GU J Sci 31(2): 416-435 (2018)

Gazi University

Journal of Science

http://dergipark.gov.tr/gujs

Comparison of Two Metaheuristic Algorithms on Sizing and Topology Optimization of Trusses and Mathematical Functions

Ali MORTAZAVİ[1,*], Vedat TOĞAN[2], Ayşe DALOĞLU[3], Ayhan NUHOĞLU[4]

[1]*Uşak University, Faculty of Engineering, Department of Civil Engineering, 64000, Uşak, Turkey*

[2,3]*Karadeniz Technical University, Faculty of Engineering, Department of Civil Engineering, 61080, Trabzon, Turkey*

[4]*Ege University, Faculty of Engineering, Department of Civil Engineering, 35100, Izmir, Turkey*

Article Info

Received: 26/12/2016
Accepted: 26/02/2018

Keywords

Size and topology optimization,
Particle swarm optimization,
Teaching learning based optimization,
Metaheuristic algorithms.

Abstract

Optimal solution of a desired optimization problem is mostly obtained via minimizing or maximizing a real function considering several predefined limitations. Selection of proper optimization algorithm as an optimizer tool plays a key role on the solution process. In this respect, current study intends to compare the performances of two different prevalent metaheuristic optimization algorithms. These are integrated particle swarm optimizer (iPSO) and teaching and learning based optimizer (TLBO). The former method is a single-phase algorithm while the latter one is the double-phase algorithm. Capabilities of both algorithms were compared separately on some mathematical benchmark test problems. Furthermore, to exhibit and compare their performance on solving more complex problems, size and topology optimization of the structural systems are also examined. Achieved results demonstrate the superiority of iPSO in comparison with TLBO in both search capability and convergence rate.

1. INTRODUCTION

Mathematically, the optimization process is defined as the selection of the best elements considering certain criteria from a set of existing choices. Nowadays optimization process is widely applied in the different fields of engineering in the form of maximization or minimization problems [1-3]. Consequently, in the structural optimization process the main goal is to find the minimum weight and/or production cost of the structure [4-9]. Related quantities are definable through an appropriate mathematical function. In case of being more than one objective, they are represented by a number of mathematical functions depending on the total number of purposes [10].

For a structural optimization, depending on the requirements of the designer the cross sectional areas of the members, the nodal coordinates and the connectivity pattern of members might be considered individually or in combination as the decision variables [11-16]. It is possible to obtain more optimal results via implementing simultaneously size, shape and topology optimization. However, in such a case the optimization problem becomes very complex due to increasing the number of design variables and/or non-convexities of the search space. Additionally, components of these three types of design variables can belong to discrete and/or continuous search spaces [7]. Two different approaches can be utilized for

[*] Corresponding author, e-mail: ali.mortazavi@usak.edu.tr

optimization; these aspects are unimodal and multimodal [7,17]. In unimodal approach all variables are taken into account simultaneously while in the multimodal approach variables are optimized separately. It is notable that since the corresponding design variables (i.e. size, shape and topology) are not linearly independent and so they should be taken into account all together. Due to this fact, the solution process for this class of problems requires using a robust algorithm as the optimization tool. Metaheuristic algorithms, which copycat the natural phenomena or physics rules, have been applied widely to attain the optimal solution of complex problems.

Unlike the classical form of Particle Swarm Optimizer (PSO) [18], Integrated Particle Swarm Optimizer (iPSO) offers enhanced version of PSO with robust and novel techniques such as improved fly-back method and weighted particle concept [19]. On the other hand, TLBO [20] is conceptually modeled on the two types of pedagogies within a classroom: class-level learning from a teacher and individual learning between students [21]. It is notable, TLBO is the two-phase algorithm, it means in each iteration objective function should be evaluated twice (i.e. once in the teaching phase and once in the learning phase). Indeed, exploration and exploitation search strategies are separately performed in each iteration in this method, and as shown in different work studies [5, 20-30] such an approach can provide a proper search capability to find an optimal solution. However, such methods (two-phases or higher) seems inappropriate for solving several types of engineering problems (e.g. structural optimization problems). Because in this type of problems objective function evaluation (OFE) is the most time consuming part of the optimization process. In this respect, although the iPSO demonstrates adequate ability on handling the different types of complex optimization problems [4, 31-33], in this study we precisely tested capabilities of iPSO, as the single-phase optimizer, in comparison with TLBO as a double-phase algorithm in more details. It is notable that since TLBO have been widely applied in the different fields of science and engineering [5, 20-30] this comparison can provide illustrative and explanatory outcomes on capabilities of these two class of optimizers. Based on this argue, current study intends to compare the performances of the Integrated Particle Swarm Optimizer (iPSO) and Teaching Learning Based Optimizer (TLBO) over minimizing both the mathematical benchmark functions and obtaining the optimal topology and size parameters of truss structures. The results show that iPSO as single-phase algorithm, outperforms TLBO as a double-phase approach in terms of convergence rate and solution's accuracy.

2. INTEGRATED PARTICLE SWARM OPTIMIZATION (iPSO)

The particle swarm optimization (PSO) mimics the behavior of animals (e.g. the colony of fish and birds) to find food sources or to avoid from enemies in the nature. This method is utilized in variety application areas and based on reported results it is observed that the traditional PSO has some drawbacks such as staggering of the convergence in later stage of the process. To relieve these burdens, the different forms of PSO are developed to improve its performances [34-37]. The iPSO is one on these methods. Initially the fly-back approach and weighted particle definitions are presented. Subsequently, the corresponding formulations for iPSO are given in this section.

2.1. Improved Fly Back Method to Handle Constraints of the Problem

In many engineering problems it is necessary to apply some proper constraints to acquire feasible solutions. The fly-back method was introduced by He et al. [38] to handle the relevant constraints. This method satisfies the constraints by constantly keeping all particles in the feasible region during the optimization process.

To enhance the performance of standard fly back method, it is modified by considering type of the violated constraints. This new approach is called improved fly-back mechanism and it works in three main steps [19]. These steps can be expressed logically as: (i) determine whether the updated particle violates the constraints or not; (ii) for violation, find which components cause the violation. Then, replaces them with corresponding components; (iii) determine whether the new particle obeys the constraints and produces a better fitness function value than old particle or not. If so, change it with the old one.

2.2. Weighted Particle

In standard PSO a drawback occurs when a particle stands very close to its own best prior position, $\mathbf{X}_i^P$, and the location of the global best particle, $\mathbf{X}^G$, or both of them. In such a case, the effect of one or even both of these important points on guiding the current particle is highly reduced or even vanished and the probability of trapping into the local minimum(s) increases. Li et al. [35] defined a new particle so called weighted particle to avoid the aforementioned situation. Weighted particle actually is the weighted gravity center of all particles available in the swarm, so it can be defined as below:

$$\mathbf{X}^W = \sum_{i=1}^{M} \bar{c}_i^W \mathbf{X}_i^P \tag{1}$$

$$\bar{c}_i^W = \hat{c}_i^W \Big/ \sum_{i=1}^{M} \hat{c}_i^W \tag{2}$$

$$\hat{c}_i^W = \frac{\max\limits_{1\le k\le M}\left(f(\mathbf{X}_k^P)\right) - f(\mathbf{X}_i^P) + \theta}{\max\limits_{1\le k\le M}\left(f(\mathbf{X}_k^P)\right) - \min\limits_{1\le k\le M}\left(f(\mathbf{X}_k^P)\right) + \theta} \qquad i = 1, 2, \dots, M \tag{3}$$

in which, M denotes the number of particles, $\mathbf{X}^W$ is the position of weighted particle, $\hat{c}_i^W$ is the weighted constant of each particle. Also, $f(.)$ indicates the objective function of the problem, while $\max\limits_{1\le k\le M}\left(f(x_k^P)\right)$ and $\min\limits_{1\le k\le M}\left(f(x_k^P)\right)$ are represent the worst and the best fitness values in the Pbest, respectively. Finally, θ specifies a small positive value just to prevent division-by-zero condition. In this study, it is taken as 0.001.

