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Research Article

Effect of Virtual Manipulatives on Students Academic Success in Teaching Integers

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Abstract

This study investigates the effect of virtual manipulatives on 7th-grade students' academic achievement in the multiplication and division of integers. A quasi-experimental design with a post-test control group was employed. The study involved 30 students assigned to experimental and control groups based on readiness level equivalence. The experimental group received instruction supported by virtual manipulatives, while the control group was taught using traditional teacher-centered methods. Data were collected through a readiness test and two researcher-developed achievement tests on multiplication and division of integers. Independent samples t-tests were used to analyze the data. The findings indicated a statistically significant improvement in the multiplication performance of the experimental group compared to the control group, whereas no significant difference was found in division performance. The results suggest that virtual manipulatives can effectively enhance students' conceptual understanding and success in specific mathematical operations.

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Introduction

Mathematics has been seen as a multi-story structure whose bricks are built carefully and systematically on each other (Biber, 2019). By its nature, mathematics consists of abstract structures, which can be seen as areas where students have difficulty teaching mathematics (Harel, 2008). Numbers, which form the essence of mathematics, are essential at all learning levels. The abstract structure of integers within the set of numbers may cause some students to be unable to correctly structure the concept of integers in their minds (Cantimer & Şengül, 2018). Students cannot easily understand negative numbers using out-of-school contexts or concrete objects (Schindler et al ., 2017). Therefore, it is crucial to determine which model or real-world contexts will be most beneficial to support students' construction of integers (Stephan & Akyüz, 2012). For this reason, the issue of integers, which students frequently encounter in daily life, has been examined in various studies, and it has been observed that students have difficulties (Barut & Ünüvar, 2020; Berkant & Yaren, 2020). Recent studies emphasize that virtual manipulatives significantly improve students' understanding and performance in integer operations, particularly addition and multiplication (Moyer-Packenham & Bolyard, 2002; Camp, 2018). Comparative research has shown that virtual manipulatives often outperform physical tools by providing dynamic visual representations, interactive feedback, and flexible pacing, which increase student motivation and engagement (Goh et al., 2017). Furthermore, studies incorporating number lines and multiple representations indicate that such tools support conceptual development by making abstract concepts more accessible (Vallejo-Vargas, 2024).

Since the abstract structure of mathematics causes students to have learning difficulties and to be distracted from the course, various teaching methods have been developed to prevent these. With the development of technology, education and technology have become compatible with each other, and the resources that have been developed have begun to be used in the field of education (Escueta et al., 2017). The United States National Council of Teachers of Mathematics (NCTM) stated that if technological tools are used adequately and validly in mathematics teaching, an environment will be created in which students will be able to perceive the abstract structure of mathematics better (NCTM, 2000). Among these developed resources is computer-assisted instruction. According to Baki (2002), computer-assisted teaching uses computers during teaching to enable students to interact with computers to determine their own performance and knowledge deficiencies, control their learning individually, and increase their interest in the course. Multiple representations of concepts in mathematics are essential in learning mathematical concepts, stating that each one emphasizes a different aspect of the concept and offers the opportunity to look at mathematical concepts from a wider window (Akkoç, 2008). For this reason, as long as reasonable conditions are provided when computers are used in mathematics teaching, students should be like researchers and experts in mathematics by focusing on their thinking, designing, judging, and decision-making skills (Karataş, 2003). Otherwise, using computers in mathematics teaching will contradict a researcher's identity, as students see them as tools used in simple operations.

Manipulatives have been produced as learning-teaching tools to be widely used in education from technological tools (Moyer-Packenham & Bolyard, 2016; Van De Walle et al ., 2012). Virtual manipulatives, used in mathematics by different developers towards the end of the 1990s, were proposed as a new version of all manipulatives (Moyer-Packenham & Bolyard, 2016). Different definitions were found in the literature on virtual manipulatives. Dorward (2002) defines virtual manipulatives as a digital version of familiar tools and manipulatives used in mathematics teaching. Kay and Knaack (2007) defined virtual manipulatives as reusable, interactive web-based tools that guide and develop students' cognitive processes and support learning specific concepts. Fitzallen (2008) defines virtual manipulatives as dynamic visual representations of concrete resources commonly used in mathematics classes and internet-based (small) applications that students can use to develop an understanding of certain mathematical concepts. Moyer-Packenham and Bolyard (2016) define virtual manipulatives as "an interactive, technology-compatible visual representation of a dynamic mathematical object that provides opportunities to create mathematical knowledge and includes all programmable features that allow (convenience) to be manipulated." Given the challenges students face in grasping the abstract nature of integers, especially negative values, there is a clear need for instructional tools that can provide visual and interactive support. Virtual manipulatives have emerged as a technological advancement to meet this need by offering dynamic and accessible representations of mathematical ideas. These tools enable learners to explore, manipulate, and visualize abstract concepts in ways that traditional instruction or static materials often cannot. As digital versions of physical objects like counters or number lines, virtual manipulatives are designed to enhance conceptual understanding and engagement in mathematics learning (Moyer-Packenham & Bolyard, 2016; Dorward, 2002; Fitzallen, 2008). Their ability to provide immediate feedback and support individualized learning further reinforces their pedagogical value in teaching integers.

