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An Investigation of Fractional-Order Respiratory System Dynamics

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Keywords

Feedback circuits, RLC circuits, Respiratory system modeling, Mathematical model, Biomedical instrumentation, Stability analysis.

Abstract

In order to represent a respiratory system, a linear model has been applied and for comparison, the same linear model has been represented in terms of the fractional operators. In order to obtain fractional order simulations, a fractional order modeling toolbox, FOMCON, is used. By this manner, integer order and fractional order modeling performances are compared. For different parameter values that correspond to different scenarios, corresponding simulations are obtained and a comparison of parameter values and the integer and fractional order models are investigated. Then, a static model is used and simulated for different parameter values. Also, a stability analysis is implemented by investigating the bode plots of the static model.

1. Introduction

The human respiratory system is a complex network of organs and tissues responsible for the exchange of oxygen and carbon dioxide, essential for cellular respiration. It consists of the upper respiratory tract, including the nasal cavity, pharynx, and larynx, which filter, warm, and humidify incoming air, and the lower respiratory tract, comprising the trachea, bronchi, bronchioles, and lungs. Within the lungs, alveoli—tiny air sacs surrounded by capillaries—facilitate gas exchange via diffusion, where oxygen enters the bloodstream and carbon dioxide is expelled. The diaphragm and intercostal muscles regulate breathing through rhythmic contraction and relaxation, controlled by the medulla oblongata in response to blood CO_2 levels. This intricate system ensures oxygen delivery to tissues and the removal of metabolic waste, maintaining homeostasis and supporting life.

The human respiratory system, responsible for the vital exchange of oxygen and carbon dioxide, has been extensively modeled to enhance our understanding of its intricate mechanics and physiological functions. These models range from simplified representations to complex, integrated systems that incorporate cardiovascular interactions and control mechanisms [16], [12]. Simplified models, for instance, focus on gas exchange dynamics within the lungs, exploring the relationship between molecular-level changes and alveolar partial pressure [12], [17]. Such models often revisit established assumptions, including the equivalence of airflow through the mouth and the rate of lung volume change, and the equilibrium status of oxygen-hemoglobin binding [12]. More comprehensive models aim to capture the interactions between the cardiovascular and respiratory systems, crucial for understanding the cardiopulmonary interplay [16]. These integrated models have been validated against human and animal data, providing a robust platform for studying short-term control mechanisms [16].

Computational models play a crucial role in consolidating experimental data obtained from diverse animal studies under varying conditions [13]. These models facilitate the investigation of respiratory network architecture and neural mechanisms, crucial for generating respiratory rhythm and patterns, and allow an understanding of how

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these patterns reorganize under different physiological conditions [13]. The models are developed in parallel and iteratively with experimental studies and provide predictions guiding new experiments [13].

Electrical circuit models offer a powerful and intuitive framework for simulating the complex dynamics of the human respiratory system, allowing researchers and clinicians to gain insights into its behavior under various physiological and pathological conditions [11]. By representing different components of the respiratory system, such as the lungs, airways, and respiratory muscles, as electrical elements like resistors, capacitors, and inductors, these models can mimic the flow of air and the exchange of gases in a way that is mathematically tractable and computationally efficient [12]. This approach facilitates the investigation of respiratory mechanics, gas exchange dynamics, and the neural control of breathing [13]. Such models provide a mathematical framework to link between "molecular-level" and "systems-level" models [12]. The transformation of the respiratory system into analogous electrical circuits is remarkably beneficial, facilitating the employment of well-developed circuit analysis methodologies to anticipate system responses across a spectrum of conditions, thus presenting a flexible instrument for both investigative pursuits and clinical implementations [12]. Furthermore, the ability to simulate respiratory function using electrical circuit models enables the exploration of different scenarios, such as mechanical ventilation, airway obstruction, and respiratory muscle fatigue. These simulations can help optimize ventilator settings, predict the effects of therapeutic interventions, and improve the management of respiratory diseases [13].

