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Research Article

Neutrosophic $\mathscr{I}$-Statistical Convergence of a Sequence of Neutrosophic Random Variables In Probability

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Abstract

This paper presents a novel perspective on established neutrosophic statistical convergence by utilizing ideals and proposing new ideas. Specifically, we explore the neutrosophic $\mathscr{I}$-statistical convergence of sequences of neutrosophic random variables (briefly, *NRVs*) in probability, as well as the neutrosophic $\mathscr{I}$-lacunary statistical convergence and neutrosophic $\mathscr{I}$-λ-statistical convergence of such sequences in probability. Additionally, we investigate their interconnections and examine some fundamental properties of these concepts.

1. Introduction

Smarandache [1] proposed a novel philosophical perspective that examines the origin, nature, and extent of neutralities, along with their interactions in diverse contexts. The central tenet of neutrosophy asserts that any concept possesses not only a degree of truth—as typically considered in many-valued logic systems—but also independent degrees of falsity and indeterminacy. In this approach, Smarandache appears to interpret indeterminacy from both subjective and objective standpoints, including aspects such as uncertainty, imprecision, vagueness, and error. Neutrosophy constitutes a recent mathematical theory that extends classical logic and fuzzy logic, encompassing constructs such as neutrosophic set theory, neutrosophic probability, neutrosophic statistics, and neutrosophic logic.

Recently, Bisher and Hatip in 2020 [2] employed the concepts of random variables and the indeterminacy associated with neutrosophic sets, offering an initial formulation of NRVs by introducing several fundamental notions. Subsequently, in 2021, Granados [3] established additional theoretical developments related to NRVs, and later, Granados and Sanabria [4] investigated the concept of independence within the context of NRVs. Furthermore, in 2020, Granados et al. [5, 6] examined certain neutrosophic probability distributions, both discrete [5] and continuous [6], based on the structure of NRVs.

On the other hand, the concept of statistical convergence can be traced back to A. Zygmund in 1935, and it gained further attention following its reintroduction by Steinhaus [7] and Fast [8] in 1951 for sequences of real numbers. Since then, various generalizations and applications have been explored. These developments have also been employed in neutrosophic theory. For instance, Kirişci and Şimşek [9] proposed the concept of neutrosophic normed space (briefly NNS), where they investigated statistical convergence and statistically Cauchy sequences, along with statistical completeness. Granados and Dhital [10] extended these results to double sequences in NNS, defining notions such as double statistically Cauchy sequences and their associated completeness. Kişi [11] explored the notions of ideal convergence and ideal Cauchy sequences in NNS. Khan et al. [12] defined lacunary statistically Cauchy sequences and examined the relationship between statistical completeness and classical completeness in NNS. In a subsequent work, Khan et al. [13] utilized λ-statistical convergence to generalize these concepts, presenting λ-statistically Cauchy sequences and completeness results in NNS, along with relevant inclusion relations. Ali et al. [14] investigated statistical convergence and statistically Cauchy sequences in neutrosophic metric spaces, inspired by analogous definitions in fuzzy metric spaces, and provided several characterizations. Al-Hamido [15] proposed a new generalized neutrosophic topological space that goes beyond both classical and crisp neutrosophic topologies, introducing novel types of neutrosophic sets and

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related concepts. Granados and Choudhury [16] formulated the notion of quasi-statistical convergence for triple sequences in NNS as an extension of statistical convergence, investigated key properties, and demonstrated that quasi-statistical Cauchy sequences are equivalent to quasi-statistical convergent sequences in this setting.
Inspired by the aforementioned studies and the growing interest in the investigation of neutrosophic statistical convergence, this paper aims to introduce three novel types of statistical convergence for sequences of NRVs, which are as follows:

1. Neutrosophic $\mathscr{I}$-statistical convergence in probability.
2. Neutrosophic $\mathscr{I}$-lacunary statistical convergence in probability.
3. Neutrosophic $\mathscr{I}$-λ-statistical convergence in probability.

In the present study, all the existing results provided in [17–20] are generalized and refined.

2. Preliminaries

In this section, we outline key concepts that are essential for the progression of the study.

Definition 2.1. *(see [21]) Let $\mathscr{T}$ denote a non-empty, fixed set. A neutrosophic set $\mathscr{A}$ is characterized by the expression $\{t,(\mu\mathscr{A}(t),\delta\mathscr{A}(t),\gamma\mathscr{A}(t)):t\in\mathscr{T}\}$, where $\mu\mathscr{A}(t),\delta\mathscr{A}(t)$ and $\gamma\mathscr{A}(t)$ represent the respective degrees of membership, indeterminacy, and non-membership of each element $t\in\mathscr{T}$ within the set $\mathscr{A}$.*

Definition 2.2. *(see [22]) Let $\mathscr{K}$ represent a field. The neutrosophic field associated with $\mathscr{K}$ and I is represented as $\langle\mathscr{K}\cup I\rangle$, where the operations are those of $\mathscr{K}$, and I is the neutrosophic element satisfying the property $I^2=I$.*

Definition 2.3. *(see [23]) A classical neutrosophic number takes the form $a+bI$, where a and b are real or complex numbers and I represents the indeterminacy satisfying $0.I=0$ and $I^2=I$, which implies that $I^n=I$ for all positive integers n.*

Definition 2.4. *(see [23]) The neutrosophic probability associated with the occurrence of event $\mathscr{A}$ is given by*

$$NP(\mathscr{A})=(ch(\mathscr{A}),ch(neut\mathscr{A}),ch(anti\mathscr{A}))=(T,I,F)$$

where T,I,F signify standard or non-standard subsets of the non-standard unitary interval $]^-0,1^+[$.

Now, we put forward some notions of NRVs [2].

Definition 2.5. *Let $\mathscr{T}$ be a real-valued deterministic random variable, with the mapping:*
$\mathscr{T}:\Omega\to\mathbb{R}$
where Ω is the event space. An NRV $\mathscr{T}_U$ is defined by:
$\mathscr{T}_U:\Omega\to\mathbb{R}(I)$
and
$\mathscr{T}_U=\mathscr{T}+I$
where I denotes indeterminacy.

