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A New Fractional Order Mathematical Model of Obesity and Dynamic Effects

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Abstract – Obesity represents a significant public health concern in both developed and developing countries in the present day. According to the World Health Organization, 1.9 billion people worldwide are overweight, with 600 million of these being obese. Obesity, which is linked to a number of negative health consequences, including diabetes, cardiovascular diseases, infertility and cancer, is a manageable condition. In this study, we performed a stability analysis by considering the fractional obesity model. The model is composed of three compartments: normal weight individuals (S), obese individuals (O) and recovered individuals (R). The notion of the fractional derivative as defined by Caputo is utilised herein. The fractional obesity model was analysed mathematically, and numerical results were obtained using Euler's method. Graphs were then generated to illustrate the findings.

1. Introduction

Obesity is defined as the accumulation of excessive and abnormal fat in the body to the extent that it impairs health. The calculation of body mass index (BMI) is based on the measurement of height and weight. This is a widely used method of determining levels of obesity. The body mass index (BMI) is a metric used to assess the relationship between height and weight. The BMI is calculated by dividing weight (in kilograms) by the square of height (in metres). An individual with a body mass index (BMI) calculation value in excess of 30 is categorised as obese, while a value in excess of 40 is indicative of morbid obesity (Santonja at al., 2010). Adipose tissue secretes hormonal and chemical substances that affect the whole system. Some secretions cause the appetite to increase, the satiety limit to be pushed up and obesity to progress. Excessive weight gain can greatly threaten your health, while at the same time restricting your movements and causing you to develop serious diseases (Ejima at al., 2013). The consumption of a diet that is both irregular and imbalanced, as well as the ingestion of a fast-food-style diet, can result in weight gain and the accumulation of fat in vital organs. The ingestion of substantial quantities of food in the aftermath of extended periods of fasting, in conjunction with the consumption of carbohydrate-rich foods and beverages with high levels of sucrose, constitutes a series of nutritional missteps that have been demonstrated to result in the development of obesity. The probability of obesity in children is 80 per cent if both parents are overweight. It is evident that a number of hormonal factors, including but not limited to diabetes, thyroid gland diseases and adrenal gland diseases, may be contributing factors to the development of obesity. Predisposing factors for obesity include inactivity, a sedentary lifestyle, extended periods spent in front of a computer or television, a lack of safe areas for walking and exercise, unhealthy cooking habits, and the adoption of unhealthy eating behaviours within one's immediate environment (Wang at al., 2008). Mathematical modelling is one of the effective tools used in applied mathematics. It aims to express what we know about complex systems, the interactions between the elements of the system and system dynamics through mathematics. In this context, the mathematical model can be considered as a bridge between the real world and the world of mathematics. Here, mathematical techniques or computer-aided numerical calculations are used to analyse the model and find mathematical solutions to the problem (Demirci, 2017; Podlubny, 1999).

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In the field of control theory, particularly in the context of physical, biological and dynamical systems, fractional derivative models have been demonstrated to be a preferred alternative to integer step models. The utility of fractional operators in describing the memory and hereditary properties of substances and processes has been demonstrated; these properties are frequently disregarded in integer stepwise derivatives. In population models, the future state of a population is contingent on its past state, a phenomenon referred to as the memory effect. The memory effect of a given population can be analysed through the utilisation of either a delay term or fractional differentiation. Recent years have seen a surge of interest among researchers in the field of fractional calculus, a subject which encompasses both fractional derivatives and fractional integrals. It has been determined that fractional operators offer a more precise and efficient characterisation of system behaviour in comparison to integer order derivatives. In consideration of the significant benefits offered by fractional derivatives with respect to their memory properties, the present system has been adapted by replacing the integer-order time derivative with the Caputo fractional derivative. (Akman Yıldız at al., 2018; Demirci, 2017; Öztürk at al., 2024).

The condition of obesity arises from the consumption of excessive caloric intake in conjunction with insufficient physical activity. This phenomenon is associated with an individual's lifestyle, and obesity can thus be regarded as a disease that disseminates. (Santonja et al., 2010) analyse the obesity epidemic in the United States. Santonja et al., (2010) and Ejima et al., (2013) consider obesity an epidemic disease and present mathematical models explaining its spread. Both models use integer order differential equations, assuming a constant total population size. The consequences of obesity extend beyond the individual to impact the broader society. These consequences encompass a range of domains, including health, social, psychological and economic factors (Albalawi at al., 2023; Driessche and Watmough, 2002; Gonzalez-Parra at al., 2010; Jodar at al., 2008; Kermack and McKendrick, 1927; Öztürk at al., 2023).