2.3. Integrated Particle Swarm Optimization (iPSO) Formulation

The iPSO proposed by Mortazavi and Toğan [4] is different form of the classical PSO, which is strengthened with powerful concepts of improved fly-back method and weighted particle. Both the concepts and remarkable characteristics of them are explained below. This new formulation enhances the swarm search ability to find the optimal solution via providing more chance to escape from local minima. Accordingly, the iPSO mathematically can be formulated as:

$$\left.\begin{array}{l} {}^{t+1}\mathbf{v}_i = 0 \\ {}^{t+1}\mathbf{X}_i = {}^{t}\mathbf{X}_i + \varphi_{4i}\left({}^{t}\mathbf{X}^{\mathrm{w}} - {}^{t}\mathbf{X}_i\right) \qquad i \geq 1, \;\; \varphi_{4i} = C_4 \times \mathrm{rand}_{4i} \end{array}\right\} \text{ if } \mathrm{rand}_{0i} \leq \alpha$$

$$\left.\begin{array}{l} {}^{t+1}\mathbf{v}_i = w_i \times {}^{t}\mathbf{v}_i + \left(\varphi_{1i} + \varphi_{2i} + \varphi_{3i}\right)\left({}^{t}\mathbf{X}_j^{P} - {}^{t}\mathbf{X}_i\right) \\ \\ +\varphi_{2i}\left({}^{t}\mathbf{X}^{G} - {}^{t}\mathbf{X}_j^{P}\right) + \varphi_{3i}\left({}^{t}\mathbf{X}^{w} - {}^{t}\mathbf{X}_j^{P}\right) \end{array}\right\} \text{ if } \mathrm{rand}_{0i} > \alpha \qquad (4)$$

$${}^{t+1}\mathbf{X}_i = {}^{t}\mathbf{X}_i + {}^{t+1}\mathbf{v}_i \qquad i \geq 1, \;\; j \leq M, \;\; \varphi_{1i} = C_1 \times \mathrm{rand}_{1i},$$
$$\varphi_{2i} = C_2 \times \mathrm{rand}_{2i}, \;\; \varphi_{3i} = C_3 \times \mathrm{rand}_{3i}$$

where, φ_{1i} , φ_{2i} , φ_{3i} and r_i are the vectors of coefficients contain random numbers uniformly selected from interval of U (0, 1). Superscripts of "t" and "t+1" indicate current and updated time steps, respectively. Also, C1= $-(\varphi_{1i} + \varphi_{2i})$, C2 = 2, C3 =1, and C4=2 are accelerator factors, and rand_{ki} where $k \in \{0,1,2,3,4\}$ is a value which randomly selected from [0, 1] interval; ${}^{t}\mathbf{X}_j^{P}$ is the randomly selected from previous best memory and ${}^{t}\mathbf{X}^{G}$ is the global best particle while ${}^{t+1}\mathbf{X}_i$ and ${}^{t}\mathbf{X}_i$ respectively give the updated location and current location of the *i*th particle. Also, ${}^{t}\mathbf{X}^{\mathrm{w}}$ is the weighted particle of the current colony. Inertia factor (w_i) is randomly selected between [0.5, 0.55] and α is taken as 0.4 correspondingly [4].

3. TEACHING LEARNING BASED OPTIMIZATION (TLBO)

Rao et al. [20] introduced a new approach so called Teaching Learning Based Optimization (TLBO) which simulates the interaction arisen within a classroom between the teacher and the students and between the students themselves as well. This process is also known as the teaching-learning process, pedagogically. The main goal in this process is to increase the performance level of the students and overall performance of the class towards the optimal point of knowledge level. The TLBO algorithm has two main phases: the teacher phase, where the average knowledge of the class is moved towards teacher; and the learner phase, where students share their knowledge with each other. Several studies over the structural optimization were carried out using the TLBO [5, 28, 29].

The TLBO applies two different phases as teaching phase and learning phase to conduct agents through the search domain. Teaching phase is mathematically described as bellow:

$$\begin{array}{lll} \mathbf{x}_i^{*} = x_i + r.(\mathbf{x}_{Teacher} - T_F \mathbf{x}_{mean}) & & \\ if \quad f(\mathbf{x}_i^{*}) < f(\mathbf{x}_i) & \mathbf{x}_i = \mathbf{x}_i^{*} & \\ if \quad f(\mathbf{x}_i^{*}) \geq f(\mathbf{x}_i) & \mathbf{x}_i = \mathbf{x}_i & \end{array} \qquad (5)$$

in which,

$$\mathbf{x}_{\text{mean}} = \bar{\mathbf{x}}_{i,j} = \frac{\sum_{i=1}^{np} x_{i,j}}{np} \tag{6}$$

where, x^* shows the renewed form of x_i, r is a random number varying [0,1], T_F is a teaching factor, T_F = *round* [1 + *rand* (0,1)] and is defined randomly [39].
Also, learning phase is given as follows:

$$\begin{aligned} \mathbf{x}_i^* &= \mathbf{x}_i + r.(\mathbf{x}_i - \mathbf{x}_j) \qquad \mathit{if} \qquad f(\mathbf{x}_i) < f(\mathbf{x}_j) \\ \mathbf{x}_i^* &= \mathbf{x}_i + r.(\mathbf{x}_j - \mathbf{x}_i) \qquad \mathit{if} \qquad f(\mathbf{x}_i) > f(\mathbf{x}_j) \end{aligned} \tag{7}$$

where, $\mathbf{x}^*$ and $\mathbf{x}_i$ are the new and existing solution of i, $\mathbf{x}_j$ is the any solution to be different from $\mathbf{x}_i$. If the solution, $\mathbf{x}^*$, provides better objective function value than $\mathbf{x}_i$ then $\mathbf{x}_i$ is eliminated and replaced with $\mathbf{x}^*$, otherwise $\mathbf{x}_i$ is preserved. For more clarity, the corresponding pseudo code for TLBO is provided in Table 1.

Table 1. *The pseudo code for TLBO*

Generate initial random population (i.e. class)
while one of termination criteria is not occurred **do**
  evaluate each student (solution) in the population
  determine the best learner and assign him/her as the teacher ($\mathbf{x}_{Teacher}$ *)*
  obtain mean of population based on Eq. (6)
  perform teaching phase using Eq. (5)
  perform the Learning phase using Eq. (7)
end do
report the results

4. THE FORMULATION OF THE OPTIMUM DESIGN PROBLEM

Structures can be optimized for size, shape and topology aspects and any combination of them. Each form serves for different task and only the definition of the design variables and the constraints of the problem are changed. A classical structural optimization problem is generally formulated as follows:

$$\begin{aligned} &\text{find} \quad \mathbf{X} = \{x_1, \ldots, x_{nd}\} \\ &\text{min.} \quad f(\mathbf{X}) = \sum_{e=1}^{ne} L_e \rho_e A_e \\ &\text{subject to} \quad g_k(\mathbf{X}) \le 0 \\ &\qquad\qquad x_{\min,i} \le x_i \le x_{\max,i} \end{aligned} \tag{8}$$

where $\mathbf{X}$ is a vector of the design variables; *nd* represents total number of design variables; $f(\mathbf{X})$ is the objective function, which is taken as the weight of the structure; L_e, ρ_e, and A_e are the length, material density and cross-sectional area of the *e*th element while *ne* demonstrates the total number of elements in the structure; $g_k(\mathbf{X})$ is the *k*th constraint function, x_i is the *i*th design variable, while $x_{min,i}$ and $x_{max,i}$ are its lower bound and upper bound, respectively. In engineering applications, design variables might be linked so as to acquire a symmetric structure and/or to reduce the search space. For this reason, *nd* actually shows the total independent design variables.