Theorem 2.6. *Consider the NRV $\mathscr{T}_U=\mathscr{T}+I$, with the cumulative distribution function of $\mathscr{T}_U$ given by $F_{\mathscr{T}_U}(x)=P(\mathscr{T}_U\le t)$. The following expressions hold:*

1. *$F_{\mathscr{T}_U}(t)=F_{\mathscr{T}}(t-I)$,*
2. *$f_{\mathscr{T}_U}(t)=f_{\mathscr{T}}(t-I)$.*

In these, $F_{\mathscr{T}_U}$ and $f_{\mathscr{T}_U}$ denote the cumulative distribution function and the probability density function of $\mathscr{T}_U$, respectively.

Theorem 2.7. *Consider the NRV $\mathscr{T}_U=\mathscr{T}+I$. The expected value is given by:*
$E(\mathscr{T}_U)=E(\mathscr{T})+I.$

Next, we provide some definitions concerning ideal spaces, as defined by Kuratowski [24]:

Definition 2.8. *A ideal I on a set X, as defined by [24] is a collection of non-empty subsets of X that meets the following requirements.*

1. *When $A\subset B$ and $B\in I$, then $A\in I$.*
2. *When $A\in I$ and $B\in I$, then $A\cup B\in I$.*

3. Neutrosophic $\mathscr{I}$-Statistical Convergence in Probability

Definition 3.1. *Consider a sequence $\{\mathscr{T}_{U_\alpha}\}_{\alpha\in\mathbb{N}}$ of NRVs, where each $\mathscr{T}_{U_\alpha}$ is constructed on a same event space $\mathscr{S}$, along with a specified class Λ of subsets of $\mathscr{S}$, and a probability function $\mathscr{P}:\Lambda\to\mathbb{R}$. This sequence is regarded as neutrosophic $\mathscr{I}$-statistically convergent (N-$\mathscr{I}$-stat-convergent) in probability to an NRV $\mathscr{T}_U$, where $\mathscr{T}:\mathscr{S}\to\mathbb{R}$, if for any $\rho,\varsigma,\rho>0$,*

$$\left\{\alpha\in\mathbb{N}:\frac{1}{\alpha}|\{\beta\le\alpha:\mathscr{P}(|\mathscr{T}_{U_\beta}-\mathscr{T}_U|\ge\rho)\ge\varsigma\}|\ge\rho\right\}\in\mathscr{I},$$

or equivalently,

$$\left\{\alpha\in\mathbb{N}:\frac{1}{\alpha}|\{\beta\le\alpha:1-\mathscr{P}(|\mathscr{T}_{U_\beta}-\mathscr{T}_U|<\rho)\ge\varsigma\}|\ge\rho\right\}\in\mathscr{I}.$$

This convergence is represented by $\mathscr{T}_{U_\alpha}\xrightarrow{S^{P_U}(\mathscr{I})}\mathscr{T}_U$. The collection of all sequences of NRVs that are N-$\mathscr{I}$-stat-convergent in probability is referred to as $S^{P_U}(\mathscr{I})$.

Theorem 3.2. *If $\mathscr{T}_{U_\alpha} \overset{S^{p_U}(\mathscr{I})}{\to} \mathscr{T}_U$ and $\mathscr{T}_{U_\alpha} \overset{S^{p_U}(\mathscr{I})}{\to} \mathscr{Y}_U$, then $\mathscr{P}\{\mathscr{T}_U = \mathscr{Y}_U\} = 1$.*

Proof. Let $\rho, \varsigma > 0$ and $0 < \rho < 1$, then

$$\mathscr{U} = \left\{\alpha \in \mathbb{N} : \frac{1}{\alpha}\left|\left\{\beta \leq \alpha : \mathscr{P}\left(|\mathscr{T}_{U_\beta} - \mathscr{T}_U| \geq \frac{\rho}{2}\right) \geq \frac{\varsigma}{2}\right\}\right| < \frac{\rho}{3}\right\} \in \mathscr{F}(\mathscr{I}),$$

and

$$\mathscr{V} = \left\{\alpha \in \mathbb{N} : \frac{1}{\alpha}\left|\left\{\beta \leq \alpha : \mathscr{P}\left(|\mathscr{T}_{U_\beta} - \mathscr{Y}_U| \geq \frac{\rho}{2}\right) \geq \frac{\varsigma}{2}\right\}\right| < \frac{\rho}{3}\right\} \in \mathscr{F}(\mathscr{I}).$$

Since $\mathscr{U} \cap \mathscr{V} \in \mathscr{F}(\mathscr{I})$ and $\emptyset \notin \mathscr{F}(\mathscr{I})$ implies that $\mathscr{U} \cap \mathscr{V} \neq \emptyset$. Now, let $\gamma \in \mathscr{U} \cap \mathscr{V}$. Then,

$$\frac{1}{\gamma}\left|\left\{\beta \leq \gamma : \mathscr{P}\left(|\mathscr{T}_{U_\beta} - \mathscr{T}_U| \geq \frac{\rho}{2}\right) \geq \frac{\varsigma}{2}\right\}\right| < \frac{\rho}{3}$$

and

$$\frac{1}{\gamma}\left|\left\{\beta \leq \gamma : \mathscr{P}\left(|\mathscr{T}_{U_\beta} - \mathscr{Y}_U| \geq \frac{\rho}{2}\right) \geq \frac{\varsigma}{2}\right\}\right| < \frac{\rho}{3}.$$

This implies,

$$\frac{1}{\gamma}\left|\left\{\beta \leq \gamma : \mathscr{P}\left(|\mathscr{T}_{U_\beta} - \mathscr{T}_U| \geq \frac{\rho}{2}\right) \geq \frac{\varsigma}{2} \text{ or } \mathscr{P}\left(|\mathscr{T}_{U_\beta} - \mathscr{Y}_U| \geq \frac{\rho}{2}\right) \geq \frac{\varsigma}{2}\right\}\right| < \rho < 1.$$

