

A Discrete Element Method Investigation of the Mechanical Response at The HDPE Geomembrane-Sand Interface

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Abstract

Geosynthetics made from synthetic polymers are widely used in geotechnical engineering for soil reinforcement, separation, erosion prevention, and drainage. These materials have applications in landfills, foundations, retaining walls, and dams. The interaction between geosynthetics and the surfaces they meet needs to be investigated to ensure that geosynthetics are efficient in their function. The present study examined the interface shear behaviour between granular soil and a High-Density Polyethylene (HDPE) geomembrane. A cylindrical direct shear test based on the discrete element method (DEM) was conducted on HDPE geomembranes with thicknesses of 1.5 mm and 3.0 mm. Preliminary experiments were performed solely on granular soil, after which a concrete block was placed in the lower jaw of the shear box with the geomembrane positioned on top, while the soil in the upper jaw formed the soil–geomembrane interface. Various normal stresses and shear rates were applied to analyse geomembrane behaviour. According to the DEM results, the interface friction angle for the 1.5 mm HDPE–soil configuration was reduced by approximately 51–59% compared to granular soil. For the 3.0 mm HDPE–soil interface, the reduction ranged from 42% to 48%, depending on the shear rate. These reductions, representing decreases in internal friction angle from roughly one-third to two-thirds, were found to be consistent with ASTM standards.

Keywords: Discrete Element Method, Geomembrane, Geosynthetic, High-Density Polyethylene, Shear Behaviour.

1. Introduction

Geosynthetics (GSs) are polymeric products widely used in geotechnical engineering to strengthen soil, improve drainage, reinforce, and as barriers. They are economical and environmentally friendly and contribute to extending the life of structures. Some of the typical applications include subgrades, embankments, slopes, foundations, and earth-fill dams. The primary geosynthetic (Gs) types are geotextiles, geomembranes, geogrids, geonets, and geo-composites [1]. According to ASTM D4439, a geomembrane (GM) is defined as a synthetic membrane liner or barrier exhibiting very low permeability. It is employed together with geotechnical engineering materials to control the movement of fluids or gases within engineered projects, structures, or systems [2]. GMs are widely used to provide impermeability in various fields of engineering, including geotechnical, hydraulic, environmental, and transportation engineering. Their use offers notable advantages in terms of operational efficiency, accessibility, and overall economy [3-4].

There are various types of geomembranes (GMs), classified based on their production methods and the types of polymers used. Among these, HDPE is one of the most common. HDPE is



widely chosen due to its status as one of the most extensively used geomembrane types worldwide. HDPE is one of the top choices for GMs and is used widely in most applications. HDPE is a thermoplastic polymer derived from petroleum with the general chemical formula (C_2H_4). The HDPE formula is representative of its repeating unit, ethylene, that creates a chain of long length called polyethylene. A distinguishing characteristic of HDPE among other polyethylene is that it has fewer side chains. This GMs is the favourite for coating projects since it is extremely durable, boasts high UV resistance, and is cost-effective. HDPE geomembranes (GMs) are used for lining in several projects, including mine waste landfills, tank fields, municipal solid waste landfills, acid storage tanks, ponds, and irrigation canals. HDPE geomembranes are commonly manufactured in a range of thicknesses, typically from 0.5 mm to 3.0 mm, depending on the specific application. Thicker membranes generally exhibit greater puncture resistance; however, this is accompanied by a reduction in flexibility [5].

Research on interfaces in geotechnical engineering is necessary to understand material interaction under different stress conditions with direct effects on structural stability and performance. Interfaces such as soil-geomembrane or soil-soil interfaces play a crucial role in load transfer and shear resistance behaviour. The investigation of these interactions enables engineers to provide safer, more efficient structures, reduce material use, and minimize failure risks. Moreover, an understanding of interface mechanics facilitates improved construction techniques and the development of advanced geotechnical materials [6-7]. Various test methods, including direct shear, simple shear, vane, triaxial, and ring shear tests, are utilized to study interface behaviour. This shear resistance is usually expressed in terms of adhesion and the interface friction angle (δ).

