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EVALUATION OF GRINDING PARAMETERS AND MICROWAVE PRE-TREATMENT ON THE BREAKAGE BEHAVIOR AND ENERGY EFFICIENCY OF COLEMANITE ORE

Serdar YILMAZ [1], **Mehmet BİLEN [1,*]**

[1] *Zonguldak Bülent Ecevit University, Mining Engineering Department, Zonguldak, Türkiye*

**** Corresponding Author:*** *mehmet.bilen@beun.edu.tr*

ABSTRACT

This research systematically examined the grindability behavior of colemanite ore by analyzing how variations in mill rotation speed and the geometry of grinding media influence the specific breakage rate. The ore, sourced from the Emet Colemanite Mine, was separated into narrow particle size fractions and subjected to grinding tests at speeds between 65% and 90% of the mill's critical velocity. Findings indicated that higher rotational speeds resulted in increased specific breakage rates and finer particle size distributions. Moreover, cylindrical media consistently delivered better grinding performance than spherical media across all particle size intervals tested.

A predictive model was developed to estimate the specific breakage rate based on particle size, rotation speed, and media shape, and was further validated under various operational conditions. Furthermore, the study examined the impact of microwave pre-treatment on colemanite grindability and energy consumption. Microwave irradiation was applied at 180 W for 5 minutes, resulting in internal thermal stress and microcracking that facilitated improved breakage, particularly in coarser fractions. Bond Work Index (BWI) values decreased by approximately 3–4% following microwave treatment—for instance, from 9.02 kWh/ton to 8.72 kWh/ton for the +3350 µm size fraction. However, the π breakage parameter remained virtually unchanged, at approximately 0.53 for both treated and untreated samples, indicating that the fundamental breakage characteristics of the material were not significantly altered.

Although microwave pre-treatment enhanced grindability in specific cases, the total energy requirement for comminution—including both microwave exposure and subsequent grinding—did not demonstrate a substantial reduction. These findings suggest that the energy benefits of microwave-assisted grinding are marginal unless the microwave process itself is optimized for energy efficiency.

Keywords: Colemanite, Specific breakage rate, Grinding media shape, Mill rotational speed, Microwave pre-treatment.

1 INTRODUCTION

Turkiye holds more than 60% of the world's boron reserves, making it the leading country in terms of global boron resources. Boron compounds are extensively utilized across nearly all industrial sectors in a variety of applications. Among the major boron-bearing minerals in Türkiye—colemanite, ulexite, and tincal—colemanite stands out due to its monoclinic crystalline form and specific chemical composition ($2CaO{\bullet}3B_2O_3{\bullet}5H_2O$). It serves as a principal domestic raw material for the manufacturing of borax and boric acid. This mineral is essential in various industrial sectors, including glass and fiber production, high-temperature resistant materials, refining technologies, nuclear applications, flame retardants, catalysis, and cleaning products [1, 2].

Grinding constitutes a critical operation in mineral processing, particularly for reducing particle size to prepare raw materials for subsequent industrial applications. Despite its importance, the grinding process is known for its substantial energy consumption, as a significant portion of the supplied energy is lost as heat rather than being utilized for actual particle breakage. Consequently, enhancing the energy efficiency of grinding through precise control and optimization of its parameters has become a key focus in improving the overall performance of milling systems.

In light of these energy challenges, microwave energy has emerged as a promising auxiliary technique. As a form of electromagnetic radiation with frequencies ranging from 0.3 to 300 GHz and wavelengths between 1 mm and 1 m, microwave energy has traditionally found widespread use in communication technologies such as radar, satellite systems, and mobile devices [3]. For heating applications, standardized frequencies of 915 MHz and 2.45 GHz are commonly employed to prevent interference with telecommunication systems [4, 5]. Unlike conventional thermal methods, microwave heating enables volumetric and uniform energy transfer, allowing materials to be heated internally and externally at the same time. This mechanism bypasses the limitations posed by thermal conductivity and contributes to significantly reduced processing durations [5, 6].

Building upon these thermal properties, recent developments in mineral processing have increasingly focused on microwave-assisted comminution. In this approach, minerals are subjected to targeted microwave heating followed by rapid quenching, which induces microcracks and weakens the crystal lattice. This, in turn, facilitates subsequent grinding by reducing resistance to breakage. Compared to conventional thermal treatment—often criticized for its high energy demand and low efficiency—microwave processing selectively heats the mineral phases most responsive to electromagnetic energy, thereby reducing overall energy losses from unnecessary bulk heating. Furthermore, this selectivity may allow for lower processing temperatures without compromising the effectiveness of comminution [6, 7, 8, 9].

