http://www.newtheory.org ISSN: 2149-1402

Journal of New Theory

Received: 27.02.2018 *Year*: **2018**, *Number*: **23**, *Pages*: **1-12**
Published: 29.05.2018 *Original Article*

Fuzzy Sub Implicative Ideals of KU-Algebras

Samy Mohammed Mostafa* <dr_samymostafa46@yahoo.com>
Ola Wageeh Abd El- Baseer <olawageeh@yahoo.com>

Department of Mathematics, Faculty of Education, Ain Shams University, Roxy, Cairo, Egypt

Abstract – We consider the fuzzification of sub-implicative (sub-commutative) ideals in KU-algebras, and investigate some related properties. We give conditions for a fuzzy ideal to be a fuzzy sub-implicative (sub-commutative) ideal. We show that any fuzzy sub-implicative (sub-commutative) ideal is a fuzzy ideal, but the converse is not true. Using a level set of a fuzzy set in a KU-algebra; we give a characterization of a fuzzy sub-implicative (sub-commutative) ideal.

***Keywords* –** KU-algebras - fuzzy sub implicative ideals- fuzzy sub-commutative

1. Introduction

BCK-algebras form an important class of logical algebras introduced by Iseki [2] and was extensively investigated by several researchers. It is an important way to research the algebras by its ideals. The notions of ideals in BCK-algebras and positive implicative ideals in BCK-algebras (i.e Isekis implicative ideals) were introduced by Iseki [2]. The notions of commutative (sub-commutative) ideals in BCK-algebras, positive implicative and implicative (Sub-implmicative), ideals in BCK-algebras were introduced by [4,5]. Zadeh [15] introduced the notion of fuzzy sets. At present this concept has been applied to many mathematical branches, such as group, functional analysis, probability theory, topology, and so on. In 1991, Xi [14] applied this concept to BCK-algebras, and he introduced the notion of fuzzy sub - algebras (ideals) of the BCK-algebras. Prabpayak and Leerawat [12,13] introduced a new algebraic structure which is called KU-algebra. They gave the concept of homomorphisms of KU-algebras and investigated some related properties. Mostafa et al. [8] introduced the notion of fuzzy KU-ideals of KU-algebras and then they investigated several basic properties which are related to fuzzy KU-ideals. Senapati et al. [6,7] introduced the notion of fuzzy KU-subalgebras (fuzzy *KU*-ideals) of KU-algebras with respect to a given *t*-norm, intuitionistic fuzzy bi-normed KU-ideals of a KU-algebra and obtained some of their properties. Mostafa et al. [10] introduced the notion of sub implicative (sub-commutative) ideals of KU-algebras and investigated of their properties.

Corresponding Author.

In this paper, the notion of fuzzy sub implicative (sub commutative) ideals of KU-algebras are introduced and then the several basic properties are investigated.

2. Preliminaries

Now we will recall some known concepts related to KU-algebra from the literature which will be helpful in further study of this article.

Definition 2.1. [12,13] Algebra(X, $*$, 0) of type (2, 0) is said to be a KU -algebra, if it satisfies the following axioms:

(ku_1) $(x*y)*[(y*z))*(x*z)]=0$,
(ku_2) $x*0=0$,
(ku_3) $0*x=x$,
(ku_4) $x*y=\mathbf{0}$ and $y*x=0$ implies $x=y$,
(ku_5) $x*x=\mathbf{0}$, for all $x, y, z \in X$.

On a KU-algebra $(X,*,\mathbf{0})$ we can define a binary relation $\leq$ on X by putting:

$x \leq y \Leftrightarrow y*x=0$.

Thus a KU - algebra X satisfies the conditions:

$(ku_{1^\backslash})$: $(y*z)*(x*z) \leq (x*y)$
$(ku_{2^\backslash})$: $0 \leq x$
$(ku_{3^\backslash})$: $x \leq y, y \leq x$ implies $x=y$,
$(ku_{4^\backslash})$: $y*x \leq x$.

