Black Holes with Anisotropic Fluid in Lyra Scalar-Tensor Theory

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Abstract: In this paper, we investigate distribution of anisotropic fluid which is a resource of black holes in regard to Lyra scalar-tensor theory. As part of the theory, we obtain field equations of spherically symmetric space-time with anisotropic fluid. By using field equations, we suggest distribution of anisotropic fluid, responsible for space-time geometries such as Schwarzschild, Reissner-Nordström, Minkowski type, de Sitter type, Anti-de Sitter type, BTZ and charged BTZ black holes. Finally, we discuss obtained pressures and density of the fluid for different values of arbitrary constants, geometrically and physically.

Lyra Skaler-Tensör Teoride Anizotropik Akışkanlı Karadelikler

Özet: Bu çalışmada, Lyra skaler-tensör teorisine göre karadeliklere kaynak olacak anizotropik akışkan dağılımı araştırılmıştır. Bu teori kapsamında, anizotropik akışkanlı küresel simetrik uzay-zaman için alan denklemleri elde edilmiştir. Alan denklemlerinin çözümleri kullanılarak, Schwarzschild, Reissner-Nordström, Minkowski tipi, de Sitter tipi, Anti-de Sitter tipi, BTZ ve yüklü BTZ karadelikleri gibi uzay-zaman geometrilerinden sorumlu olabilecek anizotropik akışkan dağılımı önerilmiştir. Sonuçta, anizotropik akışkan için elde edilmiş olan basınç ve yoğunluklar farklı keyfi sabit seçimleri için fiziksel ve geometrik açıdan tartışılmıştır.

1. Introduction

There are wide range of efforts to combine electromagnetical and gravitational fields within the scope of unification theories. For this purpose, Weyl suggested a new geometry, in which Lagrangian density directly depends on metric tensor and a gauge field [1]. Weyl geometry differs from Riemannian geometry owing to the gauge field [1]. Structure of the vector length can be deformed under the coordinate transformations in the Weyl geometry [2]. The deformation poses a problem in respect to availability of the geometry [3]. Lyra resolved the problem by choosing metric potentials which depend on coordinates and gauge field. The choice allows that the vector length remains to be invariant under the parallel transformations [3]. So, new scalar-tensor theory of gravitation was constructed by using the Lyra geometry [4,5].

It is known that the general relativity loses its validity because of cosmological constant problem and behaviour of universe in early and/or late stages

[6,7,8]. Contrary to this, it is seen that alternative ways such as Brans-Dicke Theory, teleparallel gravity, f(R) gravity and other modified theories have been approved in the recent years. It can be considered that Lyra scalar-tensor theory is one of the available alternative ways. The theory has all the makings of Newtonian and general relativity limits [9]. Various space-time models have been paid attention to the theory. Particularly, spherically symmetric space-times and black holes have been gone about with perfect fluid or vacuum cases: Sen and Dunn suggested a serial solution for spherically symmetric models in Lyra scalar-tensor theory [5]. The solution has structure like as Schwarzschild black hole. Afterwards, Rahaman *et.al.* [10] proposed a different solution of field equations for the space-time. They showed that the solution, called as Lyra black hole has two singular points. Bhamra [11] proved that static cosmological models are non-realistic by using the spherically symmetric space-time together with perfect fluid in Lyra scalar-tensor theory. Reddy and Venkateswarlu [12] investigated conformal spherical symmetric cosmologies. They

showed that displacement vector in Lyra scalar-tensor theory has same role spin density in Einstein-Cartan theory. Rahaman *et.al.* [13] obtained higher dimensional spherically symmetric model without any restriction or singularity. Global monopoles were considered in non-static spherically symmetric space-times [14]. It is showed that the monopole has an attractive force on test particle [15].

Even though there are above-mentioned studies, behaviour of the anisotropic fluid and black holes, hasn't been investigated in the context of Lyra scalar-tensor theory. In this study, it is aimed to define a cosmic matter formed as anisotropic fluid, resource of Schwarzschild, Reissner-Nordström, Minkowski type, de Sitter type, Anti-de Sitter type, BTZ and charged BTZ black holes, in regard to Lyra scalar-tensor theory.

