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College Science Students' Problem-solving Approaches in the Context of Rectilinear Motion: A Phenomenographic Study

Özden Şengül[1]

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Abstract

This study examined how college science students solved different kinematics problems. The study defined students' problem-solving approaches to reveal the conceptual differences. This qualitative research design used phenomenography to examine college students' responses through an open-ended survey and semi-structured interviews on two different problems. Participants were 179 college science students from the physics, physics education, and engineering departments. The participants were among sophomore, junior, and senior students who had passed the first-year physics courses. Data was collected from the volunteer students by visiting their classrooms and collecting their written responses. The researcher invited some students to interviews to clarify their conceptions. Student responses were thematically coded through exploratory content analysis. The results showed hierarchical categories of students' problem-solving approaches for knowledge level (surface, procedural, deep) and skill type (intuitive, qualitative, mathematical, visual, blended). Some students intuitively performed the derivative operation on the graph, but the mathematical justification was incomplete. Some students exhibited a deep approach. They derived mathematical models based on position-time graphs and made velocity interpretations by associating the concept of algebraic calculations with physical meaning. The research has implications for instructional strategies and curriculum design in introductory physics.

Keywords: kinematics, reasoning, phenomenography, physics education, problem-solving

[1] ozden.sengul@bogazici.edu.tr, Boğaziçi University

Introduction

Reform efforts in science, technology, engineering, and mathematics (STEM) education aim to develop critical thinking and problem-solving skills for adapting to new and complex situations. Physics Education Research (PER) has explored how students understand and apply physics concepts, and revealing consistent misconceptions has necessitated research on how students make sense of physics concepts to enhance curriculum and teaching practices (Beichner et al., 2000). Research studies show that students often succeed with simple problems using mathematical methods and face challenges with complex, open-ended problems. This process entails a continued investigation into how students make sense of physics concepts.

In physics education, the goal is to help students relate physics concepts to real-life problems to develop as expert problem solvers. Problem-solving is a cognitive process requiring multiple approaches to solve non-standard or authentic issues (Heller & Heller, 2010). Researchers designed innovative strategies to promote deeper conceptual understanding and enhance verbal reasoning and dialogue (Maries & Singh, 2023; Reinhard & Felleson, 2022). Problem-solving requires both content knowledge and cognitive, social, and epistemic, as well as metacognitive practices, to engage in constructivist learning processes (Duschl, 2008). Students' metacognitive strategies can help them monitor, evaluate, and adjust their problem-solving approaches to reflective processes beyond the mechanical procedures. According to Miller (2023), problem-solving involves surface or deep-level thinking. Experts tend to present deep conceptual structures with multiple connected and reflective representations. In contrast, novices often rely on mechanical or formulaic approaches without a developed knowledge structure (Heller & Reif, 1984; Kohl & Finkelstein, 2008). Students' ability to engage in metacognitive practices distinguishes expert problem solvers from novices. Experts use a systematic and holistic approach to problem-solving involving planning, calculation, and evaluation. In contrast, novices rely on surface-level, formulaic approaches, struggling to apply their knowledge in complex contexts (Heller & Reif, 1984; Kohl & Finkelstein, 2008; Miller, 2023). Fostering metacognitive skills could help students move from a surface-level approach to more integrated, deep-learning strategies.

Problem-solving approaches of students in physics classrooms have significant implications for improving curriculum design, teaching, and learning. Phenomenography provides a systematic approach to help students experience and understand physics concepts, categorizing their approaches into hierarchical models based on conceptual depth (Marton & Booth, 1997). Prior research has examined students' conceptions of key physics topics, such as confusion between velocity and acceleration (Trowbridge & McDermott, 1980), displacement and velocity vectors (Nguyen & Rebello, 2011), and reference frames in kinematics (Ceuppens et al., 2019) to analyze problem-solving strategies. Clement (1982) also emphasized students' misconceptions about force and acceleration as a force that always maintains motion. Beichner (1994) addressed students' failure to distinguish scalar quantities (speed) from vectorial quantities (velocity). Many studies, such as Miller (2023), discussed expert versus novice problem solvers, but few studies connected problem-solving to phenomenography. The current study aimed to map expert versus novice problem-solving approaches using phenomenographic categories. Moreover, Walsh and colleagues (2007) investigated college physics students' approaches to quantitative and qualitative problem-solving in physics through context-rich problem-solving interviews in a phenomenographic study. Walsh's study explained the categories of students' hierarchical problem-solving strategies: scientific, plug-and-chug, and memory-based. There are other phenomenographic studies in physics education research focusing on mental models of sound propagation based on particle, wave, or blended models (Hrepic et al., 2010), electromotive force in the context of electromagnetic induction (Zuza et al., 2016), and electric and magnetic fields and their relationship with field theory (Zuza et al., 2018). Lastly, Campos et al. (2021) studied students' understanding of electric and magnetic fields and the use of the superposition principle. These studies found that students had conceptual responses and incorrect definitions. For example, some students accurately understood emf in induction through a changing magnetic field that caused an electric field; others discussed the isolated elements from a scientific framework.

These studies revealed how students' conceptualizations shifted depending on context, problem type, and instructional approach. The literature has featured learners' perpetual difficulties eliciting foundational concepts to promote deep conceptual understanding through effective instructional strategies. Some studies addressed students' conceptions as context-dependent in a progression from simple to complex models to report alternative conceptions (Campos et al., 2021; Zuza et al., 2018). Prior research on kinematics has explored alternative conceptions (Beichner, 1994; Trowbridge & McDermott, 1980), but no study has focused on the hierarchical nature of students' problem-solving strategies in rectilinear motion. This study analyzed college science students' problem-solving approaches in kinematics to determine their cognitive approaches. Kinematics understanding often included an analysis of speed, velocity, forces, and energy misconceptions. The current phenomenographic study aimed to contribute to understanding students' problem-solving processes in a Turkish research university with second-year or higher grade physics and engineering students. This study focused on problem-solving in rectilinear motion structured within a phenomenography framework, particularly in a non-Western education context. The study aimed to reveal how students reached the correct answer, established physical meaning, and integrated multiple representations (graphical, verbal, algebraic, etc.). The research questions were: How do college science students solve problems in rectilinear motion? Which cognitive approaches do students use when solving problems? The study aimed to capture the diversity of students' experiences using phenomenography to provide insights into how students qualitatively conceptualize, regulate, and adapt their approaches to solving kinematics problems.

