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Evaluation of 6th Grade Students' Mathematical Communication Levels within the Framework of Realistic Mathematics Education Approach[1]

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Abstract

The purpose of this study is to investigate the levels of mathematical communication demonstrated by sixth-grade students in the context of instructional activities grounded in the principles of Realistic Mathematics Education (RME). There are several skills that mathematics education aims to develop in students. Mathematical communication skills are among these essential skills. Among the aims of realistic mathematics education is to encourage students to actively participate in the learning process and to make sense of mathematical knowledge in real-life situations. In this study, the case study method, one of the qualitative research methods, was used. The study was carried out during the 2022–2023 academic year at a public middle school located in the Southeastern Anatolia Region, involving 12 students who were selected through convenience sampling. Data were collected through RME problems, interviews, audio recordings, and observations, and were subjected to descriptive analysis and content analysis in line with the problems of the study. In individual applications, students' mathematical communication levels were found to be below zero, inadequate, and partially adequate, while in group work, mathematical communication levels were found to be partially adequate, adequate, and constructive. As a result of the study, it was seen that students' mathematical communication levels were at lower levels in realistic mathematics education problems in which they worked individually, while students' mathematical communication levels were at higher levels in group work. In individual tasks, students' levels of mathematical communication were observed to range from below zero to partially adequate, whereas in collaborative group activities, their communication levels ranged from partially adequate to constructive. The findings indicate that students demonstrated lower levels of mathematical communication when engaging with RME tasks individually, while their communication skills improved notably during group-based problem-solving processes. Based on these results, suggestions were made to increase mathematical communication levels.

Keywords: mathematical communication, realistic mathematics education, middle school students

Gerçekçi Matematik Eğitimi Yaklaşımı Çerçevesinde 6. Sınıf Öğrencilerinin Matematiksel İletişim Düzeylerinin Değerlendirilmesi

Öz

Araştırmanın amacı, Gerçekçi Matematik Eğitimi (GME) yaklaşımı doğrultusunda yürütülen öğretim etkinlikleri çerçevesinde, altıncı sınıf öğrencilerinin ortaya koyduğu matematiksel iletişim düzeylerini analiz etmektir. Matematik eğitiminin öğrencilerde geliştirmeyi hedeflediği birtakım beceriler bulunmaktadır. Matematiksel iletişim becerisi de bu becerilerden biridir. GME'nin amaçları arasında öğrencilerin öğrenme sürecine aktif katılımlarını teşvik etmek suretiyle matematiksel bilgilerin gerçek yaşam durumları ile anlamlandırmalarını sağlanması bulunmaktadır. Bu çalışmada nitel araştırma yöntemlerinden durum çalışması yöntemi kullanılmıştır. Araştırma, 2022–2023 eğitim-öğretim yılında Güneydoğu Anadolu Bölgesi'nde yer alan bir devlet ortaokulunda yürütülmüş olup, çalışma grubunu kolay ulaşılabilir örnekleme yöntemiyle seçilen 12 öğrenci oluşturmaktadır. Veriler; gerçekçi matematik eğitimi problemleri, görüşmeler, ses kayıtları ve gözlemler yoluyla toplanmış olup araştırmanın problemleri doğrultusunda betimsel analize ve içerik analizine tabi tutulmuştur. Bireysel uygulamalarda öğrencilerin matematiksel iletişim düzeyleri sıfırın altında, yetersiz ve kısmen yeterli düzeylerinde bulunurken, grup çalışmalarında ise matematiksel iletişim düzeylerinin kısmen yeterli, yeterli ve yapıcı düzeylerinde olduğu tespit edilmiştir. Araştırmanın sonucunda, öğrencilerin bireysel olarak çalıştıkları GME problemlerinde matematiksel iletişim düzeylerinin alt düzeylerde olduğu, grup çalışmalarında ise öğrencilerin matematiksel iletişim düzeylerinin daha üst düzeylerde olduğu görülmüştür. Bireysel görevlerde öğrencilerin matematiksel iletişim düzeylerinin sıfırın altı ile kısmen yeterli arasında değiştiği gözlemlenirken, iş birliğine dayalı grup çalışmalarında bu düzeylerin kısmen yeterli ile yapıcı düzey arasında değiştiği belirlenmiştir. Araştırma bulguları, öğrencilerin GME yaklaşımıyla tasarlanan problemlerde bireysel çalışmalarda daha düşük düzeyde matematiksel iletişim sergilediklerini, buna karşılık grup temelli problem çözme süreçlerinde iletişim becerilerinin belirgin biçimde geliştiğini ortaya koymuştur. Bu sonuçlardan yola çıkılarak matematiksel iletişim düzeylerinin artırılmasına yönelik öneriler sunulmuştur.

Anahtar kelimeler: matematiksel iletişim, gerçekçi matematik eğitimi, ortaokul öğrencileri

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INTRODUCTION

In recent years, there has been a noticeable increase in mathematics education research focusing on the importance of mathematical skills and their development. 21st-century skills emphasized for student development are "critical thinking, creativity, research and inquiry, self-direction, initiative and persistence, information use, systems thinking, communication, and reflection" (The Organisation for Economic Co-operation and Development [OECD], 2018, p. 31). The National Council of Teachers of Mathematics (NCTM, 2000), in its book *Principles and Standards for School Mathematics*, and the Turkish middle school mathematics curriculum (Ministry of National Education [MoNE], 2013) emphasize the importance of mathematical communication skills. NCTM (2000) categorizes mathematical standards into five main areas: problem-solving, reasoning and proof, communication, connections, and representations. In contrast, the Turkish middle school mathematics curriculum identifies the targeted competencies under the following categories: problem-solving, mathematical process skills (including communication, reasoning, and connections), affective skills, psychomotor skills, and, finally, information and communication technologies (MoNE, 2018). The goal of the mathematics curriculum is to enable individuals to articulate their mathematical thoughts, master mathematical terminology and language, and interpret relationships in daily life through mathematical language (MoNE, 2013). Considering Baykul's (2020) statement that mathematics is a language composed of symbols used by humanity, a system that develops ways of thinking, and a tool for making sense of the world, the concept of communication—an essential component of any language—plays a key role in mastering this language. This is because knowing a language requires proficiency in all its components. Given that language develops through speaking, listening, reading, and writing, these components are also crucial for mathematics, which is recognized as a language. Language serves as a means of communication, and communication is the process through which individuals express themselves, convey their emotions and thoughts, and understand others. Mathematical communication, in turn, can be regarded as the means through which mathematical knowledge is conveyed to students, allowing them to explore, comprehend, and discuss mathematics (Doruk, 2011). In other words, mathematical communication skills encompass the ability to reason, explain, and evaluate mathematical ideas, terms, and concepts (Dahlan, 2011). Just as people use language for communication, self-expression, and understanding others, communication in the language of mathematics serves as a means of comprehending and conveying mathematical ideas. Zengin (2017) emphasizes the importance of communication in mathematics in terms of facilitating the understanding of concepts, enabling the exchange of mathematical ideas, and enhancing the retention of knowledge.