Remark 2.2. Substituting $z*x$ for x and $z*y$ for y in ku_1 ,we get

$[(z*x)*(z*y)]*[(z*y)*z))*[(z*x)*z)] \leq [(z*x)*(z*y)]*[(z*x)*(z*y)]=0$ by (ku_1) ,hence $(x*y)*[(z*x)*(z*y)]=0$ that mean the condition (ku_1) and $(x*y)*[(z*x)*(z*y)]=0$ are equivalent.

For any elements x and y of a KU-algebra, $y*x^n$ denotes by $(\overbrace{y*x)*x)......*x}^{n\,times}$

Theorem 2.3. [8] In a KU-algebra X , the following axioms are satisfied:
For all $x, y, z \in X$,

(1) $x \leq y$ imply $y*z \leq x*z$,
(2) $x*(y*z)=y*(x*z)$, for all $x, y, z \in X$,
(3) $((y*x)*x) \leq y$.
(4) $(y*x^3)=(y*x)$

We will refer to X is a KU-algebra unless otherwise indicated.

Definition 2.4. [12,13**]** Let I be a non empty subset of a KU-algebra X. Then I is said to be an ideal of X, if

(I_1) $0 \in I$
(I_2) $\forall y, z \in X$, if $(y * z) \in I$ and $y \in I$, imply $z \in I$.

Definition 2.5. [8] Let I be a non empty subset of a KU-algebra X. Then I is said to be an KU- ideal of X, if
(I_1) $0 \in I$
(I_3) $\forall x, y, z \in X$, if $x * (y * z) \in I$ and $y \in I$, imply $x * z \in I$.

Definition 2.6. [11] KU- algebra X is said to be implicative if it satisfies
$(x * y^2) = (x * y) * (y * x^2)$

Definition 2.7. [11] KU- algebra X is said to be commutative
if it satisfies $x \leq y$ *implies* $(x * y^2) = x$

Lemma 2.8. [10] Let X be a KU-algebra. X is KU-implicative iff X is KU-positive implicative and KU-commutative.

Definition 2.9. [10] A non empty subset A of a KU-algebra X is called a ***sub*** implicative ideal of X, if $\forall x, y, z \in X$,

(1) $\mathbf{0} \in A$
(2) $z * ((x * y) * ((y * x^2)) \in A$ and $z \in A$, imply $(x * y^2) \in A$.

Definition 2.10. [10] Let $(X, *, 0)$ be a KU-algebra, a nonempty subset A of X is said to be a ***ku*** **-** positive implicative ideal if it satisfies, for all x, y, z in X,

(1) $0 \in A$,
(2) $z * (x * y) \in A$ and $z * x \in A$ imply $z * y \in A$.

Definition 2.11. [10] A non empty subset A of a KU-algebra X is called a ***ku*** **–** sub commutative ideal of X, if

(1) $\mathbf{0} \in A$
(2) $z * \{((y * x^2)) * y^2)\} \in A$ and $z \in A$, imply $(y * x^2) \in A$.

Definition 2.12. [10] A nonempty subset A of a KU-algebra X is called a kp-ideal of X if it satisfies

(1) $\mathbf{0} \in A$,
(2) $(z * y) * (z * x) \in A$, $y \in A \Rightarrow x \in A$

Definition 2.13. [8] A fuzzy set μ in a *KU*-algebra X is called a fuzzy sub -algebra of X if $\mu(x * y) \geq \min\{\mu(x), \mu(y)\}$ $\forall x, y \in X$.

Definition 2.14. [8] Let X be a KU-algebra, a fuzzy set μ in X is called a fuzzy ideal of X if it satisfies the following conditions:

(F_1) $\mu(0) \geq \mu(x)$ for all $x \in X$.
(F_2) $\forall x, y \in X$, $\mu(y) \geq \min\{\mu(x * y), \mu(x)\}$.

3. Fuzzy Sub-Implicative Ideals

Definition 3.1. [15] Let X be a non-empty set, a fuzzy subset μ in X is a function $f : X \rightarrow [0,1]$.