In this study, since size-topology optimization of truss structures is examined, the optimization constraints and fitness function are mathematically expressed as:

$$\begin{aligned} &\text{find} \quad \mathbf{X} \\ &\text{min.} \quad W(\mathbf{X}) = \sum_{e=1}^{ne} L_e \rho_e A_e \end{aligned} \tag{9}$$

$$\begin{aligned} \text{subject to} \quad & g_1(\mathbf{X}) = d_k(\mathbf{X}) \le d_{\max,k} \\ & g_2(\mathbf{X}) = \sigma_e(\mathbf{X}) \le \sigma_{a,e} \quad \text{(for tension and compression)} \\ & g_3(\mathbf{X}) = x_{\min,e} \le x_e \le x_{\max,e} \\ & g_4(\mathbf{X}) = \text{kinematic stability} \end{aligned} \tag{10}$$

In which, *W* is the weight of structure, *ne* is the total number of members in the corresponding truss structure, ρ is the material density, *L* is the length of the members. d_k and $d_{max,k}$ show the displacement and maximum displacement allowed for node *k*, correspondingly. σ_e is the stress available in the eth member, while $\sigma_{a,e}$ is the allowable tensile or compression stress. $x_{min,e}$ and $x_{max,e}$ are the lower and upper boundary values for the cross section of *e*th member. and the kinematic stability indicates the dynamic stability and bucking criterion limitations for the structure.

In this investigation, in order to topology optimization, cross sections are considered both negative and positive. Such that, the negative values imply that the corresponding member must be removed while positive values are applied for valid elements.

5. CRITERIA OF EVALUATION FOR MATHEMATICAL FUNCTIONS

In current investigation in order to make comprehensive comparison between TLBO and iPSO one shifted unimodal and two shifted and rotated multimodal functions taken from CEC2005 [40] are minimized via two investigated methods. The dimension of all functions is taken as *D*=30. For all functions maximum number of objective function evaluations (OFEs) is taken as 10000**D* (i.e. OFEs=300000). All processes are started uniform random initialization inside the search space. Termination criteria are to reach maximum OFEs or maximum error value. The error value ($f(\mathbf{X})-f(\mathbf{X}^*)$) is taken as 1.0E-05 in which the $\mathbf{X}^*$ is the vector of decision variables which gives the optimum value of $f(\mathbf{X})$. The diversity index for all runs are reported via following formulation [41]:

$$Diversity(t) = \frac{1}{N|L|} \sum_{i=1}^{N} \sqrt{\sum_{j=1}^{D} \left(X_i^j - \bar{X}^j \right)^2} \tag{11}$$

where *t* is the current iteration, *N* is total number of agents, *L* is the search space longest diagonal length, *D* is the dimension of the problem, $\bar{X}^j$ is the mean value of all swarm over jth component.

6. EXAMPLES

This section is divided into two parts as mathematical functions and real world structural problems in order to demonstrate clearly the performances of the relevant algorithms on the different types of optimization problems from the different fields. To solve the numerical problems, the computer equipped with the Intel™ CORE i5 CPU @ 2GHz and 4 MB of installed RAM is utilized.

6.1. Mathematical Functions

This section is devoted to solve 3 mathematical functions to test the performance of the two considered metaheuristic algorithms. All of these functions are taken from problem definitions and evaluation criteria for the CEC 2005 database [40]. It is remarkable that, to provide a unified set of test problems to evaluating different methods in the common circumstances and in the more systematic manner, several mathematical functions are defined in the CEC 2005 databased. All of these test problems are defined and/or modified by experts in this field. So, all available benchmarks (e.g. shifting and rotating functions) in this selective database can efficiently applied to assess the optimization algorithms. Due to this reason the numerical benchmark functions for the current study are selected from this database.

6.1.1. Shifted Sphere Function

As shown in Figure 1 the shifted sphere function is considered as the first example. The dimension, *D*, of the function is taken as 30. The formulation of corresponding function is shown in Eq. (12). In this equation **O** indicates shifted global optimum.

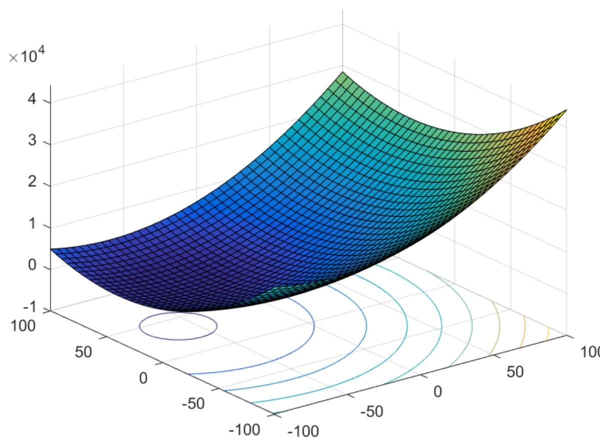


Figure 1. *The shifted sphere function*

The convergence history and diversity of the optimization process are shown in Figs.Figure **2**-Figure **3**. As can be seen from convergence history plots, the iPSO outperforms the TLBO with faster convergence rate in optimization of shifted sphere function. It is notable that the average required time for iPSO and TLBO methods for optimizing of the current example are registered as 115.98 s and 396.07 s, respectively. However, as the interaction level is much higher in TLBO due to learning phase the diversity index of TLBO is higher during whole optimization process.

$$f(\mathbf{X}) = \sum_{i=1}^{D} \mathbf{Z}_i^2 - 450 \qquad , \quad \mathbf{Z} = \mathbf{X} - \mathbf{O} \tag{12}$$

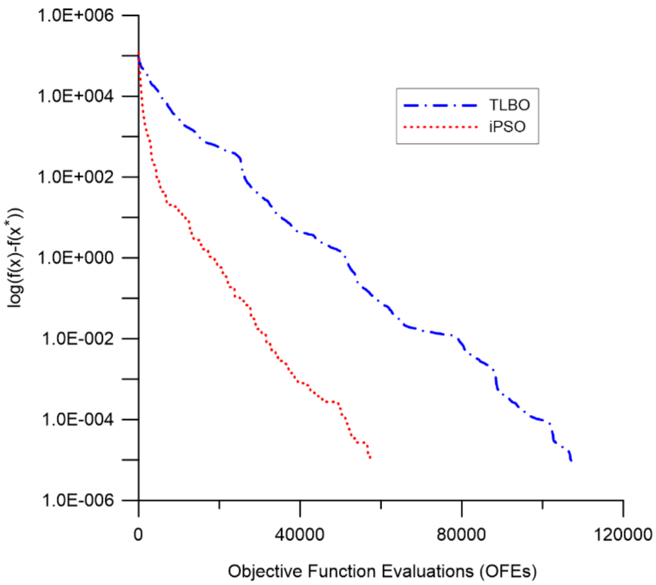


Figure 2. *Convergence history for iPSO and TLBO over shifted sphere function optimization*

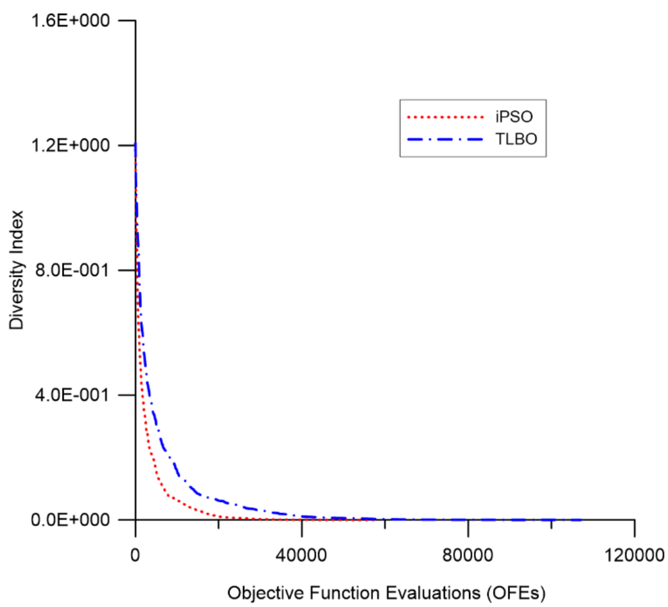


Figure 3. *Diversity indexes for iPSO and TLBO over shifted sphere function optimization*

6.1.2. Shifted Rosenbrock's Function

As shown in Figure 4 the shifted Rosenbrock's function is addressed as next example. The dimension, D, of the function is taken as 30. The formulation of this function is given in Eq. (13). In this equation **O** indicates shifted global optimum.