The human respiratory system, characterized by its intricate network of airways and alveolar structures, exhibits complex impedance behavior that is not adequately represented by classical integer-order models. Fractional-order circuits, employing elements with non-integer order derivatives and integrals, offer a more accurate and nuanced approach to capturing the system's dynamic characteristics. This enhanced accuracy stems from the ability of fractional-order models to represent the distributed nature of the respiratory system's components, such as the viscoelasticity of lung tissue and the varying diameter of the airways, with greater fidelity [14]. Fractional-order models inherently possess memory effects, which are crucial for accurately simulating the respiratory system's response to varying breathing patterns and external stimuli [14]. By incorporating fractional-order elements, such as constant phase elements, these models can better capture the frequency-dependent behavior observed in respiratory impedance measurements, providing a more complete picture of the system's dynamics. Traditional modeling approaches often rely on simplified representations of the respiratory system, such as lumped parameter models, which may not fully capture the complexities of airflow and gas exchange [15]. Fractional-order models, on the other hand, offer a more flexible framework for incorporating these complexities, allowing for a more accurate representation of the system's behavior across a wide range of conditions [14].

2. Human Respiratory System: Linear Model

As described in the introduction part, electrical model of a respiratory system has benefits in terms of its simplicity and efficiency to apply different scenarios for an individual. There exist many different models for a respiratory system. In this work, a linear model is assumed with the following parameter values; R_C, fluid mechanical resistance in central airways, R_P, fluid mechanical resistance in peripheral airways, C_L, lung compliance, C_W, chest wall compliance, C_S, shunt compliance, L_C, inertance in central airways to gas flow. The details and description of the model can be found in [1] and for a detailed biological background of the respiratory system with other systems, one may see [20]. Please see Figure 1 for a circuit diagram of the considered model. If the Kirchoff's first law is applied to the closed circuit, the following equation is obtained [1].

$$R_P Q_A + (\frac{1}{C_L} + \frac{1}{C_W}) \int (Q_A) dt = \frac{1}{C_S} \int (Q - Q_A) dt$$

The following equations are used to represent the inertance, Lc, to gas flow in the central airways:

$$P_{aw} = \frac{1}{C_S} \int (Q - Q_A) dt \tag{1}$$

$$P_{a0} = QR_C + \frac{1}{C_S}\int (Q - Q_A)dt + L_C \frac{dQ}{dt} \tag{2}$$

By rearranging equations 1 and 2, we obtain the following equation:

$$P_{a0} - P_{aw} = L_C \frac{dQ}{dt} + QR_C. \tag{3}$$

The corresponding linear model and the circuit design are illustrated in Figure 1. For a healthy adult, the parameter values are assumed as $R_C = 1\, cmH_2Oslt^{-1}$, $R_P = 0.5\, cmH_2Oslt^{-1}$, $C_L = 0.2\, ltcmH_2O^{-1}$, $C_W = 0.2\, ltcmH_2O^{-1}$, $C_S = 0.005\, ltcmH_2O^{-1}$ and $L_C = 0.01\, cmH_2O\, s^2lt^{-1}$. In order to obtain respiratory breath rates, we adjust the amplitude to 2.5cm and frequency to 1.57 $rads^{-1}$. Then, we obtain,

$$\frac{1.57}{2\pi} = 0.25\, Hz = 15\, breaths\, min^{-1}$$

Therefore, by adjusting frequency as 6.28, 12.56, 25.12, 50.24 and 100.48, we obtain 60, 120, 240, 480 and 960 $breaths\, min^{-1}$, respectively. An individual has a respiratory rate per minute depending on his/her age and health condition [19]. A study discovered that just 33% of individuals arriving at an emergency department with an oxygen saturation below 90% exhibited an elevated respiratory rate [18]. Therefore, 15 and 30 $breaths\, min^{-1}$ are considered and compared in this work.

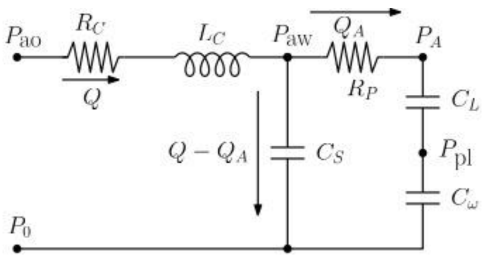

Figure 1. Circuit design representation of the linear model. The quantity P_0=0.

