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Research Article

DECISION MAKING WITH BIJECTIVE SOFT ROUGH SET MODEL

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Abstract: *Rough sets and soft sets are two important mathematical tools for dealing with uncertainty. Methods that combine properties of both sets can provide more effective solutions in certain applications. This paper aims to present an application of one such method, bijective soft rough sets. Subsequently, an algorithm based on bijective soft rough sets for decision making is developed and a real-life application of the proposed method is presented*

Keywords: *Rough sets, soft sets, bijective soft sets, bijective soft rough sets, decision making method.*

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1. Introduction

Real-life applications often require more advanced and efficient mathematical tools to deal with uncertainties. Data associated with complex problems in engineering, marketing, medical science, environmental science and other fields often contain various types of incomplete information. Therefore, various mathematical theories such as fuzzy set theory [1], intuitionistic fuzzy set theory [2], uncertain set theory [3], and rough set theory [4] have been developed to handle uncertain situations and draw meaningful conclusions. While all these theories are valuable tools for describing imprecision, each possesses inherent complexities, as highlighted by Molodtsov [5].

To address these challenges, Molodtsov [5] introduced soft set theory, which has since gained recognition as an effective approach for handling uncertainty. In recent years, substantial advancements have been made in both its theoretical underpinnings and practical implementations. Maji et al. [6] enhanced the field by formulating algebraic operations for soft sets and providing an in-depth theoretical analysis. In [7], Ali et al. contributed to the theory with the definition of new operations such as restricted union, intersection and difference. The effectiveness of soft sets in decision-making scenarios was highlighted by Maji et al. [8], and Chen et al. [9] focused on parameter reduction techniques within soft sets, drawing comparisons with feature reduction in rough set theory. Moreover, Gong et al. [10] introduced the concept of bijective soft sets, outlining their key operations. In [11], Aktaş presented the concept of bijective soft groups and discussed their potential applications. Koyuncu introduced bijective soft rings and explored their applications in [12]. Recent research has further explored the potential applications of these bijective soft sets [13–14].

Rough set theory, originally created by Pawlak [4], is another powerful mathematical instrument for dealing with vagueness and granularity in information systems. It approximates uncertain or inexact concepts using lower and upper approximation sets. This approach has proved essential in numerous domains, including data analysis, mereology, image processing, intelligent systems, and knowledge discovery in databases [15–19].

Despite their individual strengths, these theories may not always yield optimal results when used in isolation. As well, researchers have explored hybrid approaches that combine the strengths of different models. For instance, Herawan and Deris [20] revisited the rough set models of Pawlak and Iwinski through the lens of soft set theory. In [21], several hybrid models—such as soft rough sets, rough soft sets, and soft rough fuzzy sets—were introduced. Feng et al. [22] further explored the concept of soft rough sets, representing an integration of soft and rough set theories. Over time, numerous studies have examined the application of such hybrid models to decision-making problems [23–25]. Recently, a new unification of soft and rough set theories has been proposed by Bağırmaz [26] in the form of bijective soft rough sets.

This study introduces a fresh hybrid framework that is compatible with rough set theory while preserving the essential properties of soft sets. A decision methodology built on bijective soft rough sets is developed, including a detailed algorithm and a practical application that demonstrates the usefulness of the proposed model.

2. Preliminaries

Key concepts and definitions of rough sets, soft sets, soft rough sets, bijective soft sets, bijective soft rough sets and similar structures are sketched in this part.

Definition 2.1 [4] Let's take a non-empty set U and an equivalence relation θ on the set U. *In* this case, the pair (U,θ) is called the approximation space. The equivalence class of $a \in U$ is denoted by $\theta(a)$. For a subset $X \subseteq U$,

$$\underline{X} = \bigcup_{a \in U} \{\theta(a): \theta(a) \subseteq X\},$$

$$\overline{X} = \bigcup_{a \in U} \{\theta(a): \theta(a) \cap X \neq \emptyset\}.$$

The sets $\underline{X}, \overline{X}$ are called the lower and upper approximations of X with respect to (U,θ), respectively. $Bnd(X) = \overline{X} - \underline{X}$ is called rough boundary regions of X. If $Bnd(X) = \emptyset$; X is said to be rough definable; otherwise X is called a rough set.