Educational environments are inherently conducive to communication and serve as social settings where students interact with one another most frequently. In educational settings and learning processes, mathematical communication is employed both consciously and unconsciously. Therefore, education and communication are inseparable components of a unified whole. However, for communication to be effective, this process must be carried out with awareness. It is emphasized that problem-solving processes foster students' mathematical thinking and communication skills (Nufus & Mursalin, 2020). Thus, in order to facilitate mathematical communication—both as a goal and as a tool in mathematics education—communication must be consciously integrated into learning environments (Zengin, 2017). Greater emphasis should be placed on mathematical communication in classrooms. In this process, instructional activities related to mathematical symbols should be conducted, and students' communication skills should be developed to help them understand and solve real-life problems. Furthermore, the use of mathematical language in mathematical modeling should be emphasized (Baran, 2019; Toptaş, 2015).

Communication acts as a bridge in the process of understanding mathematics and mathematical concepts. Activities involving reading, writing, listening, thinking, and communicating about mathematical problems enhance students' mathematical comprehension (Freitag, 1997; Nicolas & Emata, 2018; Öztürk et al., 2020). Therefore, students should use communication as a tool for understanding and producing mathematical ideas (NCTM, 2000). Encouraging students to continuously write and speak about mathematical processes will make mathematical communication a natural part of their problem-solving processes (Kostos & Shin, 2010). Consequently, mathematical communication skills should be emphasized through textbooks, classroom practices, and curriculum design. The Programme for International Student Assessment (PISA) has highlighted that countries achieving high levels of success in mathematics place significant emphasis on mathematical communication skills within their education systems (Baran, 2019). OECD (2018) considers mathematical communication skills to be one of the most critical stages of problem-solving and includes them as a component of mathematical literacy.

In NCTM (2000), the objectives of mathematical communication skills, which are considered among the mathematical process skills, include fostering and strengthening mathematical thinking through communication, sharing mathematical ideas with others, critiquing peers' mathematical reasoning, and expressing mathematical thoughts using mathematical language. In the Turkish middle school mathematics curriculum, the indicators of communication skills can be summarized as the ability to interpret the meaning and accuracy of mathematical ideas, express mathematical thoughts both in written and oral forms, use mathematical language effectively and accurately both within mathematics itself and across disciplines, and recognize that mathematics is a language with its own terminology, formulas, and symbols (MoNE, 2013). In the context of mathematical representation, the Türkiye Century of Maarif model for middle school mathematics education emphasizes mathematical communication as one of the key domain-specific skills to be developed. Within this framework, mathematical communication is addressed as a process-oriented component of the skill of mathematical representation (MoNE, 2024). Studies have shown that mathematical communication, which is classified under mathematical process skills, is also essential for acquiring other skills. Jung and Reifel (2011) emphasize that mathematical communication supports the development of other competencies.

To equip students with these skills, various educational authorities have begun adopting foundational approaches, one of which is Realistic Mathematics Education (RME). This instructional theory was introduced in the 1970s by Dutch mathematician Hans Freudenthal and his colleagues, advocating that students should learn mathematical concepts by developing understanding and applying them in meaningful ways (Topbaş Tat, 2020). In the context of RME, Van den Heuvel-Panhuizen (1996) outlined six guiding principles for structuring and monitoring the process of teaching and learning mathematics. These principles are: *level, activity, intertwinement, reality, guidance*, and *interactivity*. The *level principle* highlights the processes that students undergo as they progress through several stages of understanding mathematical concepts. According to the *activity principle*, students are active participants in learning mathematics. The intertwinement principle emphasizes the integrated nature of mathematical subjects, rather than focusing on isolated topics. The *reality principle*, real-life problems are significant for students to notice and acknowledge the objectives of mathematics education. The *guidance principle* is related to teachers' roles and curriculum coherence in the learning process (Van den Heuvel-Panhuizen, 1996). Mathematical knowledge should be structured in alignment with real-life contexts (Uygun, 2020). Mathematics should not remain abstract; rather, students should be able to connect it with their daily lives. In addition to imparting academic knowledge, school mathematics should enable students to use mathematical terminology appropriately, actively engage them in learning, establish connections between real-life situations and academic content, and encourage inquiry-based learning through communication (Bukova Güzel, 2019). In this regard, RME is crucial in fostering the targeted competencies.

Monroe and Orme (2002) state that mathematical communication plays a significant role in relating mathematical knowledge learned in the classroom to real-life situations. Similarly, Üzel (2007) argues that instruction detached from real-life contexts and outdated assessment approaches hinder student achievement. Zengin (2017) asserts that students who develop mathematical thinking skills through mathematical communication cultivate positive attitudes toward mathematics, which in turn enhances their engagement and success in solving mathematical problems.

RME aligns with the expectations outlined in the MoNE's mathematics curriculum, which emphasizes the significance of real-life problems in mathematics instruction (Büyükikiz Kütküt, 2017; MoNE, 2018). A review of research on RME reveals that most studies focus on comparing the effectiveness of RME with traditional teaching methods (Akyüz, 2010; Bıldırcın, 2012; Çakır, 2013; Özçelik, 2015; Özdemir, 2015; Uygur, 2012; Ünal, 2008; Ünal & İpek, 2010; Üzel, 2017). These studies report that instruction based on RME is more effective than traditional teaching methods.

A review of the literature reveals various approaches to mathematical communication. Brenner (1994) pioneered research in this field with his study titled *Communication Framework for Mathematics*. Different classifications exist on this topic. Brenner (1998) categorized communication into three types: (i) communication *about* mathematics, (ii) communication *within* mathematics, and (iii) communication *through* mathematics. Meanwhile, Brendefur and Frykholm (2000) classified communication into four categories: (i) uni-directional communication, (ii) contributive communication, (iii) reflective communication, and (iv) instructive communication. Additionally, Turner et al. (2015) examined mathematical communication in terms of two components—receptive and constructive—across four levels. International groups and societies, such as the OECD aimed to assess mathematical competencies across national levels in terms of mathematical literacy as an umbrella term. Niss (2015) provided a broad description of mathematical competencies associated with mathematical literacy. PISA mathematics framework (OECD, 2018) includes an operational definition of mathematical competencies, and one of them is mathematical communication competency. Turner and others (2015) also explained the mathematising competency, which is central to the RME approach. Therefore, Turner and colleagues proposed a mathematical communication framework that best fits the current study.

This study utilizes Turner et al.'s (2015) framework for levels of mathematical communication skills. The mathematical communication skills scoring framework developed by Turner et al. (2015) allows for the assessment of students' communication levels. According to this framework, levels are categorized between 0 and 3, with mathematical communication being explained through two dimensions: the receptive component and the constructive component. The receptive component encompasses elements such as understanding the mathematical task, interpreting what the problem statement requires, the complexity of the given situation, the need to relate multiple elements, and the activation of prior knowledge. The constructive component, on the other hand, pertains to students' explanations, justifications, and reasoning during the solution process. At Level 0, in terms of the receptive component, all given information in the problem statement is directly related to the mathematical task. In terms of the constructive component, communication is limited to providing a single-word or numerical response. At Level 1, in terms of the receptive component, the problem situation consists of more comprehensive and complex statements than short sentences or simple phrases. In addition, it may also contain extraneous information that is not necessary for solving the problem. Students at this level can recognize and relate given information and representations. In terms of the constructive component, students provide brief explanations or calculations and may describe values within a specific numerical sequence. At level 2, from the perspective of the recipient component of communication, the mathematical task involves multiple components that need to be associated with one another. A student at this level can identify and relate these components. In terms of the constructive communication component, it involves presenting the computation phase through a brief and concise description. At Level 3, in terms of the receptive component, the mathematical task consists of complex relationships and interconnected expressions with multiple components. Students at this level can identify these components and interpret their relationships. From the perspective of the constructive communication component, it involves presenting mathematical proof in which the components of the problem or solution are interconnected.