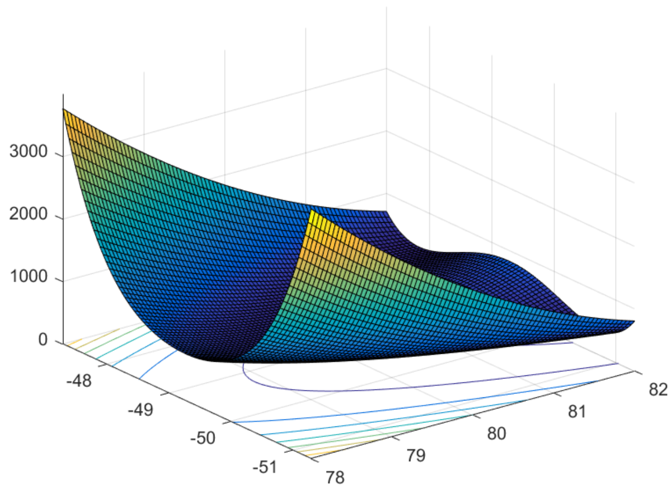


Figure 4. *The shifted Rosenbrock's function*

$$f(\mathbf{X}) = \sum_{i=1}^{D-1} \left(100\left(z_i^2 - z_{i+1}^2\right) + \left(z_i + 1\right)^2\right) + 390 \qquad , \mathbf{Z} = \mathbf{X} - \mathbf{O} + 1 \tag{13}$$

The convergence history and diversity of the optimization process for current example are given in Figs. Figure **5**-Figure **6**, respectively. As can be seen from convergence history plots, the iPSO similar to prior example outperforms the TLBO with both faster convergence rate and final solution in optimizing the Rosenbrock's function. It is remarkable that, average run time for iPSO and TLBO methods for solving the current example are 201.1 s and 502.09 s, respectively. The diversity index of TLBO is higher than iPSO nearly during whole optimization process; this can be result of learning phase of TLBO. The rationale behind this issue is that TLBO in each iteration performs both exploration and exploitation search strategies separately through the teaching and learning phases, respectively. Consequently, high number of local search performed during the learning phase (i.e. in each iteration) causes the agents continuously affected by each other rather than just going toward the best agent (i.e. teacher), subsequently, such a strategy provides higher diversity level among the agents.

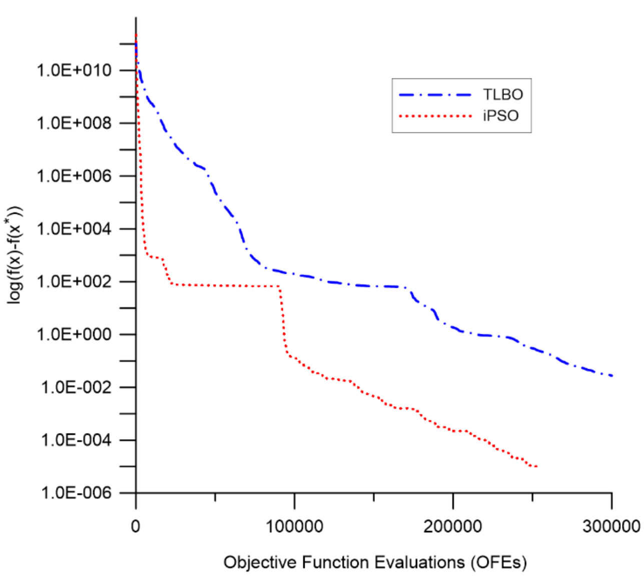


Figure 5. *Convergence history for iPSO and TLBO over Rosenbrock's function optimization*

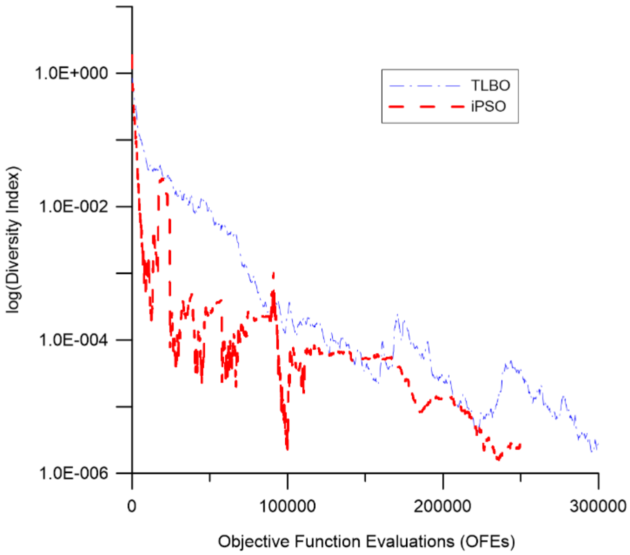


Figure 6. *Diversity indexes for iPSO and TLBO over Rosenbrock's function optimization*

6.1.3. Shifted Rotated Griewank's Function

For last mathematical problem as shown in Figure 4 the Shifted Rotated Griewank's Function is addressed as next example. The dimension (D) of the function is taken as 30.

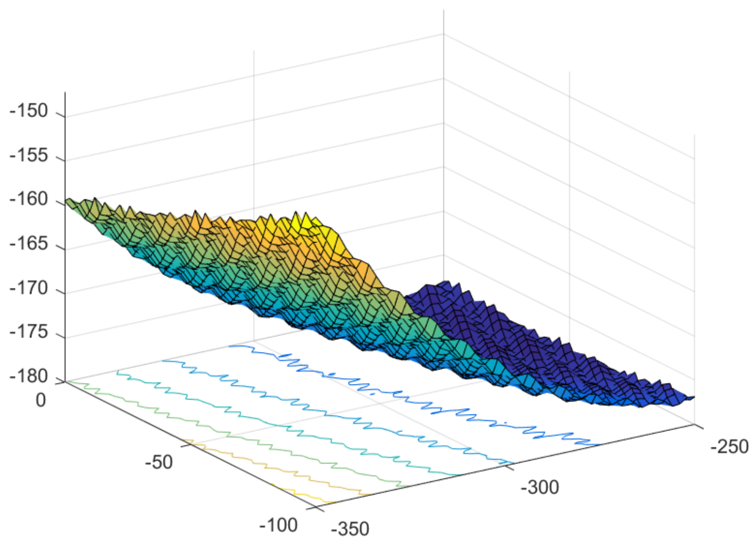


Figure 7. *The Shifted Rotated Griewank's Function*

The formulation of this function is given in Eq. (14).

$$f(\mathbf{X}) = \sum_{i=1}^{D} \frac{z_i^2}{4000} - \prod_{i=1}^{D} \cos\left(\frac{z_i}{\sqrt{i}}\right) + 1 - 180 \tag{14}$$

The convergence history and diversity of the optimization process for current example are given in Figs. Figure **8**-Figure **9**, respectively. As can be seen from convergence history plots, the iPSO similar to prior example outperforms the TLBO with faster convergence rate in optimizing the Shifted Rotated

Griewank's function. It is notable that, average required time for iPSO and TLBO methods for current example are 48.2 s and 201.87 s, respectively. Since the interaction level is higher in TLBO, due to learning phase, the diversity index of TLBO is higher during whole optimization process.