Therefore, there exists any $\beta \leq \gamma$ such that $\mathscr{P}\left(|\mathscr{T}_{U_\beta} - \mathscr{T}_U| \geq \frac{\rho}{2}\right) < \frac{\varsigma}{2}$ and $\mathscr{P}\left(|\mathscr{T}_{U_\beta} - \mathscr{Y}_U| \geq \frac{\rho}{2}\right) < \frac{\varsigma}{2}$. Hence,

$$\mathscr{P}(|\mathscr{T}_U - \mathscr{Y}_U| \geq \rho) \leq \mathscr{P}\left(|\mathscr{T}_{U_\beta} - \mathscr{T}_U| \geq \frac{\rho}{2}\right) < \frac{\varsigma}{2} + \mathscr{P}\left(|\mathscr{T}_{U_\beta} - \mathscr{Y}_U| \geq \frac{\rho}{2}\right) < \varsigma.$$

□

Theorem 3.3. *If a sequence of constants $\mathscr{T}_{U_\alpha} \overset{S^{p_U}(\mathscr{I})}{\to} \mathscr{T}_U$, we can treat each constant as an NRV with a one-point distribution at that specific value, thus expressing the convergence as $\mathscr{T}_{U_\alpha} \overset{S^{p_U}(\mathscr{I})}{\to} \mathscr{T}_U$.*

Proof. The proof is succeeded by the definition. □

The reverse of the above theorem is not always valid, as demonstrated in the following example:

Example 3.4. *Consider $\mathscr{K} = \{\alpha^2 + \mathscr{I}\}$, where $\alpha = 1, 2, 3, \ldots$ and let the neutrosophic density $\mathscr{T}_{U_\alpha}$ be defined as $f_\alpha(x - \mathscr{I}) = 1 + \mathscr{I}$ for $\mathscr{I} < x < 1 + \mathscr{I}$, and equal to $\mathscr{I}$ otherwise. If $\alpha \in \mathscr{K}$, then $f_\alpha(x - \mathscr{I}) = \frac{\alpha(x-\mathscr{I})^{\alpha-1}}{2^\alpha}$, for $\mathscr{I} < x < 2 + \mathscr{I}$; and $f_\alpha(x - \mathscr{I}) = \mathscr{I}$ if $\alpha \in \mathbb{N} - \mathscr{K}$. Now, for $\rho \in (0,1)$, we have*
(i) $\mathscr{P}(|\mathscr{T}_{U_\alpha} - 2| \geq \rho) = 1 + I$ if $\alpha \in \mathscr{K}$ and
(ii) $\mathscr{P}(|\mathscr{T}_{U_\alpha} - 2| \geq \rho) = \left(1 - \frac{\rho}{2} + \mathscr{I}\right)^n$ if $\alpha \in \mathbb{N} - \mathscr{K}$.
Thus, $\mathscr{T}_{U_\alpha} \overset{S^{p_U}(\mathscr{I}_d)}{\to} 2 + \mathscr{I}$.

Theorem 3.5. *The properties listed below hold for N-$\mathscr{I}$-stat-convergence in probability:*

1. $\mathscr{T}_{U_\alpha} \overset{S^{p_U}(\mathscr{I})}{\to} \mathscr{T}_U$ *if and only if* $\mathscr{T}_{U_\alpha} - \mathscr{T}_U \overset{S^{p_U}(\mathscr{I})}{\to} 0$,
2. $\mathscr{T}_{U_\alpha} \overset{S^{p_U}(\mathscr{I})}{\to} \mathscr{T}_U$, *then* $p\mathscr{T}_{U_\alpha} \overset{S^{p_U}(\mathscr{I})}{\to} p\mathscr{T}_U$, *where* $p \in \mathbb{R}$,
3. $\mathscr{T}_{U_\alpha} \overset{S^{p_U}(\mathscr{I})}{\to} \mathscr{T}_U$ *and* $\mathscr{W}_{U_\alpha} \overset{S^{p_U}(\mathscr{I})}{\to} \mathscr{W}_U$, *then* $\mathscr{T}_{U_\alpha} + \mathscr{W}_{U_\alpha} \overset{S^{p_U}(\mathscr{I})}{\to} \mathscr{T}_U + \mathscr{W}_U$,
4. $\mathscr{T}_{U_\alpha} \overset{S^{p_U}(\mathscr{I})}{\to} \mathscr{T}_U$ *and* $\mathscr{W}_{U_\alpha} \overset{S^{p_U}(\mathscr{I})}{\to} \mathscr{W}_U$, *then* $\mathscr{T}_{U_\alpha} - \mathscr{W}_{U_\alpha} \overset{S^{p_U}(\mathscr{I})}{\to} \mathscr{T}_U - \mathscr{W}_U$,
5. $\mathscr{T}_{U_\alpha} \overset{S^{p_U}(\mathscr{I})}{\to} t + I$, *then* $\mathscr{T}^2_{U_\alpha} \overset{S^{p_U}(\mathscr{I})}{\to} (t+I)^2$
6. $\mathscr{T}_{U_\alpha} \overset{S^{p_U}(\mathscr{I})}{\to} t + I_1$ *and* $\mathscr{W}_{U_\alpha} \overset{S^{p_U}(\mathscr{I})}{\to} w + I_2$, *then* $\mathscr{T}_{U_\alpha}\mathscr{W}_{U_\alpha} \overset{S^{p_U}(\mathscr{I})}{\to} (t + I_1)(w + I_2)$ *where* $I_1 \neq I_2$,
7. $\mathscr{T}_{U_\alpha} \overset{S^{p_U}(\mathscr{I})}{\to} t + I_1$ *and* $\mathscr{W}_{U_\alpha} \overset{S^{p_U}(\mathscr{I})}{\to} w + I_2$, *then* $\mathscr{T}_{U_\alpha}\mathscr{W}_{U_\alpha} \overset{S^{p_U}(\mathscr{I})}{\to} [tw + I(1 + t + w)]$ *where* $I_1 = I_2 = I$,
8. $\mathscr{T}_{U_\alpha} \overset{S^{p_U}(\mathscr{I})}{\to} t + I_1$ *and* $\mathscr{W}_{U_\alpha} \overset{S^{p_U}(\mathscr{I})}{\to} w + I_2$, *then* $\mathscr{T}_{U_\alpha}/\mathscr{W}_{U_\alpha} \overset{S^{p_U}(\mathscr{I})}{\to} (t + I_1)/(w + I_2)$ *provided* $w \neq -I_2$.
9. $\mathscr{T}_{U_\alpha} \overset{S^{p_U}(\mathscr{I})}{\to} \mathscr{T}_U$ *and* $\mathscr{W}_{U_\alpha} \overset{S^{p_U}(\mathscr{I})}{\to} \mathscr{W}_U$, *then* $\mathscr{T}_{U_\alpha}\mathscr{W}_{U_\alpha} \overset{S^{p_U}(\mathscr{I})}{\to} \mathscr{T}_U\mathscr{W}_U$,
10. *If* $0 \leq \mathscr{T}_{U_\alpha} \leq \mathscr{W}_{U_\alpha}$ *and* $\mathscr{W}_{U_\alpha} \overset{S^{p_U}(\mathscr{I})}{\to} 0$, *then* $\mathscr{T}_{U_\alpha} \overset{S^{p_U}(\mathscr{I})}{\to} 0$,
11. *If* $\mathscr{T}_{U_\alpha} \overset{S^{p_U}(\mathscr{I})}{\to} \mathscr{T}_U$, *then for each* $\rho, \varsigma > 0$, *there exists* $\beta \in \mathbb{N}$ *so that any* $\rho > 0$

