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# Students' algebraic approach and the geometric visualization of irrational numbers

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## Abstract

Students typically learn about irrational numbers in math class, particularly in topics such as algebra and geometry. They may encounter irrational numbers when solving equations involving square roots or other non-perfect square roots. In algebraic expressions, students may need to simplify square roots or manipulate irrational numbers to solve for variables. In geometry, irrational numbers may also be used in connection with measurements and calculations involving square roots of areas or lengths of geometric shapes. For example, students would need to work with irrational numbers to find the diagonal of a square given the side length.

The research project partially presented in this paper aims to investigate learning geometric and algebraic visualization have been studied. As discussion students in the research showed what they learn about irrational numbers and weakness in learning and inability to perform was evident with a focus on improving classroom discourse to adapt to the conceptual development of most students in 9th and 10th grade. An empirical investigation of the enacted learning environment revealed its potential to enhance classroom discourse and students' conceptual and geometric development by introducing an explicit distinction between empirical and theoretical aspects of measurement concepts. The findings include the identification of conditions and limitations that need to be addressed in the design and instructional practices of integrated mathematics interventions in general and for the next steps of the current project.

**Keywords**: Irrational numbers, Algebraic approach, Geometric visualization, Mathematics Education.

## 1. Introduction

People deal with numbers when they perform operations such as measuring and counting to solve the problems that arise in their daily lives. Real numbers are among the topics that connect mathematics to the real world. The set of real numbers, the largest known set of numbers, is a combination of the sets of rational and irrational numbers. Furthermore, the number π, which is widely used in mathematics, Euler's number e (2.7182818...), and the golden ratio, which we encounter both in our daily lives and in nature (1.6180339887...), are all special irrational numbers (Ercire... (2016)).

One of the main concerns of teaching irrational numbers is to understand the real meaning of these numbers, their use and placement on the number axis. In many cases, students in mathematics classes are not able to use irrational numbers to solve equations or inequalities or similar cases (Zazkis& Sirotic (2004)). This problem caused a comprehensive research in the field of their knowledge, considering the scope of the problem, only the geometric and sequential understanding of irrational numbers was focused in this research.

## 2. Research Questions

The following research question was formulated to guide the study:

Are they able to represent irrational numbers on the real axis?

Do they have the ability to use their knowledge in practice?

## 3. Review of Related Literature

Studies show that the Activity System can be used to analyze classroom interactions. The Activity System has 6 components: Tools, Subject, Rules, Community, and Division of Labor and Object. The result of the interaction of these 6 components is the output of the activity system. These 6 components, along with the output of the system, can be seen in Figure 1.

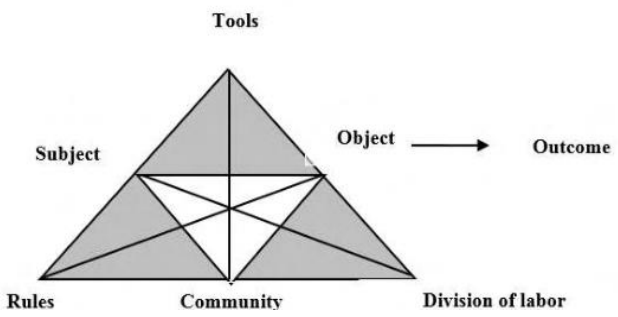


**Figure 1**.The structure of human activity (Jurdak, 2006)

When plotting irrational numbers on a number line, students can approximate the values of irrational numbers and plot them accordingly. For example, the square root of 2 is approximately 1.41, so it would be placed between 1 and 2 on the number line. Irrational numbers are often represented as points that are not evenly spaced on the number line, reflecting their non-integer and non-repeating nature.

Overall, understanding and working with irrational numbers is important for students to develop their mathematical reasoning and problem solving skills in a variety of mathematical contexts. Starting with how students learn about irrational numbers and students' understanding of irrational numbers, exploring the geometric visualization of irrational numbers on the real number axis can be an effective way to help students better understand the concept (Breda, ... (2021)).

## 4. Methodology

The main purpose of this study is to assess students' conceptual understanding of irrational numbers. The participants of the study were 60 female students. Activity Theory was used as a framework to show the learning of conceptual understanding. Modifying these tensions, has led to their students make the concept of the irrational number of the square root.