The direct shear test (DST) is a popular laboratory test, known for its simplicity, economy, and reproducibility for measuring the shear strength of soil. It also provides controlled conditions for measuring shear resistance along material interfaces. For example, Fishman and Pal [8] presented experimental data on the interface shear strength between cohesive soils and geomembranes, and also provided a comprehensive review of the factors influencing this strength and their respective impacts. Similarly, Dove and Frost [9] performed a series of direct shear tests using Ottawa 20/30 sand in contact with HDPE geomembranes. Their findings indicated that parameters such as normal stress, particle shape, and surface roughness of the materials play a critical role in determining the dominant shear mechanism governing the interface behavior. The findings show that the peak shear strength at the interface between textured geomembranes and soil is greater than that at the interface between smooth geomembranes and soil, although the residual shear strengths are generally comparable. Nevertheless, equipment and technique constraints of available tests do not allow the direct examination of micro-mechanical behaviour between geomembrane materials and soil in the laboratory. The discrete element method represents a suitable alternative to investigate geosynthetic-soil interactions. It has been successfully applied in past research, e.g., Lai et al. [10], where they concentrated on the soil-arching effect for geogrid-reinforced pile-supported embankments, and Wang & Alonso-Marroquín [11], where they investigated load transfer behaviour between geogrid and sand. Additionally, Kostkanová & Herle [12] analysed the direct shear behaviour of coal-fouled geogrid-reinforced ballast, whereas Cheng, Yamamoto, & Thoeni [13] examined stress states and fabric anisotropies in geotextile containers.

In order to study the impact of geomembrane thickness on the shear behaviour of the geomembrane-sand interface by using the DEM, a three-dimensional direct shear model was established in Altair EDEM [14]. The model allowed micro-mechanical interaction analysis between soil and geomembrane in detail. Soil-soil and soil-geomembrane interface shear strength behaviour was compared for two thicknesses of geomembranes, with tests being

conducted at various normal stresses and shear rates to establish their influence on interface behaviour.

2. The Discrete Element Method

The DEM simulates the mechanical behaviour of granular material, one of its most significant aspects using Newton's laws of motion for individual particles [15]. Particle-particle and particle-geomembrane interaction forces are calculated during the simulation cycle. Newton's laws are repeatedly applied in every cycle in order to determine the acceleration, velocity, and displacement of a single particle. The new displacement is subsequently used to derive the contact forces and torques of the particle interactions in their new positions [16]. The process is iterated several times with a view to simulating the mechanical behaviour of the bulk material [17]. DEM is particularly suited to analyse the individual behaviour and movement of granular materials [17] or analyzing the effect of interface behaviour [18].

DEM's DST model was formulated with novelty to simulate the complexities involved in the actual process of direct shear testing. The model is made up of a 52 mm radius and 75 mm height cylindrical box. The shear box is modelled in such a way that it would not be deformed by the normal and shear stresses on the ground. The lower cylinder can be moved freely underneath the stationary higher cylinder. In this setup, the vertical loading system—comprising a dead-weight sphere positioned atop the upper cylinder—applies and maintains a constant normal load on the sample throughout the test. Concurrently, the horizontal loading system is capable of applying a shearing force to the sample at a constant displacement rate, in a direction parallel to the movement of the lower cylinder. In addition, the properties of the model have been modified to mimic the properties of steel according to the requirements of the ASTM D3080 [19]. DEM parameters have been established in a trial-and-error method given in Table 1 in a way that the results of the simulation are reasonable as given in Table 1. Various components of the model, as given in Figure 1.

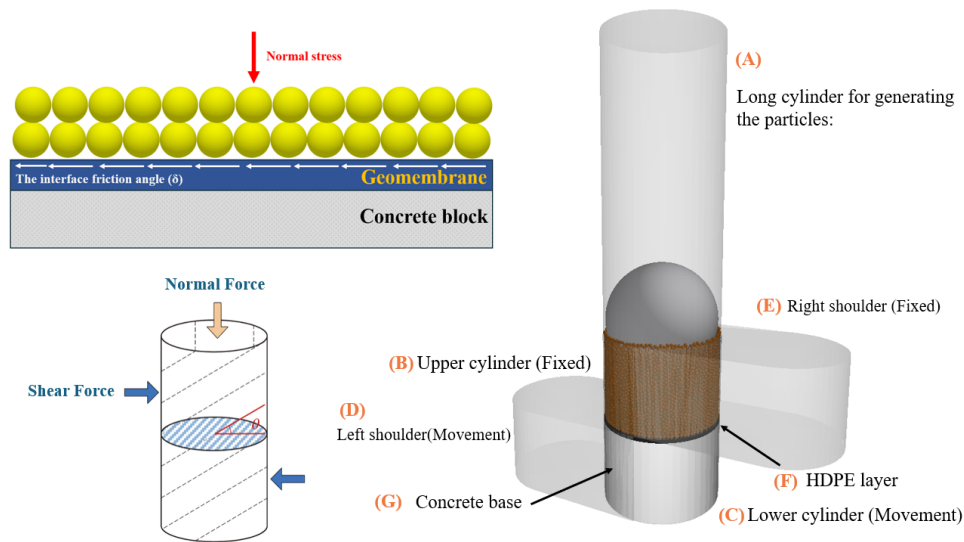


Fig. 1. Key parts of the direct shear test model

Granular materials were prepared in a cylindrical cell using 1.5-mm-radius spherical beads, in which the particles fill the shear cylinder and are compacted by normal gravity forces for all samples. For effective sample comparison, simulated tests were carried out in three consecutive steps.