Definition 2.3 [5] Let U be a certain set called the universe, and E be a set of parameters representing the properties of the elements in U. If $A \subseteq E$ and $f: A \to P(U)$ is a set-valued mapping, a pair $S = (f, A)$ is called a soft set on U.

Definition 2.4 [6] Let (f, A) and (g, B) be two soft sets over U. Then (f, A) is called a soft subset of (g, B), denoted by $(f, A) \tilde{\subset} (g, B)$, if $A \subseteq B$ and $\forall\ e \in A$, $f(e)$ and $g(e)$ have the same approximations.

Definition 2.5 [6] Let (f, A) and (g, B) be two soft sets over U. The union of (f, A) and (g, B) is defined as (h, C), where $C = A \cup B$ and for all $e \in C$,

$$h(e) = \begin{cases} f(e), if e \in A \backslash B, \\ g(e), if e \in B \backslash A, \\ f(e) \cup g(e), if \ \ e \in A \cap B \end{cases}$$

This is denoted by $(f, A) \tilde{\cup} (g, B) = (h, C)$.

Definition 2.6 [2] If (f, A) and (g, B) are two soft sets then *"(f, A) AND (g, B)"* (also denoted as $(f, A) \wedge (f, B)$) is defined by $(f, A) \wedge (f, B) = (h, A \times B)$, where $h(a, b) = f(a) \cap g(b)$ for all $(a, b) \in A \times B$.

Definition 2.7 [10] Let (f, A) be a soft set on U, where f is a set-valued mapping $f: A \to P(U)$

and A is a non-empty set of parameters. (f, A) is called a bijective soft set if it satisfies the following conditions:

1. $\cup_{e\in A} f(e) = U$,
2. For two arbitrary parameters $e_i, e_j \in A$, $e_i \neq e_j$, $f(e_i) \cap f(e_j) = \emptyset$.

Example 2.8 Let (f, E) be a soft set over the set $U = \{a_1, a_2, a_3, a_4, a_5\}$ and set of parameters

$E = \{e_1, e_2, e_3, e_4\}$. *Let* $A = \{e_1, e_2, e_3\}$, $B = \{e_2, e_4\}$.

Let the mapping (f, E) be given by

$$f(e_1) = \{a_1\}, f(e_2) = \{a_2, a_4\}, f(e_3) = \{a_3, a_5\}, f(e_4) = \{a_1, a_3, a_4\}.$$

From Definition 2.7, (f, A) is bijective soft set. Whereas (f, B) is not bijective soft set.

Proposition 2.9 [10] If (f, A) and (g, B) are two bijective soft sets over U, then $(h, C) = (f, A) \wedge (g, B)$ is also a bijective soft set.

Definition 2.10 [22] Let (f, A) be a soft set over the universe U. In this case, the triple (U, f, A) is referred to as a soft approximation space. Depending on (U, f, A), for any subset X of U, following two operators are defined

$$\underline{A}(\mathrm{X}) = \{x \in U : \exists e \in A[x \in f(e) \subseteq X]\},$$
$$\overline{A}(\mathrm{X}) = \{x \in U : \exists e \in A[x \in f(e), f(e) \cap X \neq \emptyset]\}.$$

The subsets $\underline{A}(\mathrm{X})$ and $\overline{A}(\mathrm{X})$ are called the lower and upper soft rough approximations of X on (U, f, A), respectively.

A rough set is given by a universe set U and an equivalence relation $\theta \subseteq U \times U$ in the well-known Pawlak approximation space. In the notion of a subjective soft set, each item in the universe can be mapped to only one parameter, and the union of the partitions formed by these parameters constitutes the universe of discussion. According to Definition 2.7, since the expression $\mathfrak{C} = \{f(e_1), f(e_2), \ldots, f(e_n)\}$, $e_1, e_2, \ldots, e_n \in A$, is both a partition and a cover of the universe, the expression $f(e)$ can be considered as an equivalence class of e. In this context, the structure of the universe can be analysed in the framework of a subjective soft set.