The appropriate use of mathematical language and terminology holds significant importance. When examining the universe, nature, and real-life phenomena in depth, the reflections of mathematical expressions become evident. Galileo stated that the universe is written in the language of mathematics and can only be read and understood by those who are proficient in this language (Mankiewicz, 2002). Considering these statements, the ability to speak a language and share ideas within it requires mutual interaction. This necessity has existed since the dawn of humanity and highlights the importance of communication skills, which are among the essential mathematical process skills.

Research indicates that learning processes in which mathematical knowledge and concepts are taught without mathematical communication tend to be ineffective (Toptaş, 2015). Studies on mathematical communication skills suggest that educational environments incorporating mathematical communication enable students to express themselves more effectively, manage problem-solving processes more efficiently, support implicit learning through mathematical discussions, and reduce students' anxiety levels (Chapin et al., 2009; Lee, 2006; Lomibao et al., 2016; Qohar & Sumarmo, 2013).

In the literature, there are also scientific studies involving teachers as participants in this subject (Franke et al., 2015; Uygur Kabael & Baran, 2016; Kıymaz et al., 2019; Özpınar & Arslan, 2017). Akarsu Yakar and Yılmaz (2017) emphasize the importance of teachers valuing mathematical communication skills, which help reveal students' mathematical language abilities. They argue that incorporating real-life problems into lessons and assessing students' mathematical understanding during this process is crucial for ensuring that students fully grasp and accurately express mathematical concepts. There are a limited number of studies on the topic of combining the RME approach and mathematical communication in the literature. When it is looked at the literature on mathematical communication, research has predominantly focused on areas such as the assessment of mathematical communication (Cai et al., 1996), the impact of mathematical communication on students' mathematical performance and math anxiety (Lamibao et al., 2016), and mathematical communication within classroom environments (Brendefur & Frykholm, 2000; Rushdi et al., 2020). Since the general aim of mathematics education is to make students gain mathematical competencies and skills (MoNE, 2018), the RME approach is one of the theories that emphasizes the mathematics teaching and learning process through real-life problems and situations put into action.

Williams and Baxter (1996) argue that in order to establish a high-quality learning environment where mathematical communication is emphasized, students should be engaged in mathematical tasks, either individually or in groups. During this process, the teacher is expected to take an active role by circulating among the groups, motivating students, posing thought-provoking questions, and providing hints when necessary. At the end of the process, the teacher should summarize the learning by highlighting different solution strategies and reflecting on the overall process.

The importance of mathematical communication in the classroom is underlined by several studies (Brendefur & Frykholm, 2000; Cooke & Buchholz, 2005; Huang & Normandia, 2009; Viseu & Oliveira, 2012). Students need to utilize their mathematical language to explain their mathematical thinking to others (Ginsburg et al., 1999; NCTM, 2000). For students to engage effectively in mathematical communication with both their teachers and peers, it is crucial that they receive support and encouragement within the classroom environment (Cooke & Bulchholz, 2005; NCTM, 2000). At the middle school level, students are in transition from arithmetic to algebra and begin to engage with more abstract mathematical concepts. This stage can be considered a critical period for fostering students' ability to use mathematical language effectively and to engage in the exchange of mathematical ideas, both of which are essential for developing strong mathematical communication skills. Therefore, determining students' levels of mathematical communication in mathematical applications is of great significance, and this study is expected to contribute to the related literature. This study aims to determine the mathematical communication levels of 6th-grade students within the context of RME applications.

In line with this objective, the research problem of the present study has been defined as follows: "What are the mathematical communication levels of 6th-grade students in the context of Realistic Mathematics Education (RME) applications?"

Additionally, based on the main research problem, the sub-problems of the study are specified below: a) What are the mathematical communication levels of students in individual applications of RME problems? b) What are the mathematical communication levels of students in group applications of RME problems?

METHOD

Research Design

This study employs a case study approach, a qualitative research method that aims to thoroughly investigate and explore the mathematical communication skills of 6th-grade students in a classroom environment where RME is implemented. In case studies, the phenomenon under investigation is examined holistically and comprehensively to reveal the factors influencing the process, their roles, and their effects (Yıldırım & Şimşek, 2021). Case studies are employed when the phenomenon (case) being studied cannot be distinguished from its real-life context, allowing for an in-depth investigation (Yin, 2018). The specific case at the focus of this study is the determination of students' mathematical communication levels during the implementation of activities designed within the context of Realistic Mathematics Education.

The Participants

The participants of the study consist of twelve 6th-grade students (seven males and five females) enrolled in the same classroom of a public middle school affiliated with the MoNE in a district in the Southeastern Anatolia region. The students come from low socio-economic backgrounds and are members of families engaged in farming. The participants represent a heterogeneous group in terms of academic achievement, with generally low to moderate performance levels. The participants were selected through convenience sampling, a non-probability sampling technique. The rationale for employing the convenience sampling method in this study is based on several factors: the feasibility of conducting the research at the school where one of the researchers is employed as a teacher, the researcher's familiarity with the participating students, the opportunity to spend time with students throughout the process, and the ability to collect data in various formats (Gezer et al., 2021). Although the class initially consisted of thirteen students, one student, who was a special education student with literacy difficulties, was excluded from the study.

Data Collection Tools

Considering the study's purpose, the data collection tools were determined by the researchers, who developed problems in accordance with the RME model. Developed RME problems were presented to a field expert to determine whether they met the principles of RME or not. After the expert's feedback, the last version of the problems was formed. These problems were supplemented by semi-structured interview forms prepared based on relevant literature, audio recordings of group work, and field notes. These tools were used to assess the mathematical communication levels of students, both individually and in groups. The semi-structured interview form consists of questions about unanswered problems in the individual phase of implementation, as well as questions designed to gather in-depth knowledge about the implementation process.

Implementation

The researcher administered two problems individually and two problems in group work to determine mathematical communication levels in both settings. The problems, developed in accordance with the RME approach, were first given to the same students individually. The planned problems were distributed to the students individually, with a two-week interval between each distribution, due to several factors, including curriculum planning, student suitability, and preventing students from influencing each other during individual implementation. The students were then asked to explain their solution methods along with their expressions. In this stage, students were first expected to present their solution steps to assess whether they understood the question, followed by an examination of their problem-solving processes. While determining communication levels, not only whether students reached the correct answer but also the entire process leading to the answer (including their interpretation of the problem and the strategies they planned to use) was analyzed in detail. During this process, participants expressed their ideas in written form without interacting with each other. At this stage, students' individual mathematical communication levels were determined.