In parallel with these advancements, the characteristics of the target minerals play a critical role in determining the efficiency of such thermal pre-treatment techniques. Boron minerals, for instance, comprise more than 150 naturally occurring types that contain varying levels of B_2O_3. Nonetheless, only a subset—primarily those hydrated compounds formed with calcium, sodium, or magnesium—hold economic value. Among them, tincal (sodium-based), colemanite (calcium-based), and ulexite (sodium-calcium-based) are considered the most commercially significant [10]. These minerals, detailed in Table 1, represent key candidates for further investigation in the context of microwave-assisted processing.

Table 1. Primary boron minerals of economic significance (Source: [11]).

Mineral	Chemical Formula
Tincal	$Na_2B_4O_7.10H_2O$
Ulexite	$NaCaB_5O_9.8H_2O$
Colemanite	$CA_2B_6O_{11}.5H_2O$

As previously highlighted, Turkiye holds a commanding position in the global boron market, possessing over 60% of the world's known boron reserves. This considerable geological endowment establishes Turkiye as a dominant force in boron resource potential on a global scale. Moreover, beyond mere resource abundance, Turkiye also played a pivotal role in global production, contributing approximately 60% of the total boron output in 2018—equivalent to nearly 4.1 billion tonnes [12]. Nevertheless, the economic value of these boron-bearing raw materials—primarily colemanite, ulexite, and tincal—can only be realized through extensive processing, beginning with comminution, which involves crushing and size reduction (see Table 1). Milling, in particular, is a fundamental operation aimed not only at liberating mineral phases from the host matrix but also at preparing the feed material for subsequent beneficiation processes. To optimize process performance and reduce energy demands, these units are commonly operated with grinding media. Despite their importance, such comminution processes are notoriously energy-intensive, accounting for as much as 53% of a mineral processing plant's total energy consumption [13, 14]. Indeed, comminution has been identified as the largest single consumer of energy in the mining sector, with milling operations alone estimated to represent approximately 1% of global electricity usage, amounting to roughly 80 TWh annually [14, 15].

Furthermore, the efficiency of the breakage process within mills is significantly influenced by both the intrinsic properties of the processed material and the dynamic characteristics of the grinding environment. While it is commonly believed that operational parameters—such as the diameter of the mill [16], the density of the grinding media [17], the proportion of mill and material filling [18, 19, 20], and the speed at which the mill rotates [21]—could influence the specific breakage rate, experimental evidence suggests their influence is relatively limited. Rather, the specific rate of breakage is predominantly governed by the material's intrinsic behavior under mechanical stress. Initially, this rate tends to rise almost linearly; however, as grinding progresses, a distinct reduction in the rate becomes apparent, as illustrated in Figure 1.

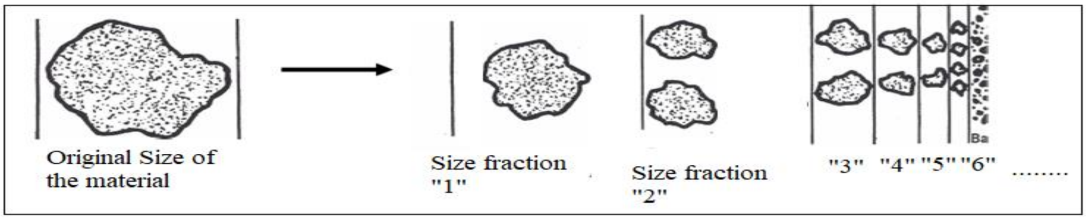

Figure 1. A gradual reduction in the specific breakage rate is observed as particle size fractions become smaller [22].

Figure 1 reveals that coarser particles tend to break at a faster rate initially, with this rate diminishing progressively as particle size decreases—a behavior commonly observed in most materials. Across all grinding conditions, finer feed sizes consistently demonstrate lower breakage rates. However, even within a constant particle size range, variations in specific breakage rates are still evident. This suggests that the breakage rate is influenced not only by the intrinsic properties of the material but also by the distinctive characteristics of the grinding environment.