In the initial step, particle assemblies were formed quickly, and each particle contacted its neighbours while still possessing kinetic energy. As a result, a brief settling period was required for the particles to stabilize and reach zero kinetic energy. In the second step, a normal load was applied to the assembly. The use of particles with nearly the same diameter as the shear testing system, along with a precisely calculated density, allowed for the proper application of vertical load. The spherical nature of the load distribution helped prevent weight tilting, a common issue in DEM shear tests [12]. The last step included performing the direct shear test through horizontal displacement of the bottom half of the shear apparatus with sustained vertical stress. The present investigation was conducted in several phases. During the first phase, internal friction angles (ϕ) of granular soil alone were found. Details of the granular soil are presented in Table 1. Then, interface friction angle (δ) of the soil-GM interface was found. HDPE geomembranes are widely used in geotechnical engineering, typically with the thicknesses ranging from 0.5 mm to 3 mm [20]. For this study, geomembranes with thicknesses of 1.5 mm and 3.0 mm were selected. The thickness 3.0 mm provides higher puncture resistance, and greater longevity, and it provides a more forceful barrier to leakage, which is very essential for high-pressure containment. Conversely, the 1.5 mm geomembrane is more pliable and thus best suited for lighter applications where installation convenience is a consideration. It also provides a less expensive option for projects with lower structural requirements [21-22].

Table 1. Material Parameters selected in DEM Simulation

Parameter	Value
Solid density of sand particle (ρ_{sand})	2100 kg/m ³
Shear modulus of sand particle (G_{sand})	1e+07 pa
Poisson ratio of sand particle (ν_{sand})	0.3
Restitution coefficient ($e_{\text{sand-sand}}$)	0.6
Static friction coefficient ($\mu_{\text{sand-sand}}$)	0.5
Rolling friction coefficient ($\mu_{r,\text{sand-sand}}$)	0.05
The number of sand particles (N_{sand})	20000
Solid density for HDPE geomembrane (ρ_{GM})	970 kg/m ³
Shear modulus for HDPE geomembrane (G_{GM})	7.5e+08 pa
Poisson ratio for HDPE geomembrane (ν_{GM})	0.35
Restitution coefficient ($e_{\text{sand-HDPE}}$)	0.7
Static friction coefficient ($\mu_{\text{sand-HDPE}}$)	0.6
Rolling friction coefficient ($\mu_{r,\text{sand-HDPE}}$)	0.05
The fixed-time step (Δt)	1.328e-05 sec.
Solid density of steel (Model) (ρ_{steel})	7500 kg/m ³
Shear modulus of steel (Model) (G_{steel})	8e+10 pa
Poisson ratio of steel (Model) (ν_{steel})	0.3

To establish the interface shear surface between the geomembrane and granular soil, the geomembrane was positioned atop a concrete block. This block was placed in the lower box of the direct shear apparatus, while the granular soil was filled into the upper box. As a result, the interface between the soil and geomembrane served as the designated shear plane [23]. Utilizing a concrete block helps minimize soil settlement during simulation and maintains the stability

of the geomembrane on the shearing surface throughout the test, as depicted in Figure 1. The DSTs were performed in two stages under three different normal stresses: 12.5 kPa, 25 kPa, and 50 kPa. Shear rates of 0.5, 1.0, and 2.0 mm/s were applied, selected based on ASTM D3080 [19] and aligned with the conditions of the related experimental program.

3. Simulation Results and Discussion

The shear behaviour at the interface between the HDPE geomembrane and granular soil was investigated using the DEM, with particular emphasis on both macroscopic and microscopic stress–strain responses. The primary outputs of the simulations included shear stress, interface friction angle, and apparent cohesion. The interface friction angle (δ) was determined in accordance with the Mohr–Coulomb failure criterion. The study was conducted in two phases: initially, the internal friction angle (ϕ) of the granular soil was evaluated; subsequently, the interface friction angle (δ) for the geomembrane–soil interface was measured.

3.1. The Soil–Soil Interface

Initially, shear tests were conducted on the soil–soil interface using granular soil subjected to three different normal stresses of 12.5 kPa, 25 kPa, and 50 kPa, at shear rates of 0.5 mm/s, 1.0 mm/s, and 2.0 mm/s, respectively. The corresponding internal friction angles of the granular soil were determined to be 30.6°, 33.4°, and 37.1°, as summarized in Table 2. Figures 2, 3, and 4 illustrate the stress–strain behavior observed for the tested samples under these conditions.