Definition 3.2. [1.15] Let μ be a fuzzy set in a set X . For t ∈ [0, 1], the set

$$\mu_t = \{x \in X \mid \mu(x) \geq t\}$$

is called upper level cut (level subset) of μ and the set $L(\mu, t) = \{x \in X \mid \mu(x) \leq t\}$ is called lower level cut of μ.

Definition 3.3. A non empty subset μ of a KU-algebra X is called a fuzzy ***sub*** implicative ideal (briefly FSI - ideal) of X , if $\forall x, y, z \in X$,

(F_1) $\mu(0) \geq \mu(x)$
(FSI_1) $\mu(x * y^2) \geq \min\{\mu(z * ((x * y) * ((y * x^2)), \mu(z)\}$

Example 3.4. Let $X = \{0,1,2,3,4\}$ in which the operation $*$ is given by the table

*	0	1	2	3	4
0	0	1	2	3	4
1	0	0	1	3	4
2	0	0	0	3	4
3	0	0	0	0	4
4	0	0	0	0	0

Then $(X, *, 0)$ is a KU-Algebra. Define a fuzzy set $\mu : X \rightarrow [0,1]$ by $\mu(0) = t_0$, $\mu(1) = \mu(2) = t_1$, $\mu(3) = \mu(4) = t_2$, where $t_0, t_1, t_2 \in [0,1]$ with $t_0 > t_1 > t_2$. Routine calculation gives that μ is FSI- ideal of KU- algebra X.

Proposition 3.5. Every FSI- ideal of a KU-algebra X is order reversing.

Proof. Let μ be FSI -ideal of X and let x, y, z ∈ X be such that x ≤ z, then z* x = 0 and by (F_1) $\mu(x*y^2)\geq \min\{\mu(z*((x*y)*((y*x^2)),\mu(z)\}$. Let y = x,

$$\mu(x*x^2)\geq \min\{\mu(z*((x*x)*((x*x^2)),\mu(z)\}$$
$$\mu(x)\geq \min\{\mu(z*x),\mu(z)\}=\min\{\mu(0),\mu(z)\}=\mu(z)$$

Lemma 3.6. Let μ be a fuzzy FSI - ideal of KU - algebra X , if the inequality y ∗ x ≤ z hold in X , Then μ (x) ≥ min {μ (y) , μ (z)} .

Proof. Let μ be FSI -ideal of X and let x, y, z ∈ X be such that y*x ≤ z, then z* (y*x)= 0 or y* (z*x)= 0 i.e $z*x\leq y$ we get

$$\mu(z*x)\geq \mu(y) \tag{a}$$

By (FSI_1): $\mu(x*y^2)\geq \min\{\mu(z*((x*y)*((y*x^2)),\mu(z)\}$. Let y = x

$$\mu(x*x^2)\geq \min\left\{\mu(z*(\overbrace{(x*x)}^{0}*\overbrace{((x*x^2)}^{x}),\mu(z)\right\}=\min\{\mu(z*x),\mu(z)\},\text{i.e}$$

$\mu(x)\geq \min\{\mu(z*x),\mu(z)\}\geq \min\{\mu(y),\mu(z)\}$ by (a) .

Definition 2.7. [9,10] KU- algebra X is said to be implicative if it satisfies

$$(x*y^2)=(x*y)*(y*x^2)$$

Lemma 3.8. If X is implicative KU-algebra, then every fuzzy ideal of X is an FSI-ideal of X.

Proof. Let μ be an fuzzy ideal of X, then by (F_2)

$$\forall y,z\in X,\ \ \mu(y)\geq \min\{\mu(z*y),\mu(z)\}.$$

Substituting $x*y^2$ for y in (F_2) $\mu(x*y^2)\geq \min\{\mu(z*(x*y^2)),\mu(z)\}$, but KU- algebra is implicative i.e $(x*y^2)=(x*y)*(y*x^2)$, hence

$$\mu(x*y^2)\geq \min\{\mu(z*(x*y^2)),\mu(z)\}=\min\{\mu(z*(x*y)*(y*x^2)),\mu(z)\}$$

Which shows that μ is FSI-ideal of X.