The research was carried out by two groups that each group was 30 students of 9th grade and 10th grade. Each student was given a questionnaire to show the numbers $\mp(\sqrt{2})/2$, $\mp(\sqrt{3})/3$ on the axis (Figure2). Some questions were raised about the principles and theorems used, and how to find the points and place them in their order.

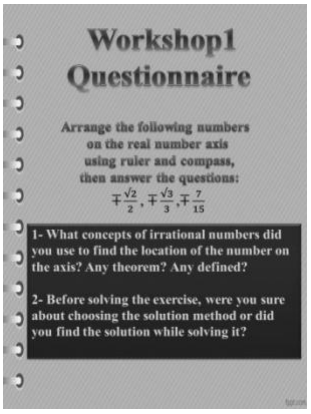


**Figure 2**.Questionnaire

**5. RESULTS AND DISCUSSION**

Incorporating geometric visualization of irrational numbers into teaching and research can improve students' understanding of this complex mathematical concept and make it more accessible and engaging for them.

As shown in Table 1, this could involve comparing traditional classroom instruction with interactive or computer-based learning tools, exploring the impact of using real-world examples in the classroom, or investigating the role of visual aids in conceptualizing irrational numbers.

Observe high school math classes where irrational numbers are taught. Take notes on the teaching methods used, the level of student engagement, and the challenges students face in understanding irrational numbers. Review high school math textbooks and curriculum standards to see how irrational numbers are introduced and taught. Look for gaps or areas that may need improvement.

By incorporating the geometric visualization of irrational numbers into your teaching and research, you can improve students' understanding of this complex mathematical concept and make it more accessible and engaging for them.

This research could help educators develop more effective strategies for teaching and supporting students as they learn about irrational numbers.

By observing classrooms that have used classical teaching methods to teach numerology. Students' understanding of irrational numbers as a number on the number axis or determining the placement of numbers and considering larger and smaller numbers was investigated. For example, students were asked to place 6 positive and negative numbers, two of which were rational and the rest irrational, on the number axis. And they were asked what knowledge they used and what tools they used.

It is very difficult, if not impossible, to have a deep understanding of numbers without doing grammar. Use technology tools such as graphing calculators, Desmos, or GeoGebra to create visual representations of irrational numbers on the real number axis. These tools can help students see more clearly the relationship between irrational numbers and other real numbers.

By creating a survey and distributing it to a sample of high school students to collect data on their understanding of irrational numbers.

| | 9th | 10th | percent |
|---|---|---|---|
| **Correct Usages of principles and theorems in right way** | **15** | **21** | **60%** |
| **Algebraic Understanding** | **18** | **23** | **67%** |
| **Geometric Understating** | **20** | **20** | **66%** |
| **Creation Connection between Algebraic and Geometric concepts** | **11** | **14** | **40%** |
| **Correct Answers** | **10** | **14** | **42%** |

**Table1. Extracting the results**

## 6. Analysis of results

In studying the results, the answers were analyzed based on the components of the activity system (figure1). Some students were able to implement the concepts well as it is shown in figure3, they finned the correct answer.In figure3 student could use the tool, learned the subject and did the rules.

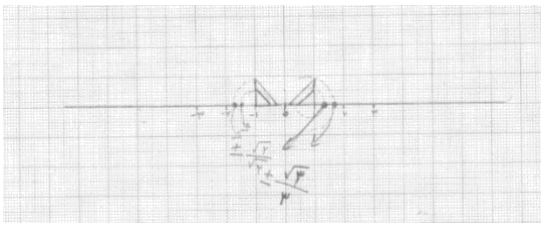

**Figure3**.Finding numbers1

As it shown in figure4, some students are not able to find the point because they understand the general idea, tools and rules, but are not able to apply it, they did not find the object. Their learning is not deep and they have many weaknesses in understanding.

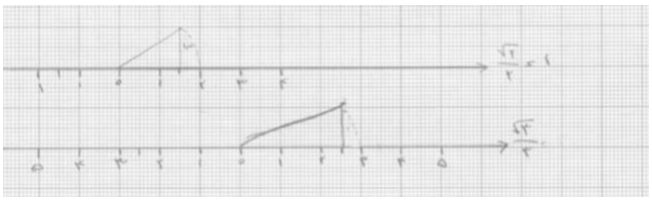

**Figure 4**.Finding numbers2

Some students were able to find the point because they understand the general idea, tools and rules, but are unable to apply it, as shown in Figure5(a,b&c), they did not find the object.

