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STABILITY ANALYSIS FOR THE KAWACHARA AND MODIFIED KAWACHARA EQUATIONS

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Abstract: *In this paper, by using the extended direct algebraic method, we obtain the exact traveling wave solutions for the Kawachara equation and the modified Kawachara equation. The exact solutions of the Kawachara and modified Kawachara equations demonstrated by graphs. The stability of these solutions and the movement role of the waves by sketching the graphs of the exact solutions are analyzed.*

Key words*: Traveling wave solutions, Extended direct algebraic method, Kawachara equation, modified Kawachara equation*

1. Introduction

Many physical, chemical and biological phenomena such as combustion waves, optical solitons, chemical reactions, propagation of dominant genes and nerve pulses etc. are modelled by nonlinear partial differential equations exhibiting traveling wave solutions [1].

Traveling wave solutions of partial differential equations are solutions of special shape which do not change in time. The existence of such solutions for parabolic equations was first studied by A.N. Kolmogorov, I.G. Petrovskii, and N.S. Piskunov in their mathematical investigations of the equation proposed by R.A. Fisher in 1937 to describe the propagation of an advantageous gene.

Traveling wave solutions of partial differential equations are solutions of the form $u(x;t) = f(x-ct)$, with x being the spatial variable, t the time variable and c the constant speed of propagation of the wave. The properties of such solutions at different moments of time are obtained from one another by means of simple translation [2]. The investigation of exact traveling wave solutions to nonlinear evolution equations plays an important role in the study of nonlinear physical phenomena. Solitons are the most important solutions among traveling wave solutions [3].

The word soliton was discovered (and named) in 1965 by Zabusky and Kruskal, who experimenting with the numerical solution by computer of KdV equation. The word soliton was coined by Zabusky and Kruskal after “photon”, “proton”, etc., to emphasize that a soliton a localized entity which may keep its identity after an interaction [4].

In this study, we applied the extended direct algebraic method to find soliton solutions of Kawachara and Modified Kawachara equations.

2. Analysis of the extended direct algebraic method

The following is a given nonlinear partial differential equations with two variables x and t as where

$$P\left(u,u_x,u_t,u_{xx},u_{xt},u_{tt},\ldots\right)=0, \tag{1}$$

P is a polynomial function with respect to the indicated variables or some functions which can be reduced to a polynomial function by using some transformations.

Step 1: Assume that Eq. (1) has the following formal solution as:

$$u\left(x,t\right)=u\left(\xi\right)=\sum_{i=0}^{m}a_i\vartheta^i\left(\xi\right), \tag{2}$$

where

$$\vartheta'=\sqrt{\alpha\vartheta^2+\beta\vartheta^4} \qquad \text{and} \quad \xi=kx+\omega t, \tag{3}$$

where α, β are arbitrary constants and k and ω are the wave length and frequency and $\vartheta'=\frac{d\vartheta}{d\xi}$.

Step 2: Balancing the highest order derivative term and the highest order nonlinear term of Eq. (1), and the coefficients of series α, β, a_0,a_1, a_m, k, ω are parameters can be determined.

Step 3: Substituting from Eqs. (2) and (3) into Eq. (1) and collecting coefficients of $\vartheta^i\vartheta^{(i)}$, then setting coefficients equal zero, we will obtain a set of algebraic equations. By solving the system, the parameters α, β, a_0, a_1, a_m, k, ω can be determined.

Step 4: By substituting the parameters α, β, a_0, a_1, a_m, k, ω and $\vartheta\left(\xi\right)$obtained in step 3 into Eq. (2), the solutions of Eq. (1) can be derived.

3. Stability analysis

Hamiltonian system is a mathematical formalism to describe the evolution equations of a physical system. By using the form of a Hamiltonian system for which the momentum is given as

$$M=\frac{1}{2}\int_{-\infty}^{\infty}u^2d\xi, \tag{4}$$

where M is the momentum, u is the traveling wave solutions in Eq. (2). The sufficient condition for soliton stability is

$$\frac{\partial M}{\partial\omega}>0, \tag{5}$$

where ω is the frequency [5-19].