$$\left\{\alpha \in \mathbb{N} : \frac{1}{\alpha}|\{\beta \leq \alpha : \mathscr{P}(|\mathscr{T}_{U_\beta} - \mathscr{T}_U| \geq \rho) \geq \varsigma\}| \geq \rho\right\} \in \mathscr{I}.$$

This will be called the neutrosophic $\mathscr{I}$-statistical Cauchy condition in probability.

Proof. Let ρ,ς,ρ denote arbitrary positive real values. Then,

Properties (1), (2), (3), and (4) directly follow from the definitions, so their proofs are omitted.
(5) If $\mathscr{Z}_{U_\alpha} \overset{S^{p_U}(\mathscr{I})}{\to} 0$, then $\mathscr{Z}^2_{U_\alpha} \overset{S^{p_U}(\mathscr{I})}{\to} 0$ for

$$\{\beta \leq \alpha : \mathscr{P}(|\mathscr{Z}^2_{U_\beta} - 0| \geq \rho) \geq \varsigma\} = \{\beta \leq \alpha : \mathscr{Z}_{U_\beta} - 0| \geq \sqrt{\rho}) \geq \varsigma\}.$$

Next, let

$$\mathscr{T}^2_{U_\alpha} = (\mathscr{T}_{U_\alpha} - (t+I))^2 + 2(t+I)(\mathscr{T}_{U_\alpha} - (t+I)) + (t+I)^2 \overset{S^{p_U}(\mathscr{I})}{\to} (t+I)^2.$$

(6) We get

$$\begin{aligned}\mathscr{T}_{U_\alpha}\mathscr{W}_{U_\alpha} &= \tfrac{1}{4}\{(\mathscr{T}_{U_\alpha} + \mathscr{W}_{U_\alpha})^2 - (\mathscr{T}_{U_\alpha} - \mathscr{W}_{U_\alpha})^2\} \\ &\overset{S^{p_U}(\mathscr{I})}{\to} \tfrac{1}{4}\{(t+I_1+w+I_2)^2 - (t+I_1-(w+I_2))^2 = (t+I_1)(w+I_2).\end{aligned}$$

(7) We have

$$\begin{aligned}\mathscr{T}_{U_\alpha}\mathscr{W}_{U_\alpha} &= \tfrac{1}{4}\{(\mathscr{T}_{U_\alpha} + \mathscr{W}_{U_\alpha})^2 - (\mathscr{T}_{U_\alpha} - \mathscr{W}_{U_\alpha})^2\} \\ &\overset{S^{p_U}(\mathscr{I})}{\to} \tfrac{1}{4}\{(t+w+2I)^2 - (t-w)^2 = [tw + I(1+t+w)].\end{aligned}$$

(8) Let $\mathscr{U}$ and $\mathscr{V}$ be events of $|\mathscr{W}_{U_\alpha} - (w+I_2)| < |w+I_2|$, $\left|\frac{1}{\mathscr{W}_{U_\alpha}} - \frac{1}{w+I_2}\right| \geq \rho$, respectively. Now,

$$\begin{aligned}\left|\frac{1}{\mathscr{W}_{U_\alpha}} - \frac{1}{w+I_2}\right| &= \frac{|\mathscr{W}_{U_\alpha} - (w+I_2)|}{|(w+I_2)\mathscr{W}_{U_\alpha}|} \\ &= \frac{|\mathscr{W}_{U_\alpha} - (w+I_2)}{|w+I_2||(w+I_2) + (\mathscr{W}_{U_\alpha} - (w+I_2)|} \\ &\leq \frac{|\mathscr{W}_{U_\alpha} - (w+I_2)}{|w+I_2||(|w+I_2| - |\mathscr{W}_{U_\alpha} - (w+I_2)|)|}.\end{aligned}$$

If $\mathscr{U}$ and $\mathscr{V}$ occur simultaneously, then

$$|\mathscr{W}_{U_\alpha} - (w+I_2)| \geq \frac{\rho|w+I_2|^2}{1+\rho|w+I_2|},$$

that follows from the above inequality. Next, let $\rho_0 = \dfrac{\rho|w+I_2|^2}{1+\rho|w+I_2|}$ and let $\mathscr{G}$ be the event $|\mathscr{W}_{U_\alpha} - (w+I_2)| \geq \rho_0$. This implies $\mathscr{U}\mathscr{V} \subset \mathscr{G}$, then $\mathscr{P}(\mathscr{B}) \leq \mathscr{C} + \mathscr{P}(\overline{\mathscr{A}})$, where the bar represents the set of complement. This implies

$$\begin{aligned}&\left\{\beta \leq \alpha : \mathscr{P}(\left|\tfrac{1}{\mathscr{W}_{U_\alpha}} - \frac{1}{w+I_2}\right| \geq \rho) \geq \varsigma\right\} \\ &\subset \{\beta \leq \alpha : \mathscr{P}(|\mathscr{W}_{U_\alpha} - (w+I_2)| \geq \rho_0) \\ &\geq \tfrac{1}{2}\varsigma \cup \{\beta \leq \alpha : \mathscr{P}(\mathscr{W}_{U_\alpha} - (w+I_2)| \geq |w+I_2|) \geq \tfrac{1}{2}\varsigma\}.\end{aligned}$$