Let (f, A) be a bijective soft set on U. Then, corresponding pair $\mathfrak{B} = (U, f, A)$ is called a bijective soft rough approximation space. Then, the set $\mathfrak{C} = \{f(e_1), f(e_2), \ldots, f(e_n)\}$, where $e_1, e_2, \ldots, e_n$ are elements of A, will be called a class of values and can be defined by the equivalence relation $\theta \subseteq U \times U$, defined as follows $(x, y) \in \theta \Leftrightarrow x, y \in f(e)$ for all $x, y \in U$ and only one $e \in A$. Thus $\theta(x) = f(e) \Leftrightarrow x \in f(e)$. Therefore, we can identify the notion of the class of values $\mathfrak{C}_A$ of the bijective soft set (f, A) and the quotient set U / θ in $\mathfrak{B} = (U, f, A)$.

Definition 2.11 [26] Let (f, A) be a bijective soft set over U and $\mathfrak{B} = (U, f, A)$ be a bijective soft rough approximation space. Let X be a nonempty subset of U. Then the sets

$$\underline{A}_{\mathfrak{B}}(X) = \bigcup_{e\in A} \{f(e) : f(e) \subseteq X\} \text{ and } \overline{A}_{\mathfrak{B}}(X) = \bigcup_{e\in A} \{f(e) : f(e) \cap X \neq \emptyset\}$$

are called, respectively, lower and upper bijective soft rough approximations of X based on $\mathfrak{B} = (U, f, A)$.

Moreover, $Bnd_{\mathfrak{B}}(X) = \overline{A}_{\mathfrak{B}}(X) - \underline{A}_{\mathfrak{B}}(X)$ is called bijective soft rough boundary regions of X. If $Bnd_{\mathfrak{B}}(X) = \emptyset$, X is said to be bijective soft rough definable; otherwise X is called a bijective soft rough set.

From this point on, throughout this study, each bijective soft set (f, A) defined over the universe U will be considered together with its corresponding bijective soft rough approximation space $\mathfrak{B} = (U, f, A)$, and for convenience will be denoted as $(f, A) \in \mathfrak{B}$.

Example 2.12 Let the soft set (f, E) be defined on the set $U = \{x_1, x_2, x_3, x_4, x_5\}$ with parameters $E = \{e_1, e_2, e_3, e_4, e_5\}$. The mapping of (f, E) is as follows:

$$f(e_1) = \{x_1, x_2\}, f(e_2) = \{x_3, x_4, x_5\}, f(e_3) = \{x_1, x_2, x_3\}, f(e_4) = \{x_4\} \text{ and } f(e_5) = \{x_5\}.$$

We can tabulate this soft set as shown in Table 1. If $x_i \in f(e)$ then $x_{ij} = 1$, otherwise $x_{ij} = 0$, where x_{ij} are the entries Table 1.

Table 1. Soft set (f, E).

	e_1	e_2	e_3	e_4	e_5
x_1	1	0	1	0	0
x_2	1	0	0	1	0
x_3	0	1	0	0	1
x_4	0	1	0	0	1
x_5	0	1	0	1	0

Let $A = \{e_1, e_2\} \subseteq E$, $B = \{e_3, e_4, e_5\} \subseteq E$. From Definition 2.7, (f, A) and (f, B) are bijective soft sets.

Then

$$\mathfrak{C}_A = \{f(e_1), f(e_2)\} = \big\{\{x_1, x_2\}, \{x_3, x_4, x_5\}\big\},$$
$$\mathfrak{C}_B = \{f(e_3), f(e_4), f(e_5)\} = \big\{\{x_1\}, \{x_2, x_5\}, \{x_3, x_4\}\big\}.$$

For $X = \{x_1, x_2, x_4\} \subseteq U$ and $Y = \{x_3, x_5\} \subseteq U$ we can write

$$\underline{A}_{\mathfrak{B}}(X) = \{x_1, x_2\},\ \overline{A}_{\mathfrak{B}}(X) = U$$

and

$$\underline{B}_{\mathfrak{B}}(Y) = \emptyset,\ \overline{B}_{\mathfrak{B}}(Y) = \{x_2, x_3, x_4, x_5\}.$$

Thus, by Definition 2.11, X and Y are bijective soft rough sets.

