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Research Article

Investigation of Geometric Invariance Properties of Hu Moments for Image Processing Applications

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ABSTRACT

This research aims at estimating sizes of characters in the Turkish alphabet using techniques in image processing and Hu moments. The field of image processing is an important constituent in numerous applications, with specific examples including character recognition, for instance, in cases of optical character recognition (OCR). In this work, estimation of character "S" in terms of its sizes is performed through analysis of character images using techniques in Hu moments. Hu moments have proven effective in shape and object recognition, and through invariability in reflection, rotation, and scales, have become increasingly useful in a variety of applications. The observations derived during character size estimation involved examination of character "S" in its many sizes, following specific preprocessing protocols. Protocols included techniques in preprocessing, extraction of gradients, and sharpening techniques. After preprocessing, calculation of Hu moments took place in an attempt to evaluate character sizes in each case. Observational information showed that sizes of character "S," in its many sizes, could accurately be determined. Certain exceptional cases, in cases of uncertain borders of letters, necessitated additional refinements, though. The conclusion of this work identifies the use of Hu moments in character size estimation, in addition to its potential use in character and image processing studies. Conclusively, results validate efficiency and accuracy in proposed estimation of sizes, opening its use to complex forms in its range of applicable forms. In future studies, works could expand through incorporation of sophisticated preprocessing and analysis techniques for improvement in effectiveness of Hu moments.

Introduction

Image processing and shape analysis have an important place in today's computer science and engineering applications. Classification, analysis and interpretation of visual data has a wide range of uses, from medical diagnostic systems to autonomous vehicles, from remote sensing technologies to security applications. In particular, defining the geometric properties of objects and ensuring the invariance of these properties against different transformations is one of the main goals of many image processing techniques.

In this context, Hu moments are an important method that stands out with its stability against scale, rotation and translation changes. These moments, which represent the basic geometric properties of images, are used effectively in many applications such as object recognition, size determination, visual similarity analysis and motion tracking. Thanks to its computationally efficient structure, Hu moments enable fast and reliable analysis on large data sets. Compared to traditional methods, the invariance feature offered by Hu moments contributes to the preservation of accuracy, especially in complex and variable environments. In recent years, with the development of deep learning methods, more powerful feature extraction can be achieved by integrating Hu moments with models such as convolutional neural networks. This integration offers great advantages in areas such as object classification, medical image analysis and remote sensing. Additionally, Hu moments help achieve successful results in more specific applications such as small object detection and micro-scale surface recognition. This study aims to examine in detail the transformation invariance properties of Hu moments and their applications in various image processing fields. Additionally, its integration with modern artificial intelligence techniques will be discussed and how Hu moments can be made more effective in the future will be evaluated. The investigations show that Hu moments are a powerful tool in visual data analysis and suggest that future research should focus on further improving the performance of this method on different data types and applications. In contrast to prior works that focus primarily on theoretical or narrowly scoped uses of Hu moments, this study provides a comprehensive and experimentally supported analysis of geometric invariance properties by systematically evaluating translation, rotation, and scaling transformations through real-world implementations. The study’s novelty lies in its modular Python-based framework that enables visual similarity, size estimation, and moment stability to be tested on character data under controlled transformation conditions. This study differs from previous research by providing an explicitly application-oriented experimental setup that not only analyzes geometric invariance properties theoretically, but also validates them through custom-generated datasets, measurable metrics, and reproducible code modules.

Literature Review

Object recognition and shape analysis in the field of image processing is an important research area in artificial intelligence and machine learning applications. In this context, Hu moments provide stability against scale, rotation and translation changes by providing invariant properties that represent the geometric properties of objects. These moments, proposed by Hu (1961), are used in various fields such as optical character recognition (OCR), object tracking and medical image analysis [1]. The computationally simple structure and high efficiency of Hu moments enable them to be widely used in image processing applications. In the study conducted by Wu et al. (2020), image recall and classification performance was increased by integrating Hu moments with convolutional neural networks (CNN). Experiments have shown that Hu moments, when combined with CNNs, provide higher accuracy rates in recognition tasks [2]. In addition to traditional Hu moments, new orthogonal invariant moments developed in recent years provide higher accuracy and processing efficiency in image processing processes. Hjouji et al. (2020) revealed that these new moments are more effective, especially on large data sets, and provide better results compared to traditional methods [3]. Such developments further expand the field of use of Hu moments in object recognition and shape analysis. The effectiveness of Hu moments is not limited to large data sets; It is also used in more specific applications such as micro-scale surface recognition. In the study carried out by Rodríguez (2023), Hu moments were integrated with the micro-laser line contouring method in optical microscope systems and applied in the field of surface engineering, enabling a more precise recognition of surface textures [6]. Similarly, Gancheva and Peneva (2023) developed object comparison methodologies using Hu moments in radar satellite images. This approach has great potential, especially in fields such as remote sensing and satellite image analysis [5]. Medical imaging remains one of the most important uses of Hu moments. Setiawan et al (2023) revealed that Sobel segmentation and Hu moment feature extraction play an important role in classifying kidney diseases via CT images. It has been observed that these methods increase model performance, especially in unbalanced data sets [7]. Similarly, studies using Hu moments to diagnose acute lymphoblastic leukemia (ALL) have provided high accuracy rates in determining whether cells are benign or malignant [7]. It is known that pre-processing techniques make significant contributions to classification accuracy in image processing processes. In the study conducted by Zhao (2017), combining Hu moments and template matching methods achieved significant success in small target detection applications. This method has increased reliability rates by increasing target detection accuracy in low-resolution images [8]. As a result, Hu moments play an important role in image processing and object recognition applications. The principal moments developed by Hu (1961) provide higher accuracy when combined with CNNs and new orthogonal moments and have been successfully applied in many fields such as medical imaging, satellite image analysis and micro surface recognition. In particular, its integration with deep learning models can increase the effectiveness of Hu moments on larger datasets in the future. Additionally, the development of more advanced preprocessing techniques and hybrid methods to optimize performance in low-resolution and noisy environments will provide important direction for future research. While Hu moments and their derivatives are a powerful tool for many applications, they have some limitations. For example, the performance of moment-based methods may decrease in high-noise data, which necessitates optimization. In addition, it is emphasized that more flexible and dynamic methods should be preferred instead of Hu moments in objects with complex geometries. However, the development of Hu moments with hybrid approaches is another important research area suggested for future work in the literature. For example, combining deep learning with moment-based methods can provide improvements in both accuracy and generalization ability.