For students who start performing mathematical operations with natural numbers at the primary school level, performing operations using integers is difficult in secondary school (Erdem et al ., 2015). Within the subject of integers, especially performing operations with negative integers is one of the areas where students have difficulty. The reason for this is that negative integers have no equivalent in the physical world (Martinez, 2006); negative numbers cannot be used as objects that exist in the real world (such as Can number of fish in an aquarium be under zero?) (Wessman-Enzinger, 2018). Thus, there are studies in the literature showing that student-centered learning methods are effective instead of learning environments where the teacher-centered presentation method is dominant (Altıparmak & Özdoğan, 2010; Barut & Ünüvar, 2020; Berkant & Yaren, 2020; Işık & Şanlı, 2020). Among these studies, Altıparmak and Özdoğan (2010), who show that teaching using animations increases students' academic success in integers, recommend using animations in students' learning of integers. Similarly, Bozkurt and Polat (2011) stated in their study that students improved in understanding integers, which aligns with teachers' opinions on using counting stamps in teaching integers. The idea that virtual manipulatives, which combine the effective use of concrete materials such as counting stamps and number lines, can also be practical in teaching integers has been a motivating factor in conducting this research.

Moyer-Packenham and Bolyard's (2016) definition of virtual manipulatives is "tools that can be created for the achievements of the mathematics course and that students can use like their physical counterparts." Clements (1999) also stated that using virtual manipulatives provides teachers with many practical and pedagogical benefits. There are numerous studies in the literature about using virtual manipulatives (Moyer-Packenham & Bolyard, 2016; Triona & Klahr, 2003). For instance, in the study by Moyer-Packenham and Bolyard (2002) the effect of virtual manipulatives on students' success in addition and subtraction operations with integers has been examined and it was concluded that the success in addition and subtraction operations in groups using virtual manipulatives increased significantly. Moyer-Packenham and Bolyard (2002), who included students' opinions in their research, stated that virtual manipulatives helped students learn addition and subtraction operations of integers. In the study by Samioğlu and Siniksaran (2016) the effect of the use of virtual manipulatives on the mathematics achievement and attitude of 8th-grade students in classes has been investigated and it was concluded that the use of virtual manipulatives had a positive effect on the student's academic success and attitudes towards mathematics. In Speer's (2009) study about the adverse effects of using virtual manipulatives in education, it was stated that "every technology may have its limitations and can be misused." In other words, he emphasized that experimental research methods should be used to examine the effects of virtual manipulatives in depth (Moyer-Packenham et al., 2008). The results obtained from these studies revealed the fact that virtual manipulatives are at least as practical as physical manipulatives. Furthermore, when used effectively, virtual manipulatives can mirror many of the activities a teacher demonstrates in an instructional setting. In addition, virtual manipulatives allow students to learn the subject at their own pace. For this reason, this study was deemed worth investigating the effect of virtual manipulatives in teaching multiplication and division of integers on students' academic success. The aim is to examine the effect of using virtual manipulatives on multiplication and division of integers on the academic achievement of 7th-grade students. In line with the research's purpose, the research problem is determined as follows: "Does the use of virtual manipulatives in multiplication and division of integers affect the academic achievement of 7th grade students?".

Method

Research Model

In this research, a post-test control group model, one of the quasi-experimental research models, was used to determine the effect of virtual manipulatives on the academic achievement of 7th-grade students in teaching multiplication and division of integers. In order for the results of scientific data to be stronger, it is recommended to use experimental models in the field of educational research (Loschiavo et al ., 2008). In this research model, no effort is made to create groups through random assignment when selecting experimental and control groups. While one or more of the groups formed by a method other than random assignment are selected as the experimental and control groups, care is taken to ensure that the groups are most similar to each other (Büyüköztürk et al., 2017). For this reason, the most critical feature that distinguishes the quasi-experimental research model from the experimental research model is the non-random sampling selection. In cases where matching will not replace random assignment, but it is not easy to make a random assignment, another matching method can be used (Büyüköztürk et al., 2017). In the post-test matched control group model, the pre-test is not applied, analysis is made, and comparison is made as a result of post-test application (Cohen et al. 2017). Since the study group encountered the subject of 'Multiplication and division of integers' for the first time, it was assumed that they had no prior knowledge of the subject and a pre-test was not required.