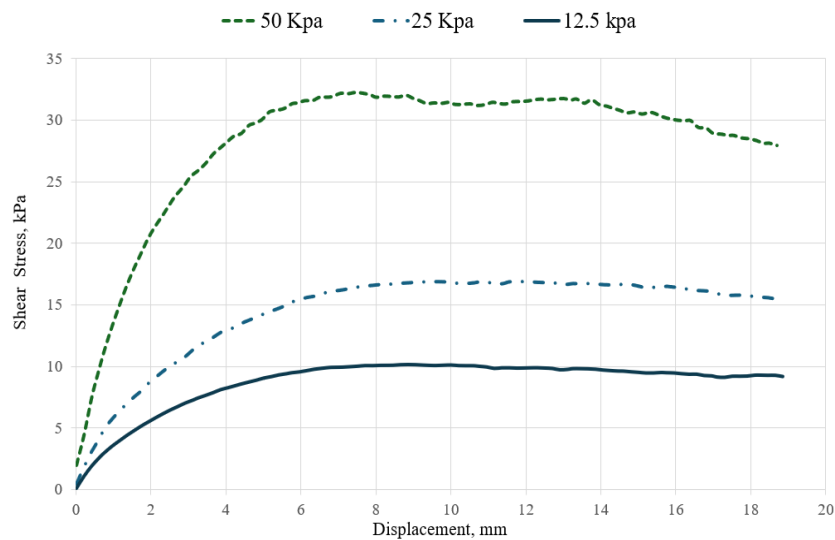


Fig. 2 Shear stress and strain curves for the GM-1 sample subjected to three normal stresses (12.5, 25, and 50 kPa) at a constant shear rate of 0.5 mm/s

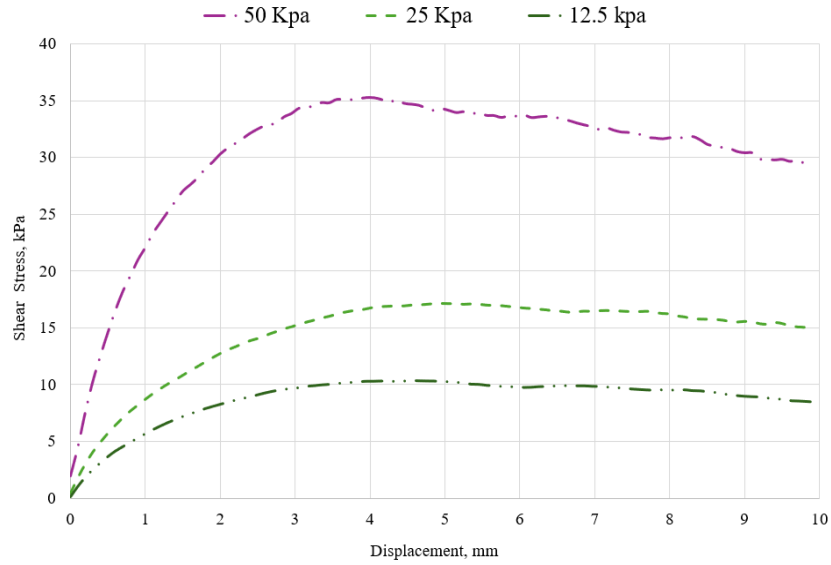


Fig. 3. Shear stress and strain curves for the GM-2 sample subjected to three normal stresses (12.5, 25, and 50 kPa) at a constant shear rate of 1 mm/s