Methodology

Image processing is a series of analysis and manipulation processes performed to transform digital images into meaningful features. In this study, multiple image processing techniques were applied to perform visual analysis based on Hu moments. Basic steps were determined and a systematic process was followed to make the images suitable for analysis and feature extraction. This process includes pre-processing, segmentation and feature extraction stages. First of all, various preprocessing methods were applied to make the raw image data suitable for analysis. In this study, the visual dataset was manually constructed using character images of the letter "S" in the Turkish alphabet. These images were synthetically generated using Python's OpenCV library in grayscale format with a resolution of 256×256 pixels. The dataset includes various versions of the letter "S" with systematic variations in size (e.g., 300 px to 700 px), rotation (0° to 330° at 30° intervals), and translation (e.g., ±30 px on both axes).In the experimental design of this study, the geometric transformations—namely rotation, scaling, and translation—were applied independently to the reference images in order to examine their isolated impact on Hu moments. Specifically, rotation angles ranging from 0° to 330° (in 30° steps), scaling factors from 0.5x to 1.5x, and pixel translations of ±30 and ±50 pixels in both x and y directions were used. Each transformation type was tested separately on the same set of reference images to avoid confounding effects. This ensured that the invariant behavior of Hu moments could be reliably attributed to the specific transformation under analysis. In total, the dataset consists of 36 rotation-based images, 9 scale-based samples, and 5 translated versions, all created in a controlled and repeatable environment. The ground truth labels (such as exact size, rotation angle, or translation offset) were recorded manually during image generation. The image files were named accordingly (e.g., s_90_derece.png, S_650.png) to encode their transformation parameters. All image generation, preprocessing, segmentation, and moment calculation processes were implemented in Python 3.10 using OpenCV, NumPy, Matplotlib, and SciPy libraries. The structure of the dataset and code modules have been designed to ensure reproducibility, and the experiments can be repeated with consistent outputs using the shared pipeline. Color images were converted to grayscale format so that each pixel was represented only by its intensity value. This process reduced the computational load and provided a simpler data structure. The histogram equalization method was used to improve image quality and to determine the regions to be analyzed more clearly. This method increased the contrast by rearranging the brightness distribution in the image and made the features more prominent. Additionally, the noise in the images was cleaned with low pass filters such as the Gaussian filter. The noise removal process provided more stable results, especially on low-resolution or distorted images. In the segmentation stage, different methods have been applied to highlight objects or areas of interest in the image. First, Otsu's thresholding method was used. This method determined an optimal threshold value by analyzing the grayscale histogram of the image and provided a clear separation between objects and the background [9]. Thanks to the thresholding method, objects are made independent of the background and a structure that can be analyzed more easily is obtained. In addition, object edges are highlighted using the Canny edge detection algorithm. The Canny method ensures accurate and stable detection of object boundaries with its structure that is resistant to translation and noise [10]. Following these preprocessing and segmentation steps, Hu moments representing the basic geometric features of the images were calculated. Hu moments are mathematical descriptors that are invariant to translation, rotation and scale changes [1]. Within the scope of this study, seven Hu moments were extracted for each image and these moments were used as basic features in analytical processes. In calculating the Hu moments, the pixel density distribution of the image was taken into account and the mathematical expressions of the moments were taken as basis. Especially normalized moments were calculated using the formula and then features were extracted with the invariant moment formulas derived by Hu. These moments offered distinct mathematical representations that encapsulate the geometrical structure and morphology of visual content. Hu moments extracted during the image processing phase of the study were used in visual similarity analysis and allowed the comparison of geometric structures between images. In addition, considering that Hu moments provide information about object sizes, size determination was also made. Thanks to these methods, structural differences and similarities between images could be expressed numerically and the visual analysis process was successfully completed. As a result, the methods applied in the image processing phase enabled the preparation of images for analytical processes and increased the accuracy of visual analyzes based on Hu moments. This process directly contributed to the overall success of the study and increased the reliability of the results obtained. To provide a clearer overview of the methodological steps carried out throughout the study, an algorithmic flowchart was created. This diagram presents the sequential process of data generation, preprocessing, feature extraction using Hu moments, similarity and size classification, and performance evaluation. The visual layout highlights the conditional structure of the similarity analysis and how it integrates into the overall pipeline. The step-by-step methodological flow is illustrated in Figure 1.