Theorem 3.9. Let μ be a fuzzy set in X satisfying the condition (FSI_1) , then μ satisfies the following inequality:

$$\mu(x*y^2)\geq \mu((x*y)*(y*x^2)) \tag{FSI_2}$$

Proof. Let μ satisfying (FSI_1) i.e $\mu(x*y^2) \geq \min\{\mu(z*((x*y)*(y*x^2))), \mu(z)\}$, then by taking $z=0$ in (FSI_1) and using (F_1) and (ku_3) we get

$$\mu(x*y^2) \geq \min\{\mu(0*((x*y)*((y*x^2)), \mu(0)\} = \mu((x*y)*(y*x^2))$$

Theorem 3.10. Every FSI-ideal is a fuzzy ideal, but the converse does not hold.

Proof. Let μ be FSI-ideal FSI-ideal of X; put x=y in (FSI_1), we get

$$\mu(\overbrace{x*x^2}^{x}) \geq \min\{\mu(z*((x*x)*((x*x^2)), \mu(z)\}\ then$$

$$\mu(x) \geq \min\left\{\mu(z*((\overbrace{x*x}^{0})*((\overbrace{x*x^2}^{x})), \mu(z)\right\} = \min\{\mu(z*x), \mu(z)\}$$

Hence μ is a fuzzy ideal of X .

The following example shows that the converse of Theorem 3.10 may not be true.

Example 3.11. Let $X = \{0,1,2,3,4\}$ in which the operation $*$ is given by the table

*	0	1	2	3	4
0	0	1	2	3	4
1	0	0	1	3	4
2	0	0	0	3	4
3	0	0	0	0	4
4	0	0	0	0	0

Then $(X,*,0)$ is a KU-Algebra. Define a fuzzy set μ : X→ [0,1] by μ (0) = 0.7 , μ (1) = μ (2) = μ (3) = μ (4) = 0.2 , we get for z=0 , x=1 and y=2. L.H.S of (FI_1)

$$\mu((1*2)*2) = \mu(1) = 0.2\ \ R.H.S\ of\ (FI_1)\ \min\left\{\mu(0*((\overbrace{1*2}^{1})*((\overbrace{2*1}^{0})*1), \mu(0)\right\} = \mu(0) = 0.7$$

i.e in this case $\mu(x*y^2) \ngeq \min\{\mu(z*((x*y)*((y*x^2)), \mu(z)\}$.

We now give a condition for a fuzzy ideal to be a FSI-ideal.

Theorem 3.12. Every fuzzy ideal μ of X satisfying the condition (FSI_2) is a FSI-ideal ideal of X .

Proof. Let μ be fuzzy ideal of X satisfying the condition (FSI_2). We get

$$\mu(x*y^2) \geq \{\mu(((x*y)*((y*x^2))\} \text{ and } \mu(x*y^2) \geq \min\{\mu(z*((x*y)*((y*x^2)), \mu(z)\}$$

by (Definition of fuzzy ideal (F_2)), hence

$$\mu(x*y^2) \geq \ \mu(((x*y)*((y*x^2)) \ \geq \min\{\mu(z*((x*y)*((y*x^2)),\mu(z)\}$$

(Definition of fuzzy ideal (F_2)) ,which proves the condition (FSI_1) . This completes the proof.

Theorem 3.13. Let μ be a fuzzy ideal of X. Then the following are equivalent

(i) μ is an FSI-ideal of X,
(ii) $\mu(x*y^2) \geq \mu(z*((x*y)*((y*x^2))$
(iii) $\mu(x*y^2) = \mu(z*((x*y)*((y*x^2))$.