The objective of this paper is twofold: firstly, to present a comprehensive analysis of potential future trends in obesity and associated healthcare costs; and secondly, to characterise the necessity for national policies and programmes. The present study employs a novel mathematical model to delineate the contemporary obesity epidemic, encompassing both social transmission and mortality risks associated with obesity. The present study is distinguished from previous research in that it considers the relative hazard of obesity among individuals who have previously been obese, a dimension that previous studies have not addressed. The model will subsequently be employed to compare the effectiveness of different types of models (Bonyah and Akgül, 2021; Preston at al., 2013; Öztürk at al., 2024; Öztürk, 2025).

The distinguishing feature of this model is its incorporation of the risk of individuals who have recovered and returned to a normal weight becoming obese again. This paper is composed of four parts. The initial section presents an overview of the literature on fractional modelling and obesity. In the second part, the fractional obesity model was investigated and the system was analysed. In addition, the Generalised Euler Method was employed and the model's stability analysis was conducted. In the third section, numerical results of the fractional order obesity model are obtained and graphs are drawn. Finally, in the fourth section, conclusions were drawn.

2. Materials and Methods

2.1. Fractional Derivation

These comparisons show that the Caputo fractional-order model presented is more representative of the system than its integer-ordered form. Mathematical modelling based on enhanced models naturally leads to differential equations of fractional order and to the necessity of the formulation of initial conditions to such equations. The main advantage of Caputo's approach is that the initial conditions for fractional differential equations with Caputo derivatives take on the same form as for integer-order differential equations, contain the limit values of integer-order derivatives of unknown functions at the terminal $t = \alpha$.

Definition 2.1. It is proposed that the function $f(t)$ be defined as one that is continuously differentiable n times. For $n-1 < \alpha < n$, Caputo fractional derivative of α-th order $f(t)$ is defined

${}_{a}^{C}Dt^{\alpha} = \frac{1}{\Gamma(n-\alpha)} \int_{\alpha}^{t} (t-x)^{n-\alpha-1} f^{n}(x)dx$ (Podlubny, 1999).

2.2. The Fractional Obesity Model

The fractional Obesity model is predicated on the classification of communities into three primary groups. The first group consists of individuals with a normal body mass index (BMI), the second group consists of individuals with an obesity classification, and the third group consists of individuals who have recovered. The expression of the obesity model as a system of fractional differential equations is as follows.

$$\frac{d^{\alpha}S}{dt^{\alpha}} = \mu N - \beta OS - \mu S$$
$$\frac{d^{\alpha}O}{dt^{\alpha}} = \beta OS + \sigma\beta OR - \gamma O - \mu O \quad (2.1)$$
$$\frac{d^{\alpha}R}{dt^{\alpha}} = \gamma O - \sigma\beta OR - \mu R.$$

Here $\frac{d^{\alpha}}{dt^{\alpha}}$ denotes the Caputo fractional derivative of order α with respect to the variable t. The initial values are specified as follows:

$$S(0) = S_0\ ,\ O(0) = O_0,\ R(0) = R_0,$$

$0 < \alpha \leq 1$. It is important to note that societal structure is divided into three distinct compartments, the following equation is established: $S + O + R = N$. It should be noted that all terms are derived in relation to the passage of time.

$$\frac{d^{\alpha}N}{dt^{\alpha}} = \frac{d^{\alpha}S}{dt^{\alpha}} + \frac{d^{\alpha}O}{dt^{\alpha}} + \frac{d^{\alpha}R}{dt^{\alpha}}$$

it is clear that. It is evident that models characterised by fractional orders are inherently endowed with a memory attribute in time-dependent occurrences, thereby ensuring the production of more realistic and accurate results in comparison to integer order models. For this reason, the model was constructed as fractional order (Arshad at al., 2019; Öztürk at al., 2023). In this system (2.1), the fractional order differential equation is reduced to a first order differential equation when $\alpha = 1$. The computations and parameters are outlined in Tables 1 and 2, respectively.

Table 1.

