerinin sayı duyusu performanslarının ve stratejilerinin incelenmesi(An Investigation of 8th Grade Students' Number Sense Performance and Strategies). *Trakya Eğitim Dergisi, 13*(3), 1799–1817. https://doi.org/10.24315/tred.1251704

Clarke, D. M., & Roche, A. (2009). Students' fraction comparison strategies as a window into robust understanding and possible pointers for instruction. *Educational Studies in Mathematics, 72,* 127–138. https://doi.org/10.1007/s10649-009-9198-9

Cramer, K. A., Behr, M. J., Post, T. R., & Lesh, R. (1997). *Rational number project: Fraction lessons for the middle grades*. Research in Mathematics Education.

Creswell, J. W. (2013). *Qualitative inquiry and research design: Choosing among five approaches (3rd ed.).* Sage Publications.

DePascale, J., & Ramani, G. B. (2024). Exploring the role of mathematics games in developing number sense in elementary students: A systematic review. *Journal of Early Mathematics Education, 15*(1), 50–75.

Durmuş, S., & Karaca, E. (2013). İlköğretim matematik öğretim programındaki kesirler konusunun öğretiminde karşılaşılan güçlükler (Challenges encountered in teaching the fractions topic in the primary school mathematics curriculum). *Milli Eğitim Dergisi, 197,* 106–123.

Fennell, F., Kobett, B., & Wray, J. (2014). Fractions are numbers too! *Mathematics Teaching in the Middle School, 19*(8), 486–493. https://www.jstor.org/stable/10.5951/mathteacmiddscho.19.8.0486

Jordan, N. C., Kaplan, D., Ramineni, C., & Locuniak, M. N. (2010). Early math matters: Kindergarten number competence and later mathematics outcomes. *Developmental Psychology, 46*(3), 850–867. https://doi.org/10.1037/a0019672

Kartal, A., & Pirasa, N. (2022). Examination of fractional number sense in eighth graders with high academic performance. *Education Quarterly Reviews, 5*(3), 297–308. https://doi.org/10.31014/aior.1993.05.03.546

Lamon, S. J. (2007). *Rational numbers and proportional reasoning: Toward a theoretical framework for research*. In F. K. Lester Jr. (Ed.), Second handbook of research on mathematics teaching and learning (pp. 629–667). Information Age Publishing.

Lin, C., & Powell, S. R. (2023). Supporting conceptual understanding in fractions through dynamic visualizations. *Mathematics Teacher Education and Development, 25*(1), 98–122.

McNamara, J., & Shaughnessy, M. M. (2015). B*eyond pizzas & pies: 10 essential strategies for supporting fraction sense, grades 3–5.* Math Solutions.

Ng, S. F. (2010). Teaching fractions through realistic problems in Chinese classrooms: A pilot study. *International Journal of Science and Mathematics Education, 8*(5), 911–935. https://doi.org/10.1007/s10763-009-9183-2

Reeder, S. L., & Utley, J. (2007). *Assessing preservice teachers' development of fraction number sense*. Issues in the Undergraduate Mathematics Preparation of School Teachers, 1.

Rochaendi, R., & Ariyani, F. (2024). Enhancing students' conceptual understanding of fractions through visual-based models in middle school mathematics. *International Journal of Educational Research, 125,* Article 102076. https://doi.org/10.1016/j.ijer.2023.102076

Siegler, R. S., Thompson, C. A., & Schneider, M. (2012). An integrated theory of whole number and fractions development. *Cognitive Psychology, 62*(4), 273–296. https://doi.org/10.1016/j.cogpsych.2011.03.001

Sowder, J. T. (1992). *Making sense of numbers in school mathematics*. In G. Leinhardt, R. Putnam, & R. Hattrup (Eds.), Analysis of arithmetic for mathematics teaching (pp. 1–51). Lawrence Erlbaum.

Suh, J. M., Moyer, P. S., & Heo, H. J. (2005). Examining technology uses in the classroom: Developing fraction sense using virtual manipulatives. *Journal of Computers in Mathematics and Science Teaching, 24*(3), 303–324.

Utley, J., & Reeder, S. (2012). Prospective elementary teachers' development of fraction number sense. *Investigations in Mathematics Learning, 5*(1), 1–24.

Yıldız, C., & Eryılmaz, A. (2018). Ortaokul matematik öğretiminde kesir sayılarının öğretimi ve öğrenci başarılarına etkisi (The effect of teaching fractional numbers in middle school mathematics on student achievement). *Kuram ve Uygulamada Eğitim Bilimleri, 18*(3), 625–648. https://doi.org/10.12738/estp.2018.3.0231

Yılmaz, E., & Yeşildere-İmre, S. (2018). Öğrencilerin sayı duyusu gelişiminde öğretim sürecinin etkisi. *Elementary Education Online, 17*(4), 2234–2250. https://doi.org/10.17051/ilkonline.2019.507441