Proof. (i)) $\Rightarrow$ (ii) Suppose that μ is an FSI-ideal of X. By (FSI_1) and (F_1) we have
$\mu(x*y^2) \geq \min\{\mu(0*((x*y)*((y*x^2)),\mu(0)\} = \mu(0*((x*y)*((y*x^2)$ *i.e*
$\mu(x*y^2) \geq \mu(((x*y)*((y*x^2))$

(ii) $\Rightarrow$ (iii) Since $(x*y)*((y*x^2) \leq x*y^2$, by Lemma 3.5 we obtain ,
$\mu(x*y^2) \geq \mu((x*y)*((y*x^2))$ Combining (ii) we have $\mu(x*y^2) = \mu((x*y)*((y*x^2))$.

(iii) $\Rightarrow$ (i) Since

$$\begin{aligned}[(z*((x*y)*((y*x^2))]*[(x*y)*((y*x^2)] &= [(x*y)*(z*((y*x^2))]*[(x*y)*((y*x^2)] \\ &\leq [(z*((y*x^2))]*[(y*x^2)] \\ &= [(z*((y*x^2))]*[0*(y*x^2)] \\ &\leq 0*z = z\,,\end{aligned}$$

by Lemma 3.6. we obtain $\mu((x*y)*((y*x^2) \geq \min\{\ \mu,((x*y)*((y*x^2),\mu(z)\}$. From (iii), we have $\mu(x*y^2) \geq \min\{\mu(z*((x*y)*((y*x^2)),\mu(z)\}$. Hence μ is an FSI-ideal of X The proof is complete.

Theorem 3.14. A fuzzy set μ of a KU-algebra X is a sub-implicative fuzzy ideal of X if and only if $\mu_t \neq \Phi$ is a sub-implicative ideal of X.

Proof: Suppose that μ is a fuzzy sub-implicative ideal of X and $\mu_t \neq \Phi$ for any $t \in (0,1]$, there exists $x \in \mu_t$ so that $\mu(x) \geq t$. It follows from (F_1) that $\mu(0) \geq \mu(x) \geq t$ so that $0 \in \mu_t$. Let $x, y, z \in X$ be such that $z*((x*y)*((y*x^2) \in \mu_t$ and $z \in \mu_t$. Using (FI_1) , we know that

$$\mu(x*y^2) \geq \min\{\mu(z*((x*y)*((y*x^2)),\mu(z)\} = \min\{t,t\} = t$$

thus $x*y^2 \in \mu_t$. Hence μ_t is a sub-implicative ideal of X.

Conversely, suppose that $\mu_t \neq \Phi$ is a sub-implicative ideal of X ,for every $t \in (0,1]$. and any $x \in X$, let $\mu(x) = t$. Then $x \in \mu_t$. Since $0 \in \mu_t$, it follows that $\mu(0) \geq t = \mu(x)$ so that

$\mu(0) \geq \mu(x)$ for all $x \in X$. Now, we need to show that μ satisfies (FI_1) . If not, then there exist $a, b, c \in X$ such that

$$\mu(a * b^2) \leq \min\{\mu(c * ((a * b) * ((b * a^2)), \mu(c)\}$$

Taking

$$t_0 = \frac{1}{2}(\mu(a * b^2) + \{\mu(c * ((a * b) * ((b * a^2)), \mu(c)\})$$

then we have

$$\mu(a * b^2) < t_0 < \{\mu(c * ((a * b) * ((b * a^2)), \mu(c)\}$$

Hence $c * ((a * b) * (b * a^2)) \in \mu_t$ and $c \in \mu_t$, but $a * b^2 \notin \mu_t$ which means that μ_t is not a sub-implicative ideal of X.this is contradiction. Therefore μ is a fuzzy sub-implicative ideal of X .

4. Fuzzy Sub-Commutative Ideals

Definition4.1. A non empty subset A of a KU-algebra X is called a sub commutative ideal of X , if

(1) $0 \in A$

(2) $z * \{((y * x^2)) * y^2)\} \in A$ and $z \in A,$ imply $(y * x^2) \in A$.

Lemma4 .2. Every fuzzy FSC ideal of a KU-algebra X is order reversing.