Variables used in the systems	Meaning
$S(t)$	the number of individuals with normal weight at time t
$O(t)$	the number of obese individuals at time t
$R(t)$	the number of individuals who have recovered in time t
$N(t)$	Total population

Table 2.

Parameters	Meaning
β	Annual obesity rate
μ	Annual birth and natural death rate
σ	Relative risk ratio of weight gain among former obese individuals per year
γ	Annual recovery rate

It assumes a constant birth and natural death rate. It is widely accepted that all births are to be considered as falling within the vulnerable class. It is evident that the parameters defined within the model remain constant over the course of time.

The population N is dimensionless and constructed as follows with the help of new variables.

$$s = \frac{S}{N}\ , o = \frac{O}{N}\ \ , r = \frac{R}{N}\ .$$

It is evident from this observation that $s + o + r = 1$. The novel formulation of the fractional obesity model is expressed as follows:

$$D^{\alpha}s(t) = \mu - \beta o(t)s(t) - \mu s(t)$$

$$D^{\alpha}o(t) = \beta o(t)s(t) + \sigma\beta o(t)r(t) - \gamma o(t) - \mu o(t) \tag{2.2}$$

$$D^{\alpha}r(t) = \gamma o(t) - \sigma\beta o(t)r(t) - \mu r(t).$$

Theorem 2.2. $\forall t \geq 0,\ S(0) = S_0\ \geq 0\ ,\ O(0) = O_0\ \geq 0\ , R(0) = R_0\ \geq 0$ the solutions of the system in (2.2) with initial conditions $(S(t), O(t), R(t)) \in R_+^3$ that the solutions are non-negative.

Proof: (Generalized Mean Value Theorem) Let $f(x) \in C[a,b]$ and $D^{\alpha}f(x) \in C[a,b]$ for $0 < \alpha \leq 1$. Then we have

$$f(x) = f(\alpha) + \frac{1}{\Gamma(\alpha)}D^{\alpha}f(\epsilon)\ (x-a)^{\alpha} \tag{2.3}$$

with $0 \leq \epsilon \leq x,\ \ \forall x \in (a,b]$.

It can be demonstrated that the existence and uniqueness of the solution in $(0, \infty)$ can be obtained via the application of the Generalised Mean Value Theorem. It is imperative to demonstrate that the domain R_+^3 is positively invariant.

$$D^{\alpha}s(t) = \mu - \beta o(t)s(t) - \mu s(t) \geq 0$$

$$D^{\alpha}o(t) = \beta o(t)s(t) + \sigma\beta o(t)r(t) - \gamma o(t) - \mu o(t) \geq 0$$
$$D^{\alpha}r(t) = \gamma o(t) - \sigma\beta o(t)r(t) - \mu r(t) \geq 0.$$

It is evident that on each hyperplane bounding the nonnegative orthant, the vector field points into R_+^3.

2.3. Existence, Uniqueness and Non-Negativity of the System

The present study investigates the existence and uniqueness of the solutions to the fractional-order system (2.1) in the region. $B \times [t_0, T]$ where

$$B = \{(S, O, R) \in\ R_+^3 \colon \max\{\ |S|\ , |O|\ , |R|\} \leq \varphi, \min\{|S|\ , |O|, |R|\} \geq \varphi_0\} \tag{2.4}$$

and $T < +\infty$.

Theorem 2.3.
For each $A_0 = (S_0, O_0, R_0) \in B$ there exists a unique solution $A(t) \in\ B$ of the fractional-order system (2.1) with intial condition A_0, which is defined for all $t \geq 0$.

Proof: We denote $A = (S\ , O, R)$ and $\bar{A} = (\bar{S}, \bar{O}, \bar{R})$. Consider a mapping $M(a)$=($M_1(a), M_2(a),\ M_3(a)$) and

$$M_1(a) = \mu N - \beta OS - \mu S$$
$$M_2(a) = \beta OS + \sigma\beta OR - \gamma O - \mu O \tag{2.5}$$
$$M_3(a) = \gamma O - \sigma\beta OR - \mu R.$$