Therefore, $\frac{1}{\mathscr{W}_{U_\alpha}} \overset{S^{p_U}(\mathscr{I})}{\to} \dfrac{1}{w+I_2}$ provided $w \neq I_2$. Consequently, $\frac{\mathscr{T}_{U_\alpha}}{\mathscr{W}_{U_\alpha}} \overset{S^{p_U}(\mathscr{I})}{\to} \dfrac{t+I_1}{w+I_2}$ provided $w \neq I_2$. If $I_2 = I_1 = I$, it can be seen that $\frac{\mathscr{T}_{U_\alpha}}{\mathscr{W}_{U_\alpha}} \overset{S^{p_U}(\mathscr{I})}{\to} \frac{t+I}{w+I}$, provided $w \neq I$.
(10) First at all, we should prove that if $\mathscr{T}_{U_\alpha} \overset{S^{p_U}(\mathscr{I})}{\to} \mathscr{T}_U$ and $\mathscr{Z}_U$ is a neutrosophic random variable then $\mathscr{T}_{U_\alpha}\mathscr{Z}_U \overset{S^{p_U}(\mathscr{I})}{\to} \mathscr{T}_U\mathscr{Z}_U$. Since $\mathscr{Z}_U$ is a neutrosophic random variable, given $\varsigma > 0$, there exists an $\kappa > 0$ such that $\mathscr{P}(|\mathscr{Z}_N| > \kappa) \leq \frac{1}{2}\varsigma$. Then, for any $\rho > 0$,

$$\begin{aligned}\mathscr{P}(\mathscr{T}_{U_\alpha}\mathscr{Z}_U - \mathscr{T}_U\mathscr{Z}_U| \geq \rho) &= \mathscr{P}(|\mathscr{T}_{U_\alpha} - \mathscr{T}_U||\mathscr{Z}_U| \geq \rho, |\mathscr{Z}_U > \kappa) \\ &+ \mathscr{P}(|\mathscr{T}_{U_\alpha} - \mathscr{T}_U||\mathscr{Z}_U| \geq \rho, |\mathscr{Z}_U \leq \kappa) \\ &\leq \tfrac{1}{2}\varsigma + \mathscr{P}(|\mathscr{T}_{U_\alpha} - \mathscr{T}_U| \geq \rho/\kappa).\end{aligned}$$

This implies,

$$\begin{aligned}&\{\beta \leq \alpha : \mathscr{P}(|\mathscr{T}_{U_\beta}\mathscr{Z}_U - \mathscr{T}_U\mathscr{Z}_U| \geq \rho) \geq \varsigma\} \\ &\subset \{\beta \leq \alpha : \mathbb{P}(|\mathscr{T}_{U_\beta} - (t+I)| \geq \rho/\kappa) \geq \tfrac{1}{2}\varsigma\} \in \mathscr{I}.\end{aligned}$$

Thus, $(\mathscr{T}_{U_\alpha} - \mathscr{T}_U)(\mathscr{W}_{U_\alpha} - \mathscr{W}_U) \overset{S^{p_U}(\mathscr{I})}{\to} 0$. Therefore, this implies $\mathscr{T}_{U_\alpha}\mathscr{W}_{U_\alpha} \overset{S^{p_U}(\mathscr{I})}{\to} \mathscr{T}_U\mathscr{W}_U$.
(11) Proof is straightforward and hence omitted.
(12) Take $\beta \in \mathbb{N}$ such that $\mathscr{P}(|\mathscr{T}_{U_\beta} - \mathscr{T}_U| \geq \frac{1}{2}\rho) < \frac{1}{2}\varsigma$. Then,

$$\begin{aligned}&\{\beta \leq \alpha : \mathscr{P}(|\mathscr{T}_{U_\alpha} - \mathscr{T}_{U_\beta}| \geq \rho) \geq \varsigma\} \geq \rho\} \\ &\subset \{\beta \leq \alpha : \mathscr{P}(|\mathscr{T}_{U_\beta} - \mathscr{T}_U| \geq \tfrac{1}{2}\rho) \geq \tfrac{1}{2}\varsigma\} \geq \rho\} \in \mathscr{I}.\end{aligned}$$

□

4. Neutrosophic $\mathscr{I}$-Lacunary Statistical Convergence in Probability

Fridy [25] defined a lacunary sequence is an increasing integer sequence $\theta = \{\mathfrak{s}_\upsilon\}_{\upsilon \in \mathbb{N}\cup\{0\}}$ such that $\mathfrak{s}_0 = 0$ and $\mathfrak{h}_\upsilon = \mathfrak{s}_\upsilon - \mathfrak{s}_{\upsilon-1} \to \infty$, as $\upsilon \to \infty$; and $I_\upsilon = (\mathfrak{s}_{\upsilon-1}, \mathfrak{s}_\upsilon]$ and $\mathfrak{q}_\upsilon = \dfrac{\mathfrak{s}_\upsilon}{\mathfrak{s}_{\upsilon-1}}$.

Next, we define neutrosophic $\mathscr{I}$-lacunary statistical convergence in probability:

Definition 4.1. *The sequence $\{\mathscr{T}_{U_\alpha}\}_{\alpha\in\mathbb{N}}$ is regarded as neutrosophic $\mathscr{I}$-lacunary statistically (N-$\mathscr{I}$-S_θ) convergent in probability to a NRV $\mathscr{T}_U$, where $\mathscr{T} : \mathscr{S} \to \mathbb{R}$, if for any $\rho, \varsigma, \rho > 0$,*

$$\left\{\upsilon \in \mathbb{N} : \frac{1}{\mathfrak{h}_\upsilon}|\{\beta \in I_\upsilon : \mathscr{P}(|\mathscr{T}_{U_\beta} - \mathscr{T}_U| \geq \rho) \geq \varsigma\}| \geq \rho\right\} \in \mathscr{I}.$$

This convergence is represented by $\mathscr{T}_{U_\alpha} \overset{S_\theta^{P_N}(\mathscr{I})}{\to} \mathscr{T}_U$. The collection of all sequences of NRVs that are N-$\mathscr{I}$-S_θ-convergent in probability is referred to as $S_\theta^{P_N}(\mathscr{I})$.