While the grindability of coal has attracted significant attention in the literature [23, 24, 25], several other studies have extended this focus to explore the grinding characteristics and kinetics of alternative materials such as clinker, colemanite, and vitrified sanitary ware wastes [26, 27]. These works have predominantly centered on grinding kinetics, investigating how material behavior evolves under controlled milling conditions. For instance, Samanli et al. [28] studied the particle size evolution in stirred mills with a particular emphasis on breakage kinetics [29].

In parallel, Prusti et al. [30] examined the influence of limestone and dolomite fluxes on pellet quality, providing a detailed chemical characterization of the raw materials used, including iron ore fines [29]. Despite these diverse research efforts focusing either on grindability or breakage kinetics, the influence of chemical composition on the particle size distribution (PSD) of the milled product under identical grinding environments has largely been overlooked. Given that PSD variation can be attributed to multiple factors, compositional differences among samples are likely to play a non-negligible role. Addressing this gap, the present study aims to investigate not only the effects of sample chemistry but also the impact of microwave pre-treatment on grinding behavior. Although previously considered for some materials [29], microwave-assisted methods hold strong potential for enhancing the grindability of colemanite ore, particularly by promoting microstructural weakening and reducing energy demands during comminution.

Microwave heating has recently attracted considerable interest in mineral processing due to its ability to alter material properties effectively [29]. For example, Li et al. [31] demonstrated the applicability of microwave heating in treating manganese anode mud. Similarly, Li et al. [32] emphasized the potential of microwave-assisted reduction in processing low-grade pyrolusite at larger scales. In parallel, microwave sintering has been explored as an alternative to conventional methods [33], while other studies have investigated the microwave absorption behavior of vanadium slag and its use in ore calcination [34]. Numerous studies have been conducted on the combined use of microwave energy and organic materials, focusing on their role in both mineral reduction and the enhancement of liberation through microstructural modifications. These studies are comprehensively summarized in Bilen [29]. Moreover, microwave treatment has been shown to generate internal stresses and microcracks within mineral matrices, thus improving comminution efficiency [35]. Similarly, Zheng et al. [36] confirmed that microwave exposure weakens rock structures through crack formation and localized melting.

Microwave energy induces rapid internal heating through dielectric loss mechanisms, where polar molecules (such as water) and conductive mineral phases absorb electromagnetic energy and convert it into heat. This volumetric heating generates internal thermal stress gradients, which contribute to microcrack formation and intergranular weakening [4, 5].

The dielectric properties of minerals determine their ability to absorb microwave energy. Hydrated boron minerals such as colemanite exhibit relatively favorable microwave absorption behavior due to their water content and structural characteristics [6, 9]. These properties enable selective and localized heating, which facilitates crack propagation along grain boundaries and enhances subsequent breakage during grinding [7, 35].

As emphasized in the revised manuscript, microwave treatment promotes differential heating in heterogeneous mineral matrices. This thermal contrast between microwave-absorbing and transparent phases creates internal stress fields that initiate microcracking, thereby lowering the mechanical strength of the ore prior to comminution [31, 32, 36].

In general, existing research indicates that microwave pre-treatment can improve the grindability of boron-containing minerals such as colemanite by promoting selective thermal effects and enhancing energy efficiency during the size reduction process. Building on this understanding, the current study explores the grinding performance of colemanite by examining the influence of the shape of the grinding media, the distribution of feed particle sizes at the outset, and the rotational velocity of the mill. Additionally, a predictive model is introduced to estimate the specific breakage rate by integrating key grinding variables with a reference rate, providing valuable guidance.

2 MATERIAL AND METHOD

Approximately 300 kg of colemanite ore was obtained from the ETI Mine Emet facility located in Kütahya, Turkiye. The bulk sample was crushed and classified into three size fractions: −425+300 µm, −300+212 µm, and −212+150 µm. Prior to testing, the sample was subjected to chemical analysis, with the results presented in Table 2.

Table 2. The chemical constituents present in the collected colemanite ore.

B_2O_3 (%)	SiO_2 (%)	Al_2O_3 (%)	Fe_2O_3 (%)	CaO (%)	MgO (%)	K_2O (%)
29.40	11.05	5.75	1.70	17.05	2.95	1.48

The grinding experiments were conducted following the methodology established by Cuhadaroglu et al. [37], which was also adopted in the present study to determine the specific rates of breakage. A laboratory-scale mill, similar to the one used in their study, was employed under comparable operating conditions. Detailed grinding parameters are presented in Table 3, in accordance with Cuhadaroglu et al. [37].