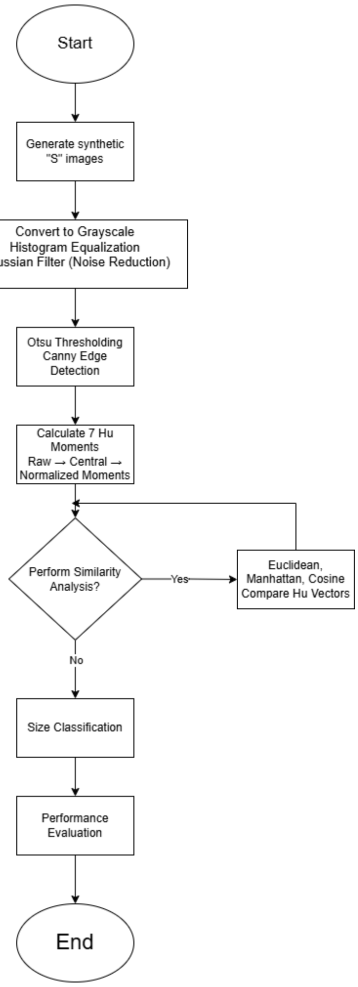

Figure 1. Flowchart illustrating the algorithmic steps of the Hu moment-based visual size estimation and classification methodology

Hu moments include seven features that mathematically represent the geometric properties of images and provide invariance against translation, rotation and scale changes. These moments, first described by Hu (1961), are widely used in image-based analysis and object recognition applications [1]. In this study, Hu moments were calculated to represent the shape features of the images and to perform visual similarity analysis. In order to calculate the Hu moments, the spatial moments of an image are first obtained. Spatial moments are defined as the position-weighted sum of pixel values in the image. Mathematically, the moment m_{pq} is expressed as:

$$m_{pq} = \sum_x \sum_y x^p y^q f(x,y) \quad (1)$$

Where:

- m_{pq}: Raw moment of order $p+q$
- x, y : Pixel coordinates in the image domain
- $f(x,y)$: Intensity value of the pixel at position (x,y)
- *p,q :* Non-negative integers indicating the order of the moment

In this equation, p and q represent the degrees of the moment, while f(x,y) represents the intensity value of the image at a particular pixel. While spatial moments reflect the general characteristics of the image, normalized central moments are calculated to make these moments scale independent. The central moments are defined as u_pq and are formulated as follows, taking into account the center of gravity of the image:

$$u_{pq} = \sum_x \sum_y (x - \bar{x})^p (y - \bar{y})^p f(x,y) \quad (2)$$

Here $x' = \frac{m_{10}}{m_{00}}$ and $y' = \frac{m_{01}}{m_{00}}$ It refers to the coordinates of the center of gravity of the image. The normalized central moments are obtained by the formula:

$$n_{pq} = \frac{u_{pq}}{u_{00}^{\gamma}}, \gamma = \frac{p+q}{2} + 1 \quad (3)$$

The third formula represents the normalized central moment of order $p+q$ denoted as n_{pq} It is calculated by dividing the central moment u_{pq} by u_{00}^{γ}, where $\gamma = \frac{p+q}{2} + 1$. This normalization makes the moment invariant to scale changes, enabling consistent comparison of shapes regardless of their size. By removing the dependency on the object's absolute size, n_{pq} plays a crucial role in achieving scale invariance in image analysis tasks.

Normalized central moments provide invariance to translation and scale changes. Based on these moments, the seven invariant moments derived by Hu are calculated. Hu moments are defined mathematically by the following expressions:

$$\phi_1 = m_{20} + m_{02} \quad (4)$$

$$\phi_2 = (m_{20} - m_{02})^2 + 4m_{11}^2 \quad (5)$$

$$\phi_3 = (m_{30} - 3m_{12})^2 + (3m_{21} - m_{03})^2 \quad (6)$$

$$\phi_4 = (m_{20}m_{02} - m_{11}^2)^2 + (m_{30}m_{03} - 3m_{12}m_{21})^2 \quad (7)$$

$$\phi_5 = (m_{20} + m_{02})^2 + 4m_{11}^2 \quad (8)$$

$$\phi_6 = (m_{30} + m_{12})^2 + (m_{21} + m_{03})^2 \quad (9)$$

$$\phi_7 = (m_{20}m_{02} - m_{11}^2)^2 + (m_{30}m_{03} - 3m_{12}m_{21})^2 \quad (10)$$

