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(M,N)-Int-Soft Generalized Bi-Hyperideals of Ordered Semihypergroups

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Abstaract − Molodtsov introduced the concept of soft set as a new mathematical tool for dealing with uncertainties that is free from the difficulties that have troubled the usual theoretical approaches. In this paper, we apply the notion of soft sets to the ordered semihypergroups and introduce the notion of (M, N)-int-soft generalized bi-hyperideals of ordered semihypergroups. Moreover their related properties are investigated. We prove that every int-soft generalized bi-hyperideal is an (M, N)-int-soft generalized bi-hyperideals of S over U but the converse is not true which is shown with help of an example. We present new characterization of ordered semihypergroups in terms of (M, N)-int-soft generalized bi-hyperideals.

Keywords − *Ordered semihypergroup, int-soft hyperideal, int-soft generalized bi-hyperideal, (M, N)-int-soft hyperideal, (M, N)-int-soft generalized bi-hyperideal.*

1 Introduction

The real world is too complex for our immediate and direct understanding. We create models of reality that are simplifications of aspects of the real word. Unfortunately these mathematical models are too complicated and we cannot find the exact solutions. The uncertainty of data while modeling the problems in engineering, physics, computer sciences, economics, social sciences, medical sciences and many other diverse fields makes it unsuccessful to use the traditional classical methods, such as fuzzy set theory [21], intuitionistic set theory [22], and probability theory are useful approaches to describe uncertainty, but each of these theories has its inherent difficulties. To overcome these problems, Molodtsov [7], introduced the concept of

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soft set that is free from the difficulties that have troubled the usual theoretical approaches. Molodtsov pointed out several directions for the applications of soft sets. Maji et al. [23], gave the operations of soft sets and their properties; furthermore, in [24], they introduced fuzzy soft sets which combine the strengths of both soft sets and fuzzy sets. As a generalization of the soft set theory, the fuzzy soft set theory makes description of the objective world more realistic, practical, and precise in some cases, making it very promising. Since its introduction, the concept of soft sets has gained considerable attention in many directions and has found applications in a wide variety of fields such as the theory of soft sets [3, 4] and soft decision making [25, 26]. Since the notion of soft groups was proposed by Aktas and Cagman [1], then the soft set theory is used as a new tool to discuss algebraic structures Feng et al. soft semirings [2], Jun et al. [5] ordered semigroups. Soft sets were also applied to structure of hemirings [6, 8]. Song et al. [10], introduced the notions of int-soft semigroups and int-soft left (resp. right) ideals. Khan et al. [19], applied soft set theory to ordered semihypergroups and introduced the notions of (M, N)-int-soft hyperideals and (M, N)-int-soft interior hyperideals.

Algebraic hyperstructures represent a natural extension of classical algebraic structures and they were originally proposed in 1934 by a French mathematician Marty [9], at the 8^{th} Congress of Scandinavian Mathematicians. One of the main reason which attracts researches towards hyperstructures is its unique property that in hyperstructures composition of two elements is a set, while in classical algebraic structures the composition of two elements is an element. Thus algebraic hyperstructures are natural extension of classical algebraic structures. Since then, hyperstructures are widely investigated from the theoretical point of view and for their applications to many branches of pure and applied mathematics. Especially, semihypergroups are the simplest algebraic hyperstructures which possess the properties of closure and associativity. Nowadays many researchers have studied different aspects of semihypergroups (see [12, 13, 14, 15, 16, 17, 18]).

In this paper, we study the notion of (M, N)-int-soft generalized bi-hyperideals of ordered semihypergroups and give some related examples of this notion. We show that every int-soft generalized bi-hyperideals is an (M, N)-int-soft generalized bi-hyperideals but the converse is not true in general. We characterize ordered semihypergroups in terms of (M, N)-int-soft generalized bi-hyperideals.

2 Preliminaries

By an ordered semihypergroup we mean a structure $(S, \circ, \leq)$ in which the following conditions are satisfied:

(i) $(S, \circ)$ is a semihypergroup.

(ii) $(S, \leq)$ is a poset.

(iii) $(\forall a, b, x \in S)$ $a \leq b$ implies $x \circ a \leq x \circ b$ and $a \circ x \leq b \circ x$.

For $A \subseteq S$,we denote $(A] := \{t \in S : t \leq h \text{ for some } h \in A\}$.

For $A, B \subseteq S$, we have $A \circ B := \bigcup\{a \circ b : a \in A,\, b \in B\}$.

A nonempty subset A of an ordered semihypergroup S is called a subsemihypergroup of S if $A \circ A \subseteq A$.

A nonempty subset A of S is called a left (resp. right) hyperideal of S if it satisfies the following conditions:

(i) $S \circ A \subseteq A$ (resp. $A \circ S \subseteq A$).