Proposition 2.13 [26] Let $S = (f, A) \in \mathfrak{B}$. Then, for every $X, Y \subseteq U$ following properties hold:

1. $\underline{A}_{\mathfrak{B}}(X) \subset X \subset \overline{A}_{\mathfrak{B}}(X)$,

2. $\underline{A}_{\mathfrak{B}}(\emptyset) = \overline{A}_{\mathfrak{B}}(\emptyset) = \emptyset$,

3. $\underline{A}_{\mathfrak{B}}(U) = \overline{A}_{\mathfrak{B}}(U) = U$,

4. $\underline{A}_{\mathfrak{B}}(X \cap Y) = \underline{A}_{\mathfrak{B}}(X) \cap \underline{A}_{\mathfrak{B}}(Y)$,

5. $\overline{A}_{\mathfrak{B}}(X \cup Y) = \overline{A}_{\mathfrak{B}}(X) \cup \overline{A}_{\mathfrak{B}}(Y)$,

6. $X \subset Y \Rightarrow \underline{A}_{\mathfrak{B}}(X) \subset \underline{A}_{\mathfrak{B}}(Y)$,

7. $X \subset Y \Rightarrow \overline{A}_{\mathfrak{B}}(X) \subset \overline{A}_{\mathfrak{B}}(Y)$,

8. $\overline{A}_{\mathfrak{B}}(X \cap Y) \subseteq \overline{A}_{\mathfrak{B}}(X) \cap \overline{A}_{\mathfrak{B}}(Y)$,

9. $\underline{A}_{\mathfrak{B}}(X \cup Y) \supseteq \underline{A}_{\mathfrak{B}}(X) \cup \underline{A}_{\mathfrak{B}}(Y)$.

Definition 2.14 [26] Let $(f, A), (g, B) \in \mathfrak{B}$. Then we can define the following two operations on bijective soft rough set $(h, C) = (f, A) \wedge (g, B)$; for every subset $X \subseteq U$

$$\underline{C}_{\mathfrak{B}}(X) = \bigcup_{e \in C} \{h(e): h(e) \subseteq X\},$$
$$\overline{C}_{\mathfrak{B}}(X) = \bigcup_{e \in C} \{h(e): h(e) \cap X \neq \emptyset\},$$

where $h(e) = f(a) \cap g(b), \forall e = (a, b) \in A \times B$.

Proposition 2.15 [26] Let $(f, A), (g, B) \in \mathfrak{B}$. Then approximation on $(h, C) = (f, A) \wedge (g, B)$, for every $X \subseteq U$ following properties hold:

1. $\underline{C}_{\mathfrak{B}}(X) \supseteq \underline{A}_{\mathfrak{B}}(X) \cap \underline{B}_{\mathfrak{B}}(X)$,

2. $\overline{C}_{\mathfrak{B}}(X) \subseteq \overline{A}_{\mathfrak{B}}(X) \cap \overline{B}_{\mathfrak{B}}(X)$.

Example 2.16 We reconsider the bijective soft sets (f, A) and (f, B) given in Example 2.12. For

$X = \{x_1, x_2, x_4\} \subseteq U = \{x_1, x_2, x_3, x_4, x_5\}$, we obtain

$$\underline{A}_{\mathfrak{B}}(X) = \{x_1, x_2\},\ \overline{A}_{\mathfrak{B}}(X) = U$$

and

$$\underline{B}_{\mathfrak{B}}(X) = \{x_1\},\ \overline{B}_{\mathfrak{B}}(X) = U.$$