This paper has contents as follows: In Section.2, field equations of Lyra scalar-tensor theory is repeated. The equations and their solutions for spherically symmetric space-time and anisotropic fluid, are obtained. By using the solutions, various black hole models, full of anisotropic fluid are suggested in Section.3. Finally, obtained results are discussed in Section.4.

2. Material and Method

By using the principle of least action, field equation of Lyra scalar-tensor theory can be written as

$$R_{ij} - \frac{1}{2}g_{ij}R = -T_{ij} - \frac{3}{2}\varphi_i\varphi_j + \frac{3}{4}g_{ij}\varphi_k\varphi^k = -T_{ij}^{(eff)} \quad (1)$$

where R_{ij}, R and g_{ij} symbolize Ricci curvature tensor, Ricci scalar curvature and metric tensor in Riemannian geometry, respectively [4,5]. Furthermore, φ_i is displacement vector in Lyra geometry. The vector is the reason of gauge function in Lyra scalar-tensor theory. T_{ij} and $T_{ij}^{(eff)}$ respectively sign energy-momentum tensor of cosmic matter and effective energy-momentum tensor. In this study, cosmic matter is considered as anisotropic fluid. Energy-momentum tensor of anisotropic fluid is commonly defined as

$$T_{ij} = (p_\perp + \rho)u_iu_j + p_\perp g_{ij} + (p_r - p_\perp)x_ix_j \quad (2)$$

where ρ, p_r and $p_\perp$ are energy density, radial pressure and tangential pressure of the fluid. u_i is four-velocity vector and x_i is unit spatial vector. The vectors must satisfy some conditions such as $u^ix_i = 0$ and $x^ix_i = -1$. In this study, the vectors are chosen in confirmity with conformal motion. So, line element for statical and spherically symmetric space-time can be noted that

$$ds^2 = F^2(r)dt^2 - \frac{dr^2}{F^2(r)} - r^2d\theta^2 - r^2\sin^2\theta d\varphi^2 \quad (3)$$

where $F(r)$ is a function which depends on radial coordinate. The relationship between line element and metric tensor is $ds^2 = g_{ij}(x^n)dx^idx^j$. So, components of metric tensor can be easly obtained as follows:

$$g_{ij} = \begin{bmatrix} -\frac{1}{F^2(r)} & 0 & 0 & 0 \\ 0 & -r^2 & 0 & 0 \\ 0 & 0 & -r^2\sin^2(\theta) & 0 \\ 0 & 0 & 0 & F^2(r) \end{bmatrix} \quad (4)$$

It is widely-known that Ricci tensor depends on metric tensor such as

$$R_{ij} = \frac{\partial\Gamma_{ij}^n}{\partial x^n} - \frac{\partial\Gamma_{in}^n}{\partial x^j} + \Gamma_{ij}^k\Gamma_{kn}^n - \Gamma_{in}^k\Gamma_{kj}^n \quad (5)$$

where connection coefficient of space-time is $\Gamma_{\mu\nu}^\alpha = \frac{1}{2}g^{\alpha\beta}\left(\frac{\partial g_{\nu\beta}}{\partial x^\mu} + \frac{\partial g_{\mu\beta}}{\partial x^\nu} - \frac{\partial g_{\mu\nu}}{\partial x^\beta}\right)$. By using the connection coefficients together with Equations 4 and 5, required components of Ricci tensor are calculated as follows:

$$R_{11} = \frac{F_{r,r}}{F} + \frac{F_r^2}{F^2} + \frac{2}{r}\frac{F_r}{F} \quad (6)$$

$$R_{22} = 2rFF_r + F^2 - 1 = \frac{R_{33}}{\sin^2\theta}, \quad (7)$$

$$R_{44} = -F^3F_{r,r} - F^2F_r^2 - \frac{2}{r}F^3F_r. \quad (8)$$

where F_r and $F_{r,r}$ symbolize first and second derivatives of $F(r)$ function regarding to radial coordinate, respectively. Ricci scalar curvature is defined as constructions of Ricci tensor and metric tensor ($R = R_{ij}g^{ij}$). From Equations 4, 5, 6, 7 and 8, one can obtain Ricci scalar curvature:

$$R = -\frac{2}{r}(r^2FF_{r,r} + r^2F_r^2 + 4rFF_r + F^2 - 1). \quad (9)$$

Also, the displacement vector can be considered as $\varphi_i = (0,0,0,\beta)$ and β is a constant in Equation 1. From Equations 1, 2, 3, 4, 5, 6, 7, 8 and 9, field equations for spherically symmetric space-time fulled with anisotropic fluid in Lyra scalar-tensor theory, are obtained in the following form:

$$p_r(r) = -\frac{2FF_r}{r} + \frac{1}{r^2} - \frac{F^2}{r^2} - \frac{3}{4}\frac{\beta^2}{F^2}, \quad (10)$$

$$p_\perp(r) = -\frac{2FF_r}{r} - F_r^2 + F_{r,r}F - \frac{3}{4}\frac{\beta^2}{F^2}, \quad (11)$$

$$\rho(r) = \frac{2FF_r}{r} - \frac{1}{r^2} + \frac{F^2}{r^2} - \frac{3}{4}\frac{\beta^2}{F^2} \quad (12)$$

From Equations 10, 11 and 12, it is seen that energy density, radial and tangential pressures of the fluid ride on radial coordinate and $F(r)$ function. Line element in Equation 3 could be described various

black holes such as Schwarzschild, Reissner-Nordström, Minkowski type, de Sitter type, Anti-de Sitter type, BTZ and charged BTZ black holes in the use of different $F(r)$ functions.

3. Results

3.1. Anisotropic fluid as recourse of Schwarzschild black hole

If the function is chosen as $(r) = (1 - \frac{2M}{r})^{1/2}$, line element in Equation 3 represents Schwarzschild black hole geometry [16]. It is known that the geometry is gained recognition as fundamental black hole model without any charged and angular momentum [16]. By using the function together with Equations 10, 11 and 12, one get dynamical parameters of anisotropic fluid in the following functions:

$$p_r(r) = p_\perp(r) = \rho(r) = -\frac{3}{4}\beta^2(1 - \frac{2M}{r})^{-1}. \quad (13)$$

Equation 13 represents distribution of anisotropic fluid which generates Schwarzschild black hole geometry. It is seen from the Equation 13 that density, radial and tangential pressures are equal. If dynamical parameters of cosmic fluid form as $p_r = p_\perp = \rho$, the matter could be called as "stiff", spatially [17,18]. Then, it can be infered that stiff matter in spherically symmetric space-time can be resource of Schwarzschild black holes according to Lyra scalar-tensor theory. In this case, dynamical components of the stiff matter change as in the Figure 1.

It is known that $r = 0$ and $r = 2M$ are singular points of Schwarzschild black holes [16]. $r_h = 2M$ also defines event horizon. From Figure 1., it is noted that dynamical components increase around the event horizon. In the case of physically meaningful cosmic matter forms, all of the pressures and density haven't

got negative magnitudes. Furtermore, the solution can be limited by $0 < r < r_h$ cause of identification of normal matter.

3.2. Anisotropic fluid as recourse of Reissner-Nordström black hole

If the function is chosen as $F(r) = (1 - \frac{2M}{r} - \frac{Q^2}{r^2})^{1/2}$, line element in Equation 3 represents Reissner-Nordström black hole geometry with total electrical charge amounting Q [19, 20]. By using the function together with Equations 10, 11 and 12, we get dynamical parameters of anisotropic fluid in the following functions:

$$p_r(r) = -\frac{Q^2}{r^4} - \frac{3}{4}\beta^2(1 - \frac{2M}{r} - \frac{Q^2}{r^2})^{-1}, \quad (14)$$