A patient with emphysema is represented with parameter values $C_L = 0.4\, ltcmH_2O^{-1}$ and $R_P = 7.5\, cmH_2Oslt^{-1}$. As expected, and investigated from the simulation results with 15 $breaths\, min^{-1}$, airflow increases with the rapid breaths and then the patient cannot breath anymore and the tidal volume is not measured when the breathing stops (see Figure 2).

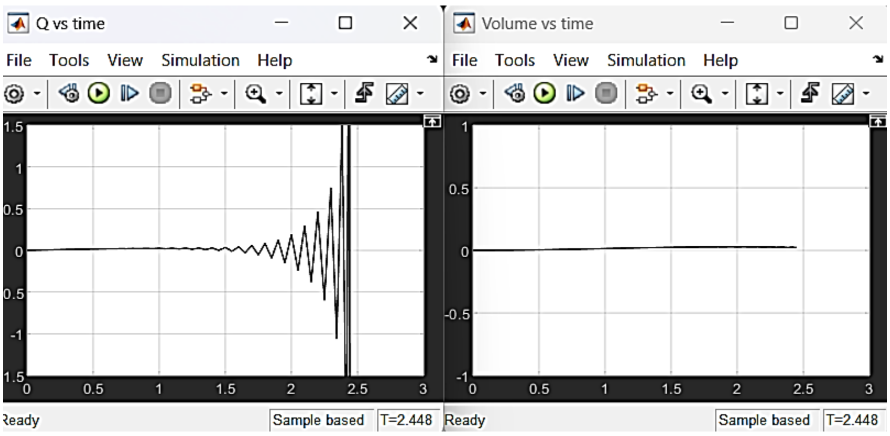

Figure 2. Airflow and tidal volume graphs for a patient with emphysema with 15 breaths min^{-1}

In order to compare, the same model has been designed in terms of the fractional operator. In order to achieve this goal a fractional order modeling toolbox, FOMCON, [2] is used. FOMCON toolbox uses Caputo type fractional derivative which is defined as follows: [3],[4],[5]

Definition 1: Let $f \in C_{-1}^{m}$, m ∈ N ∪ {0}. Then the left-sided Caputo fractional derivative of f is defined as

$$D^{\mu} f(x) = \begin{cases} I^{m-\mu} f^{m}(x), & m-1 < \mu \leq m, \quad m \in N \\ \frac{d^{m} f(x)}{dx^{m}}, & \mu = m \end{cases} \tag{4}$$

Moreover, it is stated that [3],[4],[5],

$$\begin{aligned} &\mathcal{I}^{\mu}\mathcal{I}^{\nu} f = \mathcal{I}^{\mu+\nu} f, \quad \mu,\nu \geq 0, f \in C^{\alpha}, \quad \alpha \geq 1 \\ &\mathcal{I}^{\mu} x^{\gamma} = \frac{\Gamma(\gamma+1)}{\Gamma(\lambda+\mu+1)} x^{\gamma+\mu}, \quad \mu > 0, \quad \gamma > -1, \quad x > 0. \end{aligned} \tag{5}$$

Fractional order counterparts of the same model is simulated. In the literature, there exist some works that investigate the respiratory system dynamics using fractional order derivatives. To name a few of them, respiratory compartmental models [6], [7], data fitting [8] impedance [9] or state-space versions [10] of the electrical models can be mentioned. Therefore, the simulations can be repeated with an updated version of the toolbox and with different fractional order derivatives. In Figures 6 and 7, a comparison of the integer order derivatives and fractional order derivatives are represented. As can be seen from the figures, the healthy individual and the patient individual data differ in the cases where the integer order derivative and the fractional order derivative is used. When the real world case is considered, as can be compared from the plots, more realistic results are obtained.