Participants

The study group of this research consists of 30 students enrolled in two different classes in the 7th grade of a public school in the central district of a province in the northern of Türkiye in the 2022-2023 academic year. Demographic information of the study group is given in Table 1.

Table 1. Demographic information of the study group

		Female	Male	Total
Experimental group	7-B	8	7	15
Control Group	7-A	7	8	15
Total		15	15	30

The study group selection in the research was carried out by the group matching method, although the groups were close to each other in terms of their academic achievements at school. The group matching method is a method that can be used for research to be carried out on existing groups when it is not possible or difficult to assign subjects to groups for research randomly. In this case, the most essential point to consider is to select equal group averages of the groups to be included in the research among the existing groups (Büyüköztürk et al., 2017). In this research, the equivalence of the groups was determined by the "Readiness Test" prepared for the addition of integes, which was applied to both groups before the research. Table 8 shows that the experimental and control groups comprising the study group are equivalent according to the readiness test.

Data Collection Tools

In order to compare the effect of virtual manipulatives on the academic achievement of 7th-grade students, two achievement assessment tests were used as data collection tools, developed by the researcher, covering the sub-learning areas of the operations with numbers learning area, multiplication, and division with integers. In addition a readiness test is used to compare the equivalence of the experimental and control group before the intervention.

Readiness Test

The readiness test is composed of 10 multiple choice questions about the addition of integers and these questions were taken directly from "learning outcomes comprehension questions" published by Ministry of National Education (MoNE) (URL-1, 2022). As seen in the sample question of readiness test in Figure 1, the questions have 4 choices, only one of which is correct. Therefore students graded in multiples of 10, according to the correct number of answers. Expert review was conducted to ensure content validity. Two mathematics educators evaluated the items for alignment with the intended learning objectives and grade-level appropriateness. To assess reliability, the internal consistency of the readiness test was examined using Cronbach's alpha, and the coefficient was found to be .81, indicating a good level of reliability for group-level comparisons

[(□3) + (□12)] + [(□4) – (□5)] = 0

Yukarıdaki eşitliğin doğru olması için □'lerin yerine (+) ve (–) sembollerinden hangileri sırasıyla yazılabilir?

A) +, –, +, – B) +, –, –, –

C) +, +, –, – D) –, +, +, –

Figure1. A sample question of readiness test

[Translation: Which of the following sequences of plus (+) and minus (–) signs can be placed in the boxes (□) to make the equation true]

Achievement Evaluation Test

Achievement assessment tests measure how much an individual has learned during education (Tekin, 2004). In order to prepare the achievement evaluation tests, the outcome comprehension questions published by MoNE and the questions in the textbooks used in classrooms are examined. After the preparation of outline for questions, 15 open-ended questions about the multiplication of integers and 15 open-ended questions about the division of integers were written. Then, these questions were presented to two mathematics educators and one mathematics teacher for the expert opinion. Therefore the final versions of achievement evaluation tests were applied to the participant students.

Multiplication of Integers Achievement Test

The table of specifications taken into consideration when creating open-ended questions in the integer multiplication achievement assessment test is given in Table 2.

Table 2. Table of specifications for the multiplication of integers achievement evaluation test

No.	**Learning outcomes**	**Aim**
1	Performs multiplication with integers.	Multiplying integers with appropriate models to make sense of multiplication with integers.
2		To practice the multiplication of two positive integers, two negative integers or two positive-negative integers.
3		Practicing the multiplication of two or more integers.
4		Practicing multiplication of more than two integers by displaying integers on a number line.
5		To practice multiplication of two integers by comparing integers.
6		To practice the multiplication of two integers with the same or different signs on the table.
7		Finding the factors in a given multiplication operation.

8	Solves problems that require multiplication with integers.	To use multiplication of integers in problems encountered in daily life.
9		To practice solving problems related to the multiplication of two integers.
10		To use multiplication of integers in problems encountered in daily life.

The scoring table for the questions of the multiplication of integers achievement test is shown in Table 3 and a sample question taken from the multiplication of integers achievement test is shown in Figure 2

Table 3. Achievement evaluation test scoring table

Correct Answer	Partially Correct Answer	Incorrect Answer
10 points	5 Points	0 Points

5) -7 , 3 , -4 , 6 , -1 tam sayılarının en büyüğü ile en küçüğünün çarpımı kaçtır?