Theoretical Framework

Phenomenography is a qualitative research approach that examines how individuals perceive, experience, and interpret a specific phenomenon (Marton & Booth, 1997). This approach aims to reveal individuals' qualitatively different ways of thinking and understanding the same phenomenon. This framework does not address the types of knowledge individuals possess in a static structure, but it refers to the ways that vary depending on context and experience (Marton, 1992; Marton & Booth, 1997). Phenomenography supports the development of conceptual models to make relationships from abstract to concrete or straightforward to complex phenomena in hierarchically organized categories. This focus on variation and collective meaning-making differentiated phenomenography from other qualitative approaches, such as grounded theory or ethnography, which may prioritize generating a theory or exploring cultural context. Phenomenography is appropriate for examining students' conceptual schemes in diverse physics complexes for curriculum development or instructional practices through interviews, open-ended surveys, or reflective reports (Trigwell, 2012).

This approach aims not to reveal how individuals think about a particular subject but how a group of individuals experience the same phenomenon in different ways. This study preferred phenomenography to examine how students perceive and make sense of specific physics problems (e.g., inclined plane) (Marton, 2004). Although physics education generally stands out with its quantitative and mathematical aspects, revealing students' qualitative ways of thinking about these concepts is critical for instructional interventions. In this context, phenomenography allows the development of categories to contribute to instructional design by determining students' meaning in problem-solving.

This study's phenomenographic approach directly guided the data collection and analysis process. Open-ended physics problems were used in the data collection; students were asked to provide written solutions and oral explanations regarding the problems. The answers were obtained and analyzed based on the principles of phenomenography. For example, the analysis began with a detailed reading of a student's answer and the determination of meaningful units. These units included basic thought patterns that reflected how students perceived the problems. These units were then grouped to form conceptual categories representing qualitative differences in students' problem-solving approaches. These categories were hierarchically organized based on logical relationships describing students' conceptual understanding levels and different solution strategies. This iterative process allowed us to capture how students systematically understood the phenomenon.

Method

The study aimed to utilize a curriculum material entitled “Physics by Inquiry” developed by McDermott and colleagues (1996) in the United States to address students’ misconceptions. McDermott (1991) suggested teaching physics should address students’ alternative conceptions using innovative approaches. In this study, the author chose two problems to examine each student’s approach to rectilinear motion. Students worked individually on these questions that addressed position and speed in different contexts without delving into more complex concepts like acceleration and energy. These questions were essential for college science education students who passed introductory physics courses. These questions helped justify and assess variations in students’ problem-solving approaches. Figure 1 represents these problems.

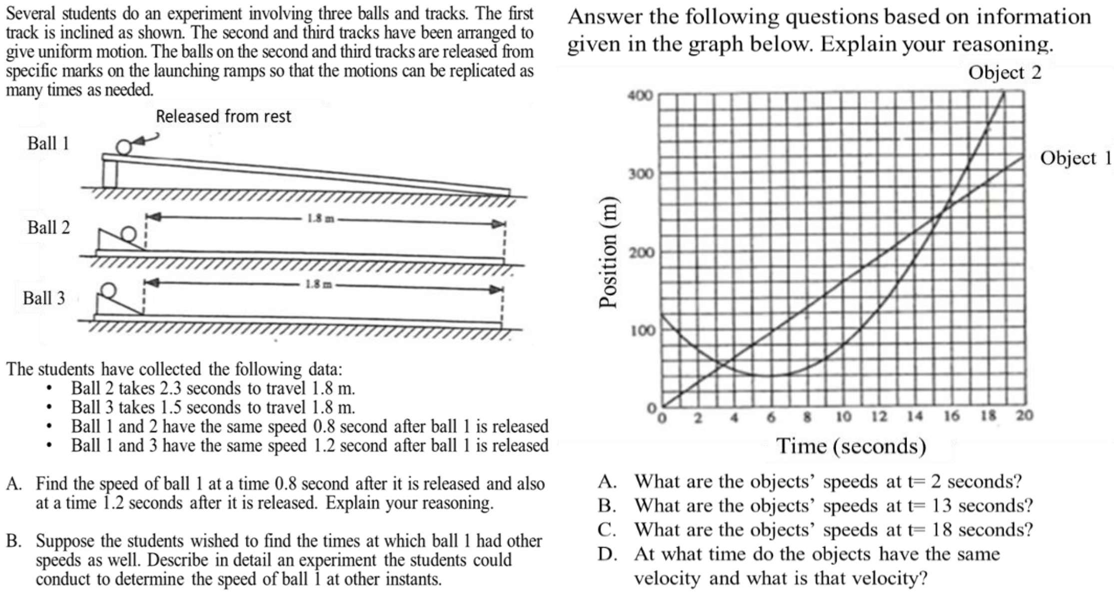

Figure 1. First physics question (at left) and second physics question (at right)

The first question refers to rectilinear motion or motion along a straight line to determine the speeds of balls in three different scenarios based on time and distance travelled. The data is about the motion of balls on three tracks, specifying their speed at given times, and designing an experimental procedure to find the speed of ball one at different time points. The question emphasizes balls two and three’s uniform motion or constant speed compared to ball one. The ball one undergoes accelerated motion due to gravity. The question also asks to describe an experimental procedure or a way to determine the speed of ball one at other time points. Students are expected to verify the experiment by repeating it several times for accuracy and consistency.