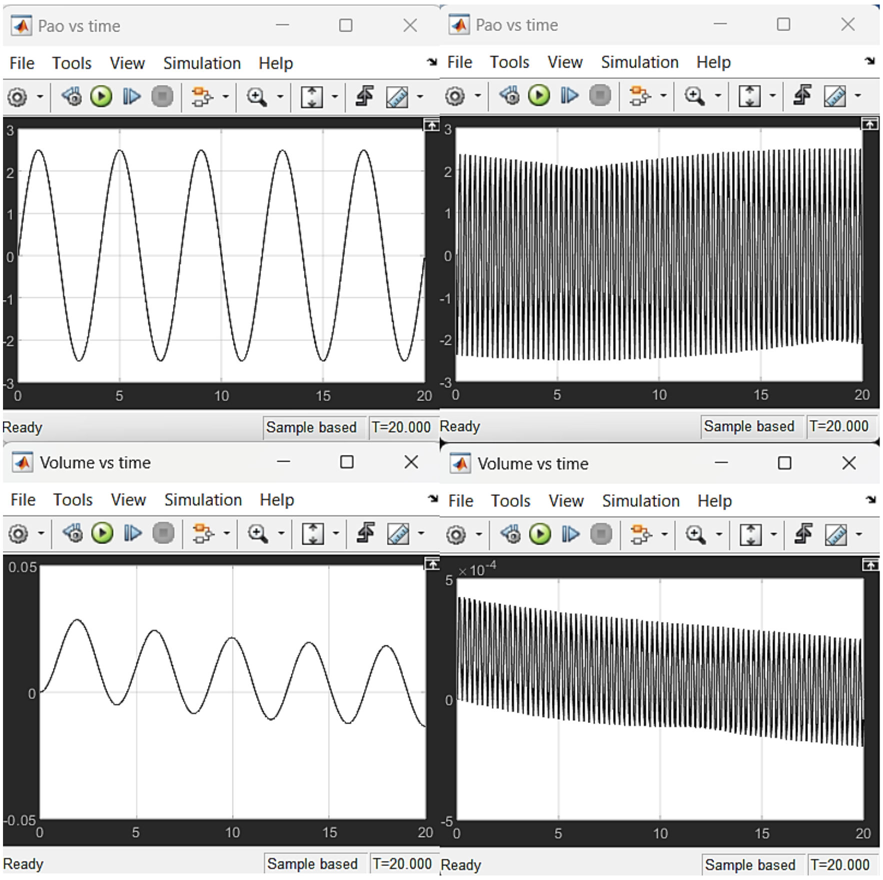

Figure 3. Pressure at the airway opening and volume graphs (from up to down) without proportional feedback effect for 15 and 960 $breaths\ min^{-1}$ (from left to right), respectively.

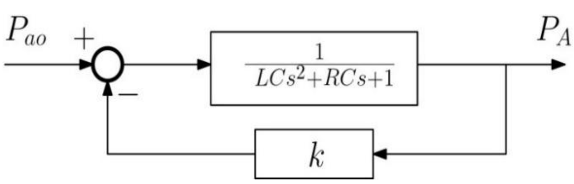

Figure 4. Proportional feedback of the ventilator system [1].