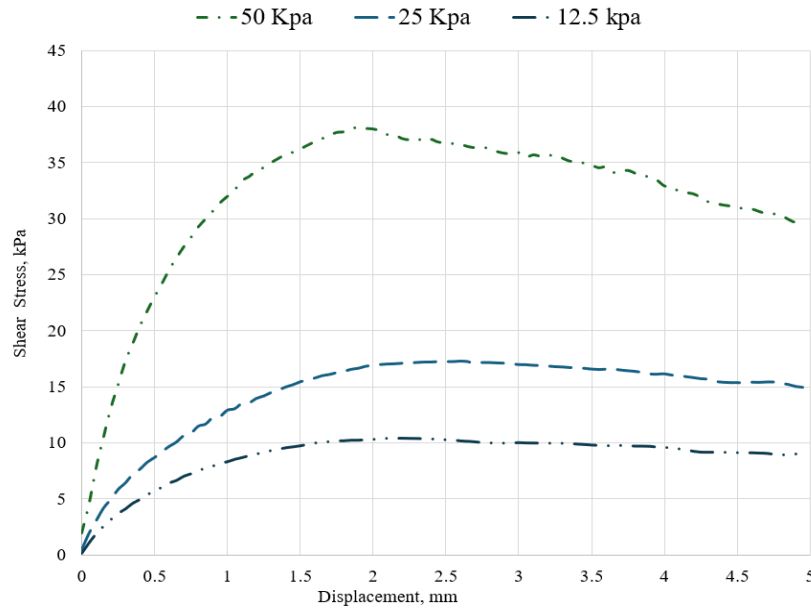


Fig. 4. Shear stress and strain curves for the GM-3 sample subjected to three normal stresses (12.5, 25, and 50 kPa) at a constant shear rate of 2 mm/s

Table 2. Shear strength parameters (internal friction angle and cohesion) of the soil–soil interface

No.	Shear rate (mm/sec)	Internal Friction angle (ϕ), ($^{\circ}$)	Cohesion (c) (kPa)
GM-1	0.5	30.6	1.3
GM-2	1.0	33.4	2.5
GM-3	2.00	37.1	0.07

3.2. The Soil-HDPE Interface Behaviour

The interface behaviour was first studied for a 1.5 mm thick high-density polyethylene geomembrane (HDPE-GM), and then for a 3.0 mm thick HDPE-GM. For the 1.5 mm HDPE-GM, the tested samples were S-GM1.5-0.5R, S-GM1.5-1.0R, and S-GM1.5-2.0R, whereas for the 3.0 mm HDPE-GM, they were S-GM3.0-0.5R, S-GM3.0-1.0R, and S-GM3.0-2.0R. The sample nomenclature is as described: "S" for sand, "GM" for geomembrane thickness in mm, and "R" for shear rate—where, for example, S-GM1.5-0.5R refers to a sand–geomembrane interface with a 1.5 mm thick HDPE geomembrane and a shear rate of 0.5 mm/sec.

Fig. 5 illustrates interface behaviour, with higher red point density found in the 1.5 mm HDPE interface, showing more extreme shear deformation, that decreases with geomembrane thickness [28]. Figures 6 and 7 then further present shear stress-strain curves for S-GM1.5-0.5R and S-GM3.0-0.5R, respectively, providing insight into the shear response at different thicknesses and rates. Table 3 presents the shear strength of the sand–HDPE geomembrane interface under various normal stresses, while Table 4 summarizes the corresponding internal friction angle and cohesion values.

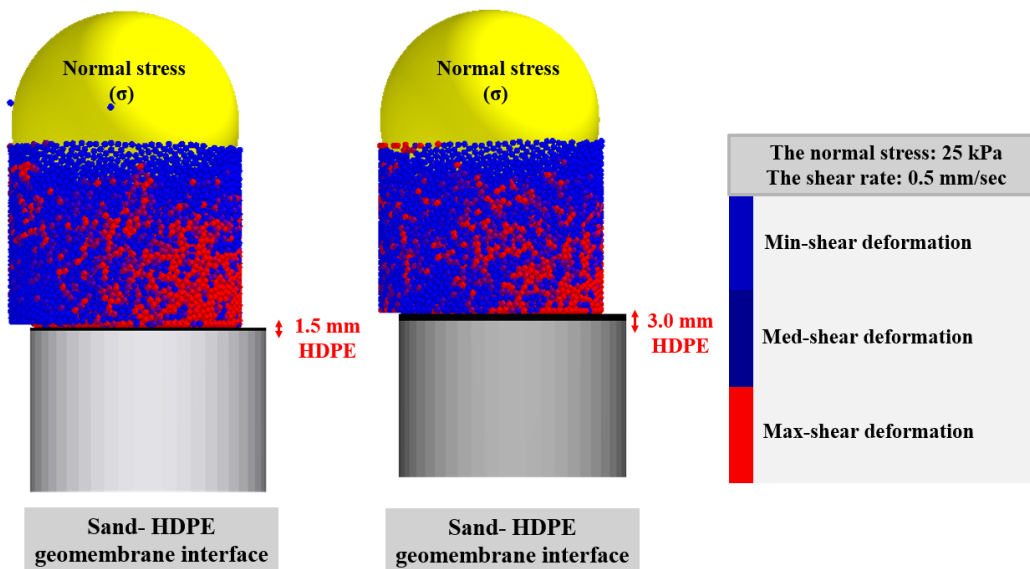


Fig. 5. The shear behaviour between soil and HDPE geomembrane for HDPE thickness 1.5 and 3.0 mm in the DEM model

The simulation result shows the shear response at the interface of granular soil and HDPE geomembrane. A new model of the DST was created in the DEM platform in this study to simulate the process of actual DST so that the result can be more credible. One of the major problems addressed was that it was hard to replicate low shear rates with DEM because a large amount of computational time was required. Even with this, though, the research was very precise because it closely replicated the parameters of the most commonly used granular soil and geomembrane in geotechnical engineering. Moreover, the modelled interface response of sand and HDPE geomembrane was in the range of results from previous experimental studies as shown in the Figure 9, justifying the model. The study also included the effect of the thickness of the geomembrane on shear behaviour and provided further insight into the dominant factors in interface performance.