Based on the communication levels identified in the individual applications, three groups of four students each were formed, and students were asked to work on different RME problems in a group setting. When forming these groups, attention was paid to including students with varying mathematical communication levels, as identified in their individual assessments. The RME problems designed for group work were also administered at two-week intervals (Table 1). Each group was provided with A4 papers to express their solution processes, and an audio recording device was placed on each group's desk. Students were given 20 minutes to read the problem statement and respond to the questions. At the end of the session, students' solution papers and audio recordings were collected. To comply with ethical rules, students' real names were not used, and they were assigned pseudonyms. Once all applications were completed, the entire process was analyzed and evaluated.

Table 1. Implementation Process of Problems

Weeks	The Content of the Application
01.03.2023-08.03.2023 (1st week)	The Application of the Grocery Problem
15.03.2023-22.03.2023 (2nd week)	The Application of the Goat Problem
03.04.2023-10.04.2023 (3rd week)	The Application of the Mitosis Division in Cells Problem
17.04.2023-24.04.2023 (4th week)	The Application of the Wardrobe Problem

Data Analysis

To determine the mathematical communication skill levels of sixth-grade students, the competency definitions and level descriptions for mathematical communication developed by Turner et al. (2015) was used to analyze students' solution processes in RME problems step by step, using pseudonyms. While determining participants' mathematical communication levels, students were initially asked to explain their solution steps in their individual works. The interview transcripts were examined to determine the mathematical communication levels of students who did not use clear expressions, yet accurately described the solution steps but made errors in the solution. In the group work, in addition to analyzing the solution sheets, audio recordings were transcribed and documented for further analysis.

Communication levels were examined under four categories (Table 2). The data obtained from group work were analyzed based on the codes established by Turner et al. (2015). Initially, 10% of the data were coded collectively by all researchers. After reaching a consensus, the remaining data were coded separately by the first and second researchers. Subsequently, the inter-rater agreement was calculated as 87%.

Table 2. Codes and Explanations Corresponding to Students' Mathematical Communication Levels

Levels	Codes	Explanations
Level 0	Inadequate	The problem-solving process was correctly expressed by the student with a single word or numerical result.
Level 1	Partially Adequate	In the problem-solving process, the student was able to provide a short explanation and correctly relate two sets of data.
Level 2	Adequate	In the problem-solving process, the student correctly presented the calculation steps and was able to establish multiple connections where necessary.
Level 3	Constructive Level	In the problem-solving process, the student was able to provide proof for his/her solution, relate the situation to daily life by giving different examples of the problem situation.

Students who were unable to comprehend any problem and could not express the solution process were categorized below Level 0. Here, the designation of Level 0 is relative. For correct answers that required no interpretation and could be expressed with a single operation or a single word, students' communication levels were classified as 0. The mathematical communication level of students at Level 0 was considered insufficient.

The communication level of problem situations requiring a brief explanation was determined to be level 1, while the mathematical communication level was found to be partially adequate. The communication level of problem situations requiring multiple associations was determined to be level 2, while the mathematical communication level was deemed adequate. The highest level (Level 3) is the stage at which the questions related to the lower levels of the problem situation are solved, and the student is expected to provide various examples related to the given problem. At this level, proof and explanations are presented regarding the relationship of the topic in the problem situation to mathematics. This level was coded as the constructive level.

Similarly, the communication levels of the problem situations were determined, and while examining the students' papers, their mathematical communication levels were identified based on the problem situations. The communication levels of the problem situations provided in the example are shown in Table 3

Table 3. Mathematical Communication Levels of RME Problems

Problem Name	Sub-problems	Communication Level	Explanation
Goat Problem	2a	0	The solution to the problem is expressed with a single word or a single numerical result.
	2b	1	The solution to the problem requires relating two pieces of data or providing a brief explanation.
	2c	2	The solution to the problem requires presenting multiple connections and calculation steps.
Mitosis Division in Cells Problem	3a	0	The solution to the problem is expressed with a single word or a single numerical result.
	3b	0	The solution to the problem is expressed with a single word or a single numerical result.
	3c	2	The solution to the problem requires presenting multiple connections and calculation steps.
	3d	3	The solution to the problem requires presenting proof and illustrating the problem situation with various examples by relating it to everyday life.

Reliability and Validity

In qualitative research, the concepts of validity and reliability are considered important criteria for demonstrating the credibility of the results (Yıldırım & Şimşek, 2021). Validity in qualitative studies refers to the researcher's ability to objectively observe and analyze the phenomenon in its current state (Kirk & Miller, 1986). Validity is generally examined under two categories: internal validity and external validity. Internal validity pertains to the extent to which the researcher's interpretations of the observed phenomenon accurately reflect the actual situation. External validity, on the other hand, concerns the generalizability of the research findings to similar contexts and environments (Miles & Huberman, 1994).

Lincoln and Guba (1985) proposed several strategies to enhance the quality of qualitative studies. In this regard, ensuring credibility and transferability is necessary for establishing validity, while consistency and confirmability are essential criteria for reliability. In this context, to enhance the validity and reliability of this study, the researcher adopted a flexible role, conducted additional interviews, employed multiple data collection methods to ensure data confirmability, and carried out the study in a natural setting. Moreover, data triangulation was implemented to ensure the validity of the research. Individual interviews were conducted with participating students to verify the findings, and experts in the field of mathematics education also reviewed the results, assessing their suitability in light of the interview conditions and ethical protocols. For reliability, both confirmability audits and consistency checks were utilized.

Research Ethics

In order to conduct this study, the necessary approvals were obtained from both the Ordu University Ethics Committee and the relevant Provincial Directorate of National Education. Since the participants were under the age of 18, parental consent forms were prepared and the required permissions were obtained from the parents.

FINDINGS

The findings obtained in the study were presented considering the sub-problems of the research.

Findings Related to the First Sub-Problem

When the mathematical communication levels of students in individual tasks were examined based on the mathematical communication levels defined by Turner et al. (2015), it was determined that four students were below Level 0 (Table 4). These students were unable to present the solution steps for the given problems and did not provide correct answers. Initially, it was assessed whether the students understood what the real-life problems were asking from them, followed by an analysis of their solution processes. It was observed that students with communication levels below 0 did not understand short sentences related to mathematical concepts in problem situations directly connected to the mathematical task, where all the information was provided, and they lacked the communication level necessary to obtain a word or numerical result related to the outcome.

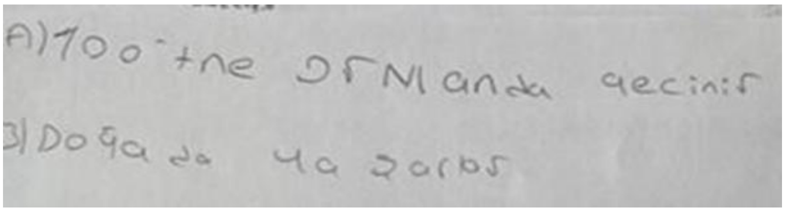
Table 4. Students' Mathematical Communication Levels in Individual Work

Participants	Mathematical communication level
Ersin	-*
Naz	-
Berfin	-
Gökberk	-
Esin	0
Ebrar	0
Kaan	0
Doruk	1
Salih	1
Birkan	1
Nur	1
Melih	1

*Minus sign means that there is no attempt to solve the problem, no level is determined.