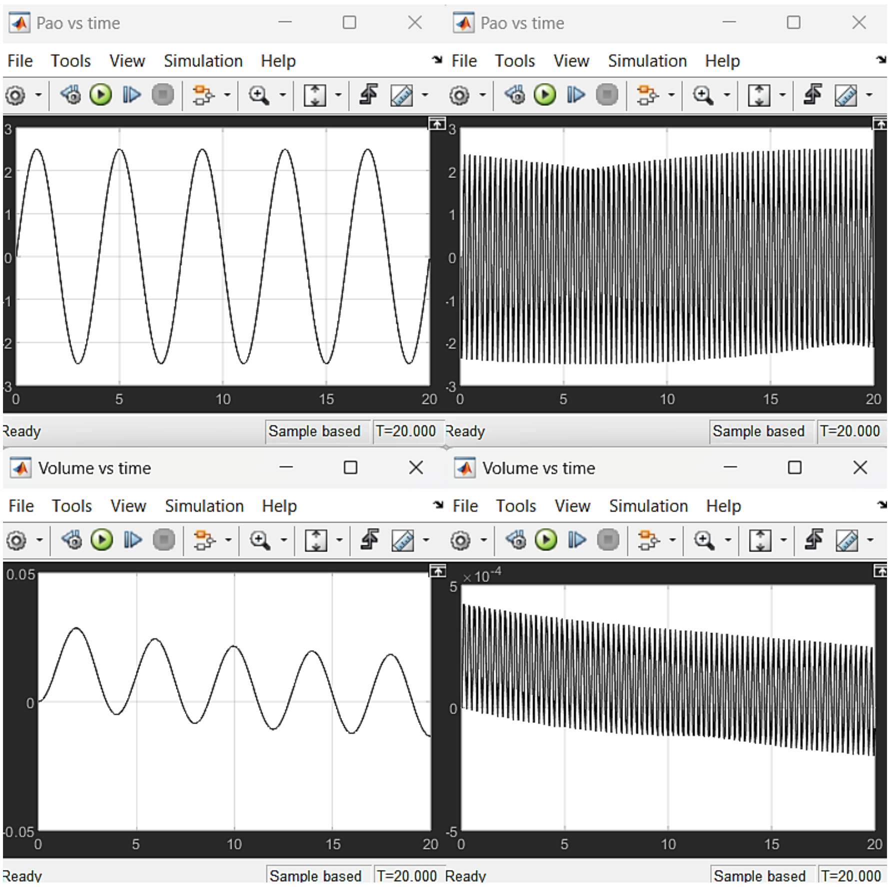

Figure 5. Pressure at the airway opening and volume graphs (from up to down) with proportional feedback effect for 15 and 960 $breaths\ min^{-1}$ (from left to right), respectively.

In this work, a fractional derivative of order 0.5 is simulated and compared with the integer order derivative. In Figures 7 and 8, one may see the simulation results of integer order derivative and fractional order derivative.