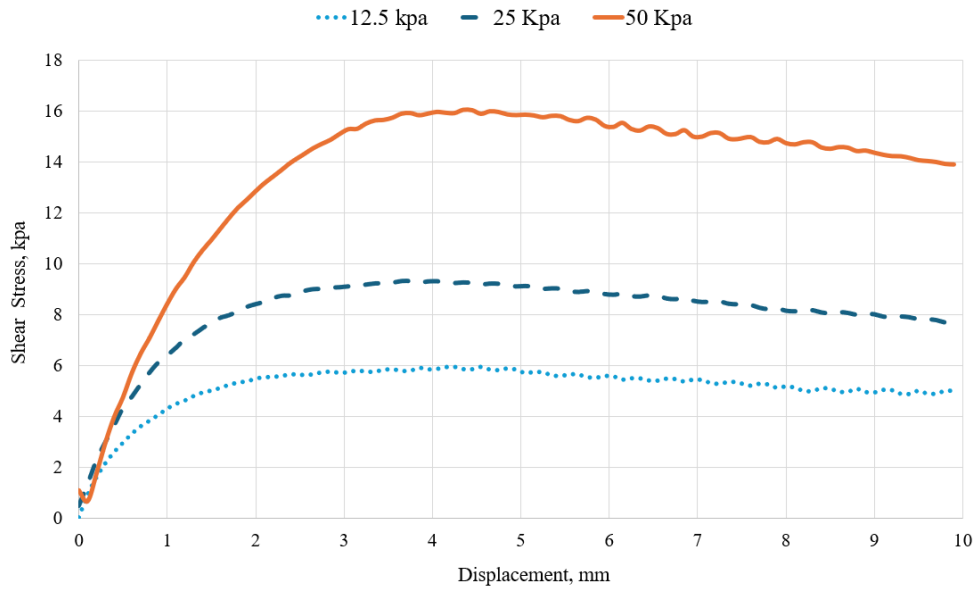


Fig. 6 Shear stress and strain curve of S-GM1.5-0.5R sample subjected to the normal stresses (12.5, 25, and 50 kPa) at a constant shear rate of 0.5 mm/s

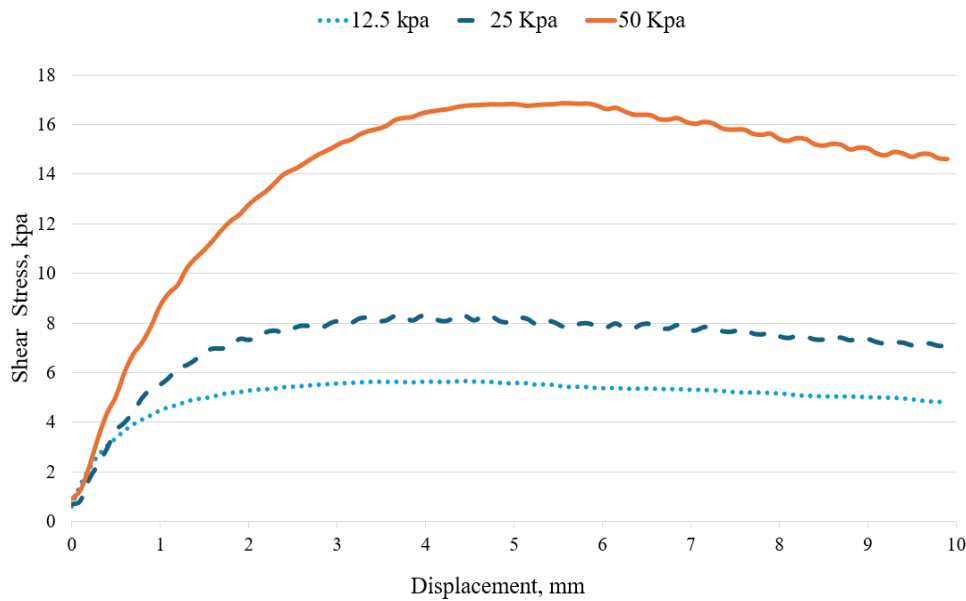


Fig. 7. Shear stress and strain curve of S-GM3.0-0.5R sample subjected to the normal stresses (12.5, 25, and 50 kPa) at a constant shear rate of 0.5 mm/s