Figure2. A sample question of multiplication of integers achievement test *[Translation: What is the product of the largest and the smallest integers among –7, 3, –4, 6, and –1?]*

As seen in Figure 2, this question is used to determine how students practice multiplication of two integers by comparing integers. For the calculation of the grade of a student in this question, 10 points are given if a student selects correctly the biggest integer and the smallest one and performs correctly their multiplication. If a student selects incorrectly one of the two integers or makes mistakes in multiplication, the student takes 5 points. But if a student selects incorrectly both integers, 0 point is given.

To ensure the content validity of the "Multiplication of Integers Achievement Test," all questions were developed based on the learning outcomes stated in the official mathematics curriculum and textbooks published by the Ministry of National Education. After initial drafting, the test items were reviewed by two mathematics education experts and one experienced mathematics teacher to confirm their appropriateness in measuring the targeted skills and alignment with grade-level expectations. Based on their feedback, necessary revisions were made to improve clarity and coverage.

Regarding reliability, a pilot implementation of the test was conducted with a group of students having similar characteristics to the study group. The internal consistency of the test was calculated using Cronbach's alpha, yielding a reliability coefficient of **.84**, indicating a high level of reliability for measuring students' achievement in integer multiplication

Division of Integers Achievement Test

The table of specifications taken into consideration when creating open-ended questions in the integer division achievement assessment test is given in Table 4.

Table 4. Table of specifications for the division of integers achievement evaluation test

No.	**Learning outcomes**	**Aim**
1	Performs division with integers.	To perform division of integers with appropriate models to make sense of division with integers.
2		To practice the division of two positive integers, two negative integers or two positive-negative integers.
3		To practice the division of integers by taking advantage of the "subtraction of integers" pre-learning.
4		To find the divisor or divisor in a division operation given the division result.
5		Practicing dividing two integers by remembering the "Compares integers" pre-achievement from 6th grade.
6		To perform division with integers by using the preliminary knowledge related to the multiplication of two or more integers with the same or different signs.
7		Finding the quotient by dividing integers and finding the factor not given in multiplication of integers.
8	Solves problems that require division with integers.	To use multiplication and division of integers in problems encountered in daily life, by taking advantage of the pre-learning of "Subtraction of integers".
9		To practice problem solving on the division of two integers, remembering the "Compares integers" pre-achievement in the 6th grade.
10		To find the divisors that can divide the divisor exactly when dividing integers.

The same scoring table as Table 3 is used for the questions of the division of integers achievement test and same procedure for grading a student is applied. To ensure the content validity of the "Division of Integers Achievement Test," the questions were designed in alignment with the official mathematics curriculum and textbook objectives regarding integer division. Draft items were reviewed by two mathematics education experts and one experienced teacher to confirm that the questions appropriately addressed the intended learning outcomes and cognitive levels. Necessary revisions were made based on expert feedback to enhance clarity and coverage.

The reliability of the test was assessed through a pilot application with a sample group similar in characteristics to the study group. Cronbach's alpha was calculated for internal consistency, resulting in a reliability coefficient of **.87**, indicating a high degree of reliability for evaluating students' performance in integer division

Intervention Process

The intervention carried out by the researcher was planned to last two weeks and cover 10 lesson hours. The subject and learning outcomes of the study are shown in Table 5.

Table 5. Scope of the study

Subject	Duration	Learning Outcomes
Numbers and Operations	3 lesson hours	Performs multiplication with integers.
	3 lesson hours	Performs division with integers.
	4 lesson hours	Solves problems that require multiplication and division with integers.

One week before the start of the lesson process for the experimental group, the suitability of the smart board of the school where the application would be carried out was examined together with the school's information technologies course teacher.

Manipulatives were used along with the textbook in accordance with the existing teaching lesson plan in the classroom selected for the experimental group, and before starting the lesson, virtual manipulatives were opened from the interactive board in the classroom and made ready for the lesson. The virtual manipulatives used in the experimental group include subject explanations, student studies, questions with solved video explanations, printable activities, subject screening tests, and theme-unit exams. In the first part of the course, information was given about the course's objectives and questions related to the objectives were asked to attract the student's attention to the subject. After drawing attention to the subject, the subjects learned in previous lessons were repeated, the students were asked questions about the addition of integers, and the subject was introduced by connecting them with the subject to be covered and by watching the videos in the lecture section on the virtual manipulatives made ready for use on the interactive board, with the students. In this section, the researcher explains the parts of the subject that need to bedding to fully understood. In the second part of the course, students were allowed to apply the student work section included in the virtual manipulatives on the interactive board. In the third part of the course, the students examine the solved questions section in the virtual manipulatives. Their solutions were compared with the solutions in the video. In the 4th part of the course, the students answered the test questions in the subject screening test section in the virtual manipulatives. In the 5th part of the course and in the last lesson where the study was carried out with the experimental group, the study questions in the assignments section of the virtual manipulative were printed and photocopied and then distributed to the students and allowed to be solved in the course.