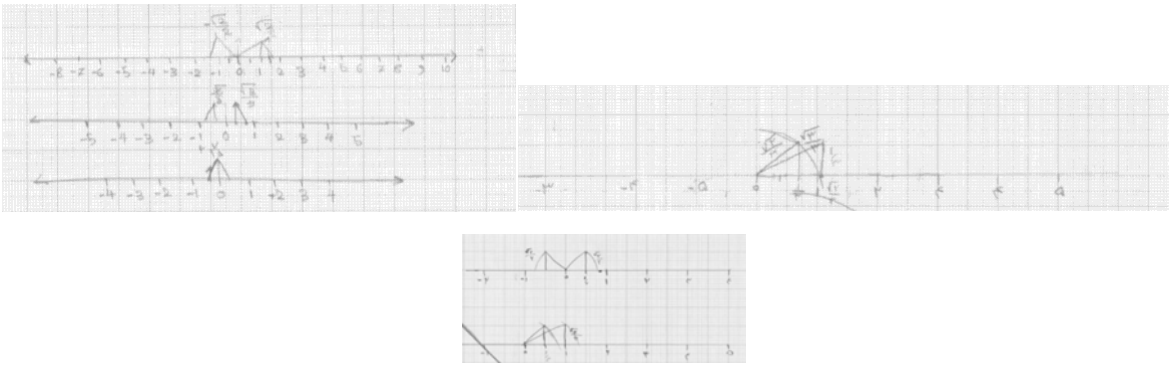

**Figure5**.Finding numbers (a,b&c)

## 7. Discussion and Conclusion

The present study investigated students' skills and knowledge of the algebraic and geometric definition of irrational numbers.

The first research question investigated the students' ability to represent irrational numbers on the axis. It was found that half of the student responses were correct, while the other half were either incorrect or incomplete. When the correct student responses were examined, it was found that the most common rule used in the response was "The Pythagorean Theorem" and some students' responses included incorrect and incomplete expressions. The second research question, it seems that most of the students had the ability to use their knowledge, but they didn't have enough practice to master the subject.

A possible research topic could be to examine the effectiveness of different teaching methods in helping high school students understand and work with irrational numbers. This could include comparing traditional classroom instruction with interactive or computer-based learning tools, exploring the impact of using real-world examples in the classroom, or investigating the role of visual aids in conceptualizing irrational numbers. Another potential research topic could be to study the factors that influence students' attitudes and beliefs about irrational numbers, such as math anxiety or prior mathematical experiences. Another potential research topic could be to study the factors that influence students' attitudes and beliefs about irrational numbers, such as math anxiety or prior math experiences. This research could help educators develop more effective strategies for teaching and supporting students as they learn about irrational numbers.

Here are some ideas on how to research and teach the geometric visualization of irrational numbers:

I. Visual Aids: Use visual aids such as number lines, graphs, and geometric shapes to help students visualize the position of irrational numbers on the real number axis. Encourage students to plot irrational numbers such as $\sqrt{2}$, $\sqrt{3}$, or $\pi$ on a number line and discuss their placement in relation to other numbers.

II. Interactive Activities: Engage students in interactive activities that allow them to explore the geometric representation of irrational numbers. For example, you might create a hands-on activity where students cut out squares representing the square roots of different numbers and arrange them on a number line.

III. Technology Tools: Use technology tools such as graphing calculators, Desmos, or GeoGebra to create visual representations of irrational numbers on the real number axis. These tools can help students see more clearly the relationship between irrational numbers and other real numbers.

IV. Collaborative Learning: Encourage collaborative learning among students by having them work in groups to discuss and visualize the geometric representation of irrational numbers. They can compare their visualizations, share their findings, and learn from each other's perspectives.

V. Assess student understanding: Develop assessment tools such as quizzes, problem sets, or projects that require students to demonstrate their understanding of the geometric visualization of irrational numbers. This will help you evaluate their learning and identify areas for improvement.

VI. Research Impact: Conduct research on the effectiveness of using geometric visualization to teach irrational numbers to high school students. Collect data on students' attitudes, understanding, and performance before and after implementing visualization strategies to see the impact on their learning outcomes.

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