4. The applications of the method

4.1. The Kawachara equation

The Kawachara equation describing nonlinear wave processes in dispersive system [5] as

$$u_t+uu_x+u_{xxx}-u_{xxxxx}=0. \tag{6}$$

Consider the traveling wave solutions (2) and (3), then Eq. (6) becomes

$$\omega u'+kuu'+k^3u'''-k^5u^{(5)}=0. \tag{7}$$

Balancing the nonlinear uu' and highest order derivative $u^{(5)}$ in Eq. (7) gives $m=4$. Suppose the solution of Eq. (6) in the form

$$u\left(\xi\right)=a_0+a_1\vartheta+a_2\vartheta^2+a_3\vartheta^3+a_4\vartheta^4. \tag{8}$$

By substituting (8) into Eq. (7) yields a set of algebraic equations for $a_0, a_1, a_2, a_3, a_4, \alpha, \beta, k, \omega$. The system of equations are found as

$$\begin{aligned}
k^3\alpha a_1 - k^5\alpha^2 a_1 + \omega a_1 + k a_0 a_1 = 0,\\
k a_1^2 + 8k^3\alpha a_2 - 32k^5\alpha^2 a_2 + 2\omega a_2 + 2k a_0 a_2 = 0,\\
6k^3\beta a_1 - 60k^5\alpha\beta a_1 + 3k a_1 a_2 + 27k^3\alpha a_3 - 243k^5\alpha^2 a_3 + 3\omega a_3 + 3k a_0 a_3 = 0,\\
24k^3\beta a_2 - 480k^5\alpha\beta a_2 + 2k a_2^2 + 4k a_1 a_3 + 64k^3\alpha a_4 - 1024k^5\alpha^2 a_4 + 4\omega a_4 + 4k a_0 a_4 = 0,\\
-120k^5\beta^2 a_1 + 60k^3\beta a_3 - 2040k^5\alpha\beta a_3 + 5k a_2 a_3 + 5k a_1 a_4 = 0,\\
-720k^5\beta^2 a_2 + 3k a_3^2 + 120k^3\beta a_4 - 6240k^5\alpha\beta a_4 + 6k a_2 a_4 = 0,\\
7k a_3 a_4 - 2520k^5\beta^2 a_3 = 0,\\
4k a_4^2 - 6720k^5\beta^2 a_4 = 0. \quad (9)
\end{aligned}$$

The solution of the system of algebraic equations, can be found as

$$a_0 = -\frac{36}{169} - \frac{\omega}{k},\ a_1 = 0,\ a_2 = 0,\ a_3 = 0,\ a_4 = \frac{105\beta^2}{169\alpha^2},\ \alpha = \frac{1}{52k^2},$$

$$a_0 = \frac{-4k^3\alpha + 16k^5\alpha^2 - \omega}{k},\quad a_1 = 0,\quad a_2 = \frac{280}{13}(-k^2\beta + 52k^4\alpha\beta),\quad a_3 = 0,\quad a_4 = 1680k^4\beta^2. (10)$$

Substituting from Eqs. (10) into (8), the following solutions of Eq. (6) can be obtained as

$$u_1(x,t) = -\frac{36}{169} - \frac{\omega}{k} + \frac{105}{169}\sec h^4\left(\frac{1}{2k\sqrt{13}}(kx + \omega t)\right), \quad (11)$$

$$u_2(x,t) = -\frac{36}{169} - \frac{\omega}{k} + \frac{26880\beta^2\exp\left(2/k\sqrt{13}(kx + \omega t)\right)}{169\left(1 - 4\beta\exp\left(1/k\sqrt{13}(kx + \omega t)\right)\right)^4}, \quad (12)$$

$$u_3(x,t) = -\frac{36}{169} - \frac{\omega}{k} + \frac{26880\beta^2\exp\left(2/k\sqrt{13}(kx + \omega t)\right)}{169\left(\exp\left(1/k\sqrt{13}(kx + \omega t)\right) - 4\beta\right)^4}, \quad (13)$$