The interface friction angles (δ) between soil and the HDPE geomembrane were always lower than the internal friction angles of the sand-sand interface for both geomembranes having a thickness of 1.5 mm and 3.0 mm. Specifically, in the case of 1.5 mm thick HDPE, the interface friction angles were 50.92%, 54.49%, and 58.95% less than granular soil ones at shear rates of 0.5, 1, and 2 mm/sec, respectively. Similarly, for the 3.0 mm thick HDPE, the reductions were 42.30%, 46.84%, and 48.04% at the same shear rates. As specified in ASTM D5321, the interface friction angle (δ) for HDPE geomembrane–soil interfaces generally falls within the range of one-third to two-thirds of the soil's internal friction angle (ϕ), i.e., $1/3\phi < \delta < 2/3\phi$. This indicates that the interface shear strength is lower than the internal shear strength of the soil (ϕ), yet it still contributes significantly to resistance against sliding.

Table 3. Variation of Shear Strength Parameters at the Sand–HDPE Geomembrane Interface under Different Normal Stresses

No.		Normal stress, Kpa		
		12.5	25	50
S-GM1.5-0.5R	Shear Strength, kPa	5.97	9.33	16.07
S-GM1.5-1.0R		5.99	9.38	16.17
S-GM1.5-2.0R		5.99	9.37	16.20
S-GM3.0-0.5R		5.50	8.59	17.25
S-GM3.0-1.0R		5.47	8.61	17.31
S-GM3.0-2.0R		5.48	8.98	17.43

Table 4. The internal and interface friction angles results

Shear rate (mm/sec)	No.	(ϕ), (°)	No.	(δ), (°)	one-third $\phi < \delta < \text{two-thirds } \phi$
0.5	GM-1	30.6	S-GM1.5-0.5R	15.08	10.1 < 15.08 < 20.4
1.0	GM-2	33.4	S-GM1.5-1.0R	15.19	11.1 < 15.19 < 22.3
2.0	GM-3	37.1	S-GM1.5-2.0R	15.23	12.4 < 15.23 < 24.7
0.5	GM-1	30.6	S-GM3.0-0.5R	17.65	10.1 < 17.65 < 20.4
1.0	GM-2	33.4	S-GM3.0-1.0R	17.76	11.1 < 17.73 < 22.3
2.0	GM-3	37.1	S-GM3.0-2.0R	17.82	12.4 < 17.82 < 24.7

This reduction in the friction angle is mainly due to the lack of interlocking between soil particles and the geomembrane, limiting shear resistance. Interlocking of particles in soil provides additional resistance when shearing, raising its internal friction angle. Geomembranes, however, do not permit this interlocking, hence leading to a lower interface friction angle. In addition, the contact mechanism is very different: in soil, particles are in contact at more than one point, leading to a complex shear response, whereas soil-geomembrane contact is more 2D than the 3D inter-particle contacts in soil. The shear strength at the soil-geomembrane interface is thus still less than that of undisturbed pure granular soil, as observed in findings of [29].

HDPE geomembrane thickness is a significant variable influencing interface friction behaviour. In this study, 1.5 mm HDPE samples gave friction angles (δ) in the range of 15.1° to 15.2°- and 3.0-mm samples gave friction angles in the range of 17.6° to 17.8° (Fig. 8). The thicker 3.0 mm geomembrane has a greater deformation under load, leading to more uniform stress distribution and greater interlocking of sand particles, thereby increasing resistance to movement. Conversely, the thin 1.5 mm geomembrane experiences higher localized stress with less deformation and less particle interlocking, thus a lower friction angle and frictional resistance. Also, while shear rate significantly affects the shear strength of granular soils, it affects the geomembrane interface to a small extent. Findings by [18, 24] indicate that asperity thickness, rather than surface roughness, is a more critical factor in the performance of geomembranes. The findings indicate that as geomembrane thickness increases, the interface shear strength

stabilizes and further changes in shear rate do not have a significant impact. This is contrasted with granular material like sand, where shear strength is more greatly affected by shear rate.