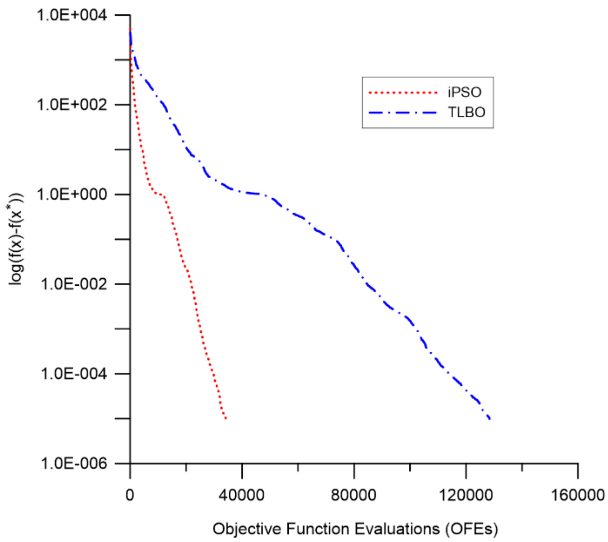


Figure 8. *Convergence history for iPSO and TLBO over Shifted Rotated Griewank's function optimization*

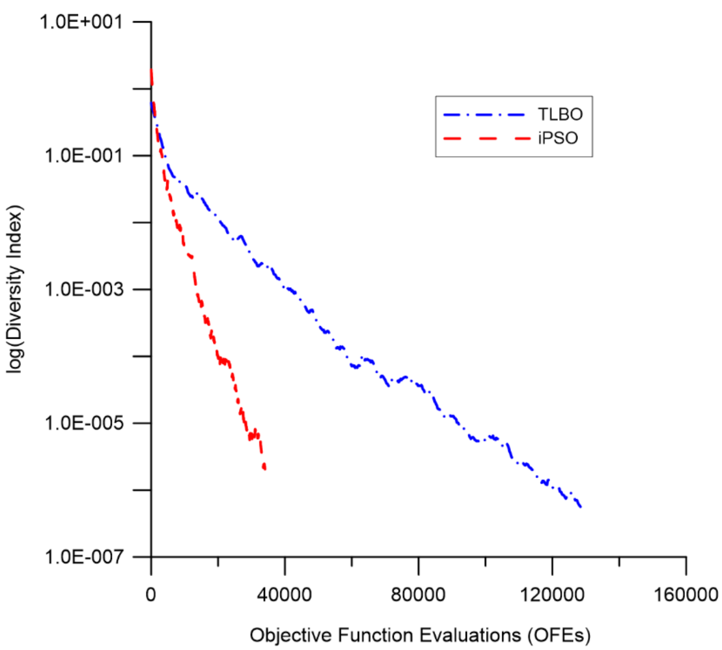


Figure 9. *Diversity indexes for iPSO and TLBO over Shifted Rotated Griewank's function optimization*

6.2. Truss Structures Optimization

Two typical truss optimization examples are considered to demonstrate the feasibility and validity of the iPSO and the TLBO for solving size-topology optimization of trusses. For each example, the optimization process is repeated 30 times. At every turn, the population used in the solution process is generated independently and randomly. The value of T_F is taken as 1 in the TLBO process.

6.2.1. 11-Bar Truss Structure

The ground structure of 10 bar truss is given in Figure 10. Size and topology optimization of this structure is already studied by Deb and Gulati [39] and Hajela and Lee [42] using genetic algorithms (GA) and Miguel et al. [7] using firefly algorithm (FA) via a multimodal approach. The design parameters adopted in the present optimization process are taken as the modulus of elasticity E=10000 ksi (68947.59 MPa), density of the material ρ=0.1 lb/in^3 (2768 kg/m^3), allowable stress in both tensile and compressive σ_a=±25 ksi (172.36 MPa), and maximum displacement d_{max}=±2 in (±5.08 cm). (in all principal directions). In addition, the cross sections can be selected from [0, 35] in^2 ([0, 225.8] cm^2) interval which specify the continuous search space for sizing variables.

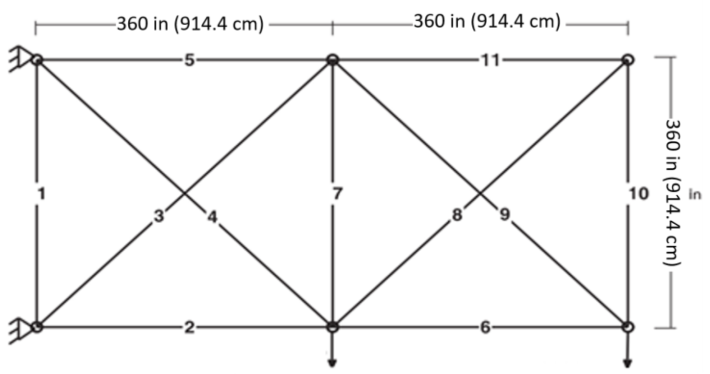


Figure 10. *Ground structure*

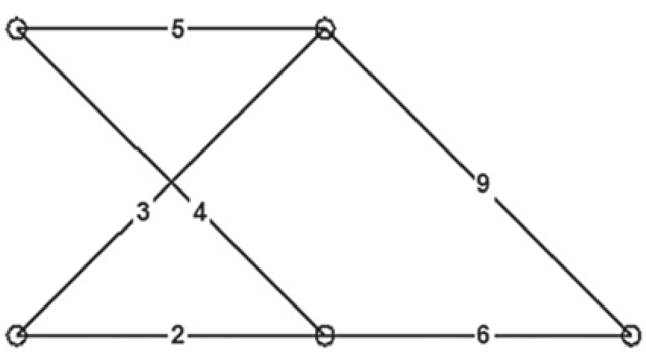
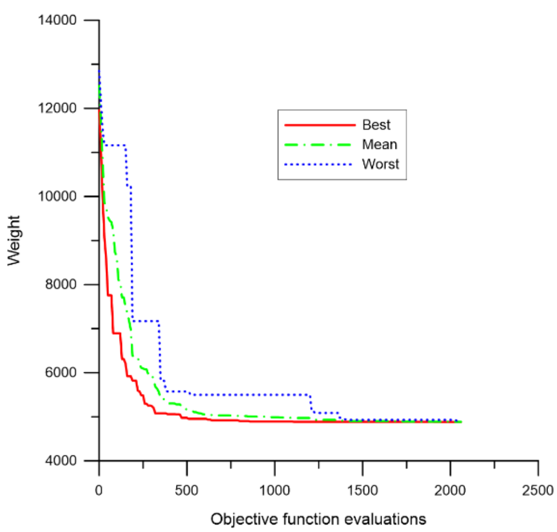


Figure 11. *Optimum solution for 11-bar truss structure*

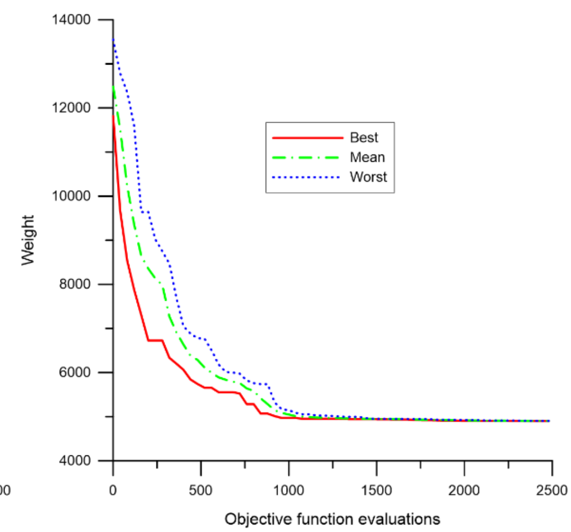
Table 2 comparatively summarizes the results found by different techniques for current example. As can be seen from this table the iPSO found the lightest structure among all other referred methods. Particularly, iPSO in comparison with TLBO could find the structure which is 16.95 lb (128.38 N) lighter. Such difference for such a simple example can be remarkable. The required duration for iPSO and TLBO for optimizing this problem are 210.22 s and 552.91 s, respectively. The convergence history diagrams for both iPSO and TLBO are given in Figure 12. In this figure the best, worst and mean diagrams indicate the history of the best agent, the worst agent and mean value of all available agents in the colony, respectively. Also, Figure 12 demonstrates that convergence rate of both algorithms nearly is the same (780 OFEs for iPSO and 1100 OFEs for TLBO) but it is astonishing that TLBO shows significant inconsistency in finding final solution. Such that standard deviation for iPSO is 90.12 lb (581.41) while this value for TLBO is 680.34 lb (4389.28 N).