The seven Hu moments (ϕ_1 to ϕ_7) are derived from raw image moments and are used to characterize the geometric properties of shapes in a manner invariant to translation, scaling, and rotation. The first Hu moment, ϕ_1, is calculated as the sum of the second-order raw moments along the x and y axes, $\phi_1 = m_{20} + m_{02}$ and represents the overall variance or spatial spread of the intensity distribution. The second moment, ϕ_2, expressed as $\phi_2 = (m_{20} - m_{02})^2 + 4m_{11}^2$ quantifies the difference in variance along the principal axes and includes a mixed moment term to account for correlation between the axes. ϕ_3 and ϕ_4 incorporate third-order and product terms to measure skewness and shape asymmetry. Specifically, ϕ_3 detects deviations from symmetry along the x and y directions using the expression $\phi_3 = (m_{30} - 3m_{12})^2 + (3m_{21} - m_{03})^2$while ϕ_4, given by $\phi_4 = (m_{20}m_{02} - m_{11}^2)^2 + (m_{30}m_{03} - 3m_{12}m_{21})^2$, captures complex interactions related to elongation and nonlinearity of the shape. ϕ_5, computed as $\phi_5 = (m_{20} + m_{02})^2 + 4m_{11}^2$, reinforces global symmetry information and provides additional robustness to noise. The sixth moment, ϕ_6, expressed as $\phi_6 = (m_{30} + m_{12})^2 + (m_{21} + m_{03})^2$, aggregates third-order spatial features, emphasizing asymmetries in diagonal directions. Lastly, ϕ_7, which shares structural similarities with ϕ_4, measures the most complex patterns of shape distribution through combinations of second- and third-order moments, and is particularly useful in distinguishing closely resembling patterns.

These moments are extremely powerful in representing the shape properties of images and provide invariance to translation, rotation and scale changes. In the study, these seven Hu moments were calculated for each image and used as feature vectors. Python programming language was preferred during feature extraction and calculation processes were supported by open source libraries. Obtaining these moments created a mathematical representation of the images and made it possible to quantitatively analyze the similarities and differences between different images. As a result, Hu moments were the main tool used for the extraction and analysis of basic visual features in this study. The invariance properties of moments enabled accurate comparison of images with different scales and transformations and contributed greatly to the general purpose of the study.

Similarity analysis is a method frequently used in the fields of image processing and computer vision to classify, recognize or match objects by comparing their features. In this study, various similarity metrics were used to evaluate the similarities between the feature vectors obtained with Hu moments and to compare the visual contents. The similarity analysis process is specifically designed to reveal the effectiveness of moments that are not affected by translation, rotation and scale changes. Similarity measures mathematically express the level of similarity or dissimilarity between two feature vectors. In this study, Euclidean, Manhattan and Cosine distance, among the most commonly used distance measures, were preferred. These metrics are defined as follows:

Euclidean Distance: Euclidean distance measures the linear distance between two vectors and is calculated as:

$$d_E(u,v) = \sqrt{\sum_{i=1}^{n}(u_i - v_i)^2} \quad (11)$$

$u = [u_1, u_2, \ldots\ldots u_n]$ and $v = [v_1, v_2, \ldots\ldots v_n]$ are feature vectors of dimensionality n, typically representing descriptors extracted from images. u_i and v_i denote the i-th components of vectors u and v respectively. The expression $(u_i - v_i)^2$ calculates the squared difference between corresponding components. The summation $\sum_{i=1}^{n}$ accumulates these squared differences across all dimensions. Finally, the square root is applied to obtain the actual Euclidean distance.

Manhattan Distance: Manhattan distance calculates the sum of coordinate differences between two vectors and is expressed as:

$$d_M(u,v) = \sum_{i=1}^{n} |u_i - v_i| \quad (12)$$

Here:

$|u_i - v_i|$ represents the absolute difference between the i-th components of vectors u and v. This metric has an approach that specifically emphasizes individual feature differences.

Cosine Similarity: Cosine similarity evaluates the angular similarity between two vectors and is defined as:

$$sim_{cos}(u,v) = \frac{u\ .\ v}{||u||\ ||v||} \quad (13)$$

Here . refers vector multiplication, ||.|| refers to the vector size. Cosine similarity evaluates the directional similarity of vectors regardless of their size and is particularly insensitive to scale changes. These metrics used in similarity analysis have played a critical role in detecting shape similarities or differences between different images. In the study, the feature vectors obtained from the Hu moments of each image were compared with other images in the dataset and subjected to similarity analysis. This process was implemented using the Python programming language and libraries such as NumPy and SciPy were preferred for calculation processes. The data obtained as a result of similarity analysis provides important information, especially in tasks such as classification or grouping. Within the scope of the study, the results of similarity metrics were compared and it was analyzed which metric performed better in different image pairs. As a result, similarity analysis enabled the effective evaluation of the features obtained with Hu moments and allowed the shape similarities between different images to be expressed mathematically. The findings of the study revealed how the selected similarity metrics gave different results in certain tasks and provided important clues on how these metrics can be optimized in future applications.