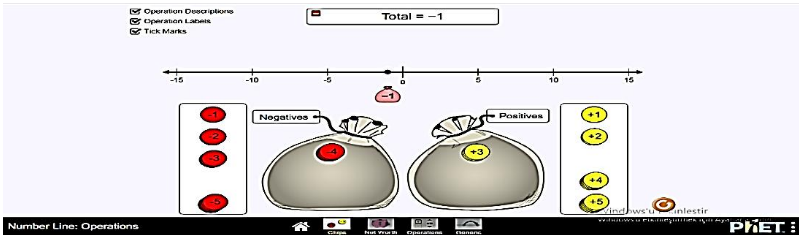


Figure 3. Example of activity applied in the experimental group – 1

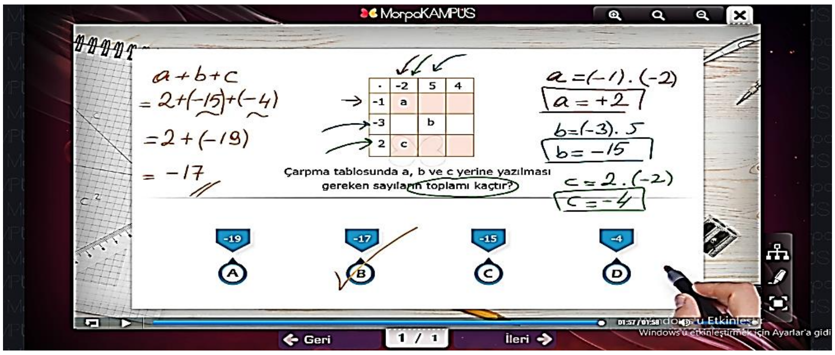


Figure 4. Example of activity applied in the experimental group – 5

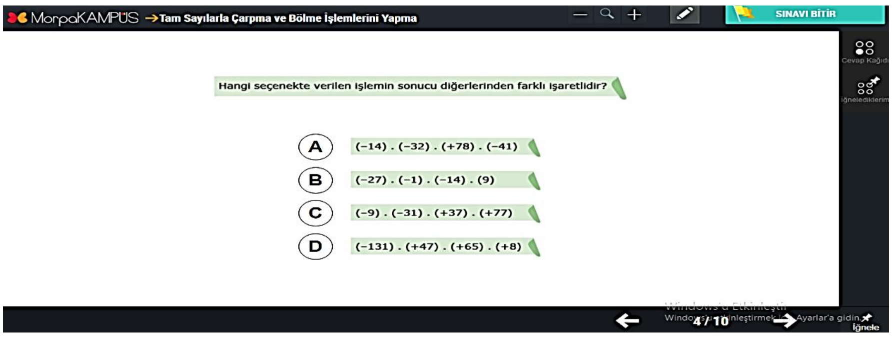


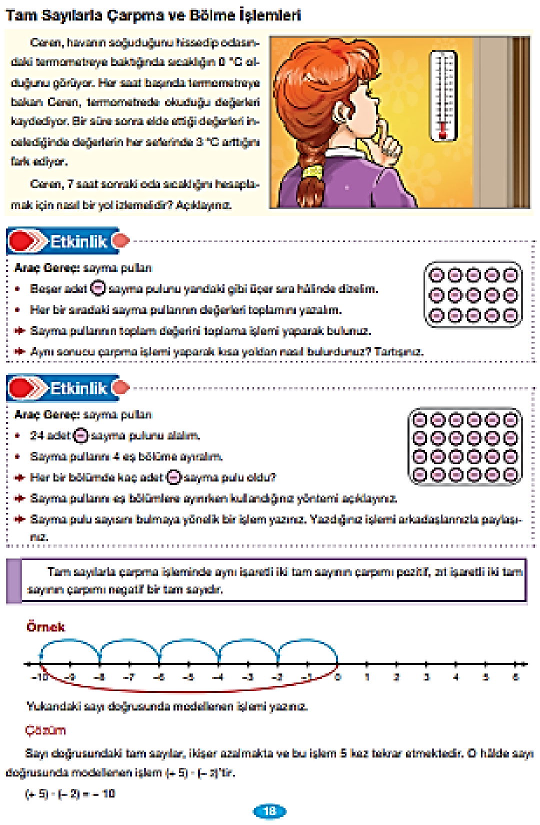
Figure 5. Example of activity applied in the experimental group – 6

The lesson plans applied to the control group during the lesson process are shown in Table 7. The researcher herself taught each lesson.