Table 2. *The results comparison*

Member number	Member areas, in^2					
					Current Study	
	FA [7]	GA[42]	GA [43]	GA [39]	iPSO (cm^2)	TLBO (cm^2)
2	24	24	30.1	29.68	21.419 (138.19)	22.271 (143.68)
3	20	21	22	22.07	21.893 (141.24)	20.947 (135.14)
4	6	6	15	15.3	6.1233 (39.51)	6.038 (38.95)
5	30	28	6.08	6.09	29.651 (191.3)	31.023 (200.15)
6	16	16	21.3	21.44	14.695 (94.81)	17.367 (112.04)
9	21	22	21.3	21.29	21.34 (137.68)	19.243 (124.15)
Weight (lb)	4912.85	4942.7	4900	4899.15	4880.36	4897.32
Weight (N)	21853.44	21986.216	21796.27	21792.50	21708.91	21784.36



(a)



(b)

Figure 12. *Convergences histories of the (a) iPSO and the (b) TLBO methods*

6.2.2. 39-bar Truss Structure

As last example a 39 bar two-layer truss structure is addressed. This example was previously optimized by Deb and Gulati [39] using GA. Configuration of the ground structure for this example is depicted in Figure 13, wherein the overlapped members are displayed with a little gap to clarify the visualization. Member's connectivity is presented in Figure 13. To maintain symmetry members are grouped symmetrically about middle vertical member. So, members are placed into 21 independent groups. The cross section variables considered within the continuous search space (A_{min}= -2.25 in^2 (-14.51 cm^2) $\leq A_i \leq$ A_{max}= 2.25 in^2 (14.51 cm^2)). A negative cross section indicates that its corresponding element should be removed. To give the equal chance for both existence and eliminations of the elements, the size search space is selected symmetric around the origin (A=0). The modulus of elasticity is 10000 ksi (68947.59

MPa) and the material density is 0.1 lb/in^3. Allowable stresses in both tensile and compressive are 20 ksi (137.89 MPa). The nodal displacements are limited to ±2in and critical area ϵ=0.05 in^2 (0.32 cm^2).

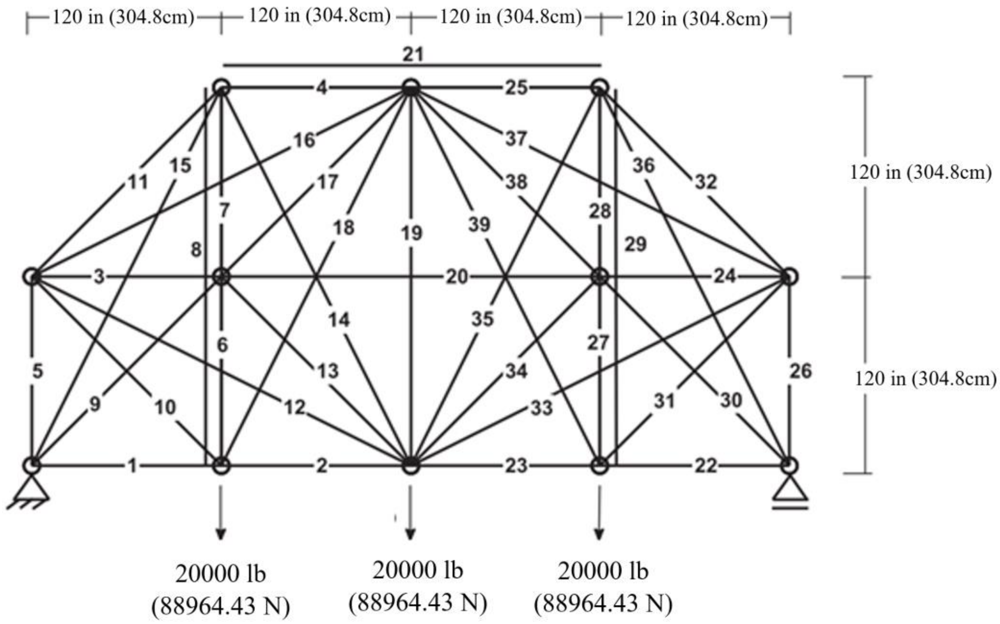


Figure 13. *Ground structure for 39-bar truss structure*

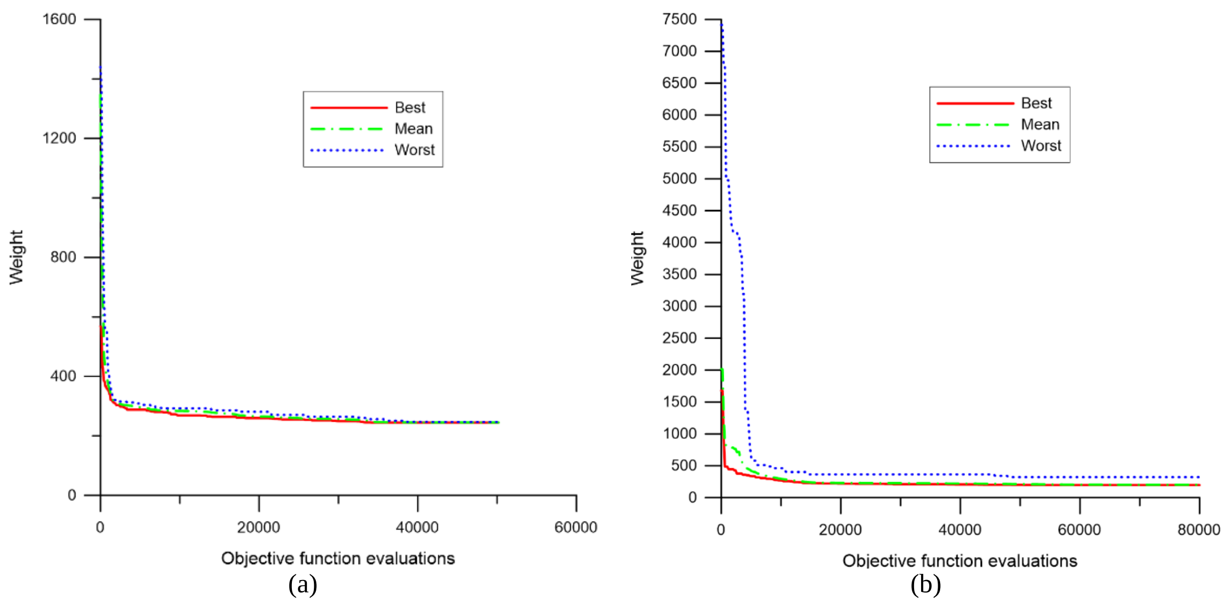


Figure 14. *Convergences histories of the TLBO with a) 30 and b)100 agents*

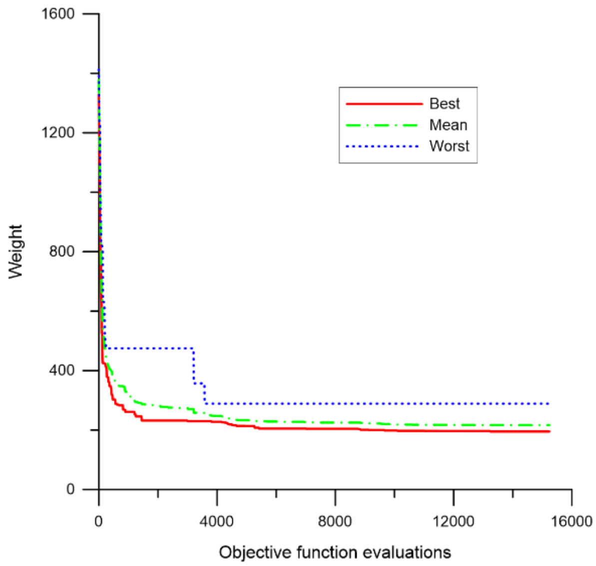


Figure 15. *Convergences histories of the iPSO*

In this example firstly, iPSO and TLBO both start from same number of initiated agents (as 30 agents) and both algorithms run for 30 times and best results are reported. The obtained optimal structures for iPSO and TLBO are shown in Figure 16 and Figure 18, respectively. In this figure the cross-sectional areas are shown in in^2, and for more clarity in parentheses they also reported in cm^2 form. As the structure is symmetric, these sections are shown for half of structure to prevent complexity in the figures and overlapped members are shown with a little gap. Based on this figure the iPSO can find highly lighter structure than TLBO. Also, the iPSO outperforms the TLBO in the number of objective function evaluations (OFEs), since iPSO required 16000 OFEs while the TLBO needs up to 60000 OFEs. Also, computational time for iPSO and TLBO on solving the current example are 640.55 s and 3025.59 s, respectively. In order to decrease the sensitivity of TLBO to the number of initial agents, the TLBO is run one more time but with a colony consist of 100 number of agents. In later run, TLBO within the 3041.95 seconds could find more optimal structure. This structure and corresponding cross-sectional areas of the members are also presented in Figure 17. As can been seen in latter case the TLBO runs with a population size nearly five time larger than the number available variables of the problem. Also, it needs 80000 OFEs which is considerably higher than required OFEs for iPSO.