$$u_4(x,t) = \frac{-4k^3\alpha + 16k^5\alpha^2 - \omega}{k} + \frac{280}{13}(\alpha k^2 - 52k^4\alpha^2)\sec h^2\left(\sqrt{\alpha}(kx + \omega t)\right) + 1680\alpha^2k^4\sec h^4\left(\sqrt{\alpha}(kx + \omega t)\right), \quad (14)$$

$$u_5(x,t) = \frac{-4k^3\alpha + 16k^5\alpha^2 - \omega}{k} + \frac{4480}{13}\alpha(-k^2\beta + 52k^4\alpha\beta)\frac{\exp\left(2\sqrt{\alpha}(kx + \omega t)\right)}{\left(1 - 4\beta\exp\left(2\sqrt{\alpha}(kx + \omega t)\right)\right)^2} + 430080k^4\alpha\beta^4\frac{\exp\left(4\sqrt{\alpha}(kx + \omega t)\right)}{\left(1 - 4\beta\exp\left(2\sqrt{\alpha}(kx + \omega t)\right)\right)^4}, \quad (15)$$

$$u_6(x,t)=\frac{-4k^3\alpha+16k^5\alpha^2-\omega}{k}+\frac{4480}{13}\alpha(-k^2\beta+52k^4\alpha\beta)\frac{\exp\left(2\sqrt{\alpha}\left(kx+\omega t\right)\right)}{\left(\exp\left(2\sqrt{\alpha}\left(kx+\omega t\right)\right)-4\beta\right)^2}+$$

$$430080k^4\alpha^2\beta^2\frac{\exp\left(4\sqrt{\alpha}\left(kx+\omega t\right)\right)}{\left(\exp\left(2\sqrt{\alpha}\left(kx+\omega t\right)\right)-4\beta\right)^4}. \qquad (16)$$