After the compositional characterization and after the abovementioned grinding experiments previously employed by Cuhadaroglu et al. [37], Hardgrove Grindability Index (HGI) tests were performed to assess the grindability of the colemanite ore. The HGI measurements were carried out in accordance with the procedure outlined by Austin et al. [21]. Hardgrove grindability tests are conducted for −1190 + 590 µm sized materials according to the ASTM Standard [38]. A representative 50 g sample within the -16+30 mesh size range was placed in the ball mill component of the HGI testing apparatus, along with eight standardized iron balls. The material was ground for exactly 60 revolutions. Subsequently, the ground material was sieved using a 75 µm sieve for approximately 10 minutes. The portion of the sample that passed through the 75 µm sieve but was retained on a 300 µm sieve was collected and weighed. The HGI was then calculated using the following formula (Eqn. (1)).

$$HGI = 13{,}6 + 6{,}93.\mathrm{w} \tag{1}$$

where w is the weight of the test sample passing through the 75 µm sieve and retained on the 300 µm sieve after grinding in the HGI machine [21, 23].

Numerous studies have investigated the relationship between breakage parameters and the Hardgrove Grindability Index (HGI), aiming to better understand material comminution behavior [23, 39]. In this regard, Soni et al. [40] developed an empirical expression that correlates the breakage parameter π with HGI, as presented in Equation (2). Based on this equation, HGI values can be plotted against the corresponding π values to visualize their interdependence, as illustrated in Figure 2.

$$\pi = \frac{8{,}46.\mathrm{HGI} - 84{,}72}{1000 - 6{,}69.\mathrm{HGI} + 0{,}11.HGI^2} \tag{2}$$

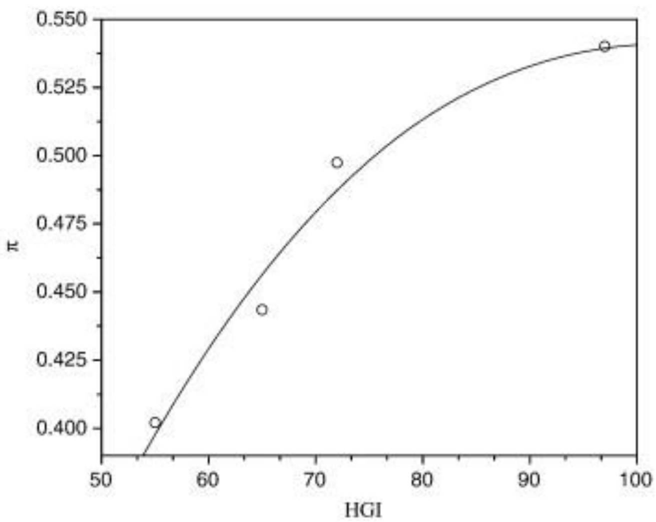

Figure 2. π versus HGI [23, 40].

In this study, a laboratory-scale mill, identical to that employed by Cuhadaroglu et al. [37], was used for all grinding experiments. The grinding media—cylindrical and spherical bodies—also shared the same physical characteristics as those defined in their work (see Table 2 in Cuhadaroglu et al., [37] for detailed properties). This study is primarily concerned with the numerical analysis of the specific rate of breakage, particularly in relation to variations caused by starting particle size, mill rotation speed, and grinding media configuration.

Complementing the breakage tests, Hardgrove Grindability Index (HGI) analyses were also performed on the colemanite samples. The data obtained from these tests were used to estimate the breakage parameter π, based on the empirical correlation proposed by Soni et al. [40]. By applying this relationship, the variation of π with respect to HGI was evaluated and illustrated graphically (Figure 2), providing insight into the material's comminution characteristics. This integrative approach allowed for a comprehensive understanding of colemanite breakage behavior under different grinding conditions, forming the methodological foundation of the current study.

Microwave pretreatment was applied to colemanite samples using a domestic microwave oven operating at a frequency of 2.45 GHz and a maximum power output of 800 W. The experiments were conducted on three distinct size fractions: +3350 µm, −3350+2360 µm, and −2360 µm. Each fraction was subjected to microwave irradiation for 5 minutes at a fixed power level of 180 W. Following the pretreatment, Hardgrove Grindability Index (HGI) tests were performed on both microwave-treated and untreated samples to assess the influence of microwave exposure on the grindability of colemanite, while it was previously employed for limestone samples in the context of Bilen [29] study.

Table 3. Laboratory milling conditions adopted in accordance with the methodology of Cuhadaroglu et al. [37].