Since several topologies can be obtained for this example, rather than any detailed table just the final solutions (final value of objective function) obtained by different methods are tabulated in Table 3. The achieved optimal topologies and cross-sectional areas are addressed in Figs Figure **16**-Figure **18**. Based on the reported results in Table 3, FA method acquires the lightest structure in comparison with all other cited methods. However, it should be noted that in the related study [7] the accuracy for continuous sizing variables for FA is accepted up to nine digits after decimal, while to make a fair comparison, this accuracy is taken up to two digits after decimal in this study. For current example iPSO is considerably superior than TLBO in both convergence rate and final solution. The standard deviation for iPSO is 180.3 lb and this value for TLBO is 199.22 lb (886.17 N) and 197.21 lb (877.23 N) for the first case (i.e. TLBO with population size=30 and the second case (i.e. TLBO with population size=100), respectively. Thus, the stability of iPSO nearly is higher than TLBO. It is notable that via implementing the proper velocity control strategies for iPSO lighter structure can be founded however to provide an impartial comparison condition any velocity control is not applied.

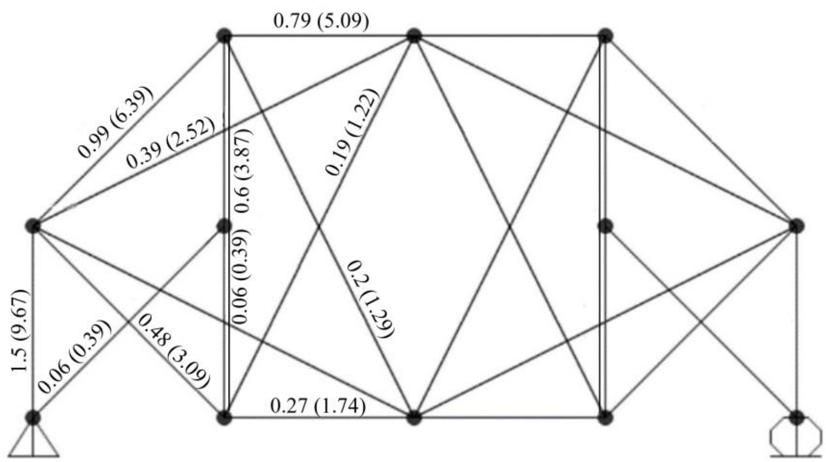


Figure 16. *Optimum solution obtained by TLBO with 30 agents in form of in^2 (cm^2)*

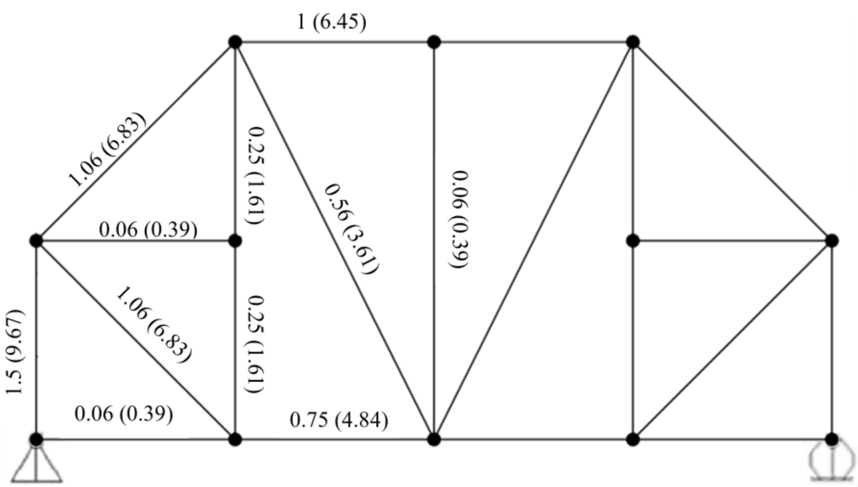


Figure 17. *Optimum solution obtained by TLBO with 100 agents in form of in^2 (cm^2)*

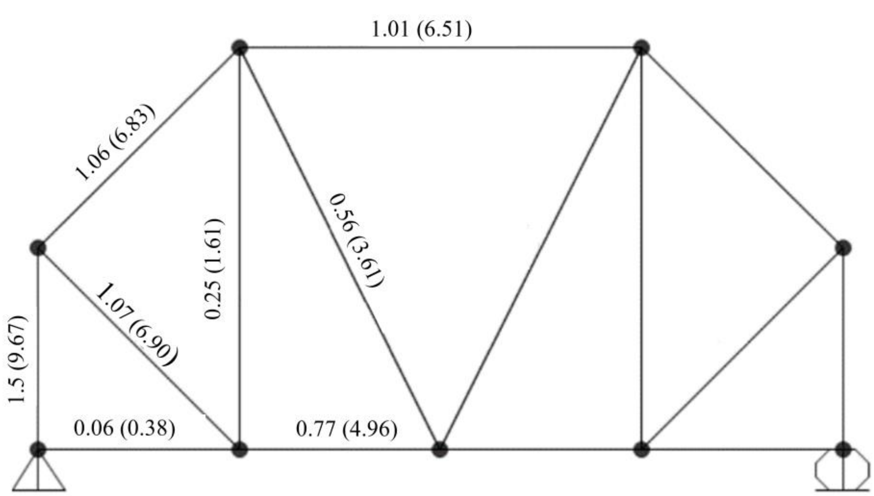


Figure 18. *Optimum solution obtained by iPSO in form of in^2 (cm^2)*

Table 3. *The results comparison*

			Current Study		
Method	GA [39]	FA [7]	iPSO	TLBO (Pop=30)	TLBO (Pop=100)
Weigh (lb)	196.54	193.54	194.99	244.83	197.31
Weigh (N)	874.25	860.90	867.35	1089.05	877.67

7. CONCLUSIONS

The current study deals with comparison of two metaheuristic optimization algorithms, teaching and learning based optimizer (TLBO), and integrated particle swarm optimizer (iPSO). TLBO and iPSO respectively are the two-phase and single-phase algorithms. To test their performances one unimodal, two multimodal mathematical functions and two combined size and topology truss structures optimization problems are addressed as test problems.

Based on the attained results for mathematical functions optimization, the iPSO outperforms TLBO on both convergence rate and final solution. However, due to learning phase of TLBO the diversity of this algorithm is higher than iPSO. The efficiency and applicability of iPSO and TLBO on truss optimization problems indicate that iPSO demonstrates superiority in comparison with TLBO in size and topology optimization of truss structures. Based on the all results achieved for all test cases the single-phase integrated particle swarm optimization (iPSO) overlay is superior to double-phase teaching and learning based optimizer (TLBO).

CONFLICT OF INTEREST

No conflict of interest was declared by the authors.

Notice

Current work is an extended version of the study presented at the 12th International Congress on Advances in Civil Engineering (ACE 2016) organized by Boğaziçi University between 21-23 September 2016.

REFERENCES

[1] Örkcü, H.H., Doğan, M.İ., Örkcü, M., “A hybrid applied optimization algorithm for training multi-layer neural networks in data classification”, Gazi University Journal of Science, 28(1): 15-32, (2015).

[2] Ozsoy, V.S., Orkcu, H., “Estimating the parameters of nonlinear regression models through particle swarm optimization”, Gazi University Journal of Science, 29: 187-99, (2016).