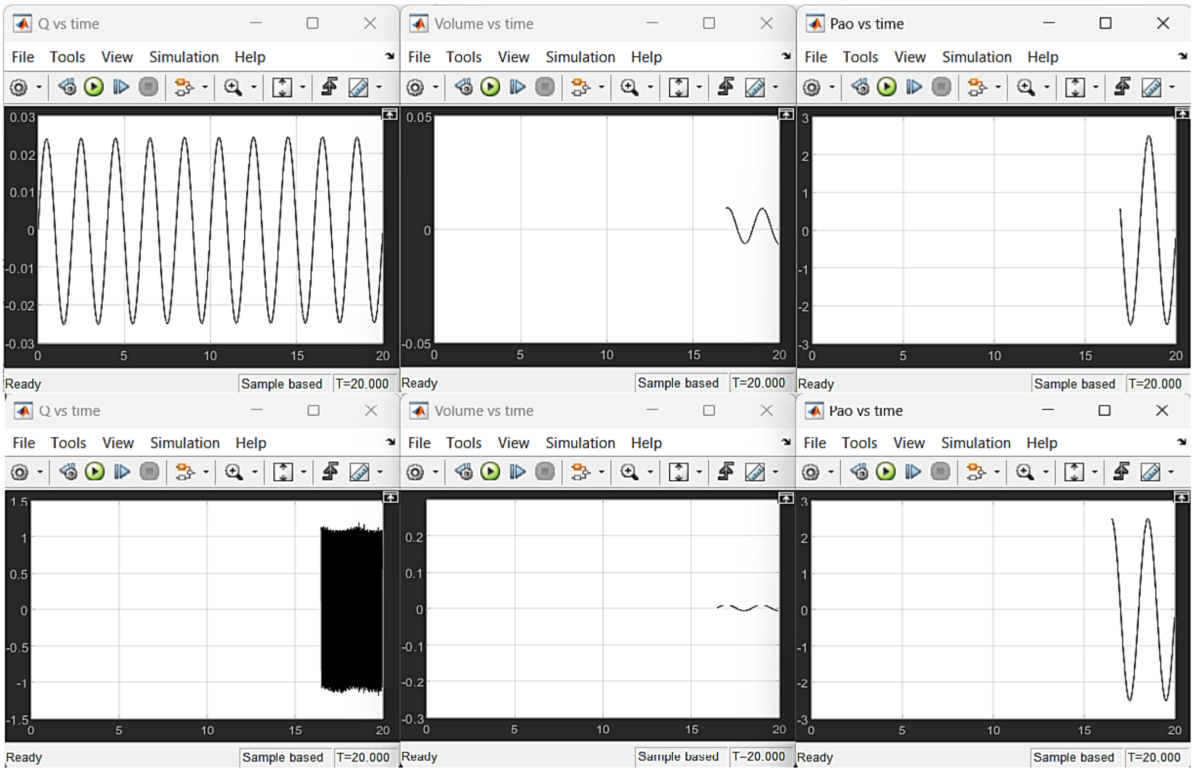

Figure 6. Air flow, tidal volume, pressure at the airway opening graphs of 30 $breaths\ min^{-1}$ for a healthy individual (up) and a patient with esophageal balloon (bottom) when integer order derivative is used.

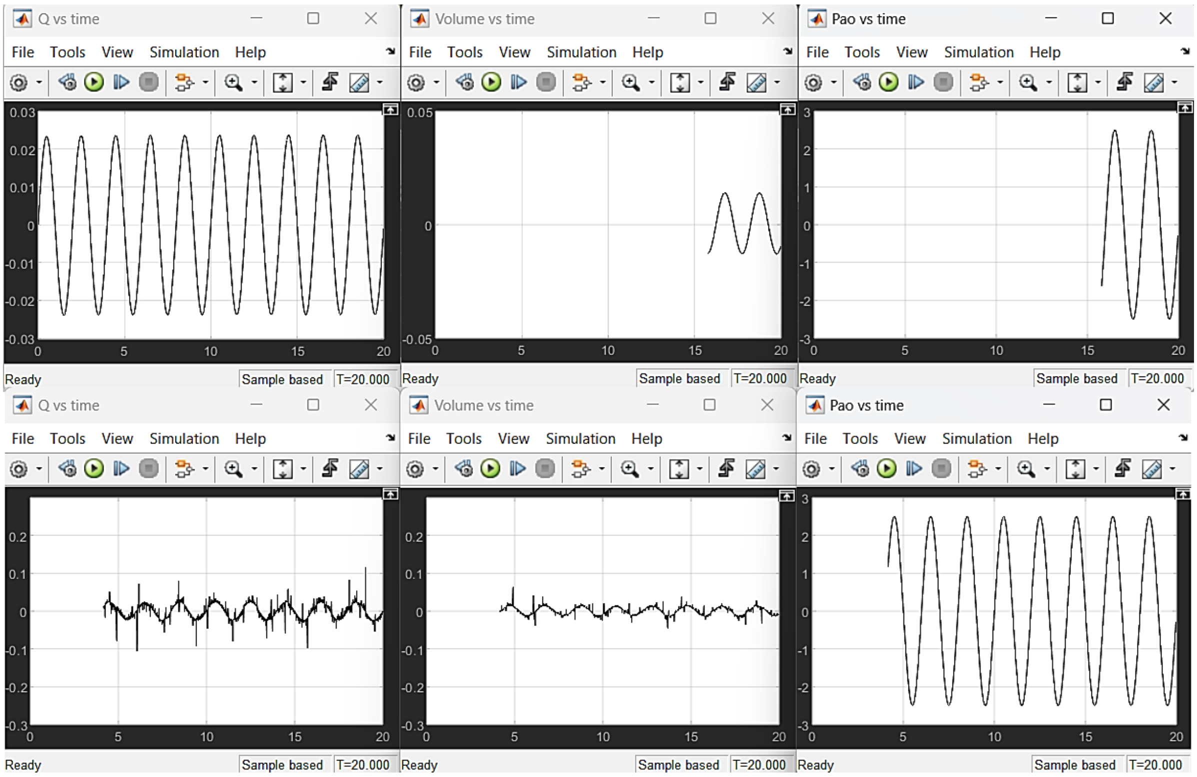

Figure 7. Air flow, tidal volume, pressure at the airway opening graphs of 30 $breaths\ min^{-1}$ for a healthy individual (up) and a patient with esophageal balloon (bottom) when fractional-order derivative 0.5 is used.