The second question gives position-time graphs for objects one and two and involves concepts from rectilinear motion or motion in a straight line. Object one presents a linear position between 0-20 seconds to take 320 meters in the positive direction. However, object two shows a parabolic motion to encourage students to analyze a parabolic position versus time graph. The question asks about the speeds of objects at different time points to guide students’ thinking when reading and working with graphs. Students use the graph to find the slope of the curve for object two at given times and determine the speed when their slopes (rate of change in position) are equal.

The researcher received ethical approval from the researcher’s university since data collection involved human subjects and required addressing human research ethics. The researcher received permission from physics education (PEDU), physics (PHYS), and engineering (ENG) departments to visit the science classrooms and invite students to participate in the study. A total of 179 students (121 male, 58 female) from the PHYS, PEDU, and ENG departments consented to participate in the study at a large public Turkish research university (Table 1). Participating students were sophomore (SOP), junior (JUN), or senior (SEN) grade students who completed first-year physics courses consisting of three hours of lectures (two-hour block and one separate lecture) and a two-hour laboratory component per

Table 1

Demographic information

		SOP	JUN	SEN	total
PHYS	Female	7	16	6	29
	Male	10	22	23	55
ENG	Female	8	2		10
	Male	35	19	3	57
PEDU	Female	1	12	6	19
	Male		4	5	9
Total	Female	16	30	12	58
	Male	45	45	31	121

week. The courses were in English, following a traditional, lecture-based format with teacher-centered strategies and regular assessments. The author collected data from students via an open-ended questionnaire with two questions. The researcher gave the task to students to work on individually in silence. The author aimed to accurately capture individual conceptions and identify qualitative variations in students' experiences. After the initial analysis of the paper-pencil questionnaire, the researcher selected 40 students for semi-structured interviews based on the diverse responses to probe students' conceptual understanding. Interview participants had a range of knowledge and skill approaches. The interviews were conducted separately, audio-taped, and transcribed for analysis. The researcher asked questions to guide the students in explaining their solutions. Sample interview questions were: How do you approach this problem? What did you understand from this question? Which physics topics do you need to consider when dealing with this problem? Participating students were encouraged to analyze and revise their solutions. The interviews did not have a time limit but ended when students said they would not continue.

Students' responses were analysed using a phenomenographic approach to explore how students conceptualized position and speed in rectilinear motion. In this study, the researcher examined students' problem-solving approaches in two dimensions: 1) Problem-solving levels (surface, procedural, deep) and 2) Cognitive skills (intuitive, qualitative, mathematical, visual, blended). The researcher thematically coded with exploratory content analysis of the expressions, conceptual transitions, and justification styles (Elo & Kyngas, 2008). First, the author developed categories to reflect the various levels of understanding and problem-solving strategies, focusing on surface, procedural, and deep knowledge. Each level defined the students' understanding of the problem, the solution process's structuring, and the solution's conceptual integrity. For example, the study wrote a formula at the surface level, used the formula directly at the procedural level, and related the formula to the conceptual explanation at the deep level. Table 2 shows definitions and examples for each knowledge level. Second, students' cognitive skills demonstrated their solutions in five categories. These categories determined whether students made intuitive inferences during the solution process, addressed their conceptual thinking levels, resorted to mathematical relationships, used visual materials, and blended skills. For example, when a student drew both a graph and made a quantitative explanation together, this was coded as a blended skill. Table 3 shows definitions and examples for these skill categories. Table 4 also represents how intuitive skills are more at the surface level, while mathematical or blended skills are in deep-level student solutions. This representation revealed that students used more diverse and conceptual skills as their solution level increased. This categorization was hierarchical, with more sophisticated conceptions of position and speed placed higher to conceptualize patterns and concepts in linear motion.

Consistency in categorization was ensured through discussion and agreement by two independent researchers on final descriptions, considering the entire answer rather than isolated statements. This iterative process allowed the author to capture how individuals systematically understood the phenomenon. Two raters, the author and a graduate student with expertise in science education, analysed 50 randomly chosen students' responses to establish consistency in the analysis process. The graduate students solved the questions, evaluated the appropriateness of categories, and discussed possible student responses as an expert outside the research team. The raters conducted the analysis independently to compare the categories and resolve disagreements. The interrater reliability agreement was 90% for both questions. The author addressed the conceptual validity by identifying themes from the codes and supporting them with representative quotes from student interviews.

Table 2
Students' problem-solving levels

Category	Definition	Sample Student Response
Surface	Students provided unstructured problem solving with separate memory-based approaches or rote memorization, and did not understand concepts.	"I feel like the object is speeding up as it goes down."
Procedural	Students tended to employ step-by-step procedures with the formula-based solution, but could not respond comprehensively.	"If there is no initial velocity, I used the formula v = a × t, as the period increases, its speed increases according to constant acceleration."
Deep	Students tended to have an extensive and integrated approach using diverse methods.	"When I drew the graph, the slope was constant, which indicates constant acceleration. If the acceleration is constant, we can explain the accelerating motion."

Table 3
Students' cognitive skill categories

Cognitive Skill	Definition	Sample Student Response
Intuitive	Students made explanations referring to their daily experiences, intuitions, and observations.	"It goes faster when the ball goes down because I feel like it."
Qualitative	Students made verbal explanations of concepts through meaningful experiences.	"The ball is accelerated as it moves down due to the constant gravitational acceleration."
Mathematical	Students used formulas and made explanations based on the quantitative relationships.	"I used v= a*t, as its initial velocity is zero, the ball's velocity is speeding up."
Visual	Students used graphs, figures, or diagrams to make explanations.	"The position-time graph is linear, its velocity is constant, and acceleration is zero."
Blended	Students used more than one of the skills together.	"The object is falling with constant acceleration. I calculated it with v= a*t, which can be shown graphically."