Example 4.2. *Let $\mathscr{T}_{U_\alpha}(\omega) = \omega + \frac{(-1)^\alpha}{\alpha}$, where ω is a random variable uniformly distributed over $[0,1]$, and define $\mathscr{T}_U(\omega) = \omega$. Let the lacunary sequence $\theta = \{k_r\}_{r\in\mathbb{N}}$ be given by $k_r = 2^r$, so that the lacunary intervals are $I_r = (k_{r-1}, k_r] = (2^{r-1}, 2^r]$, and the interval length is $\mathfrak{h}_r = k_r - k_{r-1} = 2^{r-1}$. Let the ideal $\mathscr{I}$ be the family of subsets of $\mathbb{N}$ with natural density zero. Now consider the sequence $\{\mathscr{T}_{U_\alpha}\}$. For each α, we have*

$$|\mathscr{T}_{U_\alpha}(\omega) - \mathscr{T}_U(\omega)| = \left|\frac{(-1)^\alpha}{\alpha}\right| = \frac{1}{\alpha}.$$

Then the probability that the absolute difference exceeds any fixed $\rho > 0$ is

$$\mathscr{P}(|\mathscr{T}_{U_\alpha} - \mathscr{T}_U| \geq \rho) = \begin{cases} 1, & \text{if } \frac{1}{\alpha} \geq \rho, \\ 0, & \text{otherwise}. \end{cases}$$

Let us fix $\rho = 0.1$, $\varsigma = 0.5$, and $\rho = 0.25$. In each lacunary interval I_r, the number of indices β such that $\mathscr{P}(|\mathscr{T}_{U_\beta} - \mathscr{T}_U| \geq \rho) \geq \varsigma$ is finite and gets smaller as β increases. Specifically, for large enough r, most $\beta \in I_r$ satisfy $\frac{1}{\beta} < \rho$, so the corresponding probability is zero. Therefore, the proportion

$$\frac{1}{\mathfrak{h}_r}\left|\left\{\beta \in I_r : \mathscr{P}(|\mathscr{T}_{U_\beta} - \mathscr{T}_U| \geq \rho) \geq \varsigma\right\}\right|$$

becomes less than any $\rho > 0$ for large r. Hence, the set

$$\left\{r \in \mathbb{N} : \frac{1}{\mathfrak{h}_r}\left|\left\{\beta \in I_r : \mathscr{P}(|\mathscr{T}_{U_\beta} - \mathscr{T}_U| \geq \rho) \geq \varsigma\right\}\right| \geq \rho\right\}$$

has natural density zero, so it belongs to $\mathscr{I}$.

Thus, we conclude that $\mathscr{T}_{U_\alpha} \overset{S_\theta^{P_N}(\mathscr{I})}{\to} \mathscr{T}_U$, i.e., the sequence is neutrosophic $\mathscr{I}$-lacunary statistically convergent in probability to $\mathscr{T}_U$.

Theorem 4.3. *If $\mathscr{T}_{U_\alpha} \overset{S_\theta^{P_N}(\mathscr{I})}{\to} \mathscr{T}_U$ and $\mathscr{T}_{U_\alpha} \overset{S_\theta^{P_N}(\mathscr{I})}{\to} \mathscr{Y}_U$, then $\mathscr{P}\{\mathscr{T}_U = \mathscr{Y}_U\} = 1$.*

Proof. Let $\mathscr{P}(|\mathscr{T}_U - \mathscr{Y}_U| \geq \rho) = \varsigma > 0$, where for some $\rho > 0$. Then,

$$\begin{aligned}\mathscr{P}(|\mathscr{T}_U - \mathscr{Y}_U| \geq \rho) \leq \mathscr{P}&\left(|\mathscr{T}_{U_\beta} - \mathscr{T}_U| \geq \frac{\rho}{2}\right) \\ &+ \mathscr{P}\left(|\mathscr{T}_{U_\beta} - \mathscr{Y}_U| \geq \frac{\rho}{2}\right).\end{aligned}$$

Thus,

$$\begin{aligned}&\left\{\upsilon \in \mathbb{N} : \frac{1}{\mathfrak{h}_\upsilon}|\{U \in I_\upsilon : \mathscr{P}(|\mathscr{T}_U - \mathscr{Y}_U| \geq \rho) \geq \varsigma\}| \geq \tfrac{1}{2}\right\} \\ &\subset \left\{\upsilon \in \mathbb{N} : \frac{1}{\mathfrak{h}_\upsilon}|\{\beta \in I_\upsilon : \mathscr{P}(|\mathscr{T}_{U_\beta} - \mathscr{T}_U| \geq \tfrac{\rho}{2}) \geq \tfrac{\varsigma}{2}\}| \geq \tfrac{1}{4}\right\} \\ &\cup \left\{\upsilon \in \mathbb{N} : \frac{1}{\mathfrak{h}_\upsilon}|\{\beta \in I_\upsilon : \mathscr{P}(|\mathscr{T}_{U_\beta} - \mathscr{Y}_U| \geq \tfrac{\rho}{2}) \geq \tfrac{\varsigma}{2}\}| \geq \tfrac{1}{4}\right\} \in \mathscr{I},\end{aligned}$$

where $\mathscr{N}$ is a neutrosophic set. □

Theorem 4.4. *The following statements are equivalent:*

1. $\mathscr{T}_{U_\alpha} \overset{S_\theta^{P_N}(\mathscr{I})}{\to} \mathscr{T}_U$.
2. *For all $\rho, \varsigma > 0$,*

$$\left\{\upsilon \in \mathbb{N} : \frac{1}{\mathfrak{h}_\upsilon}\sum_{\alpha\in I_\upsilon} \mathscr{P}(|\mathscr{T}_{U_\alpha} - \mathscr{T}_U| \geq \rho) \geq \varsigma\right\} \in \mathscr{I}.$$

Proof. We begin proving $(1) \Rightarrow (2)$: First, let's consider that $\mathscr{T}_{U_\alpha} \overset{S_\theta^{PN}(\mathscr{I})}{\rightarrow} \mathscr{T}_U$, then we have

$$\frac{1}{\mathfrak{h}_\upsilon} \sum_{\alpha \in I_\upsilon} \mathscr{P}(|\mathscr{T}_{U_\alpha} - \mathscr{T}_U| \geq \rho) \leq \frac{1}{\mathfrak{h}_\upsilon} |\{\beta \in I_\upsilon : \mathscr{P}(|\mathscr{T}_{U_\beta} - \mathscr{T}_U| \geq \rho) \geq \frac{\varsigma}{2}\}| + \frac{\varsigma}{2}.$$