Then, a model of the respiratory gas exchange and respiratory control system, to determine the partial pressures of O_2 and CO_2 in alveolar air for a given metabolic CO_2 production rate, metabolic O_2 consumption rate is considered. Assume the metabolic hyperbola for CO_2 and for O_2 are given as [1],

$$P_{ACO2} = P_{ICO2} + \frac{863V'_{CO2}}{V'_A} \tag{6}$$

and

$$P_{AO2} = P_{IO2} - \frac{863V'_{O2}}{V'_A} \tag{7}$$

Parameter values can be assumed as; $P_{IO2} = 150\ mmHg, P_{ICO2} = 0\ mmHg$, metabolic CO_2 production rate 200ml/min, metabolic O_2 consumption rate 200ml/min, $V'_D = 1lt/min$.

One may assume that [1],

$$P_{ACO2} = P_{aCO2} \tag{8}$$

and

$$P_{AO2} = P_{aO2} \tag{9}$$

and therefore only P_{AO2} versus P_{ACO2} is displayed (see Figure 8). Moreover, for the given parameter values, steady-state values are obtained. For the given parameter values, the person is a healthy one with P_{O2} = 99.9989. Then, the apneic threshold value is increased to 42 from 37. Then the steady-state value for P_{O2} is obtained as 94.1666. This indicates that the person is not able to take enough oxygen and the ventilation value is decreased to 5.4824 from 6.0056. If a gas mixture containing 7% CO_2 in air exist then the P_{O2} value decreases to 66.5767. For frequency response and the stability of the model, a chirp signal is applied. The model is a static model and to obtain a dynamic model, firstly an input-output model is obtained. Then in order to observe Bode plots, a chirp signal is applied. We apply the initial conditions as indicated earlier and from the plots, we observe that it is stable (see Figure 9).