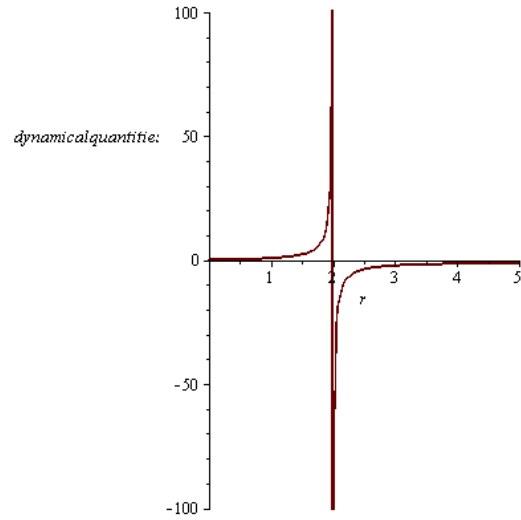$$p_\perp(r) = \rho(r) = \frac{Q^2}{r^4} - \frac{3}{4}\beta^2(1 - \frac{2M}{r} - \frac{Q^2}{r^2})^{-1}. \quad (15)$$

Figure 1. Density, Radial and Tangential Pressures of Stiff Matter in the case of Schwarzschild Black Holes.
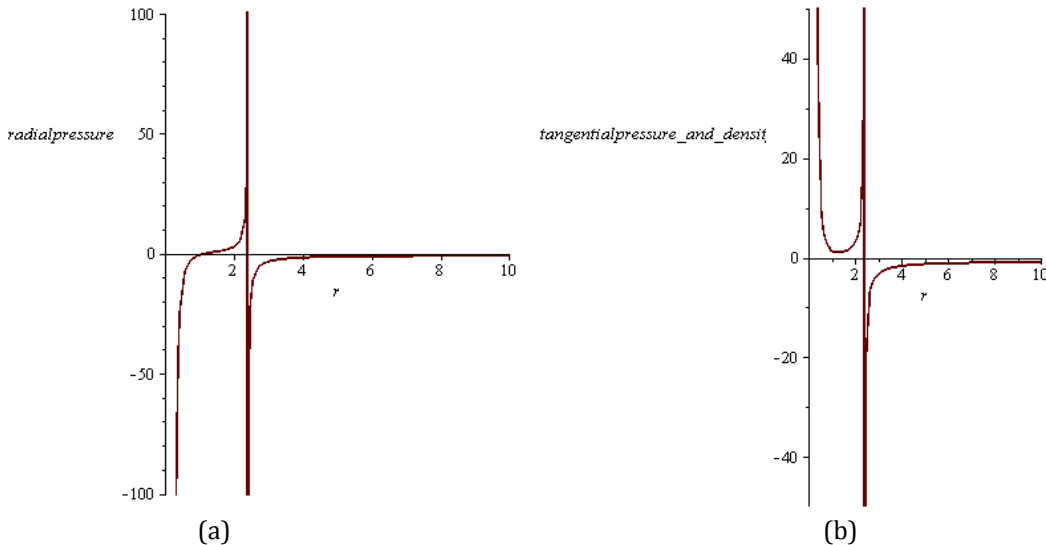

(a)

(b)

Figure 2. (a) Radial Pressure of Anisotropic Fluid in the case of Reissner-Nordström Black Holes. (b) Density and Tangential Pressure of Anisotropic Fluid in the case of Reissner-Nordström Black Holes.

Equations 14 and 15 represent distribution of anisotropic fluid which generates Reissner-Nordström black hole geometry. It is seen from the Equations 15 that density and tangential pressures are similar. If one disappears total electrical charge ($Q = 0$), the model decreases to Schwarzschild black hole, unsurprisingly. In this case, dynamical components of the anisotropic fluid change as in the Figure 2.

From Figure 2, it is clear that the solution has singularity. From Equations 14 and 15 and *F(r)* function for Reissner-Nordström black hole, the singularity is at the point of $r_h = M + \sqrt{M^2 + Q^2}$ [19, 20]. There is event horizon at the point. From Figure 2(a), it is seen that radial pressure vanishes at certain region. Also, radial pressure which has negative value outside of event horizon, tends to lose its effect. On the other hand, from Figure 2(b), both of the density and tangential pressure have negative values out of the horizon. If the matter distribution is physically meaningful, numerical values of density and pressure are not negative values, simultaneously. Because of this, the solution must be considered inner side of the horizon. The solution is valid in the range of $0 < r < r_h$.