Consequently, we obtain

$$\left\{\upsilon \in \mathbb{N} : \frac{1}{\mathfrak{h}_\upsilon} \Sigma_{\alpha \in I_\upsilon} \mathscr{P}(|\mathscr{T}_{U_\alpha} - \mathscr{T}_U| \geq \rho) \geq \varsigma\right\}$$
$$\subset \left\{\upsilon \in \mathbb{N} : \frac{1}{\mathfrak{h}_\upsilon} |\{\beta \in I_\upsilon : \mathscr{P}(|\mathscr{T}_{U_\beta} - \mathscr{Y}_U| \geq \rho) \geq \tfrac{\varsigma}{2}\}| \geq \tfrac{\varsigma}{2}\right\} \in \mathscr{I}.$$

Next, we prove $(2) \Rightarrow (1)$: Let's consider that for all $\rho, \varsigma > 0$,

$$\left\{\upsilon \in \mathbb{N} : \frac{1}{\mathfrak{h}_\upsilon} \sum_{\alpha \in I_\upsilon} \mathscr{P}(|\mathscr{T}_{U_\alpha} - \mathscr{T}_U| \geq \rho) \geq \varsigma\right\} \in \mathscr{I}.$$

supplies. Then,

$$\frac{1}{\varsigma \mathfrak{h}_\upsilon} \sum_{\alpha \in I_\upsilon} \mathscr{P}(|\mathscr{T}_{U_\alpha} - \mathscr{T}_U| \geq \rho) \geq \frac{1}{\mathfrak{h}_\upsilon} |\{\beta \in I_\upsilon : \mathscr{P}(|\mathscr{T}_{U_\beta} - \mathscr{Y}_U| \geq \rho) \geq \varsigma\}|.$$

Then, for any $\rho > 0$,

$$\left\{\upsilon \in \mathbb{N} : \frac{1}{\mathfrak{h}_\upsilon} |\{\beta \in I_\upsilon : \mathscr{P}(|\mathscr{T}_{U_\beta} - \mathscr{Y}_U| \geq \rho) \geq \varsigma\}| \geq \rho\right\}$$
$$\subset \left\{\upsilon \in \mathbb{N} : \frac{1}{\varsigma \mathfrak{h}_\upsilon} \Sigma_{\alpha \in I_\upsilon} \mathscr{P}(|\mathscr{T}_{U_\alpha} - \mathscr{T}_U| \geq \rho) \geq \varsigma\rho\right\} \in \mathscr{I}.$$

Therefore, we have $\mathscr{T}_{U_\alpha} \overset{S_\theta^{PN}(\mathscr{I})}{\rightarrow} \mathscr{T}_U$. □

5. Neutrosophic $\mathscr{I}$- λ-Statistical Convergence in Probability

In [17], Ghosal formulated the following concepts. Let $\lambda = \{\lambda_\alpha\}_{\alpha \in \mathbb{N}}$ be a non-decreasing sequence of positive real numbers such that $\lambda_1 = 1$, $\lambda_{\alpha+1} \leq \lambda_\alpha + 1$ and $\lambda_\alpha \to \infty$ as $\alpha \to \infty$. For a given sequence of real numbers $(w_\beta)_{\beta \in \mathbb{N}}$, the generalised De la Valeé-Pousin mean is demonstrated by

$$t_\alpha(x) = \frac{1}{\lambda_\alpha} \sum_{\beta \in Q_\alpha} w_\beta$$

where $Q_\alpha = [\alpha - \lambda_\alpha + 1, \alpha]$.
Next, we present the definition of neutrosophic $\mathscr{I}$-λ-statistical convergence in probability:

Definition 5.1. *The sequence $\{\mathscr{T}_{U_\alpha}\}_{\alpha \in \mathbb{N}}$ is termed neutrosophic $\mathscr{I}$-λ-statistical convergent in probability to a NRV $\mathscr{T}_U$, where $\mathscr{T} : \mathscr{S} \to \mathbb{R}$, if for any $\rho, \varsigma, \rho > 0$,*

$$\left\{\alpha \in \mathbb{N} : \frac{1}{\lambda_\alpha} |\{\beta \in Q_\alpha : \mathscr{P}(|\mathscr{T}_{U_\beta} - \mathscr{T}_U| \geq \rho) \geq \varsigma\}| \geq \rho\right\} \in \mathscr{I}.$$

This type of convergence will be written as $\mathscr{T}_{U_\alpha} \overset{S_\lambda^{PU}(\mathscr{I})}{\rightarrow} \mathscr{T}_U$.

Example 5.2. *Let $\mathscr{T}_{U_\alpha}(\omega) = \omega + \frac{1}{\sqrt{\alpha}}$ for each $\alpha \in \mathbb{N}$, where ω is a random variable uniformly distributed over $[0,1]$, and define $\mathscr{T}_U(\omega) = \omega$. Then*

$$|\mathscr{T}_{U_\alpha}(\omega) - \mathscr{T}_U(\omega)| = \frac{1}{\sqrt{\alpha}}.$$

Thus,

$$\mathscr{P}(|\mathscr{T}_{U_\alpha} - \mathscr{T}_U| \geq \rho) = \begin{cases} 1, & \text{if } \frac{1}{\sqrt{\alpha}} \geq \rho, \\ 0, & \text{otherwise}. \end{cases}$$

Now, define the non-decreasing sequence $\lambda = \{\lambda_\alpha\}_{\alpha \in \mathbb{N}}$ as $\lambda_\alpha = \alpha$ and the intervals $Q_\alpha = [\alpha - \lambda_\alpha + 1, \alpha] = [1, \alpha]$ for all $\alpha \in \mathbb{N}$. Let the ideal $\mathscr{I}$ be the family of subsets of $\mathbb{N}$ with natural density zero.
Fix $\rho = 0.05$, $\varsigma = 0.5$, and $\rho = 0.2$. Then for large α, $\frac{1}{\sqrt{\alpha}} < \rho$, so

$$\mathscr{P}(|\mathscr{T}_{U_\beta} - \mathscr{T}_U| \geq \rho) < \varsigma \quad \text{for most } \beta \in Q_\alpha.$$