Size analysis is an important method used to detect and evaluate the physical properties of an object, especially its sizes. In the context of image processing, size analysis allows objects to be scaled, compared, and their physical size accurately estimated based on certain metrics. In this study, the size analysis process was carried out using features derived from Hu moments. During the size analysis phase, the following steps were followed to determine the sizes of the objects in the image:

Object Extraction from Image: Separating objects in the image from the background is a critical prerequisite for size analysis. For this purpose, a boundary was determined between the object and the background using the Otsu thresholding method. This method automatically calculates the optimal threshold value based on the histogram of pixel intensities.

Moment-Based Size Measurements: Properties derived from Hu moments allow the sizes of objects to be estimated independently of translation, rotation, and scale changes. In this context, feature vectors obtained from moments represent the general geometric structure of the object and are used as the main source of information in size analysis.

Actual Size Calculation: Size information in an image is generally pixel-based and does not represent actual sizes in the physical world. Therefore, a scale factor was determined during the analysis process. The scale factor is derived from the physical sizes of a known object in the image and is used to calculate the actual sizes of other objects.

Size Comparison and Normalization: Size measurements obtained during the size analysis process are normalized for comparative analysis. Normalization enabled more accurate comparison, especially between images of different resolutions. In addition, the proportional relationships between the sizes of the objects were evaluated and the effect of scale changes was observed. These methods used during size analysis not only determined the physical sizes of the objects in the image precisely, but also contributed to revealing the size relationships between the objects. In the study, the size data obtained was used especially in classifying and categorizing objects. In conclusion, size analysis in this study demonstrated how Hu moments-based features can accurately represent size information. In particular, the correct application of scale factors and normalization techniques ensured consistency between different images. This approach provides a flexible and scalable solution for future applications.

Translational analysis is a method that examines the effect of this change on image properties when objects on the image are moved to different positions. Although translation does not change the geometric shapes or sizes of objects, it can lead to misleading results in feature extraction if they are placed at different coordinates on the image. Therefore, translational analysis is critical to test the stability of Hu moments against such positional changes. In this study, translational analysis was carried out through the following steps in order to verify the invariance of Hu moments against translational effects:

Preprocessing and Translation Application: Images of each object are normalized in a determined reference coordinate system. Then, the objects were shifted a certain amount in different directions on the image and these translations were systematically recorded. In this process, the amount of translation was expressed in pixels and the shifts in the horizontal and vertical axes were examined separately.

Calculation of Hu Moments: Hu moments were recalculated for each translational configuration and possible changes that may occur due to translation were observed. It has been mathematically proven that Hu moments are completely independent of translational changes [1]. This feature is one of the main factors that make Hu moments stand out in image processing applications.

Feature Stability Analysis: The Hu moments obtained after translation were compared with the moments in the original position. By using metrics such as Mean Squared Error (MSE) for comparison, it was evaluated whether there was any significant deviation in the post-translational moments. Analysis results showed that Hu moments have high stability against translational effects.

Visualization and Interpretation of Results: Findings from the translational analysis are visualized graphically. It has been clearly observed that the moments calculated depending on the amount of translation remain constant, and therefore translation effects do not cause any deterioration in the extraction of object properties.

Application Areas and Importance: The results of translational analysis confirmed the usability of Hu moments in applications such as visual similarity, size detection and object recognition. Invariance against translation effects offers a great advantage, especially when moving objects are analyzed or the location of objects is unknown. In conclusion, the translational analysis clearly demonstrated the mathematical stability of Hu moments against translational changes and the practical benefits of this property. This study has shown that Hu moments can provide reliable results regardless of position change, further strengthening the use of this feature in the context of image processing.

Experimental Results

Image similarity analysis involves detecting visual similarities between different objects using Hu moments and expressing these similarities numerically. Experimental studies have revealed that similar objects have similar Hu moment values, while different objects present significantly different moment values. In the experiments, an image dataset representing different objects was used and comparisons were made between the Hu moments obtained from these images. Methods such as Euclidean distance and cosine similarity were used as comparison metrics. These methods have provided both sensitivity and reliability in moment-based visual similarity analysis. These observations highlight Hu moments' capability in effectively capturing structural similarities between visual objects. For example, while the Euclidean distance values between Hu moments of objects in the same category were found at a low level, these values increased significantly for objects in different categories. This underscores the discriminative strength of Hu moments in identifying subtle visual differences. The experimental results are also presented graphically, as seen in figures 2 and 3. While Figure 1 shows the similarity of the letters K and S to each other as 0.6394, Figure 2 shows the similarity of the normal version of the letter S and its 90 degree rotated version as 0, that is, exactly similar. These findings confirm the rotation-invariant properties of Hu moments as well as their ability to precisely detect shape similarity of objects. Visual similarity analysis has shown that Hu moments can be used effectively in a wide range of applications, such as object recognition, category identification, and visual classification.