Mill Specifications		**Material Charge**	
Parameter	**Value**	**Material**	**Colemanite**
Diameter, *D* (cm)	30.5	Specific Gravity	2.42
Length, *L* (cm)	30.5	Powder Weight (g)	1500
Volume, *V* (cm³)	22270	Powder Filling Volume Fraction, *fc*	0.046
Critical Speed, *Nc* (rpm)	76	Powder-Ball Loading Ratio, *U*	0.06
Speed of Grinding Chamber, φc (% of *Nc*)	65, 75, 80		
Fractional Ball Filling, *J*	0.19		

Table 3 (Continued). Laboratory milling conditions adopted in accordance with the methodology of Cuhadaroglu et al. [37].

Grinding Media			
Cylinders			
Dimension (mm)	**Number**	**Total Weight (g)**	**Surface Area (cm²)**
36.50 × 36.50	30	8985.0	1882.50
30.10 × 30.10	45	7623.0	1920.15
25.40 × 25.40	7	1128.0	212.73
19.05 × 19.05	48	6080.0	922.56
12.70 × 2.70	64	800.0	485.76
Total	**194**	**20215.1**	**5321.46**
Balls			
Dimension (mm)	**Number**	**Total Weight (g)**	**Surface Area (cm²)**
36.50	36	8982.0	1832.25
30.10	69	7721.1	1963.05
25.40	10	668.0	201.63
19.05	80	5234.9	809.40
12.70	97	305.1	490.82
Total	**292**	**20124.4**	**5298.22**

3 RESULTS AND DISCUSSION

The specific breakage rates associated with varying mill speeds and particle size ranges were determined and are summarized in the corresponding data tables. For this purpose, five different mill rotational speeds were selected, corresponding to 65%, 75%, 80%, 85%, and 90% of the mill's critical speed (Nc).

Table 4. Specific breakage rates obtained using spherical (ball-shaped) grinding media across different particle size groups and mill rotational speeds.

Specific Breakage Rates					
Size Group (µm)	**Mill Rotational Speeds**				
	65% $0.65N_c$	**75% $0.75N_c$**	**80% $0.80N_c$**	**85% $0.85N_c$**	**90% $0.90N_c$**
-425+300	0.3498	0.3758	0.3838	0.4169	0.4196
-300+212	0.2687	0.2817	0.3031	0.3406	0.3648
-212+150	0.2206	0.2290	0.2412	0.3474	0.3590

Table 5. Specific breakage rates associated with different mill rotation speeds and particle size intervals, evaluated using various grinding media geometries (balls and cylinders).

Size Group (µm)	Specific Breakage Rates					
	65% $0.65N_c$		75% $0.75N_c$		80% $0.80N_c$	
	Ball	Cylinder	Ball	Cylinder	Ball	Cylinder
-425+300	0.3498	0.4031	0.3758	0.4286	0.3838	0.4550
-300+212	0.2687	0.2976	0.2817	0.3070	0.3030	0.3146
-212+150	0.2206	0.2451	0.2290	0.2891	0.2412	0.2912

Unique breakage rates were identified based on the geometry of the grinding media applied during the milling process. Nevertheless, a consistent trend is evident from both Table 4 and Table 5: as the feed particle size becomes smaller, the corresponding breakage rate diminishes. This trend is valid across all tested rotational speeds and for both ball- and cylinder-shaped media. The pattern is further illustrated in Figures 3 and 4.

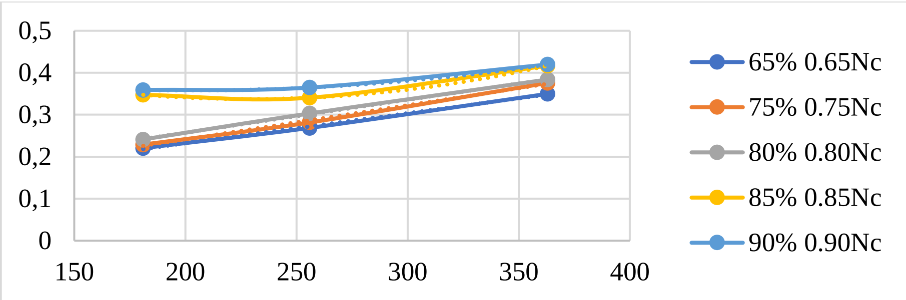

Figure 3. Specific breakage rates in relation to average feed particle size and mill rotation speeds.