An examination of students' worksheets revealed that Ersin did not attempt any calculations related to the problems, while Gökberk, Naz, and Berfin performed unrelated calculations. In this context, interviews were conducted with some students, and they were asked to explain their solution processes. These interviews confirmed that the students' mathematical communication levels were below Level 0.

For instance, when Gökberk's solution sheet for the "Goat Problem" was examined, it was observed that, similar to his approach to another problem, he wrote information unrelated to the question and performed calculations that had no connection to the problem. While the problem was about goats, he responded as if it were about birds. During the interview, when asked to reread the question, Gökberk stated that he did not understand the question and had simply written something to avoid leaving the answer blank. His solution to the "Goat Problem" is presented in Figure 1.



English translation of the Figure 1
A) 100 live in the forest.
B) They live in the nature.

Figure 1. *Gökberk's solution to the goat problem*

Similarly, when Berfin's solution sheet for the "Grocery Store Problem" was examined, no valid solution was found. A segment of Berfin's solution sheet is provided in Figure 2.

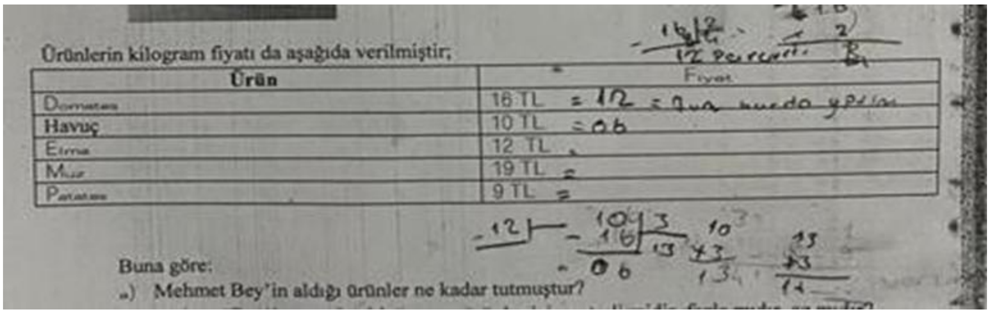
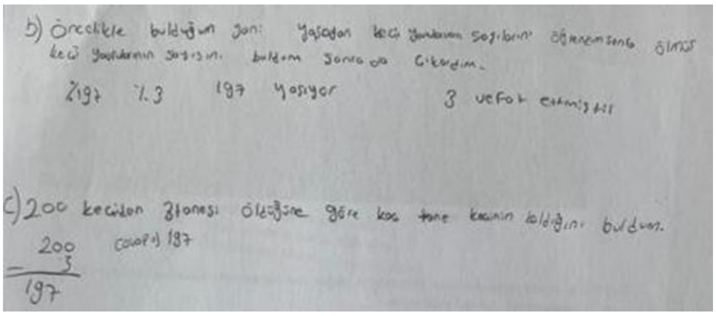


Figure 2. *Berfin's solution to the grocery problem*

According to the competency definitions and level descriptions for mathematical communication expressed by Turner et al. (2015), three students were identified as having a mathematical communication level of "0," indicating that they possess insufficient mathematical communication skills. These students were able to solve problems in which all the information was directly related to the mathematical task, expressing the solution with a single word and reaching the numerical result. However, they were unable to solve slightly more complex problems that required them to recognize and relate representations.

Based on the analysis of the students' solutions to the goat problem, they were able to express that one goat survived when three out of four goats died, indicating a ratio of 1/4. However, they could not express this fraction as a percentage. Additionally, they were unable to apply this proportion to a scenario involving 200 goats. For instance, Kaan, Ebrar, and Esin attempted to determine the number of surviving goats by subtracting 3 from 200. In addition, the students were unable to recognize and relate the representations in the problems. Additionally, it was found that the students were unable to make the mentioned connections in the similar-level questions of the greengrocer problem. Ebrar's solution to the goat problem (Level 1 and Level 2) is presented in Figure 3.



English translation of the Figure 3
b) First I found out the number of goat kids I found, that is, the number of living goat kids. Then I found out the number of dead goat kids. Then I subtracted. 197%, 3%. 197 is alive. 3% died.
c) Since 3 out of 200 goats died, I found how many goats are left. 200-3=197: 197.

Figure 3. *Ebrar's solution to the goat problem*

According to the competency definitions and level descriptions for mathematical communication developed by Turner et al. (2015), the mathematical communication level of five students was classified as '1' (partially adequate). It was observed that the students, in problem situations where all the information was provided and directly related to the mathematical task, were able to understand short sentences related to mathematical concepts and obtain a word or numerical result. Furthermore, the students were able to understand problem situations with more comprehensive information and successfully recognize and relate the given data and representations. In more complex problem situations, which required participants to handle multiple components, perform calculations, and provide explanations, they were unable to reach the correct result.

When Birkan's solutions for the "Goat Problem" and the "Grocery Store Problem" were analyzed (Figure 4), it was observed that he correctly answered the questions at Level 0 and Level 1 but was unable to correctly answer the question at Level 2. Furthermore, Birkan effectively articulated the solution steps for the Level 0 and Level 1 questions (Figure 5).

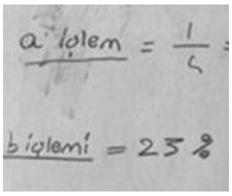
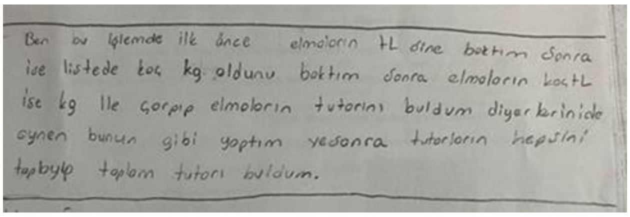


Figure 4. *Birkan's solution to the goat problem*



English Translation of Figure 5
In this process, I first looked at the price of the apples. Then I looked at how many kg there were in the list. Then I multiplied the price of the apples by the kg and found the amount of the apples. I did the others in the same way. And then I added all the amounts together and found the total amount.

Figure 5. *Birkan's interpretation to grocery problem 1a*

Findings Related to the Second Sub-Problem

In this section, the findings related to the groups' mathematical communication levels in the *Mitosis Division in Cells* and *Wardrobe* problems, as defined by Turner et al. (2015), are presented based on the analysis of solution sheets and audio recordings. Table 5 presents the mathematical communication levels of students identified during the group work.

Table 5. Students' Level of Mathematical Communication in Group Work

Participants	Group Mathematical Communication Level
Ersin	3
Naz	1
Berfin	1
Gökberk	2
Esin	2
Ebrar	3
Kaan	1
Doruk	3
Salih	2
Birkan	3
Nur	2
Melih	1

Note: Students in the same group were coded at the same level of mathematical communication.