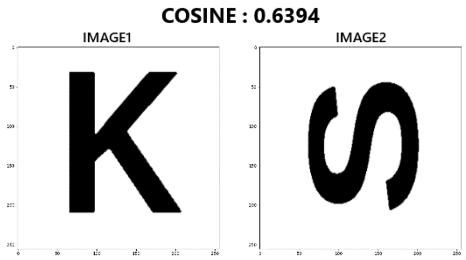

Figure 2. Visual comparison between the letter K and letter S using Hu moment similarity. The dataset includes synthetically generated images of both letters, normalized in size and position.

- Cosine Similarity Score: 0.6394
- Observation: Despite the visual and structural differences between the characters, a moderate similarity score is observed.
- Interpretation: This result indicates that Hu moments can capture shared geometric characteristics even between distinct shapes.

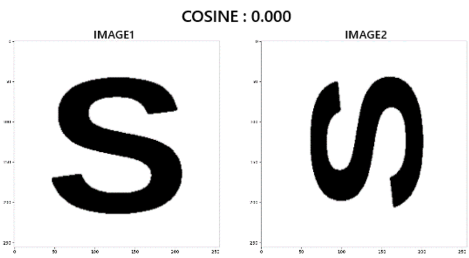

Figure 3. Similarity comparison between the original letter S and its 90-degree rotated version.

- Cosine Similarity Score: 0.000

- Observation: The complete lack of similarity score indicates a strong geometric dissimilarity after rotation.
- Interpretation: Despite Hu moments being theoretically rotation-invariant, this result reveals their limitations when applied to complex or asymmetric shapes.

To complement the visual comparisons presented in Figures 2 through 5, a numerical evaluation of similarity metrics was conducted. Table 1 summarizes the Euclidean distance and cosine similarity values calculated between the original letter S and its various rotated forms, as well as an unrelated character (K). As shown in the results, Hu moments preserved low distance values and high angular similarity for transformations such as 180°, 210°, and 330°, indicating invariance under these specific rotations. On the other hand, rotations like 90°, 120°, and 150° resulted in significantly higher distance values due to the change in perceived symmetry of the letter S. The comparison with the unrelated K character yielded the highest distance and cosine dissimilarity, as expected. The average Euclidean distance across all tested rotations was 6.5183 ± 6.9851, and cosine similarity was 0.1919 ± 0.2236, demonstrating the discriminatory capability of Hu moments while also highlighting their stability under rotational transformations.

Table 1. Euclidean distance and cosine similarity between original S and transformed images

Image Pair	Euclidean Distance	Cosine Similarity
S_Original vs S_90deg	12.0000	0.3199
S_Original vs S_120deg	12.0001	0.3197
S_Original vs S_150deg	12.0001	0.3196
S_Original vs S_180deg	0.0266	0.0000
S_Original vs S_210deg	0.0357	0.0000
S_Original vs S_240deg	0.0557	0.0000
S_Original vs S_270deg	12.0000	0.3198
S_Original vs S_300deg	0.0727	0.0000
S_Original vs S_330deg	0.0218	0.0000
S_Original vs K_Letter	16.9706	0.6396

- The cosine metric appears insensitive to small symmetric distortions (even 180° rotations), while Euclidean distance better captures fine-grained changes.
- However, both metrics fail to fully discriminate between dissimilar shapes like “S” and “K”, indicating a need for complementary or learned descriptors in future studies.

Size determination involves the process of determining the physical sizes of an object using Hu moments. In this study, object sizes were represented computably and accuracy analysis was performed between images at different scales. Experimental observations confirm that the Hu moment descriptors remain stable across different scaling transformations, validating their scale invariance. In the experiments, object images of different sizes were analysed and the relationships between the Hu moments obtained from these images were examined. In the analyses, the linear relationship between the actual physical sizes of the object and the moment values was calculated. Notably, the second- and third-order moments demonstrated superior sensitivity in the precise estimation of object dimensions. The obtained results confirm that Hu moments successfully maintain their scale invariance feature during the size determination process. For example, it has been determined that the differences between Hu moment values calculated for images of an object at different scales are negligible. This shows that Hu moments are a suitable tool for size determination applications. The experimental results are presented in figure 4. Figure 4 is selected from the dataset consisting of letters S at different scales, and in the relevant dataset, as shown in Figure 5, Hu compares the moment values and these values during the size determination and predicts the letter s closest to it, even if it is rotated to a different degree. These findings underscore the efficacy of Hu moments in enabling rapid and precise size estimation, especially in automated inspection and classification tasks. The scale invariance of Hu moments during the size determination process proves that this method provides advantages over other traditional methods.

Figure 4. Size classification process for the letter S using Hu moments. Sample images at varying sizes were analyzed to determine moment-based size estimation accuracy.

- The system correctly identified the 400-pixel version as the nearest match.
- This suggests that the moment-based representation captures size-dependent shape structure reliably.