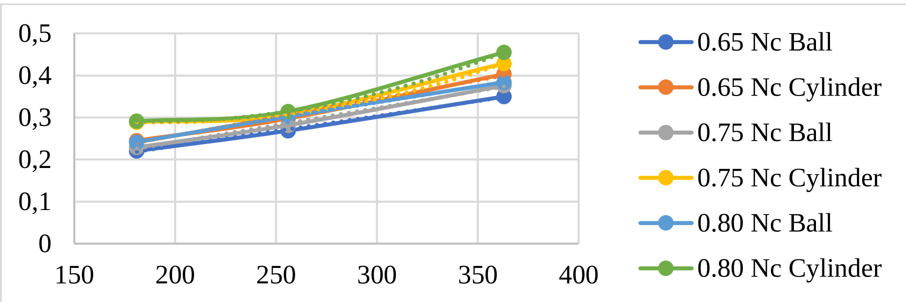

Figure 4. The relationship between specific breakage rates and the average feed particle size is shown at different mill rotation speeds, using various grinding media types (i.e., balls and cylinders).

Despite being carried out for three separate particle size groups, the observed patterns in specific breakage rates remain interpretable. Specifically, at 85% and 90% of the critical rotational speed, breakage rates show a rapid, exponential rise. In contrast, at slower speeds, i.e. such as 80% of the critical threshold or less—the increase follows a more linear pattern.

HGI (Hardgrove Grindability Index) experiments were performed on three distinct particle size fractions: +3350 µm, −3350+2360 µm, and −2360 µm. As described in the Materials and Methods section, microwave energy was applied to the samples prior to testing in order to assess its impact on grindability. For each of the three size fractions, HGI tests were conducted both with and without microwave pre-treatment. Additionally, corresponding Bond Work Index (BWI) values were calculated for each sample to provide a more comprehensive evaluation of grindability. Since the HGI method allows for simpler and less time-consuming laboratory procedures, Bond [41] proposed an empirical formula to estimate BWI from HGI values [42]. This calculation approach, which was used in the present study, is given in Equation 3 [42]. The results, including both HGI and BWI values for microwave-treated and untreated samples, are presented in Table 6.

$$BWI = \frac{88}{HGI^{0,5}} \tag{3}$$

Table 6. HGI and BWI values of microwave-treated and untreated colemanite samples for different particle size fractions.

Sample (colemanite)	HGI (Mwp)	HGI (wMwp)	BWI (Mwp)	BWI (wMwp)
+3350 µm	101,87	95,19	8,72	9,02
−3350+2360 µm	98,27	92,26	8,88	9,16
−2360 µm	91,16	88,13	9,22	9,37

*Mwp: Microwave pretreated

*wMwp: without Microwave pretreatment

A comparison of the Bond Work Index (BWI) values obtained from microwave-pretreated (Mwp) and untreated (wMwp) colemanite samples reveals a consistent and noteworthy decrease in energy consumption required for comminution across all size fractions. The BWI values for microwave-treated samples were found to be 8.72 kWh/ton, 8.88 kWh/ton, and 9.22 kWh/ton for the +3350 µm, −3350+2360 µm, and −2360 µm fractions, respectively. In contrast, the untreated counterparts exhibited slightly higher BWI values of 9.02 kWh/ton, 9.16 kWh/ton, and 9.37 kWh/ton.

Although the absolute differences between the pretreated and untreated samples may seem modest, the reduction in BWI values for the microwave-treated samples indicates a meaningful improvement in grindability. Specifically, the energy requirement decreased by approximately 3–4% for each size fraction, which is not negligible, especially when considering large-scale industrial applications where even small efficiency gains can translate into significant energy savings and operational cost reductions.

These results strongly suggest that microwave pre-treatment enhances the grindability of colemanite by weakening the material structure, thereby reducing the energy needed for size reduction. Such a trend, observed consistently across all tested particle sizes, highlights the potential benefits of microwave-assisted processing in mineral comminution.

However, when the energy input from microwave pre-treatment is also considered on a per-ton basis, the overall energy savings become less pronounced. Although the BWI values for microwave-treated samples are lower, indicating improved grindability, the additional energy consumed during microwave irradiation offsets some of the gains achieved during the subsequent milling process. In other words, when the total energy input—comprising both microwave and grinding energies—is compared to the energy required for grinding untreated samples, the difference is relatively marginal. This suggests that while microwave treatment facilitates easier size reduction, the net benefit in terms of total energy consumption may not be substantial unless the microwave application is optimized for energy efficiency. Therefore, a holistic assessment that accounts for both microwave and grinding energy is essential to determine the true viability of this method in industrial-scale operations.