3.3. Anisotropic Fluid as Recourses of Minkowski Type, de Sitter Type and Anti de Sitter Type Black Holes

If the function is chosen as $F(r) = (k + \frac{r^2}{l^2} - \frac{r_0^3}{l^2 r})^{1/2}$, line element in Equation 3 represents some black hole geometries. *k* is constant in this function. So, it is limited some specific values such as *k = 0,1,-1.* The geometry represents Minkowski type, de Sitter type or Anti-de Sitter-type black holes according to values of *k* constant, respectively [21]. Also, *l* and r_0 are other constants about the typical structures of the black holes [21]. By using the function together with Equations 10, 11 and 12, we get dynamical parameters of anisotropic fluid in the following functions:

$$p_r(r) = -\frac{k}{lr^2} - \frac{3}{l^2} + \frac{1}{r^2} - \frac{3\beta^2}{4}(k + \frac{r^2}{l^2} - \frac{r_0^3}{l^2 r})^{-1}, \quad (16)$$

$$p_\perp(r) = -\frac{3}{l^2} - \frac{3\beta^2}{4}(k + \frac{r^2}{l^2} - \frac{r_0^3}{l^2 r})^{-1}, \quad (17)$$

$$\rho(r) = \frac{k}{r^2} + \frac{3}{l^2} - \frac{1}{r^2} - \frac{3\beta^2}{4}(k + \frac{r^2}{l^2} - \frac{r_0^3}{l^2 r})^{-1}. \quad (18)$$

Equations 16, 17 and 18 represent distribution of anisotropic fluid which generates Minkowski type, de Sitter type and Anti-de Sitter type black holes. In the case of Minkowski type black hole, we consider $k = 0$ in Equations 16, 17 and 18. Dynamical components in this case have changes such as Figure 3. Minkowski black hole has singularities at the points of $r = 0$ and $r = r_0$. From Figure 3(a) and Figure 3(c), it is seen that density and radial pressure at $r = 0$ approach to

negative and positive infinity, respectively. Also, both radial and tangential pressure have negative values out of the event horizon. Contrary this, density is positive. If we recall the equation of state for anisotropic fluid, $p_r(r) = \omega_1 \rho(r)$ and $p_\perp(r) = \omega_2 \rho(r)$. Accordingly, proportionality constants for outside the event horizon must be $\omega_1 < 0$ and $\omega_2 < 0$ For the exterior solution, the fluid has properties of exotic matter in the context of Lyra scalar-tensor theory.

In the cases of de Sitter and Anti-de Sitter types black holes, we consider $k = 1$ and $k = -1$ in Equations 16, 17 and 18, respectively. Dynamical components in these cases have changes such as Figure 4 and Figure 5. As similar to Minkowski black hole, proportionality constants of equation of state must be $\omega_1 < 0$ and $\omega_2 < 0$ out of the event horizon. So, the exterior solutions sign that fluid acts role of exotic matter in these cases.

3.4. Anisotropic Fluid as Recourses of BTZ and charged BTZ Black Hole

If the function is chosen as $F(r) = (\frac{r^2}{l^2} - \frac{M}{r})^{1/2}$, line element in Equation 3 represents BTZ black hole [22]. l is a constant associated with cosmological term such as $\Lambda = -\frac{1}{l^2}$ [22]. By using the function together with Equations 4, 5 and 6, we get dynamical parameters of anisotropic fluid in the following functions:

$$p_r(r) = \frac{1}{r^2} - \frac{3}{l^2} - \frac{3}{4}\beta^2(\frac{r^2}{l^2} - \frac{M}{r})^{-1}, \quad (19)$$

$$p_\perp(r) = -\frac{3}{l^2} - \frac{3}{4}\beta^2(\frac{r^2}{l^2} - \frac{M}{r})^{-1}, \quad (20)$$

$$\rho(r) = -\frac{1}{r^2} + \frac{3}{l^2} - \frac{3}{4}\beta^2(\frac{r^2}{l^2} - \frac{M}{r})^{-1}. \quad (21)$$