[3] Satman, M.H., “Machine coded genetic algorithms for real parameter optimization problems”, Gazi University Journal of Science, 26: 85-95, (2013).

[4] Mortazavi, A., Toğan, V., “Simultaneous size, shape, and topology optimization of truss structures using integrated particle swarm optimizer”, Structural and Multidisciplinary Optimization, 54(7): 15-36, (2016).

[5] Dede, T., Togan, V., "A teaching learning based optimization for truss structures with frequency constraints", Structural Engineering and Mechanics, 53(8): 33-45, (2015).

[6] Mortazavi A, Toğan V. "Particle Swarm Optimizer integrated with fly-back mechanism and weighted particle for optimization of the truss structures". Proceedings of 11st International congress on advances in civil engineering. Istanbul, Turkey, (2014).

[7] Miguel, L.F.F., Lopez, R.H., Miguel, L.F.F., "Multimodal size, shape, and topology optimisation of truss structures using the Firefly algorithm", Advances in Engineering Software, 56: 23-37, (2013).

[8] Sonmez, M., "Discrete optimum design of truss structures using artificial bee colony algorithm", Structural and Multidisciplinary Optimization, 43: 85-97, (2011).

[9] Gökdağ, H., "A crack identification method for beam type structures subject to moving vehicle using particle swarm optimization", Gazi University Journal of Science, 26(4): 39-48, (2013).

[10] Türkşen, Ö., "Optimization and decision making stages for multiple responses: An application of NSGA-II and FCM clustering algorithm", Gazi University Journal of Science, 28(3): 21-30 (2015).

[11] Aydoğdu, İ., Akın, A., Saka, M.P., "Design optimization of real world steel space frames using artificial bee colony algorithm with Levy flight distribution", Advances in Engineering Software, 92: 1-14, (2016).

[12] Lamberti, L., "An efficient simulated annealing algorithm for design optimization of truss structures", Computers & Structures, 86(19): 36-53, (2008).

[13] Hasançebi, O., "Adaptive evolution strategies in structural optimization: Enhancing their computational performance with applications to large-scale structures", Computers & Structures, 86(1): 19-32, (2008).

[14] Camp, C., "Design of Space Trusses Using Big Bang–Big Crunch Optimization", Journal of Structural Engineering, 133: 999-1008, (2007).

[15] Achtziger, W., Stolpe, M., "Global optimization of truss topology with discrete bar areas—Part II: Implementation and numerical results", Computational Optimization and Applications, 44(3): 15-41, (2007).

[16] Camp, C., Bichon, B., "Design of Space Trusses Using Ant Colony Optimization", Journal of Structural Engineering, 130(7): 41-51, (2004).

[17] Noilublao, N., Bureerat, S., "Simultaneous Topology, Shape, and Sizing Optimisation of Plane Trusses with Adaptive Ground Finite Elements Using MOEAs", Mathematical Problems in Engineering, 2013: 1-9, (2013).

[18] Kennedy, J., Eberhart, R., "Particle swarm optimization", Neural Networks, 1995 Proceedings, IEEE International Conference , 4: 1942-1948, (1995).

[19] Mortazavi, A., Toğan, V., "Simultaneous size, shape, and topology optimization of truss structures using integrated particle swarm optimizer",Structural and Multidisciplinary Optimization, 1-22, (2016).

[20] Rao, R.V., Savsani, V.J., Vakharia, D.P., "Teaching–learning-based optimization: A novel method for constrained mechanical design optimization problems", Computer-Aided Design, 43(30): 3-15, (2011).

[21] Camp, C.V., Farshchin, M., "Design of space trusses using modified teaching–learning based optimization", Engineering Structures, 62–63: 87-97, (2014).

[22] Toğan, V., Mortazavi, A., "Sizing optimization of skeletal structures using teaching-learning based optimization", Optimization and Control: Theories & Applications, 7: 130-142, (2017).

[23] Savsani, V.J., Tejani, G.G., Patel, V.K., "Truss topology optimization with static and dynamic constraints using modified subpopulation teaching–learning-based optimization", Engineering Optimization, 48(11): 1990-2006, (2016).

[24] Pickard, J.K., Carretero, J.A., Bhavsar, V.C., "On the convergence and origin bias of the Teaching-Learning-Based-Optimization algorithm", Applied Soft Computing, 46: 115-27, (2016).

[25] Mortazavi, A., Togan, V., Daloğlu, A., "Investigation of the effect of teaching factor on optimization process". Proceedings of International congress on advances in civil engineering, Istanbul, Turkey, (2016).

[26] Rao, R.V., Waghmare, G.G., "Complex constrained design optimisation using an elitist teaching-learning-based optimisation algorithm", International Journal of Metaheuristics, 3: 81-102, (2014).

[27] Lim, W.H., Mat Isa, N.A., "Teaching and peer-learning particle swarm optimization", Applied Soft Computing, 18: 39-58, (2014).

[28] Degertekin, S.O., Hayalioglu, M.S., "Sizing truss structures using teaching-learning-based optimization", Computers & Structures, 119(1): 77-88, (2013).

[29] Toğan, V., "Design of planar steel frames using Teaching–Learning Based Optimization", Engineering Structures, 34(2): 25-32, (2012).

[30] Črepinšek, M., Liu, S.H., Mernik, L., "A note on teaching–learning-based optimization algorithm", Information Sciences, 212: 79-93, (2012).

[31] Mortazavi, A., Toğan, V., Nuhoğlu, A., "An integrated particle swarm optimizer for optimization of truss structures with discrete variables", Structural Engineering and Mechanics, 61(3): 59-70, (2017).

[32] Mortazavi, A., Toğan, V., "Sizing and layout design of truss structures under dynamic and static constraints with an integrated particle swarm optimization algorithm", Applied Soft Computing, 51(2): 39-52, (2017).

[33] Mortazavi, A., Togan, V., "Weight minimization of truss structures with sizing and layout variables using integrated particle swarm optimizer", Journal of Civil Engineering and Management, 23(8): 985-1001, (2017).

[34] Lim, W.H., Mat Isa, N.A., "An adaptive two-layer particle swarm optimization with elitist learning strategy", Information Sciences, 273: 49-72, (2014).

[35] Li, N.J., Wang, W.J., James Hsu, C.C., Chang, W., Chou, H.G., Chang, J.W., "Enhanced particle swarm optimizer incorporating a weighted particle". Neurocomputing, 124(2): 18-27, (2014).

[36] Fan, Q., Yan, X., "Self-adaptive particle swarm optimization with multiple velocity strategies and its application for p-Xylene oxidation reaction process optimization", Chemometrics and Intelligent Laboratory Systems, 139: 15-25, (2014).

[37] Elsayed, S.M., Sarker, R.A., Mezura-Montes, E., "Self-adaptive mix of particle swarm methodologies for constrained optimization". Information Sciences, 277(2): 16-33, (2014).

[38] He, S., Prempain, E., Wu, Q.H., "An improved particle swarm optimizer for mechanical design optimization problems", Engineering Optimization, 36: 585-605, (2004).

[39] Deb, K., Gulati, S., "Design of truss-structures for minimum weight using genetic algorithms", Finite Elements in Analysis and Design, 37(4): 47-65, (2001).

[40] Suganthan, P.N., Hansen, N., Liang, J.J., Deb, K., Chen, Y., Auger, A., "Problem definitions and evaluation criteria for the CEC 2005 special session on real-parameter optimization", Technical Report, Nanyang Technological University, Singapore, May 2005 AND KanGAL Report 2005005, IIT Kanpur, India, (2005).

[41] Tang, K., Li, Z., Luo, L., Liu, B., "Multi-strategy adaptive particle swarm optimization for numerical optimization", Engineering Applications of Artificial Intelligence, 37: 9-19, (2015).

[42] Hajela, P., Lee, E. "Genetic algorithms in truss topological optimization", International Journal of Solids and Structures, 32(33):3341-57, (1995).

[43] Ringertz, U.T., "On topology optimization of trusses", Engineering Optimization, 9(2): 09-18, (1985).