Findings Related to the Group with "Partially Adequate" Mathematical Communication Level

When the mathematical communication levels of students in group work were examined based on the mathematical communication levels defined by Turner et al. (2015), Group 2 was found to have a communication level classified as "Partially Adequate" It was determined that they could comprehend problem situations where mathematical tasks were explicitly stated and could solve problems where the solution could be expressed with a single word or a numerical result. However, in more complex problems requiring connections between concepts, the members of Group 2 were unable to reach a solution. Therefore, their mathematical communication level was classified as Level 1, indicating a partially adequate level. An analysis of the audio recordings revealed that one of the students initially made an error while solving the problem but was able to arrive at the correct solution with the intervention of their group members.

The dialogue between the students in Group 2 is as follows:

Kaan: When we look at the question, one cell becomes two, and then two cells become four.
Melih: Then, after three divisions, there will be eight cells.
Kaan: No, Melih, you misunderstood. After the third division, there will be six cells.
Melih: Look, after the second division, there were four cells. When each of them divides into two, there will be eight cells after the third division.
Kaan: Then, in the fifth division, there will be ten cells.
Berfin: No, Kaan, you are doubling the number of divisions. If we look at our previous results, the number does not always double. We should draw it out to avoid making mistakes.

Following this discussion, the students in Group 2 correctly visualized the fifth division and successfully arrived at the correct solution.

Findings Related to the Group with "Adequate" Mathematical Communication Level

When the solution sheets of Group 1 were analysed, their mathematical communication level was determined to be "Adequate". It was observed that Group 1 correctly answered the Level 1 and Level 2 problems but made an incorrect association between real-life problem situations and the related mathematical topic in Level 3 problems. Therefore, the mathematical communication level of Group 1 was classified as Level 2 (Adequate).

In the mitosis division in cells problem, Group 1 correctly answered the Level 0 problems (3a, 3b). In these problem situations where the information is provided directly, Group 1 was able to express the solution with a numerical result. Their solutions to the mitosis division in cells problem (3a, 3b) are presented in Figure 6.

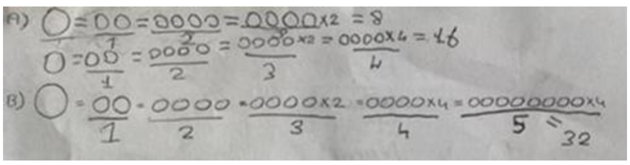
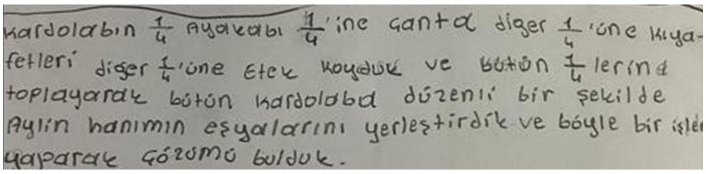


Figure 6. *Solution of group 1 to the mitosis division in cells problem*

Additionally, Group 1 correctly answered the Level 1 wardrobe problem, demonstrating that they had correctly performed the required classification. In the Level 2 mitosis division in cells problem (3c), they also successfully established the necessary relationship. However, they were unable to correctly express their solutions to the Level 3 problems.

An analysis of Group 1's solutions to the Level 3 wardrobe problem revealed that they made an incorrect association between the real-life problem situation and the relevant mathematical concept. While the problem required students to relate their classification to the concept of sets using the set builder form or notation, the students in the group mistakenly associated the problem with the topic of fractions. As a result, their mathematical communication level was classified as Level 2 (Adequate).

Group 1's solution to the wardrobe problem is presented in Figure 7.



English translation of Figure 7

We put shoes in one fourth of the wardrobe, bags in one fourth, clothes in the other fourth, skirts in the other fourth, and by adding all 1 divided by 4, we placed Mrs. Aylin's belongings in the wardrobe in an orderly manner and we found the solution by doing such an operation.

Figure 7. *Group 1's solution to the wardrobe problem*

Findings Related to the Group with "Constructive Level" of Mathematical Communication

When the solution sheets of Group 3 were analysed, their mathematical communication level was determined as the "Constructive Level". It was observed that they successfully solved all problem situations at Levels 0, 1, and 2. In Level 3 problem situations, they were able to analyse complex components and reach the relevant mathematical concept through accurate associations. Therefore, according to Turner et al.'s (2015) mathematical communication levels, Group 3's mathematical communication level was classified as Level 3 (Constructive Level).

An analysis of the audio recordings revealed that initially, the group struggled with interpreting the images representing the stages in the problem. However, following an intervention by their peer, Birkan, they recognized that the problem required identifying the formation of two cells from one. Through this realization, they correctly visualized the process and arrived at the correct solution. Additionally, the members of Group 3 identified the underlying mathematical relationship in the problem, and even though it was not explicitly required, they extended their analysis to determine the number of cells formed as the number of divisions increased. By generalizing their findings, they successfully reached the correct conclusion.

For other Level 2 and Level 3 problems (Mitosis division in cells: 3c, 3d), the audio recordings showed that they initially approached the problem using different strategies. When searching for the relationship between the number of divisions and the number of resulting cells, they first fell into the misconception of simply multiplying the number of divisions by two. Initially, they reasoned that one division results in two cells, two divisions result in four cells, and three divisions result in six cells. However, when Doruk's explanation contradicted that of his peers, they realized their mistake in establishing the relationship. The dialogue among the group members is presented below:

Birkan: These numbers have a connection to 2, what could it be?

Ebrar: Are they multiples, factors, or divisors?

Doruk: No, no! These numbers progress the powers of 2.

Birkan: Ahh, then the number of divisions corresponds to the exponent of 2, and the result represents the number of cells formed.

It was observed that Group 3 first identified the required relationship within the problem situation and then successfully associated it with the relevant mathematical concept in real life. Analysing their discussions and solution sheets revealed that the students engaged in idea-sharing and guided each other to achieve the highest mathematical communication level (Level 3). Group 3's solution to the mitosis division in cells problem is presented in Figure 8.

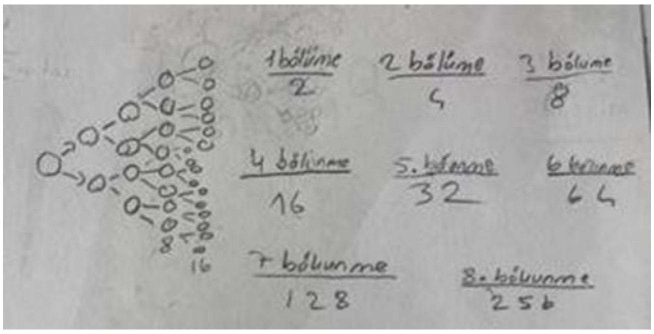
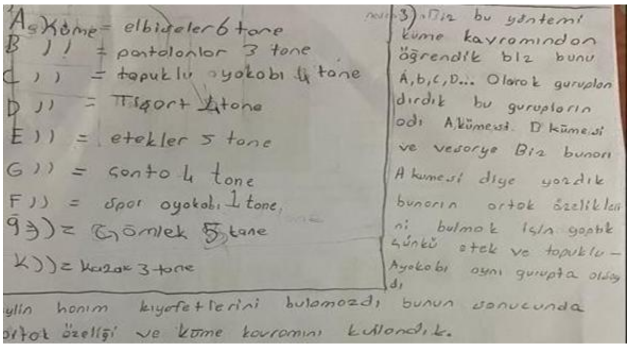


Figure 8. *Group 3's solution to the mitosis division in cells problem*

In the Wardrobe Problem, Group 3 correctly completed all steps and reached the highest level of mathematical communication (Level 3). They collaboratively explored and discovered the mathematical concept that needed to be connected to a real-life situation. As evidenced by their solution sheets and audio recordings, students successfully identified mathematical correspondences in real-world situations during the RME activities and provided various examples of these correspondences.