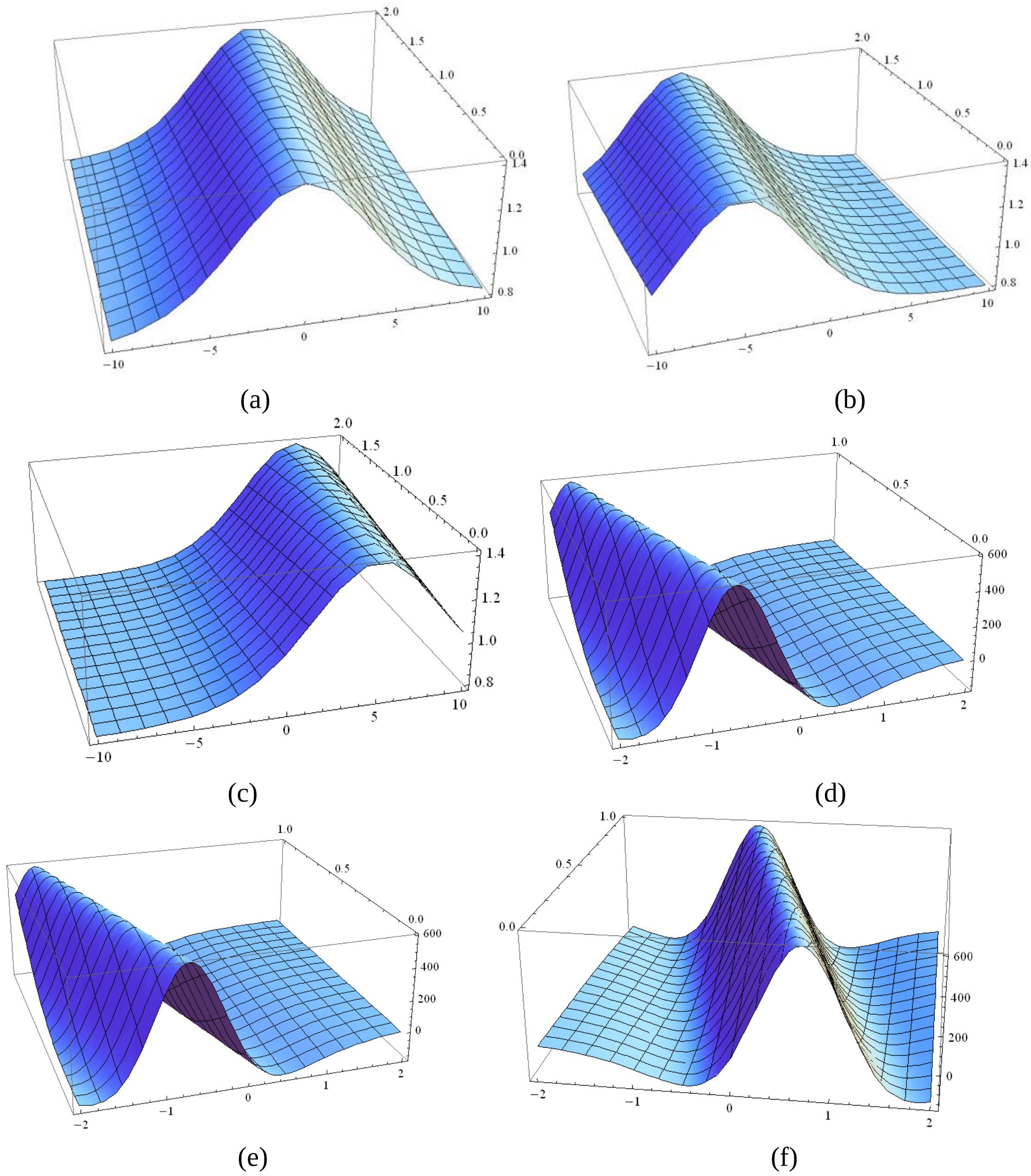


Fig 1. Traveling waves solutions (11)-(16) are plotted.

The traveling wave solutions (11)-(16) are shown in Fig. 1a-c with $\omega=\beta=-1,\ k=1$ and $\alpha=1/52$ in the interval [-10,10] and [0, 2]; the traveling wave solutions (14)-(16) are shown in

Fig.1d-f with $\omega = \alpha = k = 1$ and $\beta = -1$ in the interval [-2,2] and [0, 1]. According to the conditions of stability (4) and (5), the traveling wave solutions (11)-(13) are stable in the interval [-10,10] and [0, 2], the traveling wave solutions (14)-(16) are stable in the interval [-2,2] and [0, 1].

4.2. The Modified Kawachara equation

The modified Kawachara equation, which describes the motion of a water waves with surface tension as

$$u_t + u_x + u^2 u_x + p u_{xxx} + q u_{xxxxx} = 0, \tag{17}$$

p and q are constants [10]. Consider the traveling wave solutions (2) and (3), then Eq. (17) becomes

$$u^2 u' + k^3 p u''' + q k^5 u^{(5)} = 0. \tag{18}$$

Balancing the nonlinear $u^2 u'$ and highest order derivative $u^{(5)}$ in Eq. (18) gives $m = 2$. Suppose the solution of Eq. (18) in the form

$$u(\xi) = a_0 + a_1 \vartheta + a_2 \vartheta^2. \tag{19}$$

By substituting (19) into Eq. (18) yields a set of algebraic equations for $a_0, a_1, a_2, \alpha, \beta, k, \omega, p, q$. The system of equations are found as

$$\begin{aligned}
k a_1 + k^3 p \alpha a_1 + k^5 q \alpha^2 a_1 + \omega a_1 + k a_0^2 a_1 &= 0,\\
2 k a_0 a_1^2 + 2 k a_2 + 8 k^3 p \alpha a_2 + 32 k^5 q \alpha^2 a_2 + 2 \omega a_2 + 2 k a_0^2 a_2 &= 0,\\
6 k^3 p \beta a_1 + 60 k^5 q \alpha \beta a_1 + 6 k a_0 a_1 a_2 + k a_1^3 &= 0,\\
24 k^3 p \beta a_2 + 480 k^5 q \alpha \beta a_2 + 4 k a_1^2 a_2 + 4 k a_0 a_2^2 &= 0,\\
120 k^5 q \beta^2 a_1 + 5 k a_1 a_2^2 &= 0,\\
720 k^5 q \beta^2 a_2 + 2 k a_2^3 &= 0.
\end{aligned} \tag{20}$$