For any $A, \bar{A} \in B$ substituting into equation (2.5)

$$\|M(A) - M(\bar{A})\| = |M_1(A) - M_1(\bar{A})| + |M_2(A) - M_2(\bar{A})| + |M_3(A) - M_3(\bar{A})| \tag{2.6}$$

$$|M_1(A) - M_1(\bar{A})| = |\mu N - \beta OS - \mu S - \mu N + \beta\bar{O}\bar{S} + \mu\bar{S}|$$
$$= |-\beta(OS - \bar{O}\bar{S}) - \mu(S - \bar{S})|$$
$$\leq \beta\varphi|S - \bar{S}| + \beta\varphi|O - \bar{O}| + \mu|S - \bar{S}|$$

$$|M_2(A) - M_2(\bar{A})| = |\beta OS + \sigma\beta OR - \gamma O - \mu O - \beta\bar{O}\bar{S} - \sigma\beta\bar{O}\bar{R} + \gamma\bar{O} + \mu\bar{O}|$$
$$= |\beta(OS - \bar{O}\bar{S}) + \sigma\beta(OR - \bar{O}\bar{R}) - \gamma(O - \bar{O}) - \mu(O - \bar{O})|$$
$$\leq \beta\varphi|S - \bar{S}| + \beta\varphi|O - \bar{O}| + \sigma\beta\varphi|O - \bar{O}| + \sigma\beta\varphi|R - \bar{R}| + \gamma|O - \bar{O}| + \mu|O - \bar{O}|$$

$$|M_3(A) - M_3(\bar{A})| = |\gamma O - \sigma\beta OR - \mu R - \gamma\bar{O} + \sigma\beta\bar{O}\bar{R} + \mu\bar{R}|$$
$$= |\gamma(O - \bar{O}) - \sigma\beta(OR - \bar{O}\bar{R}) - \mu(O - \bar{O})|$$
$$\leq \gamma|O - \bar{O}| + \sigma\beta\varphi|O - \bar{O}| + \sigma\beta\varphi|R - \bar{R}| + \mu|O - \bar{O}|.$$

Then equation (2.6) becomes,

$$\|M(A) - M(\bar{A})\| \leq \beta\varphi|S - \bar{S}| + \beta\varphi|O - \bar{O}| + \mu|S - \bar{S}| + \beta\varphi|S - \bar{S}| + \beta\varphi|O - \bar{O}| + \sigma\beta\varphi|O - \bar{O}|$$
$$+\sigma\beta\varphi|R - \bar{R}| + \gamma|O - \bar{O}| + \mu|O - \bar{O}| + \gamma|O - \bar{O}| + \sigma\beta\varphi|O - \bar{O}| + \sigma\beta\varphi|R - \bar{R}| + \mu|O - \bar{O}|$$
$$\leq (\mu + 2\beta\varphi)\ |S - \bar{S}| + (2\beta\varphi + 2\sigma\beta\varphi + \mu + 2\sigma)|O - \bar{O}| + (2\sigma\beta\varphi + \mu)|R - \bar{R}|.$$

$\|M(A) - M(\bar{A})\| \leq L\|A - \bar{A}\|$ where

$L = \max(\mu + 2\beta\varphi\ , 2\beta\varphi + 2\sigma\beta\varphi + \mu + 2\sigma, 2\sigma\beta\varphi + \mu\ )$.

It can thus be concluded that $M(A)$ obeys the Lipschitz condition. Consequently, this result signifies the existence and uniqueness of a solution to the fractional-order system (2.4).

2.4. Stability Analysis of the Fractional Order Obesity Model

The objective of this study is to determine the equilibrium point of the system (2.2) in the system, $D^\alpha s = 0, D^\alpha o = 0, D^\alpha r = 0$ it is considered to be. E_0=(s_0,o_0, r_0) including,

$$E_0 = (\ 1\ ,0\ ,0) \tag{2.7}$$

the disease-free equilibrium point is achieved. The Jacobian matrix at the diseased equilibrium point is

$$J(E_0) = \begin{pmatrix} -\mu & -\beta & 0 \\ 0 & \beta-\mu-\gamma & 0 \\ 0 & \gamma & -\mu \end{pmatrix} \quad (2.8)$$

The object of our study has been successfully procured. This section shows the eigenvalues from the Jacobian matrix (2.8).

$$\lambda_1 = -\mu$$

$$\lambda_2 = -\mu$$

$$\lambda_3 = \beta - \mu - \gamma$$

where $\beta, \mu, \sigma, \gamma$ are the parameters of positively defined real numbers. It is clear that $\lambda_1 < 0$ and $\lambda_2 < 0$. If $\lambda_3 < 0$, the equilibrium point of the system is locally asymptotically stable. If $\lambda_3 > 0$, the equilibrium point of the system is unstable.