Proof. Let μ be FSC -ideal of X and let x, y, z ∈ X be such that x ≤ z, then z* x = 0 and by $(FSCI_1)$ $\mu(y * x^2) \geq \min\{\mu(z * ((y * x^2)) * y^2), \mu(z)\}$. Let y = x ,then

$$\mu(x) \geq \min\{\mu(z * ((x * x^2)) * x^2), \mu(z)\} = \min\{\mu[(z * x)], \mu(z)\} = \min\{\mu(0), \mu(z)\} = \mu(z)$$

Lemma 4.3. let μ be a fuzzy FSCk - ideal of KU - algebra X , if the inequality y ∗ x ≤ z hold in X , Then μ (x) ≥ min {μ (y) , μ (z)} .

Proof. Let μ be FSC -ideal of X and let x, y, z ∈ X be such that z*x ≤ y, then z* (y*x)= 0 or y* (z*x)= 0 i.e $z * x \leq y$ [$\mu(z * x) \geq \mu(y)$]. By $(FSCI_1)$:

$$\mu(y * x^2) \geq \min\{\mu(z * ((y * x^2)) * y^2), \mu(z)\}$$

Put x =y

$$\mu(x) \geq \min\{\mu(z * ((x * x^2)) * x^2), \mu(z)\} = \min\{\mu[(z * x)], \mu(z)\} \geq \min\{\mu(y), \mu(z)\}$$

Lemma 4.4. If X is commutative KU-algebra, then every fuzzy ideal of X is an FSC-ideal of X.

Proof. Let μ be an fuzzy ideal of X, then by (F_2) $\forall y, z \in X$,

$$\mu(y) \geq \min\{\mu(z*y), \mu(z)\}.$$

Substituting $y*x^2$ for y in (F_2)

$$\mu(y*x^2) \geq \min\{\mu(z*(y*x^2)), \mu(z)\},$$

but KU- algebra is commutative i.e $(y*x)*x = (x*y)*y$, hence

$$\mu(y*x^2) \geq \min\{\mu(z*(y*x^2)), \mu(z)\} = \min\{\mu(z*(y*x^2)), \mu(z)\}$$

since

$$(y*x^2)*y^2 = ((y*x)*x)*y)*y = ((y*x)*x)*y)*(0*y) \leq (y*x)*x)$$

Then $z*[(y*x^2)*y^2] \geq z*(y*x)*x)$ by i.e $\mu[z*((y*x^2)*y^2)] \leq \mu\{z*(y*x^2)\}$ by theorem 4.2. Therefore

$$\mu(y*x^2) \geq \min\{\mu(z*(y*x^2)), \mu(z)\} \geq \min\{\mu(z*((y*x^2)*y^2)), \mu(z)\}.$$

Which shows that μ is FSIk-ideal of X.

Theorem 4.5. Let μ be a fuzzy set in X satisfying the condition $(FSCI_1)$, then μ satisfies the following inequality

$$\mu(y*x^2) \geq \mu((y*x^2))*y^2) \qquad (FSCI_2)$$

Proof. Let μ satisfying $(FSCI_1)$ i.e $\mu(y*x^2) \geq \min\{\mu(z*((y*x^2))*y^2), \mu(z)\}$, then by taking z = 0 in (FI_1) and using (F_1) and (ku_3) we get

$$\mu(y*x^2) \geq \min\{\mu(0*((y*x^2))*y^2), \mu(0)\}.$$

Hence $\mu(y*x^2) \geq \mu((y*x^2))*y^2)$

Theorem 4.6. Every fuzzy SCI is a fuzzy ideal, but the converse does not hold.

Proof . Let μ be fuzzy fuzzy SCI of X; put x=y in $(FSCI_1)$, we get

$$\mu(x*x^2) \geq \min\{\mu(z*((x*x^2))*x^2), \mu(z)\} = \min\{\mu(z*x), \mu(z)\}$$

for all x, z □ X . Hence μ is a fuzzy ideal of X .

The following example shows that the converse of Theorem 4.6 may not be true.