The solution of the system of algebraic equations can be found as

$$a_0 = \mp i p \frac{i}{\sqrt{10q}}, \quad a_1 = 0, \quad a_2 = \frac{6 a_0}{\alpha}, \quad \alpha = -\frac{p}{10 q k^2}, \quad \omega = -k - 4 k^3 p \alpha - 16 k^5 q \alpha^2 - k a_0^2. \tag{21}$$

Substituting from Eqs. (19) into (18), the following solutions of Eq. (17) can be obtained as

$$u_1(x,t) = \frac{ip}{\sqrt{10q}} - \frac{6ip}{\beta\sqrt{10q}} \sec h^2 \left(\frac{i\sqrt{p}}{k\sqrt{10q}} (kx + \omega t) \right), \tag{22}$$

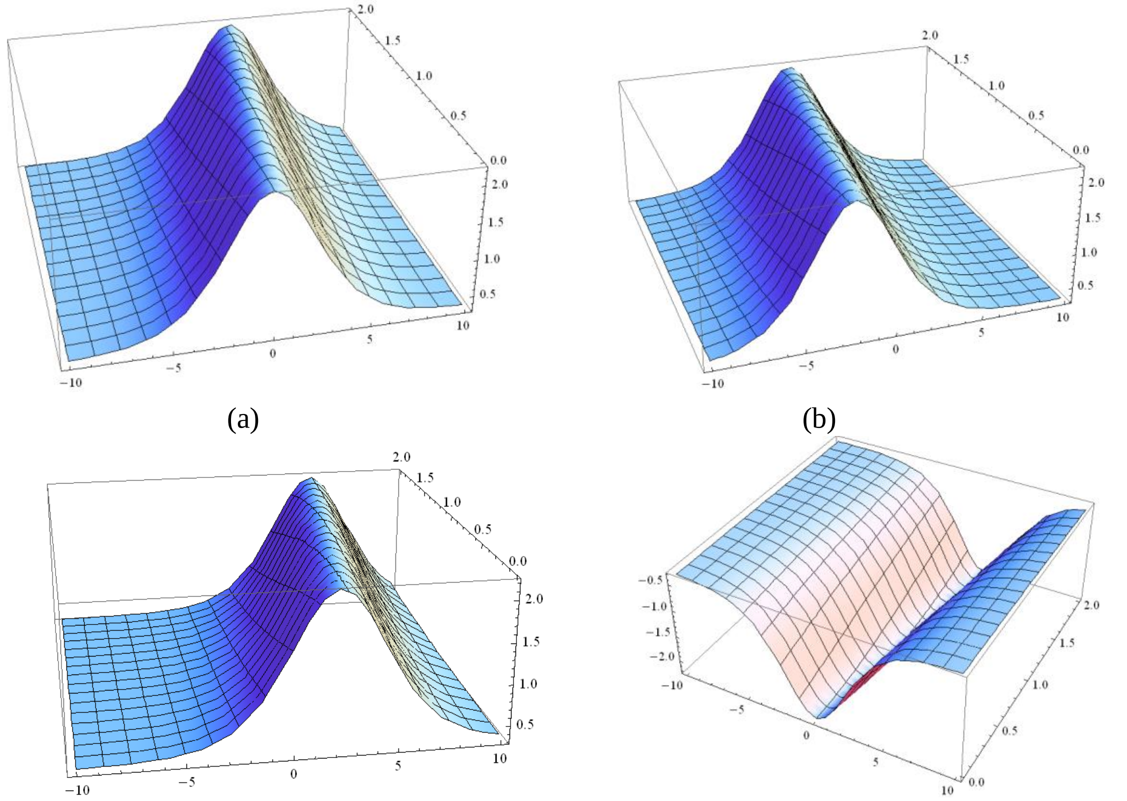
$$u_2(x,t) = \frac{ip}{\sqrt{10q}} + \frac{96ip}{\sqrt{10q}} \frac{\exp\left(\frac{2i\sqrt{p}}{k\sqrt{10q}} (kx + \omega t) \right)}{\left(1 - 4\beta \exp\left(\frac{2i\sqrt{p}}{k\sqrt{10q}} (kx + \omega t) \right) \right)^2}, \tag{23}$$