While the students were using communication skills, they were also able to perform the mathematization required by the GME practices. When the group solutions are analyzed, it is seen that the students used the concept of set, although there is no mathematical concept in the problem text. It is understood that they associate grouping with the concept of set. Therefore, the phenomenon expressed as vertical mathematization in RME was observed here. Group 3's solution and interpretation of the Wardrobe Problem are presented in Figure 9 and Figure 10.

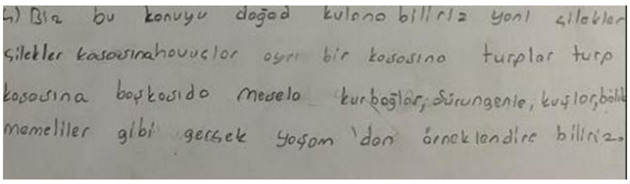


English Translation of Figure 9

Set A= Clothes (Six items)
Set B= Trousers (Three items)
Set C= Shoes (Four items)
Set D= T-Shirts (Four items)
Set E= Skirts (Five items)
Set G= Bags (Four items)
Set F= Sneakers (One item)
Set Ğ= Shirts (Five items)
Set K= Sweater (Three items)

3) We learned this method from the concept of set. We grouped it as A, B, C, D, ... We wrote the names of these groups as Set A, Set B, ... We did this to find their common properties because if the skirt and shoes were in the same group, Mrs. Aylin could not find her clothes. As a result, we used the common property and the concept of set.

Figure 9. *Group 3's solution to the wardrobe problem*



English translation of Figure 10

We can use this subject in nature. Strawberries in the strawberry crate, carrots in a separate crate, radishes in a separate crate. For example, we can give examples from real life such as frogs, reptiles, birds, fish, mammals.

Figure 10. *Group 3's interpretation on the wardrobe problem*

The students in Group 3 have reached a constructive level of mathematical communication by correctly analysing all stages. Table 6 presents the mathematical communication levels of students in both individual and group work. It is seen that there are differences in individual and group mathematical communication levels. For example, although Ersin's individual mathematical communication level could not be determined, he was at Level 3 in the group's mathematical communication level. Similarly, although Esin was at level zero individually, her mathematical communication level with the group was determined as level 2.

Table 6. Students' Mathematical Communication Levels in Both Individual and Group Work

Participants	Mathematical Communication Level in Individual Work	Mathematical Communication Level in Group Work
Ersin	-	3
Naz	-	1
Berfin	-	1
Gökberk	-	2
Esin	0	2
Ebrar	0	3
Kaan	0	1
Doruk	1	3
Salih	1	2
Birkan	1	3
Nur	1	2
Melih	1	1

DISCUSSION & CONCLUSION

In this section, discussions and conclusions are presented in light of the findings obtained during the process of determining the mathematical communication levels of sixth-grade middle school students within the context of RME applications.

Upon examining the mathematical communication levels of participating students in the context of individual implementation processes of RME problems, it was determined that four students were unable to complete any of the stages related to comprehending and applying short statements directly related to the mathematical problem situation. These students were also unable to express the problem situation with even a single word or reach a numerical solution. Interviews revealed that three of these students did not even attempt to read the question, stating that "mathematics problems are already difficult". Parallel to this finding, Zeybek and Açıl (2018) reported that students tend to avoid understanding mathematical expressions and using mathematical language. Interviews with the participants indicate that one of the primary reasons for this negative situation is students' difficulty in comprehending problems. It was observed that students struggled with reading comprehension and interpretation, which in turn hindered their ability to engage with the process of solving the RME problems presented in this study. It can be stated that the fundamental issue here is the students' deficiencies in Turkish language skills, which negatively impact their mathematical problem-solving processes. In other words, a student's inability to comprehend what she or he reads negatively affects her or his mathematical problem-solving process. In their study, Öztop and Toptaş (2022) identified a strong positive correlation between academic achievement in mathematics and reading comprehension skills. They found that as students' reading comprehension skills improved, their mathematics achievement increased, whereas a decline in reading comprehension skills was associated with lower mathematics achievement. Following the principles of realistic mathematics education, students' mathematical communication skills can be improved by presenting them with real or realistic problem situations and allowing them to discuss these situations. Therefore, teachers can provide their students with opportunities by using real or realistic problem situations in their lessons, which will contribute to the development of mathematical communication skills.

The findings of this study indicate that the mathematical communication levels of the three students were at level 0. These students were able to understand short statements expressed within the problem context and convey the result using either a single word or a numerical response. However, their mathematical communication levels remained limited to the direct information provided in RME applications, and they failed to reach conclusions in tasks requiring skills such as making connections and reasoning. As observed in this study, students' difficulties in associating multiple data sets were also highlighted as a finding in the research by Yeşildere and Türnüklü (2007). The study results revealed that students were less successful in problem situations requiring reasoning and interpretation compared to problems where the solution was explicitly requested. This suggests that such difficulties stem from the interrelated nature of different cognitive skills. Chasanah et al. (2020) also emphasized that mathematical communication skills have an impact on other mathematical abilities.

Another finding of the study revealed that the mathematical communication level of five students was classified as Level 1. These students were able to correctly solve RME tasks that required recognizing representations and identifying relationships between variables, demonstrating a broader understanding beyond simple statements. However, they were unable to analyse problem situations involving interwoven components due to difficulties in distinguishing them. When these results are examined, it is evident that students' overall mathematical communication levels are relatively low. Similarly, Chasanah et al. (2020) found that elementary school students exhibited low mathematical communication skills and that these skills needed to be further developed. Given that each educational stage influences subsequent levels, addressing this deficiency from the early childhood education stage is crucial. Although the middle school mathematics curriculum (MoNE, 2018) emphasizes the importance of mathematical communication skills, it is believed that the curriculum does not effectively facilitate the acquisition of these skills at the desired level. It can be said that this is a significant factor in the lack of development in the students' mathematical communication skills.

Positive changes in the students' mathematical communication skills were also observed throughout the process. Students who initially stated that they didn't know what to do, were reluctant to even read the problem and expressed that they didn't understand anything, were seen by the end of the process to be reading real-life problems and attempting to understand them. In the beginning, students expressed fear of such applications and believed they couldn't do them, but by the end of the process, they expressed a desire to continue studying these problems.

Furthermore, by the end of the process, the students stated that they approached the problems not with the prejudice of "I won't be able to solve it because it is a mathematical question," but by thinking, "If we encountered such a problem in daily life, how would we solve it?" and thus approached the problems in a problem-solving manner. Baran's (2019) study also supports a similar finding. Baran (2019) noted that students with lower levels of understanding initially struggled to comprehend and interpret the problems due to their lack of familiarity with such problem situations. However, in environments similar to the learning environment designed in the current study, they gradually adapted to the types of tasks by the end of the process.