Example 4.7. Let $X = \{0,1,2,3,4\}$ in which the operation $*$ is given by the table

*	0	1	2	3	4
0	0	1	2	3	4
1	0	0	1	3	4
2	0	0	0	3	4
3	0	0	0	0	4
4	0	0	0	0	0

Then $(X,*,0)$ is aKU-Algebra. . Define a fuzzy set μ : X→ [0,1] by μ (0) = 0.7 , μ (1) = μ (2) = μ (3) = μ (4) = 0.2 , we get for z=0 , x=1 and y=3, L.H.S of $(FSCI_1)$
$\mu((3*1)*1)=\mu(1)=0.2$

$R.H.S$ *of* $(FSCI_1)$ $\min\left\{\mu(0*(((\overbrace{3*1}^{0})*1)*3)*3),\mu(0)\right\}=\mu(0)=0.7$, i.e in this case

$\mu(y*x^2)\ngeq\ \min\{\mu(z*((y*x^2)*y^2),\mu(z)\}$

We now give a condition for a fuzzy ideal to be a fuzzy sub- commutative ideal.

Theorem 4.8. Every fuzzy ideal μ of X satisfying the condition $(FSCI_2)$ is a fuzzy FSC of X .

Proof. Let μ be fuzzy ideal of X satisfying the condition $(FSCI_2)$. We get

$$\mu(y*x^2)\geq\mu((y*x^2))*y^2)$$

and by (Definition (F_2) fuzzy ideal), hence

$$\mu(y*x^2)\geq\ \mu((y*x^2)*y^2)\geq\min\{\mu(z*((y*x^2)*y^2),\mu(z)\}$$

by F_2 which proves the condition $(FSCI_1)$. This completes the proof.

Theorem 4.9. Let μ be a fuzzy ideal of X. Then the following are equivalent

(i) μ is an FSC-ideal of X,
(ii) $\mu(y*x^2)\geq\ \mu((y*x^2)*y^2)$
(iii) $\mu(y*x^2)=\ \mu((y*x^2)*y^2)$.

Proof. (i)$\Rightarrow$(ii) Suppose that μ is an FSC-ideal of X. By $(FSCI_1)$ and (F_1) we have
$\mu(y*x^2)\geq\min\{\mu(z*((y*x^2))*y^2),\mu(z)\}=\min\{\mu(0*((y*x^2))*y^2),\mu(0)\}=$
$\mu((y*x^2))*y^2)$

(ii) $\Rightarrow$ (iii) Since $(y*x^2)*y^2\leq y*x^2$, we have $\mu(y*x^2)\geq\mu((y*x^2)*y^2)$
Combining (ii) we have $\mu(y*x^2)=\mu((y*x^2)*y^2)$.

(iii) $\Rightarrow$ (i) Since $[(z*((y*x^2)*y^2))]*[0*((y*x^2)*y^2] \leq 0*z = z$, by Lemma 4.3 we obtain $\mu((y*x^2)*y^2) \geq \min\{\mu(z*((y*x^2)*y^2),\mu(z)\}$

Hence μ is an FSC-ideal of X The proof is complete.

Theorem 4.10. A fuzzy set μ of a KU-algebra X is a fuzzy sub- commutative ideal of X if and only if $\mu_t \neq \Phi$ is a sub- commutative ideal of X.

Proof: Suppose that μ is a fuzzy sub- commutative ideal of X and $\mu_t \neq \Phi$ for any $t \in (0,1]$, there exists $x \in \mu_t$ so that $\mu(x) \geq t$. It follows from (F_1) that $\mu(0) \geq \mu(x) \geq t$ so that $0 \in \mu_t$. Let $x, y, z \in X$ be such that $z*((y*x^2)*y^2) \in \mu_t$ and $z \in \mu_t$. Using $(FSCI_1)$, we know that $\mu(y*x^2) \geq \min\{\mu(z*((y*x^2)*y^2)),\mu(z)\} = \min\{t,t\} = t$, thus $y*x^2 \in \mu_t$. Hence μ_t is a sub- commutative ideal of X.