Table 6. Control group lesson process

Dur.(les.h.)	Control Group Lesson Processing
1	Talk to students about what they think about when "Integers" are mentioned. They are asked to talk about situations in which they use integers in daily life. While performing operations related to multiplication with integers, preliminary information about how to operate addition with integers, which is the previous topic, is obtained through the question-answer method. In order to draw the student's attention to the subject, the questions in the preparation and activity sections on page 18 of the textbook are examined and answered by the students in a way that is related to the outcome.
1	The solved questions on page 20 of the textbook are examined, and similar questions are written on the board by the teacher, and the students are given the necessary time to solve them. Students determined by the teacher are asked to solve the questions written on the board and explain them to their friends.
1	After the study questions about the outcome available online are printed and photocopied, they are distributed to the students and asked to solve them in class. Questions solved incorrectly by students are corrected and solved again by the teacher. The teacher summarizes the subject of multiplication of integers.
1	The teacher briefly reviews the previous lesson.The solved examples on pages 23 and 24 of the textbook will be examined with the students so that they can obtain information about the solution methods of the examples related to the relevant outcome.
1	The solved questions on pages 25 and 26 of the textbook are examined, and similar questions are written on the board by the teacher, and the students are given the necessary time to solve them. Students determined by the teacher are asked to solve the questions written on the board and explain them to their friends.
1	After the study questions about the outcome available on the Internet are printed and photocopied, they are distributed to the students and asked to solve them in class. Questions solved incorrectly by students are corrected and solved again by the teacher. The teacher summarizes the subject of the division of integers.
1	The teacher briefly reviews previous lessons on multiplication and division with integers. The introductory question on page 33 of the textbook is asked to the students and answered together.
1	The problem solving steps in the solved problems on pages 33 and 34 of the textbook are examined and reviewed, and similar problems are written on the board by the teacher and the students are given the necessary time to solve them. Students determined by the teacher are asked to solve the problems written on the board and explain them to their friends.
1	After the study questions about the outcome available on the Internet are printed and photocopied, they are distributed to the students and asked to solve them in class. Questions solved incorrectly by students are corrected and solved again by the teacher.
1	Students are asked to give examples of problems they encounter daily that require operations with integers and share the solutions they find in the course. The teacher summarizes the problems that require operations on integers.

The courses were carried out using the textbook and activity pages prepared by the researcher in accordance with the existing teaching plan in the classroom selected for the control group. In the control group, in the first part of the course, the students examined and answered the questions in the preparation and activity sections in the textbook to attract the student's attention to the subject. In the second part of the course, the researcher had the students write the rules of the signs related to the outcome in their notebooks. In the third part of the course, the students examined the solved examples in the textbook, similar questions were written on the board by the researcher, and the students were given the necessary time to solve them. In the 4th part of the course, the study questions about the outcome available on the Internet were printed and photocopied and then distributed to the students, and they were asked to solve them in the course. Questions solved incorrectly by the students were corrected and solved again by the researcher. In the 5th part of the course, the researcher made a summary of the subject. Finally, after the teaching of the achievements related to the subject was completed, the questions in the exercises section in the textbook were given as homework. After the necessary time was given to complete the assignment, the researcher checked the students' answers in the next lesson and any questions that were not understood were answered.



Tam Sayılarla Çarpma ve Bölme İşlemleri

Ceren, havanın soğuduğunu hissedip odasındaki termometreye baktığında sıcaklığın 0 °C olduğunu görüyor. Her saat başında termometreye bakan Ceren, termometrede okuduğu değerleri kaydediyor. Bir süre sonra elde ettiği değerleri incelediğinde değerlerin her seferinde 3 °C arttığını fark ediyor.

Ceren, 7 saat sonraki oda sıcaklığını hesaplamak için nasıl bir yol izlemelidir? Açıklayınız.

Etkinlik

Araç Gereç: sayma pulları

- Beşer adet sayma pulunu yandaki gibi üçer sıra hâlinde dizelim.
- Her bir sıradaki sayma pullarının değerleri toplamını yazalım.
- Sayma pullarının toplam değerini toplama işlemi yaparak bulunuz.
- Aynı sonucu çarpma işlemi yaparak kısa yoldan nasıl buldunuz? Tartışınız.

Etkinlik

Araç Gereç: sayma pulları

- 24 adet sayma pulunu alalım.
- Sayma pullarını 4 eş bölüme ayıralım.
- Her bir bölümde kaç adet sayma pulu oldu?
- Sayma pullarını eş bölümlere ayırırken kullandığınız yöntemi açıklayınız.
- Sayma pulu sayısını bulmaya yönelik bir işlem yazınız. Yazdığınız işlemi arkadaşlarınızla paylaşınız.

Tam sayılarla çarpma işleminde aynı işaretli iki tam sayının çarpımı pozitif, zıt işaretli iki tam sayının çarpımı negatif bir tam sayıdır.