And also, If the function is chosen as $F(r) = (\frac{r^2}{l^2} - \frac{M}{r} - \frac{2\sqrt{2}Q^3}{r} \ln(\frac{r}{l}))^{1/2}$, line element in Equation 3 represents charged BTZ black hole [23]. Q is electrical charge of the black holes. l is a length parameter similar nature in the case of BTZ black hole [23]. By using the function together with Equations 10, 11 and 12, we get dynamical parameters of anisotropic fluid in the following functions:

$$p_r(r) = \begin{bmatrix} -\frac{3}{l^2} + \frac{2\sqrt{2}Q^3}{r^3} + \frac{1}{r^2} \\ -\frac{3}{4}\beta^2(\frac{r^2}{l^2} - \frac{M}{r} - \frac{2\sqrt{2}Q^3}{r} \ln(\frac{r}{l}))^{-1} \end{bmatrix}, \quad (22)$$

$$p_\perp(r) = \begin{bmatrix} -\frac{3}{l^2} + \frac{2\sqrt{2}Q^3}{r^3} \\ -\frac{3}{4}\beta^2(\frac{r^2}{l^2} - \frac{M}{r} - \frac{2\sqrt{2}Q^3}{r} \ln(\frac{r}{l}))^{-1} \end{bmatrix}, \quad (23)$$

$$\rho(r) = \begin{bmatrix} \frac{3}{l^2} - \frac{2\sqrt{2}Q^3}{r^3} - \frac{1}{r^2} \\ -\frac{3}{4}\beta^2(\frac{r^2}{l^2} - \frac{M}{r} - \frac{2\sqrt{2}Q^3}{r} \ln(\frac{r}{l}))^{-1} \end{bmatrix}. \quad (24)$$
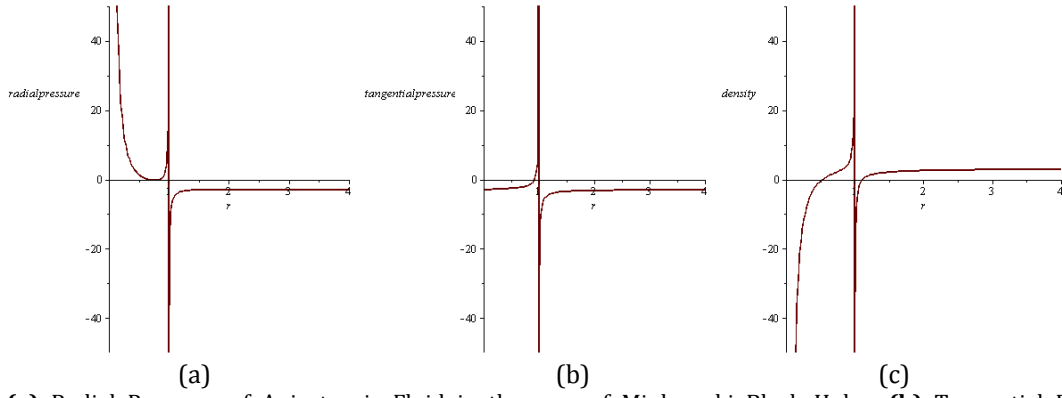
(a) (b) (c)