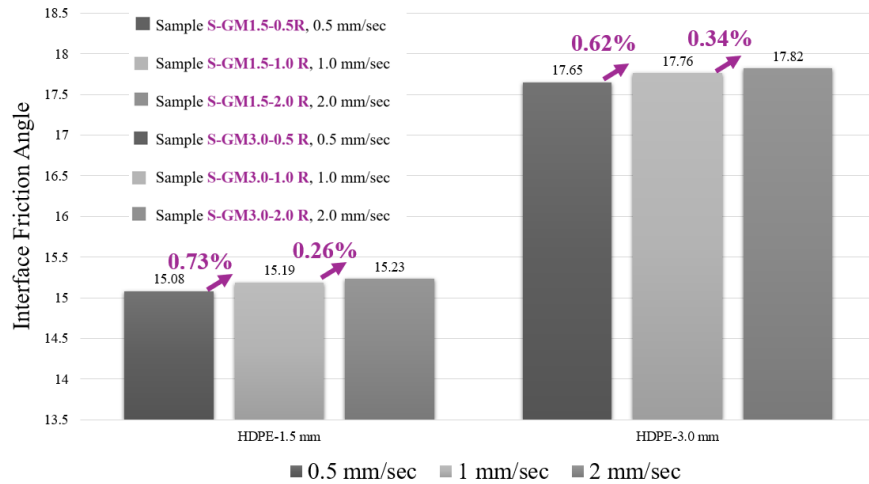


Fig. 8. Comparison of interface friction angles for 1.5 mm and 3.0 mm HDPE geomembranes across various shear rates

Outcomes of this study fell within laboratory data of the majority of studies, as reflected in Figure 9. explored where the interface between HDPE geomembrane and the dry granular soil thickness of 1.5 to 3.0 mm has been studied [25,26,27, 28]. This alignment demonstrates the high efficiency of our study in analysing geotechnical soil interfaces.

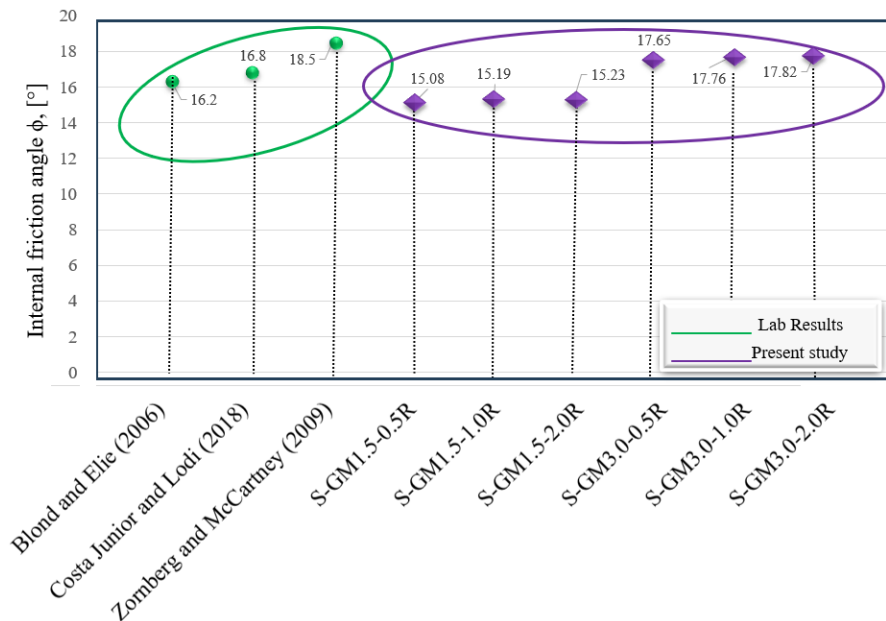


Fig. 9. Comparison of the study's findings with laboratory results from various studies HDPE geomembrane interface

4. Conclusion

This study examined the shear behaviour at the interface between granular soil and HDPE geomembranes with thicknesses of 1.5 mm and 3.0 mm using the DEM. The results showed that the internal friction angles of the soil were consistently higher than those measured at the

soil–geomembrane interfaces. This supports the hypothesis that mechanical interlocking among soil particles is inherently stronger than the interlocking between soil and the geomembrane surface. Moreover, variations in shear rate exhibited minimal influence on the interface friction angles, suggesting that shear rate is not a critical factor governing the shear strength of the soil–geomembrane interface. These findings enhance the understanding of geosynthetic interface behavior in geotechnical applications and underscore the significance of soil–geomembrane interaction in ensuring the stability and performance of engineered systems.

Author Contribution

Mohammad Bairoti: Conceptualization; development of the DEM model; execution of numerical analyses; data processing; preliminary interpretation of the results; preparation of the initial draft of the manuscript.

Hasan Firat Pulat: Conducting the literature review, optimization of experimental and numerical parameters, contribution to the validation of the results, assistance in preparing figures and tables, and critical review of the manuscript draft.

Muath S. Talafha: Overall study design and supervision; development and refinement of the methodology; engineering interpretation of the findings; restructuring of the discussion section; final proofreading, academic language editing, and preparation of the manuscript for submission.

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