$$u_3(x,t)=\frac{\mathrm{i}p}{\sqrt{10q}}+\frac{96\mathrm{i}p}{\sqrt{10q}}\frac{\exp\left(\frac{2\mathrm{i}\sqrt{p}}{k\sqrt{10q}}(kx+\omega t)\right)}{\left(\exp\left(\frac{2\mathrm{i}\sqrt{p}}{k\sqrt{10q}}(kx+\omega t)\right)-4\beta\right)^2}, \tag{24}$$

$$u_4(x,t)=-\frac{\mathrm{i}p}{\sqrt{10q}}+\frac{6ip}{\beta\sqrt{10q}}\ \sec h^2\left(\frac{\mathrm{i}\sqrt{p}}{k\sqrt{10q}}(kx+\omega t)\right), \tag{25}$$

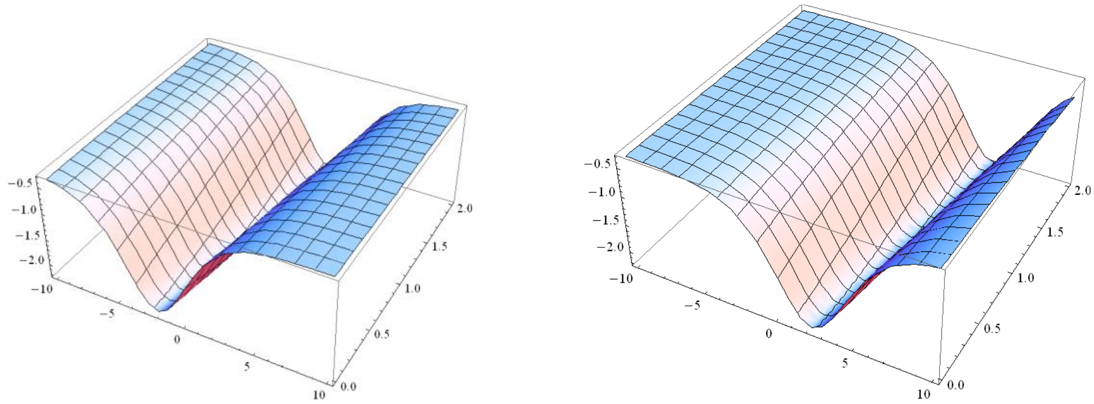
$$u_5(x,t)=-\frac{\mathrm{i}p}{\sqrt{10q}}-\frac{96\mathrm{i}p}{\sqrt{10q}}\frac{\exp\left(\frac{2\mathrm{i}\sqrt{p}}{k\sqrt{10q}}(kx+\omega t)\right)}{\left(1-4\beta\exp\left(\frac{2\mathrm{i}\sqrt{p}}{k\sqrt{10q}}(kx+\omega t)\right)\right)^2}, \tag{26}$$

$$u_6(x,t)=-\frac{\mathrm{i}p}{\sqrt{10q}}-\frac{96\mathrm{i}p}{\sqrt{10q}}\frac{\exp\left(\frac{2\mathrm{i}\sqrt{p}}{k\sqrt{10q}}(kx+\omega t)\right)}{\left(\exp\left(\frac{2\mathrm{i}\sqrt{p}}{k\sqrt{10q}}(kx+\omega t)\right)-4\beta\right)^2}. \tag{27}$$



(a) (b)

(c) (d)



(e) (f)

Fig 2. Traveling waves solutions (22)-(27) are plotted.

The traveling wave solutions (22)-(27) are shown in Fig. 2a-f with $p = k = 1,\ \beta = q = -1,\ \alpha = 0.1$ and $\omega = -1, 34$ in the interval [-10,10] and [0, 2]. According to the conditions of stability (4) and (5), the traveling wave solutions (22)-(27) are stable in the interval [-10,10] and [0, 2].

5. Conclusion

We have applied the extended direct algebraic method by using symbolic software (Mathematica) to construct a series of traveling wave solutions. Implementing the proposed method, we have demonstrated the traveling wave solutions of Kawachara and modified Kawachara equations. Also triangular, periodic, rational, Weierstrass and Jacobi doubly periodic solutions that could be obtained with applied method have been ignored. Obtained all these solutions are distinct and stable.

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