Örnek

Yukarıdaki sayı doğrusunda modellenen işlemi yazınız.

Çözüm

Sayı doğrusundaki tam sayılar, ikişer azalmakta ve bu işlem 5 kez tekrar etmektedir. O hâlde sayı doğrusunda modellenen işlem (+ 5) · (− 2)'tir.

(+ 5) · (− 2) = − 10

18

Figure 6. Example of activity applied in the control group – 1

Analysis of Data

There are 10 questions in the achievement assessment tests used in the research. Achievement assessment tests consist of open-ended questions. Correct answers in the tests were evaluated as 10 points, partially correct answers as 5 points, and incorrect answers as 0 points. SPSS 22.0 (Statistical Package) was used to analyze the data obtained in the quantitative dimension of our research. First, normality tests were used to determine the data's normal distribution. Then, using normally distributed data, parametric tests were used to determine the difference between the experimental and control groups.

Findings

The research problem of the study is "Is there a statistically significant difference between the academic achievements of the experimental group students and the control group students who used virtual manipulatives in teaching multiplication and division with integers in the 7th-grade mathematics course of secondary school?". For this reason, giving the findings regarding the equality of the groups in this section was deemed appropriate before carrying out the experimental application.

Equivalence of Experimental and Control Groups

Before starting the experimental application of the research, a test consisting of 10 questions about the subject of "Addition and subtraction of integers," which came before the subject of "Multiplication and division of integers," was administered to the students in order to determine whether the readiness levels of the students participating in the research were equal. The analysis of these questions was scored as in the data analysis section. Kolmogorov-Smirnov and Shapiro-Wilk tests were applied to check whether the students' readiness test results were normally distributed. Analysis results are given in Table 8.

Table 8. Normality distribution of readiness test

Normality Tests			
	Shapiro-Wilk		
	Statistics	SD	p
Readiness	0.917	30	0.083

As seen in Table 8, the scores obtained from the readiness test of the students in the experimental and control groups participating in the research were subject to normality conditions according to both Kolmogorov-Smirnov ($p=0.200>0.05$) and Shapiro-Wilk ($p=0.083>0.05$) normality tests. It was concluded that the readiness test averages of the experimental and control groups had a normal distribution. Since the data showed normal distribution according to the normality test, independent samples t-test was used from parametric tests. The analysis results of this test are given in Table 9.

Table 9. Equivalence of experimental and control groups before the research

Group	N	Cover.	SS	sd.	t	p
Experiment	15	74.60	20.89	28	0.913	0.369
Control	15	66.47	27.47			

When Table 9 is examined, the independent samples t-test results determined that the difference between the average scores of the students in the experimental and control groups from the readiness test was not statistically significant (p=0.369>0.05). Therefore, it can be said that the groups are equivalent.

Comparison of Experimental and Control Groups According to Post-Test Results

Before starting the experimental application of the research, it was determined that the readiness levels of the students participating were equal. Therefore, in order to examine the difference between the academic achievements of the groups after the experimental application, it was first determined whether the results of the "Integer Multiplication Achievement Test" and "Integer Division Achievement Test," which were the data collection tools of the research, were normally distributed. Analysis results are given in Table 10.

Table 10. Normality distribution of post-test results

Normality Tests			
	Shapiro-Wilk Statistics	SD	p
Multiplication	0.935	30	0.066
Division	0.971	30	0.562

As can be seen in Table 10, the scores of the experimental and control group students participating in the research from the integer multiplication test were subject to normality conditions according to Shapiro-Wilk (p=0.066>0.05) normality tests. It was concluded that the integer multiplication test averages of the experimental and control groups had a normal distribution.

Similarly, the scores obtained by the experimental and control group students participating in the research from the integer division test were met with the normality condition according to both the Kolmogorov-Smirnov (p=0.200>0.05) and Shapiro-Wilk (p=0.562>0.05) normality tests. It was concluded that the division test averages in integer numbers belonging to the experimental and control groups had a normal distribution. Since the data showed normal distribution according to the normality test, independent samples t-test was used from parametric tests. The analysis results of this test are given in Table 11.

Table 11. Comparison of post-test results of experimental and control groups

	Group	N	Cover.	SS	sd.	t	p
Multiplication	Experiment	15	77.80	20,14	28	3,001	0.006*
	Control	15	51.27	27.68			
Division	Experiment	15	70,40	20.96	28	0.702	0.488
	Control	15	65.26	19.02			

When Table 11 is examined, according to the independent samples t-test results, it is seen that the difference between the mean scores of the students in the experimental and control groups from the multiplication posttest is statistically significant in favor of the experimental group ($t_{(28)}=3.001$; $p=0.006<0.05$). Considering the average values of the experimental and control groups, the average post-test success score of the students in the experimental group on the multiplication of integers is 77.8000, while the average post-test success score of the students in the control group in mathematics is 51.2667, and the score difference between the tests is 26.5333. According to the results of the research analysis, it can be said that the experimental group students were more successful in multiplication with integers than the control group students. This result can be interpreted as the use of virtual manipulatives was effective in making the experimental group more successful after the application on multiplication of integers.