If $\beta - \mu - \gamma < 0$, $\beta < \mu + \gamma$ is. $R_0 = \frac{\beta}{\mu+\gamma} < 1$ is the basic reproduction rate. If $R_0 < 1$, obesity will decrease over time. If $R_0 > 1$, obesity will increase.

2.5. Generalised Euler Method

The Generalised Euler method was used to solve the initial value problem with the Caputo fractional derivative. A significant proportion of mathematical models are constituted of nonlinear systems, the solution to which can often be a formidable challenge to ascertain. Analytical solutions aren't found in most cases, so a numerical approach must be considered. The Generalised Euler method is one approach (Yaro at al., 2015).

$D^{\alpha}y(t) = f(t, y(t)), y(0) = y_0$, $0 < \alpha \le 1$, $0 < t < a$ for the initial value problem $h = \frac{a}{n}$ impending $[t_j, t_{j+1}]$ is divided into n sub with $j = 0,1, \dots, n-1$. It is hypothesised that $y(t), D^{\alpha}y(t)$ and $D^{2\alpha}y(t)$ are continuous on the interval $[0, a]$. Utilising the generalized Taylor's formula results in the following equation being obtained. (Yaro at al., 2015).

$$y(t_1) = y(t_0) + \frac{h^{\alpha}}{\Gamma(\alpha+1)} f(t_0, y(t_0)).$$

The process will be iterated until an array is obtained. The following equation is to be solved: Let $t_{j+1} = t_j + h$

$$y(t_{j+1}) = y(t_j) + \frac{h^{\alpha}}{\Gamma(\alpha+1)} f(t_j, y(t_j)), j = 0,1, \dots, n-1$$

the generalised formula has been obtained. For each integer k ranging from 0 to $n-1$, inclusive.

$$S(k+1) = S(k) + \frac{h^{\alpha}}{\Gamma(\alpha+1)}(\mu N - \beta O(k)S(k) - \mu S(k))$$

$$O(k+1) = O(k) + \frac{h^{\alpha}}{\Gamma(\alpha+1)}(\beta O(k)S(k) + \sigma\beta O(k)R(k) - \gamma O(k) - \mu O(k))$$

$$R(k+1) = R(k) + \frac{h^{\alpha}}{\Gamma(\alpha+1)}(\gamma O(k) - \sigma\beta O(k)R(k) - \mu R(k))$$

is obtained.

3. Results and Discussion

3.1. Numerical Simulation of Fractional Obesity Model

The subsequent section will present the fractional obesity model through graphical and simulation-based illustrations. The subsequent stage in the methodological approach will entail the numerical simulations of the fractional obesity model, leveraging the Generalised Euler method. The following parameters will be considered in this study.

The parameters of the model are as follows: $N = 100$, $S = 50$, $O = 40$, $R = 10$, $\beta = 2{,}96.10^{-7}$, $\sigma = 8$, $\mu = 0.01$, $\gamma = 0.02$. The size of the step is taken to be $h = 0.1$. The ensuing results and tables are thus obtained. Tables are derived from this using the Euler method.

Table 3: The following table presents the data relating to S, O and R at the instance t for $\alpha = 1$.

t	$S(t)$	$O(t)$	$R(t)$
0	50,00	40,00	10,00
1	50,04	39,88	10,06
2	50,09	39,76	10,13
3	50,14	39,64	10,20
4	50,19	39,52	10,27
5	50,24	39,40	10,34
6	50,29	39,28	10,41
7	50,34	39,16	10,48
8	50,39	39,05	10,55
9	50,44	38,93	10,61
10	50,49	38,81	10,68
11	50,54	38,70	10,75
12	50,59	38,58	10,81
13	50,64	38,46	10,88
14	50,69	38,35	10,95

Table 4: The following table presents the data relating to S, O and R at the instance t for α = 0.9.

t	$S(t)$	$O(t)$	$R(t)$
0	50,00	40,00	10,00
1	50,06	39,84	10,09
2	50,13	39,68	10,18
3	50,19	39,53	10,27
4	50,26	39,37	10,36
5	50,32	39,22	10,45
6	50,39	39,06	10,54
7	50,45	38,91	10,62
8	50,52	38,76	10,71
9	50,58	38,61	10,80
10	50,64	38,45	10,89
11	50,71	38,30	10,97
12	50,77	38,15	11,06
13	50,84	38,00	11,14
14	50,90	37,85	11,23