Microwave irradiation causes rapid internal heating within the mineral structure, particularly in phases with high dielectric loss. This sudden thermal expansion leads to the formation of microcracks along grain boundaries and within the crystalline lattice. These microcracks weaken the structural integrity of the ore, reducing the mechanical energy required for fracture during grinding [8, 36].

The localized thermal stress created by microwave exposure acts as a pre-fracture mechanism. As the colemanite matrix is weakened internally, subsequent mechanical comminution becomes more efficient, as the propagation of cracks requires less external energy. This directly contributes to the reduction of the Bond Work Index (BWI), which quantifies the energy demand of size reduction processes [31, 35].

In heterogeneous ores such as colemanite, microwave energy preferentially heats certain mineral constituents, leading to differential thermal expansion between phases. This selective heating further promotes internal stresses and disintegration along phase boundaries, facilitating easier breakage during milling [6, 29].

Additionally, the development of microvoids and increased porosity as a result of microwave treatment reduces particle strength and increases brittleness. These microstructural changes allow for more efficient energy transfer during impact or attrition events in the mill, thereby decreasing the BWI [4, 7].

3.1 Proposed Model and Validation

This research presents preliminary mathematical models designed to estimate specific breakage rates. The developed expressions capture the distinctive breakage behavior of the examined mill, derived from patterns observed in relation to the mean particle size and the rotational speed of the mill. Furthermore, a correction component was added to the model to consider the effect of the shape of the grinding media. The resulting mathematical expression is shown below (Eqn. (4)).

$$SBR = C_1 . \ln(AS * MRS) + C_2 \qquad \text{Eqn. (4)}$$

In this context, SBR denotes the specific breakage rate, AS represents the average particle size, MRS refers to the mill rotational speed, and C1 and C2 are constants. While the initial mathematical expression, incorporating fixed values for C1 and C2, is applicable at lower mill speeds, it becomes insufficient at higher rotational speeds. Therefore, the model requires refinement—specifically, C1 should be reformulated as a function of mill rotational speed. A linear relationship may be suitable for mill speeds up to 80% of the critical speed. Nevertheless, this study proposes a more generalized expression to account for a wider range of operating conditions. The generalized form is presented as in the following equations, i.e. Eqn. (5) and Eqn. (6).

$$SBR = C_1(MRS) . \ln(AS * MRS) + C_2 \qquad \text{Eqn. (5)}$$

$$SBR = (a * MRS + b) . \ln(AS * MRS) + C_2 \qquad \text{Eqn. (6)}$$

The modified general equation suggests that specific breakage rates increase with mill rotational speed, showing higher values at coarse size fractions and lower values at finer ones. In this context, the parameter C1 is highly sensitive to changes in mill speed. This study proposes that C1 varies linearly with mill rotational speed (MRS), and can be expressed in the form of a linear equation, y = ax + b, where x represents MRS. Since the effect of average particle size is already incorporated into the equation, the model should be calibrated individually for each combination of mill speed and feed size. Once the constants are derived from experimental data, the model enables the prediction of specific breakage rates across a range of mill speeds and particle sizes. However, it's important to note that the influence of average particle size is inherently limited due to the logarithmic form used in the equation, which constrains its overall contribution.

3.2 π Breakage Parameter

As previously discussed, the breakage parameter π can be estimated using the empirical correlation proposed by Soni et al. [40], which relates π to the HGI values (see Eqn. 2 and Figure 2). The corresponding equation allows for π values to be estimated based on HGI measurements, facilitating their graphical comparison as shown in Figure 2. In the present work, the π values were determined using the HGI results obtained from the experiments, specifically the values reported in Table 6. For all tested particle size fractions—both microwave-treated and untreated—the calculated π values were found to be approximately 0.53, indicating a consistent relationship across sample conditions. These results are presented in detail in Table 7.

Table 7. Pi Breakage Parameter values of microwave-treated and untreated colemanite samples for different particle size fractions.