In addition, students with lower communication levels stated that they initially tried to perform operations directly without understanding what was being asked in the question. However, by the end of the process, they began to first understand what the question was asking and then plan the solution process before starting. In this regard, Straker (1993) argued that rather than providing examples for students struggling with mathematics, creating a discussion environment where they can talk about mathematics would allow them to overcome these difficulties more effectively.

Compared to individual activities, students' mathematical communication levels were generally higher in group work, where they were more active and self-confident in their solution processes. Considering that students need to have confidence when using mathematical language for the development of their mathematical communication skills (MoNE, 2013), it can be suggested that effective peer communication in group work supports this process.

Zengin (2017) highlighted that communication in mathematics lessons plays a crucial role in understanding concepts and exchanging mathematical ideas, which enhances knowledge retention and fosters students' self-confidence. Similarly, the findings of Chapin et al. (2009) support this result. Their study emphasized that classroom discussions influence learning both directly and indirectly; while students gain mathematical knowledge through discussions, they also learn to respect their peers' ideas.

During the problem-solving process, it was observed that when one student in the group made an incorrect interpretation, other students identified the mistake and guided their peer toward the correct approach. Compared to individual tasks, group work encouraged students to share ideas with one another, thereby providing stronger support for mathematical communication. Pimm (1987) also emphasized that communicating with others during the process of expressing one's ideas is highly beneficial for developing clearer reasoning and explanation skills, highlighting the importance of encouraging students in this regard. An analysis of the group work papers revealed that, compared to individual work papers, students expressed their interpretations, explanations, and solution processes more clearly. Students collaborated with one another, which provided them with opportunities to refine and develop their thoughts. Pape et al. (2003) emphasized that classroom environments that foster the development of mathematical communication skills are those in which students interact with their peers, listen to their mathematical ideas, and engage in discussions. Throughout this study, efforts were made to create such an environment where students could actively use their mathematical communication skills. It can be inferred that listening to each other's thoughts and learning about different solution strategies during group work supported this process.

Furthermore, it was observed that students participated in group activities with great enthusiasm and expressed a desire to solve similar problems again after completing the tasks. Similarly, Lomibao et al. (2016) stated that discussing mathematics—mathematical communication—helps reduce students' anxiety toward the subject. Additionally, Kabael and Ata Baran (2016) found that students perceived real-life problems as making the process of learning mathematics more enjoyable, increasing their interest in the subject, and positively transforming their perceptions of mathematics by recognizing its presence in everyday life. These students also expressed a desire to engage with such problems more frequently. The positive affective impact of RME-based mathematical activities was also observed in this study, as participating students reported enjoying the tasks and solving problems with enthusiasm.

As evidenced, effective and meaningful communication is not only essential for solving real-life problems but also plays a crucial role in addressing mathematical problems. Therefore, classroom environments should be designed with a focus on mathematical communication. In this study, it was found that problems designed using RME principles were conducive to developing students' mathematical communication skills. RME problems encourage students to think critically and engage in discussions. Similarly, Altun et al. (2018) emphasized that incorporating contextual problems in mathematics lessons allows students to discuss and defend their ideas with peers, ultimately leading to greater success in problem-solving. Viseu and Oliveira (2012) stated that mathematical communication skills are supported in problem situations involving multiple solution strategies, as these encourage students to take an active role in the learning process. Various studies emphasize that establishing effective communication environments in classroom practices is a key factor in enabling students to develop a deeper understanding of mathematics (Kostos & Shin, 2010; Pape et al., 2003). This study provided students with an opportunity not only to engage in procedural operations but also to develop conceptual understanding. Through the problems they encountered, students recognized mathematical concepts such as sets and exponents, which is crucial for achieving the learning objectives of contemporary mathematics education.

Considering that students who reached Level 3 were generally those who actively participated in lessons and engaged in critical thinking, the findings support the conclusion of Paridjo and Waluya (2017) that individuals with well-developed mathematical communication skills also tend to be effective problem solvers. The same study also emphasized that students who worked individually often failed to reach higher levels due to uncertainty about where to begin solving a problem, which led to a biased approach. However, during group work, they were able to discuss and collaboratively plan the solution process. Güçler (2016) described the mathematics learning process as students engaging in mathematical discussions and refining their thinking throughout these interactions. This process was effectively facilitated through group work in the present study.

Students with a mathematical communication level below zero reported that they struggled to comprehend RME problems when working individually. However, within group discussions, these students actively participated, contributed to the problem-solving process, and shared their ideas with peers. It was observed that students who initially expressed mathematical ideas solely through numerical results and had a mathematical communication level of zero eventually became capable of interpreting questions within a group setting and expanding their perspectives. Consistent with the findings of Baran (2019), students who initially attempted to perform calculations without fully understanding the mathematical expressions in problem situations were later able to interpret problems involving multiple components. The process of solving real-life problems positively influenced their ability to comprehend and mathematically interpret problem situations.

While students working individually tended to focus solely on calculations, those participating in group discussions engaged in brainstorming and dialogue, which allowed them to interpret different solution approaches more effectively. Therefore, group work was found to support the development of students' mathematical communication skills. Additionally, some students who were more passive during the process supported the ideas of peers with higher mathematical communication levels. Among the students who consistently arrived at correct solutions in the RME activities, those with the highest mathematical communication levels maintained ongoing discussions at every stage and expanded their perspectives through collaboration.

Considering all these findings, it is evident that group work plays a significant role in fostering the development of mathematical communication skills.

In light of the discussions presented thus far, it is essential for teachers to be aware of mathematical communication skills and to create opportunities that foster their development in educational settings. In addition, teachers should actively support group work and establish learning environments that encourage students to question, discuss, and communicate about mathematical concepts.

To achieve this, mathematics teachers are advised to prioritize context-based problems that promote conceptual understanding and multiple-solution strategies rather than relying solely on single-answer or procedurally oriented problem situations. Such problems should encourage students to engage in inquiry and critical thinking.

Considering that communication is one of the fundamental keys to coexisting in society and understanding others, mathematical communication emerges as a crucial prerequisite for comprehending, utilizing, and integrating mathematics into everyday life. It should not be forgotten that all problems can be effectively and meaningfully addressed through strong and healthy communication. In this study, an attempt was made to determine students' mathematical communication levels by using RME activities. In future studies, it may be considered to include applied quantitative studies to improve students' level of mathematical communication. Additionally, research can be conducted to examine the development of mathematical communication levels using various mathematics teaching approaches (e.g., mathematical modeling, STEM, technology-supported mathematics teaching).

Statements of Publication Ethics

Ethics Committee Approval of this research was taken from Ordu University Social and Humanities Sciences Research Ethics Committee on 06.10.2022 with issue number 2022-174.

Researchers' Contribution Rate

If there is one author, please delete this sub-section. Otherwise, please specify the contribution rate of each author in the manuscript. Do not write any names. You may add new columns.

Authors	Literature review	Method	Data Collection	Data Analysis	Results	Conclusion
Author 1	☒	☐	☒	☒	☒	☐
Author 2	☐	☒	☐	☒	☐	☒
Author 3	☒	☐	☐	☒	☐	☐

Conflict of Interest

There is no conflict of interest in the study.

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