Conversely, suppose that $\mu_t \neq \Phi$ is a sub- commutative ideal of X ,for every $t \in (0,1]$. and any $x \in X$, let $\mu(x) = t$. Then $x \in \mu_t$. Since $0 \in \mu_t$, it follows that $\mu(0) \geq t = \mu(x)$ so that $\mu(0) \geq \mu(x)$ for all $x \in X$. Now, we need to show that μ satisfies $(FSCI_1)$. If not, then there exist $a, b, c \in X$ such that $\mu(b*a^2) \leq \min\{\mu(c*((b*a^2)*b^2),\mu(c)\}$. Taking

$$t_0 = \frac{1}{2}(\mu(b*a^2) + \{\mu(c*((b*a^2)*b^2),\mu(c)\})$$

then we have $\mu(b*a^2) < t_0 < \{\mu(c*((b*a^2)*b^2),\mu(c)\}$. Hence $c*((b*a^2)*b^2) \in \mu_t$ and $c \in \mu_t$, but $b*x^2 \notin \mu_t$ which means that μ_t is not a sub- commutative ideal of X, this is contradiction. Therefore μ is a fuzzy sub- commutative ideal of X .

Acknowledgment

The author is greatly appreciated the referees for their valuable comments and suggestions for improving the paper.

Conflicts of Interest

State any potential conflicts of interest here or "The author declare no conflict of interest".

References

[1] Hong S.M. and Jun Y.B. , Fuzzy and level subalgebras of BCK (BCI) algebras Pusan Kyongnam Math. J. (Presently , East Asain Math . J.) , 7(2) (1991) , 185-190 .

[2] Iseki. K. On BCI-algebras. Math. Sem. Notes 8 (1980) 125 – 130

[3] Jun Y. B., Hong A.M.T, Kim A. T. and Song S. Z., Fuzzy ideals and fuzzy subalgebras of BCK-algebras , J.fuzzy Math ,7(2) (1999) , 411-418 .

[4] Liu Y. L, Meng J. and Xu Y., BCI-implicative ideals of BCI-algebras, Information Sciences (2007) 1 - 10.

[5] Liu Y. L., S. Y. Liu S. Y., Meng J. FSI-ideals and FSC-ideals of BCl-algebras Bull. Korean Math. Soc. 41 (2004), No.1, pp. 167-179
[6] Senapati, T. and Shum K. P., Atanassov's intuitionistic fuzzy bi-normed KU-subalgebrs of a KU-algebra, Missouri J. Math. Sci., Volume 29, Issue 1 (2017), 92-112.
[7] Senapati T.,T-fuzzy KU-ideals of KU-algebras, June 2018, Volume 29, Issue 3–4, pp 591–600 Afrika Matematika.
[8] Mostafa S. M., Abd-Elnaby M. and Yousef M. M., Fuzzy ideals of KU-Algebras, International Math.Forum, Vol.6, (2011), no.63, 3139-3149.
[9] Mostafa S. M., Kareem F. F., N-Fold Commutative KU-Algebras International Journal of Algebra, Vol. 8, 2014, no. 6, 267 – 275
[10] Mostafa S. M., Omar R. A. K, Abd El- Baseer, O. W., Sub implicative ideals of KU-Algebras, International Journal of Modern Science and Technology Vol. 2, No. 5, 2017. Page 223-227.
[11] Radwan A. E., Mostafa S. M., Ibrahem F. A. & Kareem F. F., Topology spectrum of a KU-Algebra. Journal of New Theory 5 (2015) Year: 2015, Number: 8, Pages: 78-91
[12] Prabpayak C. and Leerawat U., On ideals and congruence in KU-algebras, Scientia Magna Journal, Vol. 5(2009), No.1, 54-57.
[13] Prabpayak C. and Leerawat U., On isomorphisms of KU-algebras, Scientia Magna Journal, 5(2009), no. 3, 25-31
[14] Xi O. G., Fuzzy BCK-algebras, Math. Japan, 36 (1991) 935-942.
[15] Zadeh L. A., Fuzzy sets, Inform. and Control, 8 (1965) 338-353.