Figure 3. (a) Radial Pressure of Anisotropic Fluid in the case of Minkowski Black Holes. **(b)** Tangential Pressure of Anisotropic Fluid in the case of Minkowski Black Holes. **(c)** Density of Anisotropic Fluid in the case of Minkowski Black Holes.
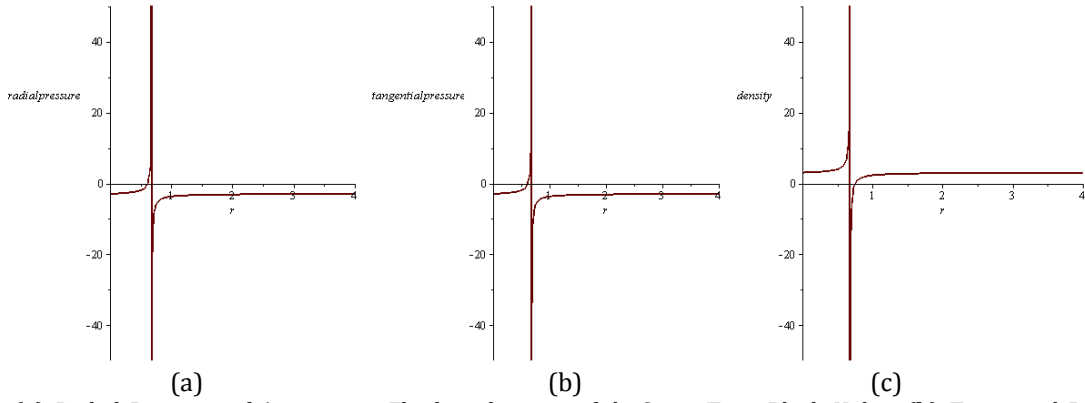

(a) (b) (c)

Figure 4. (a) Radial Pressure of Anisotropic Fluid in the case of de Sitter Type Black Holes. **(b)** Tangential Pressure of Anisotropic Fluid in the case of de Sitter Type Black Holes. **(c)** Density of Anisotropic Fluid in the case of de Sitter Type Black Holes.
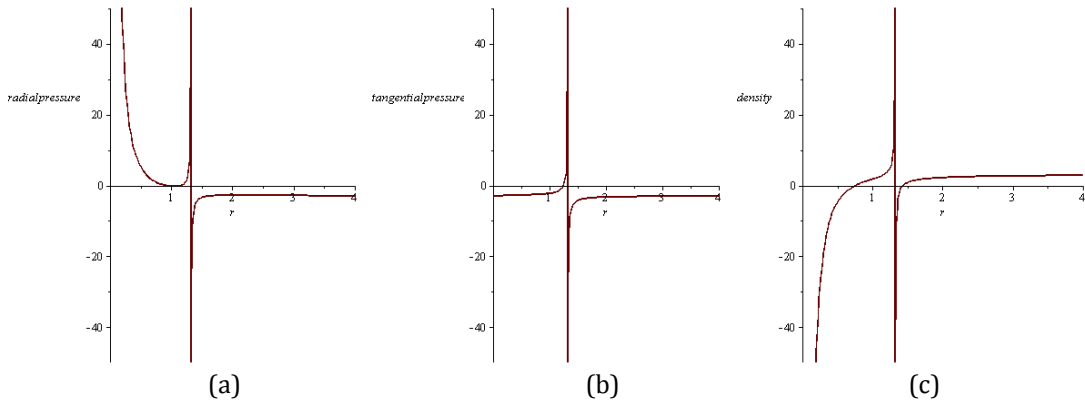

(a) (b) (c)

Figure 5. (a) Radial Pressure of Anisotropic Fluid in the case of Anti-de Sitter Type Black Holes. **(b)** Tangential Pressure of Anisotropic Fluid in the case of Anti-de Sitter Type Black Holes. **(c)** Density of Anisotropic Fluid in the case of Anti-de Sitter Type Black Holes.
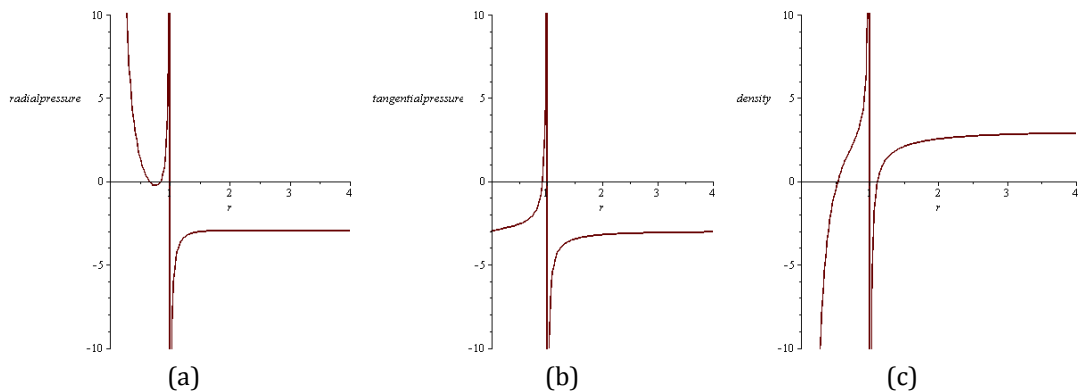