Table 4
Number of student responses in each category for both questions

Category	Knowledge					
Skill	Surface		Procedural		Deep	
	Question 1	Question 2	Question 1	Question 2	Question 1	Question 2
Intuitive	6	43	0	13	0	0
Qualitative	17	8	38	1	7	0
Mathematical	63	29	4	18	8	53
Visual	8	1	2	0	3	0
Blended	4	4	13	7	6	2
Total	98	85	57	39	24	55

Findings

The results show distinct patterns in students' problem-solving approaches involving the velocity concept in rectilinear motion. Table 4 shows a hierarchical set of approaches to problem-solving for both questions, referring to the number of students. These categories focus on students' written responses: surface, procedural, and deep knowledge. While surface and procedural approaches were more common, a smaller group of students demonstrated deep expertise in problem-solving. The responses were classified according to the type of cognitive skills and the level of problem-solving that these skills were associated with. For example, the intuitive skill was mainly used at the surface level, especially in question two, with 43 cases. Students never used intuitive skills at a deep level. This result showed that students mainly preferred intuitive skills in surface problem-solving approaches and were not used in situations requiring deep understanding. Qualitative skills were primarily seen at the procedural level, especially in question one, with 38 cases; there were quite a few evidence cases at the surface level. This result showed that students used qualitative explanations at both procedural and surface levels, but less in deep solution expertise. Moreover, mathematical skills were extensively used at the surface (63 cases in question one), procedural (18 cases in question two), and deep (53 cases in question two) levels. Mathematical skills were the most preferred at the deep level, as students used mathematical operations with a deep understanding. Visual skill was limited primarily at the surface and deep levels, and the general frequency of using this skill was low. However, it was especially noticeable at the deep level (3 cases, question one), which indicated the students' preference for visual representations at the deep level. Lastly, the blended skill was presented at all three levels, although it was used most frequently at the procedural level (20 cases total). The blended approach also played a significant role at the deep level (6 cases in question one). With this blended approach, students utilized integrated skills with more advanced strategies and multiple approaches to produce solutions. These findings illustrated that skills were hierarchically related to problem-solving levels: 1) With the surface approach, students essentially used intuitive, mathematical, and qualitative skills. 2) With a procedural approach, students utilized qualitative, blended, and mathematical skills. 3) With deep knowledge, students engaged in mathematical, blended, and visual skills.

Surface Knowledge

Surface knowledge refers to students' understanding of rectilinear motion concepts through recalling facts or definitions without applying them to new situations. In this study, students' approaches to speed varied: some students had intuitive approaches, while others sought solutions with qualitative explanations or simple mathematical operations. However, these approaches generally lacked deep analysis. In both questions, students remembered basic definitions and displayed limited calculations, although they used different forms of reasoning (intuitive, qualitative, mathematical). This situation showed that students could not handle the concept of speed in conceptual integrity, but in a fragmented manner.

An example of a student's work with a surface approach to the first problem is presented below:

> Interviewer: What do you think about this question with the inclined plane?
>
> Student: Since they have the same length, their speeds may differ due to their height. The second one is shorter, so I could proportion it accordingly. Ball one will continue to accelerate.
>
> Interviewer: For ball one and ball two, the questions included ball one and ball three.
>
> Student: Balls one and two have the same speed at 0.8 sec; ball one's speed at 1.2 sec equals ball three. Because the speed of ball two and ball three is uniform.
>
> Interviewer: How about the second part? How can we find the speed at other times for ball one?
>
> Student: We can change their height. Does the fact that the masses are different change anything? It does not change the acceleration; it changes the sum of the kinetic energy, so it does not affect the height.

This student recognized the concept of uniform motion for balls two and three, but did not demonstrate a clear understanding of its relationship with rectilinear motion. For instance, while the student identified uniform velocity, the student failed to elaborate on the conditions that led to this motion, such as the absence of acceleration for these balls or the role of gravitational force along the inclined plane. The student's statements, like "The second one is shorter, so I could proportion accordingly," did not reflect an understanding of the relationship between height, acceleration, and velocity since the student used a surface-level proportional intuition. However, the student assumed a linear relationship existed between height and velocity, and the effect of acceleration or the equations of motion was ignored. The students did not relate how the velocity of ball one changes over time due to acceleration or how energy principles were connected to the motion of objects. This result showed that the student operated intuitively and with surface reasoning but did not fully understand the physical principles. The student used the formula $V=V_o+at$, but could not contextualize it and did not explain how it was caused by the net force along the incline tied to rectilinear motion. This finding showed that the student used limited procedural knowledge.

Another student's work with a surface approach to the second problem was as follows:

> Interviewer: Can you explain your solution to the second problem?
>
> Student: Speeds are asked for at certain times for A, B, and C options. To find the speeds, we are given a position-time graph. We know the equation x= v*t. From this graph, we can interpret this as follows. The slope of the position-time graph gives us the velocity. Here, if we find the slopes of the curves given in the graph, we can find their velocities at any time we want. Object one is given as a line. If we choose any two points and divide the difference by y by x, we can find the slope of object one. It passes through the point (0,0), and if we find another point where it passes, we need to find it. For example, (320,20). 320/20 gives us 16 m/sec, which is our slope. In other words, if we write this line equation as y=mx + k, y=16x, x is time, and y is position. The slope of this graph always gives us 16. Therefore, in 2, 13, and 18 seconds, the speed of object one is the same, 16 m/sec. I see no line for object two but a quadratic structure. For this, I can find two points and perform similar operations.
>
> Student: The minimum point here is (6,40). It goes through the (0,120) points. Likewise, it passes through the (12,100) point. Using these, we can find the equation. We can arrive at velocity by finding the position equation and its derivative. It is written as $y= ax^2 + bx + c$. Here is how: if we use the points and get c = 120 in the equation and substitute the other points in the equation, we can solve the equation. If we write the equation a= 80/36= 20/9, we can find the equation $y= 20/9\ x^2 - 80/3x +120$ in this way. If we take the derivative of the position-time equation, we will get the velocity and be able to write and find the time values we want.
>
> Student: t= 2, v= -160/9; t= 13, v= 280/9; t= 18, v= 160/3. Since we have already found the equation for option D, if we equate the speed equations for both objects, then 40/9x - 80/3=16, t= -160/11.
>
> Interviewer: Can time be minus?
>
> Student: Their speeds may not be intersecting. I may have made a transaction error. 40/9x=128/3, and x=9.6 sec. When the velocities of two objects are congruent, their velocity is 16 m/sec.

Using basic mathematical operations, the student used the position-time graph to compare linear and nonlinear motions and determine the velocities. The student used familiar mathematical procedures rather than establishing cause-and-effect relationships among fundamental concepts such as the nature of motion, acceleration, constant velocity, or changing velocity. The student demonstrated qualitative reasoning by explaining the concept of velocity for rectilinear motion, x= v*t, to calculate slopes. The student correctly identified that the slope of the graph corresponds to the velocity and directly applied this information to determine the velocity of object one as constant. The student could explain the linear graph for object one at the surface level, but struggled to explain the parabolic relationship. He did not place this information in a deeper conceptual framework; for example, he did not explain why a linear graph meant constant velocity, since the student related the slope to velocity and did not emphasize the underlying meaning of this relationship. The explanations for the motion of object two were more qualitative; the student realized that the motion was nonlinear and used the points on the graph to arrive at a quadratic equation. This operation was related to mathematical knowledge at the surface level because the student's goal was to reach the velocity information using the function's formal properties. However, the student did not conceptually discuss the physical meaning of the velocity change over time (such as the effect of acceleration, the speed being zero at the minimum point, etc.). The student's explanations were incomplete since the student did not explain why a parabolic position-time curve was related to continuous velocity changes. Moreover, the student explained the time at which objects would be at the same velocity imprecisely by verbally relating to graphical features. The student equated the velocity functions and made an algebraic solution. The student made a qualitative explanation to emphasize the calculation error. The student's solution showed mathematical skills, although the student had limited conceptual awareness of the physical meaning of negative time values. The student could not establish a relationship with the experimental context to integrate concepts about constant velocity, changing velocity due to acceleration, and physical time constraints.

Procedural Knowledge

Procedural knowledge addresses the procedures by which students perform specific tasks. Students' responses typically answer "how" questions to accomplish the task in steps. Students manipulate data and apply some skills methodically through definitions, equations, and graphical interpretations. This study revealed how students expressed their conceptual and graphical understanding of linear motion through qualitative, visual, and mathematical skills. For the first question, students exhibited multifaceted approaches in explaining the instantaneous velocity by interpreting the slope of position-time graphs, applying the equations of motion, and creating visual representations. For the second question, students used procedural knowledge with the limited use of mathematical, intuitive, and blended skills. Students explained basic concepts but had difficulty accurately elaborating graphical analyses and constructing mathematical expressions.

An example of student work with a procedural approach to the first problem is presented below:

> Interviewer: What do you think about these questions?
>
> Student: Option A has three cases. In case one, the ball is released from the head of a flat ramp. In case two, there is a shorter ramp, where it is dropped from the middle. The third case is dropped from the top of the small ramp. In the first part, the ball asks about the velocities of ball one at 0.8 and 1.2 seconds. It gives us some information: it says that balls one and two have the same speed at 0.8 seconds; balls one and three have the same speed at 1.2 seconds. Because ball two has a constant speed, its speed is always the same if it goes 1.8 m in 2.3 seconds. If V_1 at 0.8 sec is the same as V_2, I equate the two speeds. Since ball three has a constant speed, its speed is always the same if it goes 1.8 m in 1.5 seconds. If V_1 at 1.2 seconds is the same as V_3, I will equate the two speeds.
>
> Interviewer: What do you think of option B?
>
> Student: I can design a setup that takes pictures every second. This can be an experiment with a photo-shooter, which takes a picture of the ball every second. I can capture how far ball one travels every second with this camera. Students will be able to find the ball's displacement at each second. We can then reach its speed by finding the path the ball one takes every second.

The response to the first question represents procedural steps in solving the rectilinear motion problem. The student described the balls' velocities and relationships based on the data. The student calculated the velocities of balls two and three using the $V= \Delta x / \Delta t$ equations. For example, the student explained that ball three moved constantly or uniformly, taking 1.8 meters in 1.5 seconds. The student equates the speeds for balls one and two and balls one and three at given time points. This method showed that the student exhibited a procedural-qualitative approach, focusing on explaining the physical concepts underlying operations in the physical system. In contrast, the application of these operations was limited. For example, the student stated that ball two moved at constant speed without referring to physical factors such as slope, friction, initial conditions, or acceleration effects. Additionally, when finding the speed of ball one by equating speeds, the student did not discuss why these speeds should be equal. The student applied the computational procedure but did not establish cause-and-effect relationships between concepts. For the second part of the question, the student could present an experimental design with a photo-shooter to record different instants. These explanations presented students' qualitative understanding of analysing motion through observation and measurement. The student applies logical reasoning but does not discuss the meaning and relationships of these operations in the physical context, showing that the knowledge level remained at the procedural-qualitative border.