Also

$$\underline{A}_{\mathfrak{B}}(X) \cap \underline{B}_{\mathfrak{B}}(X) = \{x_1\},$$

and

$$\overline{A}_{\mathfrak{B}}(X) \cap \overline{B}_{\mathfrak{B}}(X) = U.$$

Now taking $(h, C) = (f, A) \wedge (f, B), h(e) = f(a) \cap f(b), \forall e = (a, b) \in A \times B$.
Then we obtain

$$\mathfrak{C}_C = \{h(e_1), h(e_2), h(e_3), h(e_4)\} = \{\{x_1\}, \{x_2\}, \{x_3, x_4\}, \{x_5\}\}$$

and

$$\underline{C}_{\mathfrak{B}}(X) = \{x_1, x_2\},\ \overline{C}_{\mathfrak{B}}(X) = \{x_1, x_2, x_3, x_4\}.$$

Thus

$$\underline{C}_{\mathfrak{B}}(X) \nsubseteq \underline{A}_{\mathfrak{B}}(X) \cap \underline{B}_{\mathfrak{B}}(X) \text{ and } \overline{A}_{\mathfrak{B}}(X) \cap \overline{B}_{\mathfrak{B}}(X) \nsubseteq \overline{C}_{\mathfrak{B}}(X)$$

Corollary 2.17 [26] Let $(f_i, A_i) \in \mathfrak{B}$, where $(i = 1,2,3, \dots, n)$. Then approximations on $(h_n, C_n) = \wedge_{i=1}^{n} (f_i, A_i)$, for every $X \subseteq U$ following properties hold:

1. $\left(\underline{C_n}\right)_{\mathfrak{B}}(X) \supseteq \cap_{i=1}^{n} \left(\underline{A_i}\right)_{\mathfrak{B}}(X)$,

2. $\left(\overline{C_n}\right)_{\mathfrak{B}}(X) \subseteq \cap_{i=1}^{n} \left(\overline{A_i}\right)_{\mathfrak{B}}(X)$.

Proposition 2.18 [26] Let $(f_i, A_i) \in \mathfrak{B}$, where $(\mathrm{i} = 1,2,3, \dots, \mathrm{n})$. Then approximations on

$(h_n, C_n) = \wedge_{i=1}^{n} (f_i, A_i)$, for every $X \subseteq U$ following properties hold:

1. $\left(\underline{C_n}\right)_{\mathfrak{B}}(X) \supseteq \left(\underline{C_m}\right)_{\mathfrak{B}}(X)$,
2. $\left(\overline{C_n}\right)_{\mathfrak{B}}(X) \subseteq \left(\overline{C_m}\right)_{\mathfrak{B}}(X)$,

where $m \le n$.

Definition 2.19 [26] Let $(f, A) \in \mathfrak{B}$. The accuracy measure of any subset $X \subseteq U$ with respect to A is defined as

$$\beta_{\mathfrak{B}}^{A}(X) = \frac{|\underline{A}_{\mathfrak{B}}(X)|}{|\overline{A}_{\mathfrak{B}}(X)|}.$$

Obviously $0 \le \beta_{\mathfrak{B}}^{A}(X) \le 1$. If $\beta_{\mathfrak{B}}^{A}(X) = 1$, X is crisp with respect to A, and otherwise, if $\beta_{\mathfrak{B}}^{A}(X) < 1$, X is bijective soft rough with respect to A.

Let us depict above definition by examples referring to Example 2.16.

For $X = \{x_1, x_2, x_4\} \subseteq U$ and $A \subseteq E$ we have $\underline{A}_{\mathfrak{B}}(X) = \{x_1, x_2\}$, $\overline{A}_{\mathfrak{B}}(X) = U$. For this case $\beta_{\mathfrak{B}}^{A}(X) = \frac{|\underline{A}_{\mathfrak{B}}(X)|}{|\overline{A}_{\mathfrak{B}}(X)|} = \frac{2}{5}$.

It means that the parameter set A is less characteristic for X.

For $X = \{x_1, x_2, x_4\} \subseteq U$ and $B \subseteq E$ we have $\underline{B}_{\mathfrak{B}}(X) = \{x_1\}$, $\overline{B}_{\mathfrak{B}}(X) = U$. For this case $\beta_{\mathfrak{B}}^{B}(X) = \frac{|\underline{B}_{\mathfrak{B}}(X)|}{|\overline{B}_{\mathfrak{B}}(X)|} = \frac{1}{5}$.

It means that this parameter set B is much less characteristic for X.