Sample (colemanite)	Pi Breakage Parameter, Treated	Pi Breakage Parameter, Untreated
+3350 µm	0,53	0,53
−3350+2360 µm	0,53	0,53
−2360 µm	0,53	0,52

*Mwp: Microwave pretreated

*wMwp: without Microwave pretreatment

Based on the data presented in Table 7, the π Breakage Parameter values for microwave-treated (Mwp) and untreated (wMwp) colemanite samples across different particle size fractions show minimal to no variation. For both the +3350 µm and −3350+2360 µm size ranges, the π values remain identical at 0.53 for both treated and untreated samples. A marginal difference is observed only in the −2360 µm fraction, where the value slightly decreases from

0.53 to 0.52 following microwave treatment. This consistency suggests that the microwave pre-treatment has a negligible impact on the breakage behavior of colemanite in terms of the π parameter.

When considered alongside Bond Work Index (BWI) data, a clearer picture emerges. Although the BWI values for microwave-treated samples are consistently lower than those for untreated samples—indicating improved grindability and reduced energy requirements for comminution—the overall differences are relatively modest. Specifically, the energy required for grinding decreased by approximately 3–4% across all size fractions. For instance, the BWI dropped from 9.02 kWh/ton to 8.72 kWh/ton for the +3350 µm fraction, with similar reductions observed in the −3350+2360 µm and −2360 µm fractions.

These reductions, while not dramatic, could translate into meaningful energy savings in large-scale operations. However, when the energy input required for microwave irradiation is taken into account, the net decrease in total energy consumption becomes marginal. This implies that although microwave pre-treatment facilitates easier size reduction by weakening the material structure, its overall benefit in terms of total energy efficiency is limited—unless the microwave application itself is optimized for lower energy use.

Last but not the least, both the π Breakage Parameter and BWI analyses suggest that the changes induced by microwave treatment are minor. While grindability improves slightly, the total energy savings are constrained and potentially negligible, highlighting the need for a holistic and energy-conscious approach when evaluating the industrial applicability of microwave-assisted pulverization of the material.

4 CONCLUSION

This study presents an in-depth investigation into the grindability behavior of colemanite ore by examining the effects of key operational variables—namely, mill rotational speed, grinding media shape, and particle size distribution—on the specific breakage rate. In doing so, the research not only reinforces existing knowledge in the domain of mineral comminution but also proposes a novel and predictive approach for estimating the specific breakage rate. The proposed methodology integrates average particle size and mill rotational speed as fundamental parameters, while also incorporating the influence of grinding media geometry through a correction factor. Particularly at elevated mill speeds, where deviations from linear trends are observed, the study introduces an adaptive model in which the constant

C1 varies as a function of mill rotational speed. This flexible formulation enables more accurate predictions across a broader range of operating conditions and offers a valuable tool for optimizing grinding operations in practice.

In parallel, the study explores the effect of microwave pre-treatment on colemanite grindability. Microwave irradiation, by inducing microstructural weakening through internal thermal stress, was observed to enhance breakage performance, especially in coarser particle fractions. This was evidenced by consistently lower Bond Work Index (BWI) values for microwave-treated samples compared to their untreated counterparts. Despite the relatively modest decrease in BWI (approximately 3–4% across all size fractions), the observed improvements suggest that microwave pre-treatment can facilitate more efficient comminution, which may translate into operational energy savings on an industrial scale.

However, when total energy input is considered—including both the microwave treatment and the subsequent grinding—the net energy savings become less pronounced. This highlights the importance of evaluating the energy trade-offs holistically. The π Breakage Parameter analysis further supports this conclusion, as it demonstrated minimal variation between treated and untreated samples, indicating that the fundamental breakage behavior of colemanite remains largely unaffected by microwave exposure.

In conclusion, while microwave pre-treatment shows potential for enhancing grindability, its impact on overall energy efficiency is limited unless the process is optimized. The predictive model proposed in this study offers a valuable framework for forecasting breakage rates based on mill parameters and material characteristics. As such, this work not only contributes to a deeper understanding of the dynamic factors influencing grinding performance but also provides practical guidance for future studies aiming to optimize energy use and improve the sustainability of mineral processing operations.

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Conflict of Interest Statement

There is no conflict of interest between the authors.

Statement of Research and Publication Ethics

The study is complied with research and publication ethics.

Artificial Intelligence (AI) Contribution Statement

This manuscript was entirely written, edited, analyzed, and prepared without the assistance of any artificial intelligence (AI) tools. All content, including text, data analysis, and figures, was solely generated by the authors.

Contributions of the Authors

S. Yılmaz: Methodology, Investigation, Data Curation, Formal Analysis, Visualization.

M. Bilen: Investigation, Resources, Validation, Writing – Review & Editing, Supervision, Project Administration.

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