(ii) If $a \in A, b \in S$ and $b \leq a$, implying $b \in A$.

By a two sided hyperideal or simply a hyperideal of S we mean a nonempty subset of S which is both a left hyperideal and a right hyperideal of S.

A nonempty B of S is called a generalized bi-hyperideal of S if it satisfies the following conditions:

(i) $B \circ S \circ B \subseteq B$.

(ii) If $a \in B$, $b \in S$ and $b \leq a$, implying $b \in B$.

For $x \in S$, we define $A_x = \{(y, z) \in S \times S \mid x \leq y \circ z\}$.

3 Soft Sets

In what follows, we take $E = S$ as the set of parameters, which is an ordered semihypergroup, unless otherwise specified.

From now on, U is an initial universe set, E is a set of parameters, $P(U)$ is the power set of U and $A, B, C... \subseteq E$.

Definition 3.1. (see [7, 20]). A *soft set* f_A over U is defined as

$$f_A : E \longrightarrow P(U) \text{ such that } f_A(x) = \emptyset \text{ if } x \notin A.$$

Hence f_A is also called an *approximation function.*

A soft set f_A over U can be represented by the set of ordered pairs

$$f_A = \{(x, f_A(x))|x \in E, f_A(x) \in P(U)\}.$$

It is clear that a soft set is a *parameterized family* of subsets of U. Note that the set of all soft sets over U will be denoted by $S(U)$.

Definition 3.2. (see [20]). Let $f_A, f_B \in S(U)$. Then f_A is called a *soft subset* of f_B, denoted by $f_A \widetilde{\subseteq} f_B$ if $f_A(x) \subseteq f_B(x)$ for all $x \in E$.

Definition 3.3. (see [20]). Two soft sets f_A and f_B are said to be equal soft sets if $f_A \widetilde{\subseteq} f_B$ and $f_B \widetilde{\subseteq} f_A$ and is denoted by $f_A \widetilde{=} f_B$.

Definition 3.4. (see [20]). Let $f_A, f_B \in S(U)$. Then the *soft union* of f_A and f_B, denoted by $f_A \widetilde{\cup} f_B = f_{A \cup B}$, is defined by $\left(f_A \widetilde{\cup} f_B\right)(x) = f_A(x) \cup f_B(x)$ for all $x \in E$.

Definition 3.5. (see [20]). Let $f_A, f_B \in S(U)$. Then the *soft intersection* of f_A and f_B, denoted by $f_A \widetilde{\cap} f_B = f_{A \cap B}$, is defined by $\left(f_A \widetilde{\cap} f_B\right)(x) = f_A(x) \cap f_B(x)$ for all $x \in E$.

Definition 3.6. (see [11]). Let f_A and g_B be two soft sets of an ordered semihypergroup S over U. Then, the intersectional soft product, denoted by $f_A \widetilde{\odot} g_B$, is defined by $f_A \widetilde{\odot} g_B : S \longrightarrow P(U), x \longmapsto \left(f_A \widetilde{\odot} g_B\right)(x) = \begin{cases} \bigcup\limits_{(y,z) \in A_x} \{f_A(y) \cap g_B(z)\}, & \text{if } A_x \neq \emptyset, \\ \emptyset, & \text{if } A_x = \emptyset, \end{cases}$

for all $x \in S$.

Definition 3.7. (see [11]). For a nonempty subset A of S the characteristic soft set is defined to be the soft set $\mathcal{S}_{\mathcal{A}}$ of A over U in which $\mathcal{S}_{\mathcal{A}}$ is given by

$$\mathcal{S}_{\mathcal{A}} : S \longmapsto P(U). \quad x \longmapsto \begin{cases} U, & \text{if} \quad x \in A \\ \emptyset, & \text{otherwise} \end{cases}$$

For an ordered semihypergroup S, the soft set $\mathcal{S}_{\mathcal{S}}$ of S over U is defined as follows:

$$\mathcal{S}_{\mathcal{S}} : S \longrightarrow P(U), x \longmapsto \mathcal{S}_{\mathcal{S}}(x) = U \text{ for all } x \in S.$$

The soft set $\mathcal{S}_{\mathcal{S}}$ of an ordered semihypergroup S over U is called the whole soft set of S over U.

Definition 3.8. (see [11]). Let f_A be a soft set of an ordered semihypergroup S over U a subset δ such that $\delta \in P(U)$. The δ-inclusive set of f_A is denoted by $i_A(f_A, \delta)$ and defined to be the set

$$i_A(f_A, \delta) = \{x \in S \mid \delta \subseteq f_A(x)\}.$$

Definition 3.9. (see [11]). A soft set f_A of an ordered semihypergroup S over U is called *an int-soft subsemihypergroup of* S over U if:

$(\forall x, y \in S) \bigcap\limits_{\alpha \in x \circ y} f_A(\alpha) \supseteq f_A(x) \cap f_A(y)$.