For $X = \{x_1, x_2, x_4\} \subseteq U$ and $C \subseteq E$ we have $\underline{C}_{\mathfrak{B}}(X) = \{x_1, x_2\}$, $\overline{C}_{\mathfrak{B}}(X) = \{x_1, x_2, x_3, x_4\}$. For this case $\beta_{\mathfrak{B}}^{C}(X) = \frac{|\underline{C}_{\mathfrak{B}}(X)|}{|\overline{C}_{\mathfrak{B}}(X)|} = \frac{2}{4}$. It means that the set X can be characterized partially by parameter sets A and B.

Proposition 2.20 [26] Let $(f_i, A_i) \in \mathfrak{B}$, where $(i = 1,2,3,\ldots,n)$. Let $(h_n, C_n) = \wedge_{i=1}^{n} (f_i, A_i)$. Then, for every $X \subseteq U$ and $m \le n$,

$$\beta_{\mathfrak{B}}^{C_m}(X) \le \beta_{\mathfrak{B}}^{C_n}(X).$$

3. Bijective soft rough sets in decision making

Concepts for creating a decision method using bijective soft rough sets are presented in here. It also includes a decision algorithm and an application of the proposed method.

Definition 3.1 Let $(f, A) \in \mathfrak{B}$ and let $(g, B) \in \mathfrak{B}$ be a bijective soft rough set. Then, accuracy measure of parameter $e \in B$ with respect to A is defined as

$$\beta_{\mathfrak{B}}^{A}\big(g(e)\big) = \frac{|\underline{A}_{\mathfrak{B}}(g(e))|}{|\overline{A}_{\mathfrak{B}}(g(e))|}.$$

The accuracy measure of bijective soft rough set (g, B) with respect to A, denoted by $\sigma_{\mathfrak{B}}^{A}(g)$, is defined as

$$\sigma_{\mathfrak{B}}^{A}(g) = \frac{1}{|B|} \sum_{e \in B} \beta_{\mathfrak{B}}^{A}\big(g(e)\big).$$

Definition 3.2 Let $(f_i, A_i), (g, B) \in \mathfrak{B}$, where $(\mathrm{i} = 1,2,\dots,\mathrm{n})$, and $(h_n, C_n) = \wedge_{i=1}^{n} (f_i, A_i)$. If $\sigma_{\mathfrak{B}}^{C_n}(g) = \sigma_{\mathfrak{B}}^{C_m}(g)$, where $m < n$, then, we call the soft set $(f', A') = \tilde{\cup}_{i=1}^{m} (f_i, A_i)$ is a reduct of the soft set $(f, A) = \tilde{\cup}_{i=1}^{n} (f_i, A_i)$.

Definition 3.3 Let $(f, A) \in \mathfrak{B}$ and let $(g, B) \in \mathfrak{B}$ be a bijective soft rough set. Then, certainty measure of decision parameter $e_j \in B$ with the help *of* $e_i \in A$ *is defined as*

$$d^{e_i}(e_j) = \frac{|f(e_i) \cap g(e_j)|}{|f(e_i)|},$$

where $e_i \in A$ and $e_j \in B$.

Definition 3.4 Let $(f, A) \in \mathfrak{B}$ and let $(g, B) \in \mathfrak{B}$ be a bijective soft rough set. Then, a decision rule is defined as *"if* e_i then e_j*"* with the certainty measure $d^{e_i}(e_j)$, where $e_i \in A$ and $e_j \in B$.

If $d^{e_i}(e_j) = 1$, then "if e_i then e_j" will be referred to as a certain decision rule; if $0 < d^{e_i}(e_j) < 1$ the decision rule will be considered uncertain.