When Table 11 is examined, according to the independent samples t-test results, it is seen that the difference between the average scores of the students in the experimental and control groups from the post-test division test is not statistically significant ($t_{(28)}=0.702$; $p=0.488>0.05$). When we look at the average values of the experimental and control groups, the average post-test success score of the experimental group for dividing integers is 70.4000, while the average mathematics course readiness success score of the students in the control group is 65.2667. According to the independent samples t-test results, the difference between the post-test mean scores of the students in the experimental and control groups was not statistically significant. This result shows that the students in the experimental and control groups are not different from each other regarding mathematics achievement scores after the application regarding the division of integers.

Discussion and Conclusion

The research question of this study explored whether there was a significant difference in academic achievement of students who were taught with virtual manipulatives in multiplication and division of integers compared to mathematics instruction align with the curriculum and the coursebook. The findings revealed the fact that the students in the experimental group are more successful in multiplying with integers than the students in the control group. This result can be interpreted as the use of virtual manipulatives was effective in making the experimental group more successful after the application of virtual manipulatives on multiplication of integers. Even if studied concepts are different from the multiplication of integers, the majority of researches suggest that digital manipulatives can be effective in terms of improving students' mathematical achievement and understanding (Byrne et al., 2023; Guan et al., 2024; Mazo, 2024; Ng & Ye, 2022; Pavlou et al., 2024; Shen, 2023). Specifically, the studies about the effect of using of virtual manipulatives in arithmetic operations of integers emphasized also that the students' academic success were increased. For instance, in the study of Moyer and Bolyard (2002), it was seen that the students' academic success in the addition and subtraction of integers with virtual manipulatives increased. Similarly, Samioğlu and Siniksaran (2016) concluded in their study that the use of virtual manipulatives in many 8th-grade mathematics subjects contributed positively to students' academic success. In addition, Camp (2018) in the result of her study that virtual manipulatives significantly improved participants' conceptual understandings of integer multiplication. These results are similar to the results of the current study on the subject of multiplication. One possible explanation for improvements in students' achievement while using virtual manipulatives in integers multiplication is that virtual manipulatives provide students with both visual and practical insights into abstract mathematical concept (here is multiplication of integers), thereby facilitating the development of the conceptual understanding necessary to grasp these underlying abstract principles.

Furthermore, the students' achievement while using virtual manipulatives in integers multiplication might be related to immediate feedback of virtual manipulatives. Goh et al. (2017) argued that the immediacy of feedback in virtual manipulation is critical as it helps build student confidence and performance. It also motivates them to practice and correct their mistakes if they are unsure about a particular procedure. Also, in the research conducted by Bolyard and Moyer Packenham (2006), it was found that virtual manipulatives with definite features, such as dynamic virtual representations and feedback, significantly enhance student learning, especially in subtracting integers.

On the other hand, when the average values of the experimental and control groups on the division of integers are examined, it is determined that the difference between the average scores of the students in the experimental and control groups in the post-test is not statistically significant. This situation can be thought to be due to the fact that the number of virtual manipulatives created for the division of integers is not sufficient, and the content of these virtual manipulatives is generally based on the grouping method. In addition to the grouping method, it can be tested that the change in success of the experimental group compared to the control group can be significant if virtual manipulatives are used in which division is seen as repeated subtraction.

According to the results obtained in the research, the following suggestions were made for both researchers and teachers:

1. During the course on academic success, manipulatives were examined statistically, and a positive result was reached. In this case, virtual manipulatives are recommended to provide diversity in learning environments.

2. Based on the research, it has been observed that the time spent by students with different academic achievements in virtual manipulative activities varies. Therefore, the effects of the duration of virtual manipulative use during the course on students' different academic achievement levels can be investigated.

3. Since teachers do not have sufficient knowledge about virtual manipulatives and even most teachers do not use virtual manipulatives during the lesson, it can be recommended that teachers be given the necessary training by determining appropriate times for the use of virtual manipulatives during the lesson.

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Aysun YURT: *Conducted the research, collected and analyzed the data, and wrote the initial draft of the manuscript.*

İbrahim KEPCEOĞLU: *Supervised the research process, provided guidance during data analysis and interpretation, and contributed to the revision and finalization of the manuscript*

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