Therefore, the number of $\beta \in Q_\alpha$ for which the probability exceeds ς becomes negligible compared to λ_α, i.e.,

$$\frac{1}{\lambda_\alpha} \left| \left\{\beta \in Q_\alpha : \mathscr{P}(|\mathscr{T}_{U_\beta} - \mathscr{T}_U| \geq \rho) \geq \varsigma\right\} \right| < \rho$$

for all sufficiently large α.
Hence, the set

$$\left\{\alpha \in \mathbb{N} : \frac{1}{\lambda_\alpha}\left|\left\{\beta \in Q_\alpha : \mathscr{P}(|\mathscr{T}_{U_\beta} - \mathscr{T}_U| \geq \rho) \geq \varsigma\right\}\right| \geq \rho\right\}$$

is finite, and so belongs to the ideal $\mathscr{I}$.
Thus, we conclude that $\mathscr{T}_{U_\alpha} \overset{S_\lambda^{PU}(\mathscr{I})}{\rightarrow} \mathscr{T}_U$, i.e., the sequence is neutrosophic $\mathscr{I}$-λ-statistically convergent in probability to $\mathscr{T}_U$.

Definition 5.3. *The sequence $\{\mathscr{T}_{U_\alpha}\}_{\alpha\in\mathbb{N}}$ is regarded as neutrosophic $[V,\lambda]$-$\mathscr{I}$-summability in probability to a NRV $\mathscr{T}_U$, where $\mathscr{X} : \mathscr{S} \to \mathbb{R}$, provided that for any $\rho, \varsigma > 0$,*

$$\left\{\alpha \in \mathbb{N} : \frac{1}{\lambda_\alpha}\sum_{\beta\in Q_\alpha} \mathscr{P}(|\mathscr{T}_{U_\beta} - \mathscr{T}_U| \geq \rho) \geq \varsigma\right\} \in \mathscr{I}.$$

We denote this summability as $\mathscr{T}_{U_\alpha} \overset{[V,\lambda]^{PU}(\mathscr{I})}{\rightarrow} \mathscr{T}_U$.

Example 5.4. *Let $\mathscr{T}_{U_\alpha}(\omega) = \omega + \frac{1}{\alpha}$ for each $\alpha \in \mathbb{N}$, where ω is a random variable uniformly distributed on $[0,1]$, and define $\mathscr{T}_U(\omega) = \omega$. Then,*

$$|\mathscr{T}_{U_\alpha}(\omega) - \mathscr{T}_U(\omega)| = \frac{1}{\alpha},$$

so that

$$\mathscr{P}(|\mathscr{T}_{U_\alpha} - \mathscr{T}_U| \geq \rho) = \begin{cases} 1, & \text{if } \frac{1}{\alpha} \geq \rho, \\ 0, & \text{otherwise.} \end{cases}$$

Now, define $\lambda = \{\lambda_\alpha\}_{\alpha\in\mathbb{N}}$ by $\lambda_\alpha = \alpha$, and $Q_\alpha = [\alpha - \lambda_\alpha + 1, \alpha] = [1,\alpha]$. Let $\mathscr{I}$ be the family of all subsets of $\mathbb{N}$ with natural density zero. Fix $\rho = 0.01$ and $\varsigma = 0.1$. For sufficiently large α, we have $\frac{1}{\alpha} < \rho$, so $\mathscr{P}(|\mathscr{T}_{U_\beta} - \mathscr{T}_U| \geq \rho) = 0$ for most $\beta \in Q_\alpha$. Therefore,

$$\frac{1}{\lambda_\alpha}\sum_{\beta\in Q_\alpha} \mathscr{P}(|\mathscr{T}_{U_\beta} - \mathscr{T}_U| \geq \rho) \to 0 \quad \text{as } \alpha \to \infty.$$

Hence, the set

$$\left\{\alpha \in \mathbb{N} : \frac{1}{\lambda_\alpha}\sum_{\beta\in Q_\alpha} \mathscr{P}(|\mathscr{T}_{U_\beta} - \mathscr{T}_U| \geq \rho) \geq \varsigma\right\}$$

is finite and therefore belongs to the ideal $\mathscr{I}$.
Consequently, $\mathscr{T}_{U_\alpha} \overset{[V,\lambda]^{PU}(\mathscr{I})}{\rightarrow} \mathscr{T}_U$, i.e., the sequence is neutrosophic $[V,\lambda]$-$\mathscr{I}$-summable in probability to $\mathscr{T}_U$.

Theorem 5.5. *For any sequence of NRVs $\{\mathscr{X}_{N_n}\}_{n\in\mathbb{N}}$, The statements listed below are equivalent:*

1. $\mathscr{T}_{U_\alpha} \overset{S_\lambda^{PU}(\mathscr{I})}{\rightarrow} \mathscr{T}_U$.
2. $\mathscr{T}_{U_\alpha} \overset{[V,\lambda]^{PU}(\mathscr{I})}{\rightarrow} \mathscr{T}_U$.

Proof. The proof proceeds in a similar manner to that of Theorem 4.4 and is therefore omitted. □

Theorem 5.6. *If $\mathscr{T}_{U_\alpha} \overset{S_\lambda^{PU}(\mathscr{I})}{\rightarrow} \mathscr{T}_U$ and $\mathscr{T}_{U_\alpha} \overset{S_\lambda^{PU}(\mathscr{I})}{\rightarrow} \mathscr{Y}_U$, then $\mathscr{P}\{\mathscr{T}_N = \mathscr{Y}_N\} = 1$.*

Proof. Since the reasoning parallels that of Theorem 3.2, we omit the detailed proof. □

6. Conclusion

In this paper, we have introduced certain notions of statistical convergence in probability for sequences of NRVs. We also established some fundamental properties and examined their interrelations. Furthermore, it can be observed that the results presented here extend the classical framework developed in [17–19]. This aligns with the viewpoint expressed by Smarandache in [1, 20], which suggests that the neutrosophic statistical framework provides a broader generalization than its classical counterpart. For future studies, we recommend extending these notions by using [26–30].

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