Now, based on the concepts presented above, the following algorithm will be introduced:

1. Construct bijective soft rough sets (f_i, A_i) and bijective soft rough set (g, B),

2. Compute the accuracy measure $\sigma_{\mathfrak{B}}^{C_i}(g)$ for each bijective soft rough set $\wedge (f_i, A_i)$ by Definition 3.1,

3. Find a reduct bijective soft rough set $(h_m, C_m) = \wedge_{i=1}^{m} (f_i, A_i)$ by Definition 3.2,

4. Find the certainty measure $d^{e_i}(e_j)$ for all $e_j \in B$ with respect to $e_i \in C_m$ by Definition 3.3,

5. Obtain the decision rules by Definition 3.4.

Let us apply the above algorithm on a real life application as follows:

Example 3.5 Let a public health department examine "life expectancy" situation of its citizens according to their some informations to better serve the citizens. Consider the soft set (f, E) representing *"life expectancy"*. Let's assume that the set $U = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}$ consists of eight people and E denotes the parameter set, $E = A_1 \cup A_2 \cup A_3 \cup B$, A_1 describes *"sex"*, A_2 describes *"living area"*, A_3 describes *"habits"* and A_4 describes *"decision"*. The sets of these parameters are $A_1 = \{female,\ male\}$, $A_2 = \{village, city,\}$, $A_3 = \{smoke,\ smoke\ and\ drinking,\ no\ smoke\ no\ drinking\}$ and $B = \{healthy,\ drug\ addict,\ under\ stress\}$, respectively. From Definition 2.8 and Definition 2.11, (f_i, A_i) and (g, B) are bijective soft rough subsets of (f, E), where $\mathrm{i} = 1,2,3$. The mapping of each bijective soft rough sets over U defined as Table 2.

Table 2. Table of bijective soft sets for life expectancy.

	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	$\mathbf{e}_{10}$
x_1	1	0	1	0	0	0	1	1	0	0
x_2	1	0	1	0	0	0	1	1	0	0
x_3	0	1	1	0	1	0	0	1	0	0
x_4	0	1	0	1	0	1	0	0	1	0
x_5	0	1	1	0	1	0	0	0	0	1
x_6	0	1	0	1	0	0	1	1	0	0
x_7	0	1	0	1	0	1	0	0	1	0
x_8	1	0	1	0	1	0	0	0	0	1

We use abbreviation "e_1" for female, "e_2" for male, "e_3" for village, "e_4" for city,"e_5" for smoke, "e_6" for smoke and drinking, "e_7" for no smoke no drinking, "e_8" for healthy, "e_9" for drug addict and "e_{10}" for under stress in the Table 2.

Step 1. According to collected information in Table 2, the mapping of condition bijective soft rough sets (f_i, A_i), $i = 1,2,3$ and the mapping of decision bijective soft rough set (g, B) are given below:

$$f_1(e_1) = \{x_1, x_2, x_8\}, \quad f_1(e_2) = \{x_3, x_4, x_5, x_6, x_7\},$$

$$f_2(e_3) = \{x_1, x_2, x_3, x_5, x_8\}, \quad f_2(e_4) = \{x_4, x_6, x_7\},$$

$$f_3(e_5) = \{x_3, x_5, x_8\},\ f_3(e_6) = \{x_4, x_7\},\ f_3(e_7) = \{x_1, x_2, x_6\},$$

$$g(e_8) = \{x_1, x_2, x_3, x_6\}, g(e_9) = \{x_4, x_7\}, \mathrm{g}(e_{10}) = \{x_5, x_8\}.$$

Step 2. We can calculate accuracy measure of bijective soft rough set (f_4, A_4) with respect to $\wedge\,(f_i, A_i)$, i $= 1,2,3$, as Table 3 and 4.

Table 3. Table of the accuracy measure of parameters in B for each bijective soft rough set $\wedge\,(f_i, A_i)$, i $= 1,2,3$.

	$\beta_{\mathfrak{B}}^{A_1}$	$\beta_{\mathfrak{B}}^{A_2}$	$\beta_{\mathfrak{B}}^{A_3}$	$\beta_{\mathfrak{B}}^{A_{1,2}}$	$\beta_{\mathfrak{B}}^{A_{1,3}}$	$\beta_{\mathfrak{B}}^{A_{2,3}}$	$\beta_{\mathfrak{B}}^{A_{1,2,3}}$
$g(e_8)$	0	0	0	0	0,6	0,33	0,6
$\mathrm{g}(e_9)$	0	0	1	0	1	1	1
$\mathrm{g}(e_{10})$	0	0	0	0	0,33	0	0,33

Table 4. Table of the accuracy measure of bijective soft rough set (g, B) for each bijective soft rough sets $\wedge\,(f_i, A_i)$, i $= 1,2,3$.