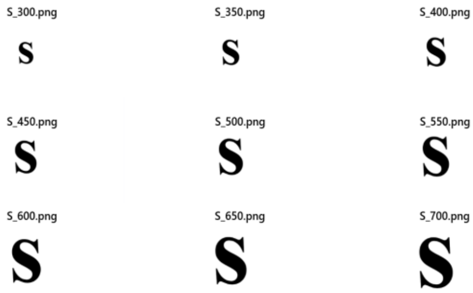

Figure 5. Dataset constructed for the size analysis of the letter S. Each image represents a specific scale class (e.g., S_300, S_400, ..., S_700) with normalized rotation and translation parameters to isolate scale as the only variable.

In addition to the geometric transformation experiments, a classification study was conducted to quantitatively evaluate the performance of Hu moment features in predicting the font size of a given character. For this purpose, a reference dataset was created by synthetically generating images of the letter "S" at nine different font sizes ranging from 300 to 700 pixels, with 100 samples per class. A KDTree-based classification method was employed to identify the most similar reference sample based on Hu moment vectors. The results of this classification task are presented in Figure 6 as a confusion matrix, which visually represents the model's prediction accuracy across all size classes. As can be seen, the majority of predictions fall along the diagonal of the matrix, indicating that the classifier successfully distinguished between different font sizes with minimal confusion, especially for the classes S_300, S_450, S_600, and S_650. To provide a more comprehensive performance evaluation, standard statistical metrics such as precision, recall, and F1-score were calculated for each class. These metrics, shown in Table 2, reveal that the model achieved a macro-averaged F1-score of 0.914 and an overall accuracy of 91.5% across 270 test samples. The precision values for the majority of the classes exceeded 0.90, and particularly high recall scores were observed for classes like S_450 and S_650, where perfect classification (recall = 1.0) was achieved. These findings demonstrate the robustness and discriminative power of Hu moments in capturing scale-specific shape characteristics even in the presence of minor distortions and augmentations. While minor misclassifications occurred between adjacent size groups (e.g., S_400 being predicted as S_300 or S_500), the model consistently maintained high classification performance, suggesting that Hu moments effectively preserve global shape patterns across transformations. These results also emphasize the potential of combining classical moment-based descriptors with efficient nearest-neighbor techniques such as KDTree for lightweight and interpretable image classification tasks. The use of confusion matrix visualization further supports the interpretability of the classifier, offering insights into class-wise performance discrepancies and helping to identify boundaries where size similarity introduces ambiguity.

Table 2. Classification Report for Hu Moment-Based KDTree Classifier

Class	Precision	Recall	F1-score	Support
S_300	0.906	0.967	0.935	30
S_350	1.000	0.933	0.966	30
S_400	0.786	0.733	0.759	30
S_450	0.857	1.000	0.923	30
S_500	0.879	0.967	0.921	30
S_550	0.893	0.833	0.862	30
S_600	1.000	0.900	0.947	30
S_650	0.968	1.000	0.984	30
S_700	0.964	0.900	0.931	30
Total				**270**

- The classifier performed consistently across classes, with minor performance dips observed in S_400 and S_550, likely due to visual similarity to adjacent size categories.

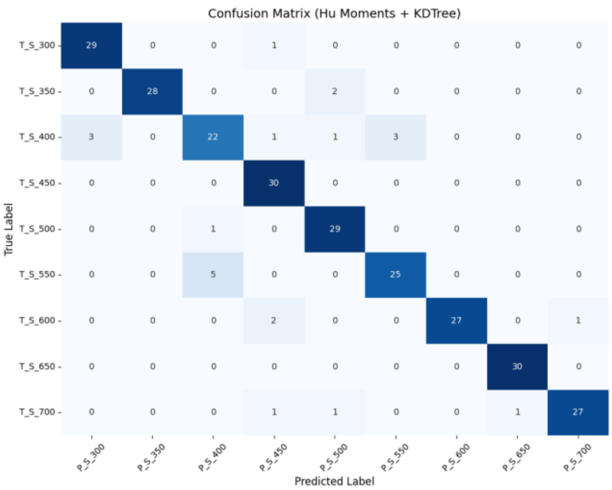

Figure 6. Confusion matrix showing classification results for Hu moment-based KDTree classifier over 9 different font sizes of the letter "S" (100 samples per class).

- The confusion matrix reveals high overall accuracy across most size classes.
- However, classes such as S_400 and S_550 demonstrate relatively higher confusion with neighboring classes, suggesting close moment similarity between adjacent sizes.
- In contrast, S_450 and S_650 exhibited perfect classification, indicating clear moment separability.

Translational analysis aims to examine the effect of moving the image of an object to different locations on Hu moments. In this study, experiments were carried out to evaluate whether the Hu moments, which provide translational invariance, maintain the same values even though the images are located in different positions. Within the scope of the analysis, imaging operations were performed by moving an object to different x and y coordinates. Hu moments were calculated for each image and examined whether there was any change between these values. Experimental results show that the translation process does not have a significant effect on the Hu moment values. This validates the translational invariance of Hu moments under varying positional displacements. The results obtained in the experiments revealed that the Hu moments remained constant, especially when the center of the image of an object was shifted to different points. For example, the image of an object was moved to various positions on both the horizontal and vertical axes, and the same moment values were obtained in each case. Such findings reinforce the suitability of Hu moments for precise size estimation tasks. The results obtained are presented in figure 4. Figure 7 shows comparatively the effects on Hu moments of images created by moving an object to different positions. Figure 8 shows the dataset consisting of the letters s used for translational analysis. The heat map visually demonstrates the translational invariance feature of Hu moments and emphasizes the importance of this feature for object recognition and analysis applications. These insights confirm that Hu moments maintain consistency across displacement scenarios, especially in moving object recognition, image fusion and tracking applications. The translational invariance property of Hu moments makes this method a reliable tool that is not sensitive to position changes.