Another student's work with a procedural approach to the second problem is presented below:

> Interviewer: What do you think about question two, the graph problem?
>
> Student: … I used to find the position change in this time interval for object two, divide it by the time square, and make x/t^2. However, since the graph is given here, I made them in such a tangent way. However, if it were used to be, I would have done it as x/t^2. It may not give a neat result. So the tangent felt more accurate...
>
> Interviewer: What did you calculate?
>
> Student: I found something like 40/9. I drew a line at two points of this curved line and took the derivative. In the flat one, whichever corresponds directly to 2 in the first object, I found the ratio by dividing it by 100 in 6 seconds. It is not quite neat...
>
> Interviewer: Did the slope 40/9 come from a tangent?
>
> Student: I think so. It says 13 seconds; it can be found by pulling a tangent from this point again. Object one always goes the same… Tangents change object two. If we find the equation, we can draw the velocity-time graph by taking its derivative.
>
> Interviewer: … And what about t= 18?
>
> Student: I already said that object one is constant; how to draw a tangent in object two is the exact endpoint. If it is drawn like this, 38 is found.
>
> Interviewer: Would you like to solve the question in another way now?
>
> Student: When I come from t=18 sec, I find the triangle's slope formed. This slope gives its speed at 18 seconds. Let me look at option D. At the points where they intersect, they are in a linear position, but their slopes may not be the same. Their speed may be different. I found the speed of object one to be 100/6 from y/x. Any tangent corresponding to 100/6 indicates that object one equals object two. Any tangent of the object two should correspond to 100/6… We already know that object one is stationary; we expect the tangent of object two to be the same.
>
> Interviewer: You can make corrections.
>
> Student: We know that object one is the same. 320/20 = 16 m/s. The velocity of object one is constant. I can go off on tangents in others. They have the same speed since it is the same tangent between 8 and 11 seconds.

This interview showed that the student made intuitive decisions using his procedural knowledge. The student prioritized intuitive evaluations over mathematical calculations while interpreting the graphical slopes. The student used the slope of the tangent line at various points to find the instantaneous velocity for object two. The tangent strategy illustrated the student's procedural-intuitive approach. The student interpreted the meaning of physical representations instead of applying formulas. For example, the student calculated the average slope by drawing a line to two points on the curve, evaluated this method as inadequate, and tried to reach the instantaneous speed by drawing a tangent. The student recognized the significance of tangents for non-linear motion but struggled with imprecise graph drawing and memory-based procedures; the student changed the problem-solving method based on the problem context by making intuitive solutions. However, the student did not provide a conceptual explanation to clarify why taking a tangent was more appropriate. The student understood the uniform rectilinear motion for object one and calculated slopes to approximate speeds for objects one and two. The student's attempt to write and differentiate a position-time equation demonstrated procedural application, though there was imprecision due to explanations based on intuition rather than a conceptual basis.

Deep Knowledge

Students in the deep knowledge category demonstrate profound and interconnected approaches to rectilinear motion concepts. Students can integrate multiple physics concepts, higher-order thinking, and innovative techniques to analyze and solve problems. Students used qualitative, visual, and mathematical approaches in various combinations in kinematics problems. Applications such as determining instantaneous velocity with position-time graphs, using motion sensors, and deriving equations with differential calculations showed that students demonstrated deep and multi-dimensional reasoning through mathematical, graphical, and qualitative explanations. These findings increased the study's contribution by reflecting the processes of students making sense of physical concepts through different cognitive pathways.

A student's work with a deep approach to the first problem is given below:

> Interviewer: What can you say about your solution?
>
> Student: What do we know here? We know how long the ball took to get this distance at this rate. If we know the ball's mass, they are all identical, so that we can say m. mgh = $0.5m*V^2$; whatever h_2 and h_3 are, these speeds will be accordingly. We know these speeds and can also find them from this height. It was said what speed the ball reached after 0.8 seconds; after this height, if the first ball reached the speed of the third ball in 1.2 seconds, it was released from a different height.
>
> Interviewer: How about the second part?
>
> Student: We find the V's; we can also find the height by calculating from the information given. We may need to add a fourth or fifth ball to the experiment by dropping them from different heights. How can we understand that they are at the same speed? … With our fundamental knowledge of physics. We know our potential energy, its behavior at two instants, and its speed. Knowing what position and speed you are in is knowing some conditions. We can find a curve for future positions based on the time graph.
>
> Interviewer: How will that graph help? What can you find by looking at the graph?
>
> Student: We can calculate the projection of the graph; its tangent at any point will give us instant velocity. We can do this by deriving.
>
> Interviewer: You drew the x-t graph as a curve; what do you think of the v-t graph?
>
> Student: We will take the derivative of the graph; our velocity will increase with time, and the acceleration will be constant over time. Concerning the initial velocity, I can write the velocity equation as $V=V_o + at$. The setup can be understood as an experiment.

The student's solution process is quite strong in mastering conceptual knowledge and integrating this knowledge with quantitative operations. The student demonstrated a blended approach, including qualitative, mathematical, and visual skills, to relate potential energy and height to velocity by establishing a logical chain consistent with physical principles. For example, the student showed the energy transformation and its relationship to motion using "$mgh = 0.5mv^2$". The student expressed how the ball's velocity dropped from different heights and was related to potential energy. The student utilized quantitative calculations and developed cause-and-effect relationships by establishing a conceptual connection between energy-velocity-height concepts. The student explained the relationship between the height and velocity of the ball at specific time points. The student also incorporated mathematical reasoning to indicate the kinematic equation $V= V_o + at$ and refer to linear motion with constant acceleration. The student also integrated equations to explain each step of the solution. Then, the student drew graphs to explain how the velocity of ball one changed at each time point. These explanations showed that the student processed the information not by rote but by establishing relationships between concepts. This was a distinctive feature of the deep-blended approach. For the second part of the question, the student illustrated an experimental thinking approach based on observation. The student made practical explanations since the student foresaw new situations and visualized experimental scenarios. This solution revealed that the student could switch between theoretical knowledge and experimental thinking.