$\sigma_{\mathfrak{B}}^{A_1}(g)$	$\sigma_{\mathfrak{B}}^{A_2}(g)$	$\sigma_{\mathfrak{B}}^{A_3}(g)$	$\sigma_{\mathfrak{B}}^{A_{1,2}}(g)$	$\sigma_{\mathfrak{B}}^{A_{1,3}}(g)$	$\sigma_{\mathfrak{B}}^{A_{2,3}}(g)$	$\sigma_{\mathfrak{B}}^{A_{1,2,3}}(g)$
0	0	0,33	0	0,57	0,44	0,57

Step 3. From step 2, we have $\sigma_{\mathfrak{B}}^{A_{1,3}}(g) = \sigma_{\mathfrak{B}}^{A_{1,2,3}}(\mathrm{g})$, and so $(f_1, A_1)\ \tilde{\cup}\ (f_3, A_3)$ a reduct bijective soft rough set of $\tilde{\cup}_{i=1}^{3} (f_i, A_i)$. Let $(h, C) = (f_1, A_1) \wedge (f_3, A_3)$. The tabular form of (h, C) is given in Table 5, where $\mathrm{c}_i \in C, i = 1,2,3,4,5$.

Table 5. Table for bijective soft rough sets $(h, C) = (f_1, A_1) \wedge (f_3, A_3)$ and (g, B)

	c_1	c_2	c_3	c_4	c_5
x_1	1	0	0	0	0
x_2	1	0	0	0	0
x_3	0	1	0	0	0
x_4	0	0	1	0	0
x_5	0	1	0	0	0
x_6	0	0	0	0	1
x_7	0	0	1	0	0
x_8	0	0	0	1	0

The parameters of C are
c_1 = (female) and (no smoke no drinking),
c_2 = (male) and (smoke),
c_3 = (male) and (smoke and drinking),
c_4 = (female) and (smoke),
c_5 = (male) and (no smoke no drinking).

$Step$ 4. We have certainty measure for e_8, e_9 and e_{10} as Table 6.

Table 6. Table of the certainty measure for e_8, e_9 and e_{10}.

	c_1	c_2	c_3	c_4	c_5
$d^{e_i}(e_8)$	1	0,5	0	0	1
$d^{e_i}(e_9)$	0	0	1	0	0
$d^{e_i}(e_{10})$	0	0,5	0	1	0

$Step$ 5. Following the steps in Definition 3.4, we can determine decision rules as follows:

1. if (female) and (no smoke no drinking) then (healthy), $(d^{e_1}(e_8) = 1)$,
2. if (male) and (smoke) then (healthy), $(d^{e_2}(e_8) = 0{,}5)$,
3. if (male) and (smoke) then (under stress), $(d^{e_2}(e_{10}) = 0{,}5)$,
4. if (male) and (smoke and drinking) then (drug addict), $(d^{e_3}(e_9) = 1)$,
5. if (male) and (no smoke no drinking) then (healthy), $(d^{e_4}(e_8) = 1)$,
6. if (female) and (smoke) then (under stress), $(d^{e_5}(e_{10}) = 1)$.

As a result of the algorithm, the following rules are defined as final decision rules: , "if c_1 then e_8", "if c_3 then e_9", "if c_4 then e_8" and "if c_5 then e_{10}" These rules express precise pairings that allow precise decisions to be made under certain conditions.

4. Conclusion

This study makes important contributions from both theoretical and practical perspectives. The primary goal of the research is to develop a novel decision algorithm based on objective soft rough sets and to test the effectiveness of this algorithm on an example. The findings show the practical applicability of this algorithm. Accordingly, this research has the potential to pave the way for many new studies on real-life applications.

Ethical statement

The author declares that this document requires no ethical approval or special permission.

Conflict of interest

The authors of the study emphasize that there are no conflicts of interest.

Generative AI statement

The author(s) declare that no Gen AI was used in the creation of this manuscript.

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