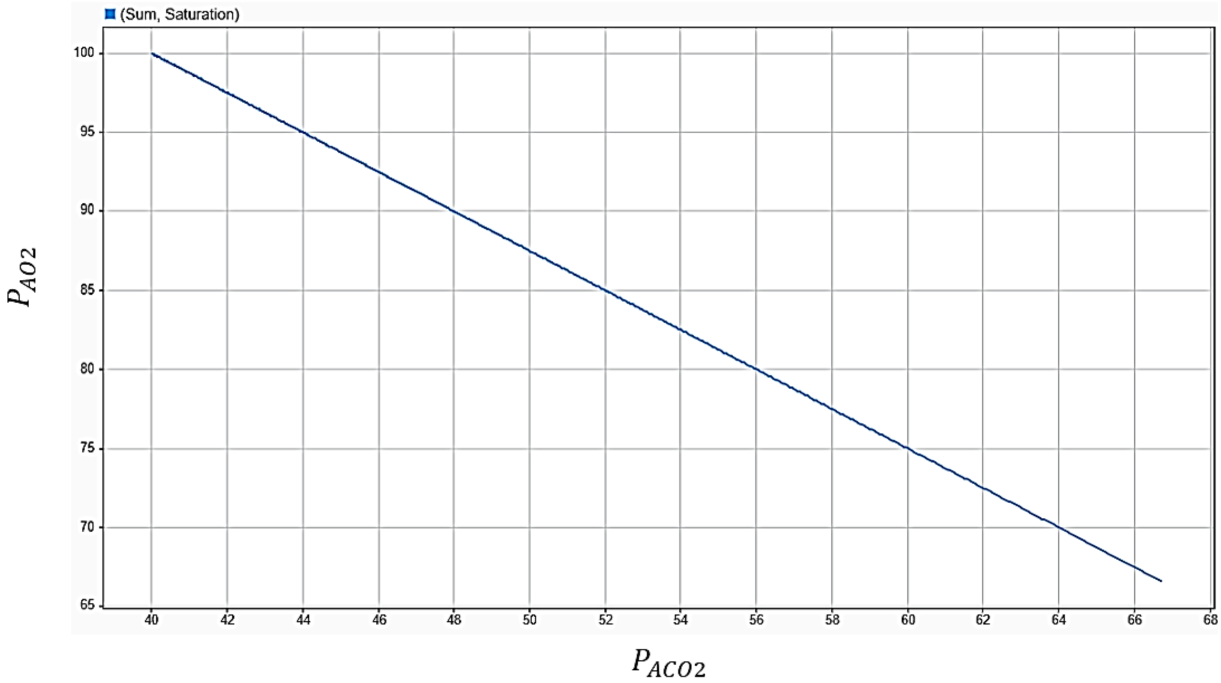

Figure 8. Graph of P_{AO2} versus P_{ACO2}.

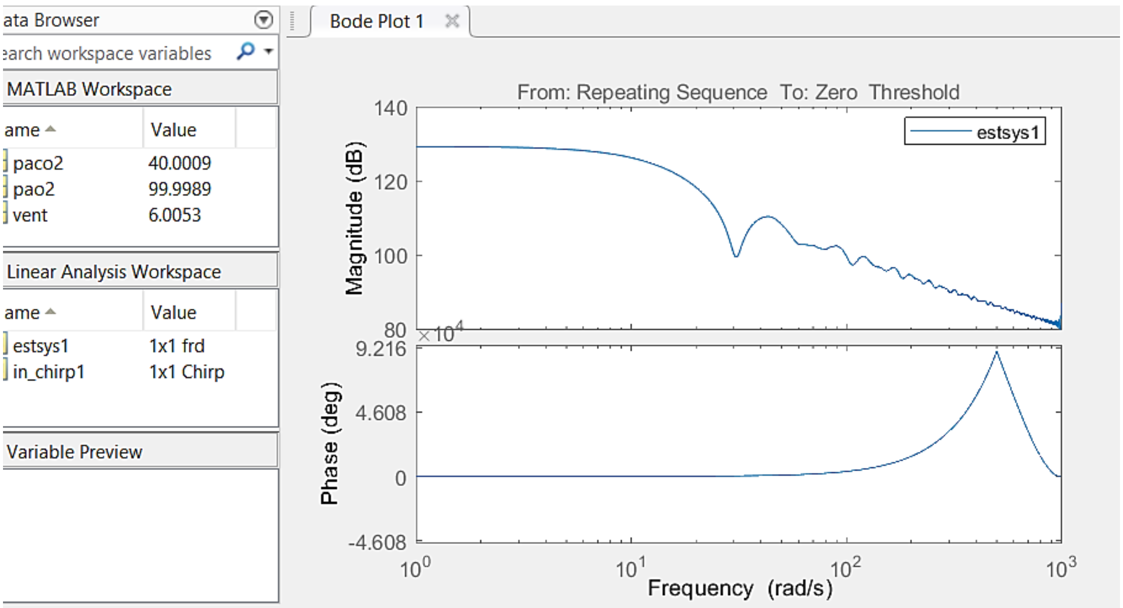

Figure 9. Bode plots to check the stability of the system.

3. Conclusions

The modeling of the human respiratory system through fractional order circuits represents an advancement in biomedical engineering, bridging the gap between classical integer-order models and the complex, non-local behavior observed in actual pulmonary mechanics. By employing fractional calculus, researchers have captured the viscoelastic properties of lung tissue, the memory effects in airway resistance, and the frequency-dependent behavior that integer-order models fail to adequately represent. Furthermore, the compact mathematical representation offered by fractional order models facilitates more efficient computational implementation while maintaining physiological relevance. As diagnostic technologies continue to advance, these models will likely play an increasingly crucial role in personalized medicine, offering clinicians powerful tools for patient-specific respiratory assessment, disease progression monitoring, and therapeutic intervention optimization. The integration of fractional order circuit models into clinical practice stands to increase our understanding and treatment of respiratory disorders by providing a more faithful representation of the underlying biophysical processes.

Declaration of Competing Interest

No conflict of interest was declared by the authors.

Authorship Contribution Statement

Nurgül Gökgöz: Writing, Reviewing, Data Preparation, Writing, Reviewing and Editing, Methodology, Supervision

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