A student's work with a deep approach to the second question is provided below:

> Interviewer: What is your approach to this question?
>
> Student: I wrote down the functions and tried to find their speed from their derivatives. First, there is a linear graph. For the second, I wrote a parabolic equation. Such an approach made sense. Since velocity is a derivative of a positive-time graph, we can find a function for graphs and take the derivative to find instantaneous velocity.
>
> Interviewer: How did you write the equation?
>
> Student: Sir, there was a peak here. On that vertex, a(x-6) 2 + b. I found a and b by choosing different points. It sounds like [20/9* (x-6) 2 + 40]. I took the derivative of their velocity and found their velocity for t = 2, 13, and 18 seconds. I found the velocity of object one, which is also constant for D, to be 9.6 seconds by equating it to object two since V_2= 40/9*(t-6). So, I can show the velocity at different time points in a table.

t (seconds)	2	13	18
V_1 m/s	16	16	16
V_2 m/s	-160/9	280/9	160/3

> Interviewer: Can the graph show that speeds are equal in 9.6 seconds without using the equation?
>
> Student: The derivative is drawn as a tangent. When the speeds are equal, their slopes must also be equal when we find that the tangents of objects one and two are parallel. Interpreting the graph can be tricky.

This solution highlights a deep understanding of rectilinear motion by accurately applying derivatives to connect position-time graphs to velocity-time equations. The student connected mathematical reasoning and the graphical representation of motion. The student followed the procedural steps and related these steps to concepts to explain the reasoning. The student conceptually explained how to obtain velocity by differentiation. The student conceptually demonstrated that velocity could be obtained from the derivative of the position-time function. The student could establish a direct relationship between graphs and physical quantities. The student also used a parabola to find the unknown coefficients using mathematical skills. This process indicated how the students established the algebraic function based on the graphical features to make transitions between concepts. The student also drew the table for the speed values at different time points and calculated the time point when two objects had the same velocity. The student made a comparative analysis to find the equal velocity time. The student integrated various motion representations into the solution to establish their relationship. For example, the student established a clear relationship between the slope as the graphical equivalent of speed and the derivative as the algebraic equivalent. The student aimed to obtain mathematical accuracy and conceptual meaning in the solution process.

Discussion, Conclusion, and Suggestions

Problem-solving is essential in physics courses to assess students' learning formatively. This study illustrated college science students' problem-solving processes based on cognitive skills (intuitive, qualitative, mathematical, visual, blended) and the knowledge levels associated with these skills (surface, procedural, deep). These findings indicated that the students used intuitive and qualitative skills at the surface level. In contrast, the students exhibited more mathematical, integrated, and blended approaches at the deep level. Students often relied on procedural or surface-level approaches, particularly when solving problems with mathematical calculations. Surface-level strategies demonstrated students' linear and disconnected approaches. Limited intuitive skills caused students to make incorrect assumptions in the problem-solving process. Only a few students demonstrated a blended skill approach by combining qualitative reasoning with mathematical modeling or visual representation to provide deep and integrated explanations. Blended skills allowed students to gain knowledge, structure, and synthesize this knowledge with different representations. Students used mathematical expressions and formulas more effectively when solving physical problems with a deep approach. This result showed that students' ability to do abstract mathematical modeling increased by going beyond just physical representations (visual or intuitive). For example, students' expression of the relationship between velocity, acceleration, and time with equations while solving kinematic problems requires a more profound understanding than a surface approach. In particular, students with a deep-blended approach could relate physical conditions to mathematical models and associate graphical representations with derivative concepts to establish cause-and-effect relationships during the solution process. In contrast, students with a procedural-intuitive approach had cognitive tendencies to process graphical information without reaching a whole conceptual level, but by developing physical intuition.

These findings showed that problem-solving processes were related to computational accuracy and conceptual representation. Students should develop a conceptual understanding and explain why specific strategies work in certain situations. Van Heuvelen (1991) argued that physicists tended to begin with a qualitative approach to constructing visual representations. In this study, college science students did not show a comprehensive or integrated approach to combine skill types and display a complete solution and sense-making (Bollen et al., 2017; Van Heuvelen, 1991). Only a few students in this study had a blended approach with a deep knowledge level to explain their sense-making processes. There were fewer visual or modeling-based approaches in both questions; students tended to depend on mathematical calculations. These results were consistent with the studies of Van Heuvelen (1991) that emphasized the significance of students' ability to switch between conceptual and mathematical representations in solving physics problems. Docktor and Mestre (2014) also addressed the students' tendency to integrate different forms of representations at the deep level. The use of blended skills in our study also supported this approach and showed that students develop and use knowledge by structuring it differently (Clement, 2000; Suthers, 2006). The failure to associate intuitive skills with the deep level is consistent with the findings of DiSessa (1993) that conceptual pieces are often limited to the surface knowledge level. This situation showed that these intuitive structures needed to be structured in the teaching processes to increase the effectiveness of students' prior knowledge. Additionally, the skill-level pattern obtained in this study can be associated with the concept of "disciplinary discourse," argued by Airey and Linder (2009) with the phenomenographic approach. How students solve problems reflects their access to physics knowledge, which also helps explain when and how the students used which representations. These findings are supported by a phenomenographic study titled Fredlund et al. (2012), since there was a relationship between students' ability to switch between skills and their knowledge levels.