Table 5: The following table presents the data relating to S, O and R at the instance t for $\alpha = 0.8$.

t	$S(t)$	$O(t)$	$R\ (t)$
0	50,00	40,00	10,00
1	50,08	39,79	10,11
2	50,16	39,59	10,23
3	50,25	39,39	10,35
4	50,33	39,19	10,47
5	50,42	38,99	10,58
6	50,50	38,79	10,70
7	50,59	38,59	10,81
8	50,67	38,39	10,92
9	50,75	38,20	11,03
10	50,84	38,00	11,14
11	50,92	37,81	11,26
12	51,01	37,62	11,36
13	51,09	37,42	11,47
14	51,17	37,23	11,58

R_0, which is the basic reproduction rate, $R_0 = \frac{\beta}{\mu+\gamma}$ according to the parameters we take, R_0=0.00000986666 is obtained. Since $R_0 < 1$, obesity will decrease over time. Thus, a threshold value has been determined that allows the control of obesity (Driessche and Watmough, 2002).

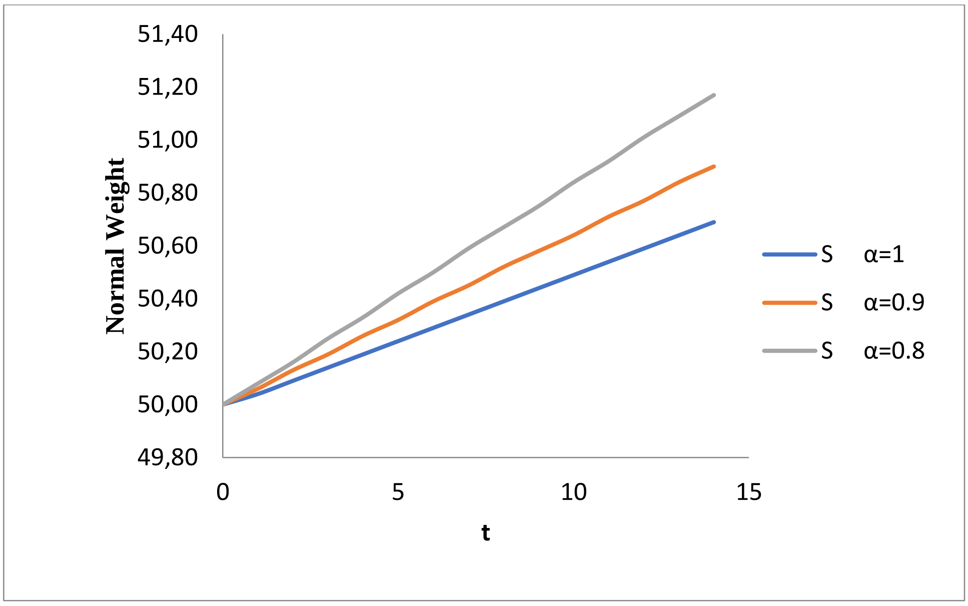

Figure 1. The graph of change of the *S* compartment model.