(a) (b) (c)

Figure 6. (a) Radial Pressure of Anisotropic Fluid in the case of BTZ Black Holes. **(b)** Tangential Pressure of Anisotropic Fluid in the case of BTZ Black Holes. **(c)** Density of Anisotropic Fluid in the case of BTZ Black Holes.
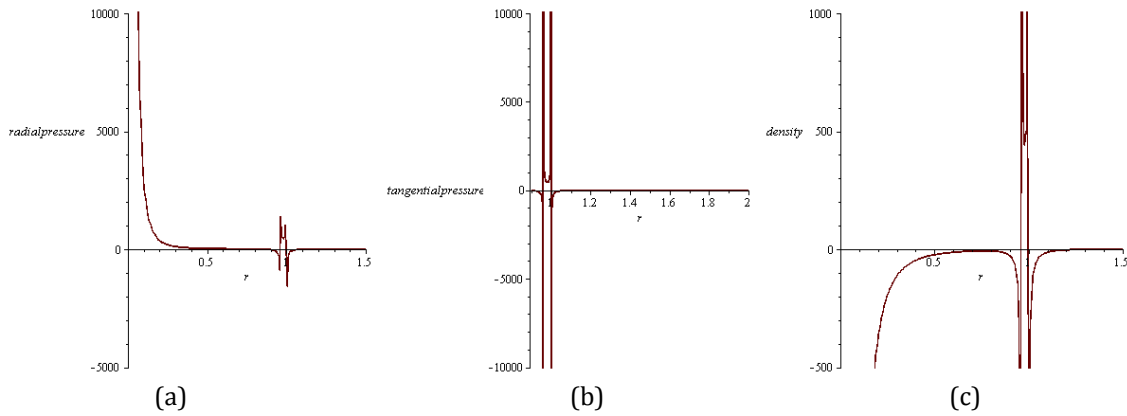
Figure 7. (a) Radial Pressure of Anisotropic Fluid in the case of Charhed BTZ Black Holes. **(b)** Tangential Pressure of Anisotropic Fluid in the case of Charhed BTZ Black Holes. **(c)** Density of Anisotropic Fluid in the case of Charhed BTZ Black Holes.

How dynamical parameters of anisotropic fluid in the cases of BTZ and charged BTZ Black holes range according to radial coordinate are shown in Figure 6 and Figure 7, respectively.

It is seen from Figure 6 and Figure 7 that both density and pressures have negative values a region inner and out of the horizon. This means that the solution denies normal and physically meaningful matter forms in the regions. So, it can be noted that obtained solution can be considered and limited at the region in where $\rho > 0, p_i > 0$ and/or $\rho > 0, p_i < 0$.

4. Discussion and Conclusion

In this paper, distribution of anisotropic fluid as a resource of black holes is investigated in regard to Lyra scalar-tensor theory. Field equations and their solutions are obtained. Results of the solutions can be generally combined in the following summary:

- In the all cases, pressures and density of anisotropic fluid have singularity at the point of geometrical singularity (event horizon).
- Anisotropic fluid must only be stiff matter form for arising Schwarzschild black hole geometry.
- In the cases of Schwarzschild black holes and Reissner-Nordström black holes, obtained solutions can be limited by $0 < r < r_h$ and admitted to interior solutions.
- In the cases of Minkowski type, de Sitter type and Anti-de Sitter type black holes, the exterior solutions sign that fluid acts role of exotic matter.
- In the cases of BTZ black holes and charged BTZ black holes, obtained solution can be considered and limited at the region in where where $\rho > 0$, $p_i > 0$ and/or $\rho > 0$, $p_i < 0$. Obtained solutions support that Lyra scalar-tensor theory can be approved useful gravitation theory.

Acknowledgment

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