The association between blended skill and deep level showed the significance of representational diversity and integrative strategies in instructional design. The findings revealed that using multiple representations in problem-solving processes improved students' cognitive skills (Suthers, 2006). Mathematical, visual, and intuitive representations increased students' analytical and conceptual thinking skills while solving problems. Moreover, effective problem-solving requires mathematical skills and epistemological awareness to make conceptual connections (Hammer, 2000). Epistemological resources examine the explanatory power of understanding the variability in students' responses, which may change depending on context (Elby & Hammer, 2010). For example, physics knowledge can be defined in a continuum from transmissionist to constructivist: knowledge as propagated stuff passed from a source to a recipient or as constructed and built from other knowledge. The constructivist approach closely aligns with the deep-blended category, while the transmissionist approach refers to surface knowledge with the mathematical or intuitive skill category. Although a few students engaged in the visual approach in this study, they preferred blending visual skills with other skill types. The lowest hierarchical category was the "surface-intuitive" category. Students tended to have a memory-based approach to predict the phenomenon without any exact solution. A study by Tuminaro and Redish (2007) addressed students' moves in the epistemic game that might involve a continuum from mathematics to pictorial to intuition skill types.

In this exploratory and descriptive study, the author recommends interpreting the findings within the context of its limitations. The analysis focuses on two kinematics problems, and we may not generalize the results to other physics topics in different instructional contexts. This study identifies a subset of skills observed in this context, which may provide a basis for future research. Moreover, the author does not collect data on other factors (demographic information, prior knowledge, etc.) that may affect students' problem-solving approaches. Therefore, future research may investigate how other factors, such as students' epistemological resources, metacognitive strategies, and physics identities, influence their problem-solving approaches. Longitudinal studies may reveal how students' approaches evolve over time and in response to different instructional designs. Exploring other factors of students' resistance to solving physics problems in other contexts may provide an understanding of problem-solving strategies. Another factor might be related to students' prior knowledge or grades from first-year courses, although this study aimed to understand how students frame their knowledge and skills while solving these problems from an innovative curriculum.

In some cases, students' responses in open-ended tasks were in novice format, but their solutions became expert-like when they solved the problem through an interview. The interview approach was more helpful in guiding students to be more reflective about their solutions. More students had a surface-knowledge approach and struggled to explain their solutions. Therefore, students should have opportunities to work on different problem contexts to activate their thinking in various situations, rather than focusing on physics concepts as accurate and stable facts. Furthermore, previous research in physics education represented students' problem-solving approaches by focusing on students' epistemic beliefs, metacognitive skills, and cognitive frameworks. For example, Hazari et al. (2010) revealed the impact of high school students' metacognitive and cognitive skills on their self-efficacy and physics identity. The findings of this study similarly showed that the cognitive and metacognitive strategies used by college science students were diverse and that these strategies were closely related to the students' knowledge levels. Students might resort to their intuition, use visual aids, monitor qualitative reasoning, use mathematical models, or develop strategic approaches by integrating different skills. While students with profound knowledge engaged in metacognitive activities, students with surface knowledge tended to engage less in such activities. Their use of different skills might differ based on the context to decide how to solve the problem. In this context, this study underlines the relationship between the cognitive strategies in problem-solving and their knowledge levels while revealing that teaching processes should be structured to diversify and deepen these strategies. Teachers or instructors play a key role in advancing students' cognitive skills to a higher level by encouraging the use of multiple representations and strategies. These results offer important implications for curriculum development and assessment practices since students need learning environments to develop procedural knowledge and the skills to transform, synthesize, and switch between representations. This approach will provide innovative and transformative contributions to instructional design and assessment procedures.

This result also showed that students who passed the first-year courses might not acquire the necessary skills and knowledge to solve the problems. Students used distinctly different skills at different knowledge levels: While intuitive and qualitative approaches were dominant at the surface level, mathematical and blended skills were related to the deep level. Students' problem-solving approach involves multiple cognitive problem-solving strategies, and teachers should analyze the correct answer and the skills and knowledge level with which students reach the answer. Therefore, this study recommends examining students' knowledge levels via context-dependent problems. Teachers should develop strategies that integrate these skills with mathematical and visual representations to support students in thinking at deeper levels. Students' skill type might be related to the kind of instruction that they received in the first-year physics courses. Collecting data about the modeling approaches instructors used in the first-year physics courses is essential; the problem-solving process should be considered as a process in which different skills work together, or the use of multiple representations should be encouraged in the lessons. Blended skills emphasize the importance of developing students' skills in transforming and synthesizing representations. These skills can be supported by content teaching and increasing metacognitive awareness. In college physics courses, students should learn how to use their metacognitive strategies to monitor and evaluate their ability and approach to solving a problem.

The instructors should provide students with the necessary assessments to monitor and evaluate their thinking. In recent years, innovative teaching approaches have assisted students in engaging in interactive dialogue to explain their reasoning differently, seek solutions, and compare ideas. For example, studio physics, peer instruction, think-aloud protocols, and context-rich problem-solving tasks may allow students to evaluate their reasoning and explore alternative approaches. Classroom assessments should also guide students to challenge and reveal their ideas by utilizing different models and solving problems with a systems thinking approach. These results emphasize the significance of instructional design or innovative curriculum materials to promote integrated reasoning (Beichner et al., 2000). These suggestions also call for the training of college science instructors on developing and using innovative curriculum materials to address students' metacognitive and cognitive knowledge and facilitate their active thinking. Further research should facilitate the cycling transition between cognitive and metacognitive approaches to develop problem-solving expertise.

As a result, this study combines a phenomenographic approach with a multi-layered analysis to explain ways of solving physics problems on the right-wrong plane and with skill-knowledge level patterns. This approach offers an original contribution to the cognitive skill-comprehension level match, which is seen only in a limited way in the literature. In this respect, it provides a new perspective on instructional design and assessment practice. These results provide insights into challenges and opportunities to explore the role of different methods (experiences, resources, etc.) in promoting students' deep understanding of problem-solving. Physics instructors and curriculum designers may benefit from strategies promoting metacognitive engagement and skill integration, helping students build practical and reflective problem-solving tools.

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