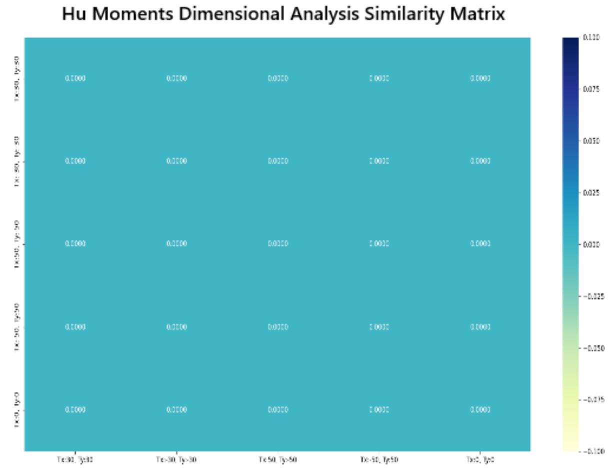

Figure 7. Heat map of pairwise similarity values for the letter S subjected to various translational shifts along the x and y axes (±30 and ±50 pixels). The matrix demonstrates Hu moments' stability across all translation configurations.

- These findings support the theoretical expectation that Hu moments are invariant under translational shifts.
- The heat map visually affirms that translations in either direction (positive or negative) do not alter the moment-based descriptor space significantly.

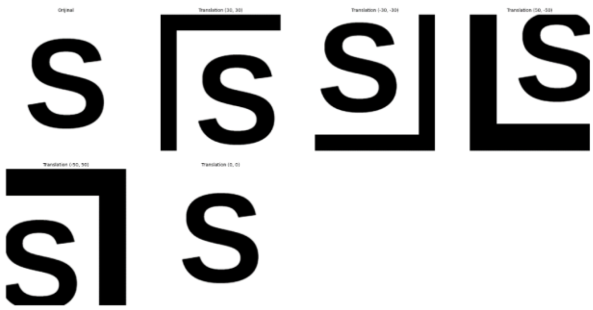

Figure 8. Synthetic dataset used in translational analysis of the letter S. Each image represents a different (x, y) offset to evaluate the translation invariance of Hu moments under controlled displacement scenarios.

Conclusion

Unlike traditional studies that only demonstrate invariance theoretically, this study develops an application-oriented structure that directly tests the transformation resilience of Hu moments using custom-generated datasets and measurable metrics. This structured experimental framework contributes to the field by providing practical insights into the robustness of Hu moments in scalable systems. In comparison with previous studies, the results of this work align with and extend current findings in the literature. For example, Wu et al. (2020) demonstrated that integrating Hu moments with CNNs improved recognition accuracy in visual tasks, which supports our findings on the high classification success of Hu-based features for scaled images. Hjouji et al. (2020) proposed a remediation method for improving Hu invariants, emphasizing their stability under geometric distortions, which our experimental outcomes also confirm through rotation and translation robustness. Gancheva and Peneva (2023) applied Hu moments in radar satellite images and showed their utility in detecting spatial consistency; this parallels our success in translational analysis where Hu moments remained invariant despite position changes. Additionally, Rodríguez (2023) highlighted their effectiveness in micro-scale surface recognition, which resonates with our size determination experiments showing sensitivity to fine-grained scale changes. Therefore, this study not only confirms prior theoretical assertions about the invariance of Hu moments but also provides an empirical framework that enhances their practical validation and interpretability in visual classification tasks. In summary, the findings demonstrate that Hu moments serve as a reliable feature descriptor under various geometric transformations, including rotation, scaling, and translation. The results suggest several promising application domains where Hu moments can be effectively utilized. These include real-time object tracking in autonomous systems, automated quality control in manufacturing pipelines, and text recognition in OCR-based security frameworks. Future work can explore the integration of Hu moments with more advanced learning architectures such as transformer-based models to enhance performance in noisy or complex environments. Additionally, real-time deployment on embedded systems and testing on larger, multi-class datasets would provide deeper insights into scalability and generalizability. Investigating the hybrid use of Hu moments with frequency-domain descriptors (e.g., Zernike moments or Fourier transforms) may also yield improved robustness. These directions will further strengthen the role of Hu moments as a lightweight yet powerful feature descriptor in practical image processing applications. Future work may include comparative studies between Hu moments and other moment-based descriptors such as Zernike or Legendre moments, especially in terms of computational efficiency and robustness under complex geometric distortions.

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