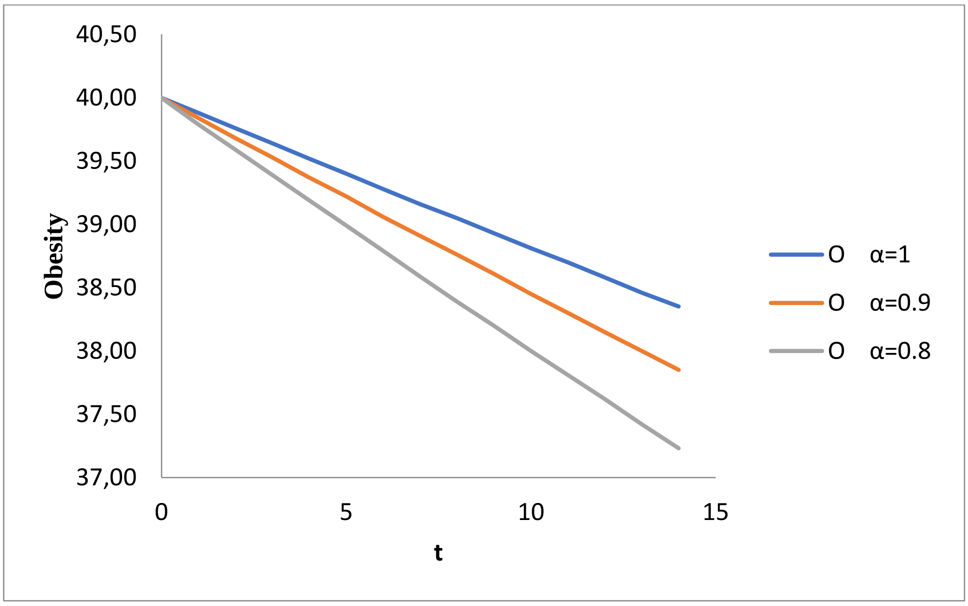

Figure 2. The graph of change of the *O* compartment model.

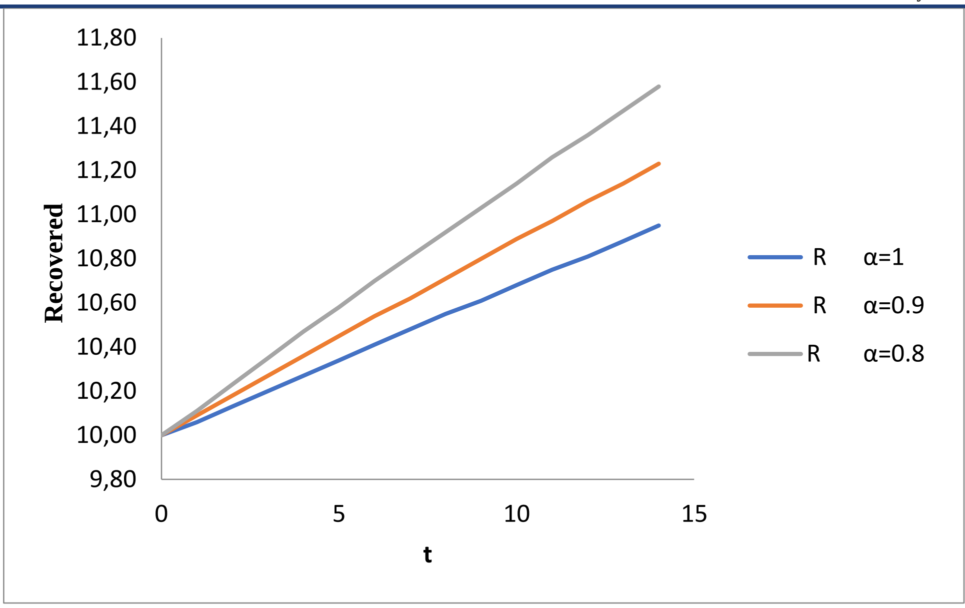

Figure 3. The graph of change of the R compartment model.

As illustrated in Table 3, Table 4 and Table 5, the alterations of S, O and R are evident for varying states of α. The following observations can be derived from the aforementioned data:

* It has been demonstrated that the proportion of individuals with normal weight increases over time (Figure 1).

* It has been established that decline in the prevalence of obesity over time (Figure 2).

* The total number of recoveries has increased steadily over time (Figure 3).

4. Conclusion

Obesity is a major global health concern. The economic consequences of obesity-related diseases have significant ramifications for national economies, rendering the dynamics of the obesity epidemic a matter of pressing concern for many countries. The selection of the most suitable intervention against obesity is contingent upon the capacity for person-to-person transmission of the condition. In order to facilitate appropriate evaluation and comparison of different public health control programmes of obesity, it is critical to quantify the epidemiological dynamics of obesity, especially its transmission potential, in advance. The present study proposes a novel implementation of the fractional obesity model, utilising numerical results derived from the model to construct graphs. A mathematical analysis was conducted to ascertain the existence, uniqueness and non-negativity of the system. A stability analysis of the disease-free equilibrium point was conducted for the fractional obesity model, resulting in the attainment of the equilibrium point. Subsequently, the fundamental reproduction rate, designated hereafter as R_0, was determined. The graphs thus obtained demonstrate a clear trend: namely, that there is an increase in the proportion of individuals with normal weight, whilst there is a decrease in the proportion of individuals with obesity and moreover, an increase in those who have recovered.

Ethics Permissions

This paper does not require ethics committee approval.

Conflict of Interest

Author declares that there is no conflict of interest for this paper.

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