Definition 3.10. (see [11]). Let f_A be a soft set of an ordered semihypergroup S over U. Then f_A is called an int-soft left (resp. right) hyperideal of S over U if it satisfies the following conditions:

(1) $(\forall x, y \in S) \bigcap\limits_{\alpha \in x \circ y} f_A(\alpha) \supseteq f_A(y)$ (resp. $\bigcap\limits_{\alpha \in x \circ y} f_A(\alpha) \supseteq f_A(x)$).

(2) $(\forall x, y \in S)\ x \leq y \Longrightarrow f_A(x) \supseteq f_A(y)$.

A soft set f_A of an ordered semihypergroup S over U is called an *int-soft hyperideal (*or int-soft two-sided hyperideal) of S over U if it is both an int-soft left hyperideal and an int-soft right hyperideal of S over U.

Definition 3.11. (see [17]). A soft set f_A of an ordered semihypergroup S over U is called an int-soft generalized bi-*hyperideal* of S over U if it satisfies the following conditions:

(1) $(\forall x, y, z \in S) \bigcap\limits_{\alpha \in x \circ y \circ z} f_A(\alpha) \supseteq f_A(x) \cap f_A(z)$.

(2) $(\forall x, y \in S)\ x \leq y \Longrightarrow f_A(x) \supseteq f_A(y)$.

4 *(M,N)*-Int-Soft Generalized Bi-Hyperideals

In this section, we introduce the notion of (M, N)-int-soft generalized bi-hyperideals of ordered semihypergroups and investigate some related properties. From now on, $\emptyset \subseteq M \subset N \subseteq U$.

Definition 4.1. (see [19]). A soft set f_A of an ordered semihypergroup S over U is called an (M, N)*-int-soft subsemihypergroup of* S over U if:

$(\forall x, y \in S)\ (\bigcap\limits_{\alpha \in x \circ y} f_A(\alpha)) \cup M \supseteq f_A(x) \cap f_A(y) \cap N$.

Definition 4.2. (see [19]). A soft set f_A of an ordered semihypergroup S over U is called an (M,N)-int-soft left (resp. right) hyperideal of S over U if it satisfies the following conditions:

(1) $(\forall x,y\in S)\ (\bigcap\limits_{\alpha\in x\circ y} f_A(\alpha))\cup M\supseteq f_A(y)\cap N$

(resp. $(\bigcap\limits_{\alpha\in x\circ y} f_A(\alpha))\cup M\supseteq f_A(x)\cap N$).

(2) $(\forall x,y\in S)\ x\le y\Longrightarrow f_A(x)\cup M\supseteq f_A(y)\cap N$.

A soft set f_A of an ordered semihypergroup S over U is called an (M,N)-*int-soft hyperideal* of S over U, if it is both an (M,N)-int-soft left hyperideal and an (M,N)-int-soft right hyperideal of S over U.

Definition 4.3. A soft set f_A of an ordered semihypergroup S over U is called an (M,N)-*int-soft generalized bi-hyperideal of* S over U if it satisfies the following conditions:

(1) $(\forall x,y,z\in S)\ (\bigcap\limits_{\alpha\in x\circ y\circ z} f_A(\alpha))\cup M\supseteq f_A(x)\cap f_A(z)\cap N$.

(2) $(\forall x,y\in S)\ x\le y\Longrightarrow f_A(x)\cup M\supseteq f_A(y)\cap N$.

Example 4.4. Let $(S,\circ,\le)$ be an ordered semihypergroup where the hyperoperation and the order relation are defined by:

$\circ$	a	b	c	d
a	$\{a\}$	$\{a\}$	$\{a\}$	$\{a\}$
b	$\{a\}$	$\{a\}$	$\{a\}$	$\{a\}$
c	$\{a\}$	$\{a\}$	$\{a,b\}$	$\{a\}$
d	$\{a\}$	$\{a\}$	$\{a,b\}$	$\{a,b\}$

$$\le:=\{(a,a),(b,b),(c,c),(d,d),(a,b)\}.$$

Suppose $U=\{p,q,r,s\}$, $A=\{a,c,d\}$, $M=\{p,q\}$ and $N=\{p,q,s\}$. Let us define $f_A(a)=\{p,q,r,s\}$, $f_A(b)=\emptyset$, $f_A(c)=\{q,r,s\}$ and $f_A(d)=\{p,s\}$. Then f_A is an (M,N)-int-soft generalized bi-hyperideal of S over U.

Remark 4.5. Every int-soft generalized bi-hyperideal is an (M,N)-int-soft generalized bi-hyperideal of S over U. But the converse is not true. We can illustrate it by the following example.

Example 4.6. Let $(S,\circ,\le)$ be an ordered semihypergroup where the hyperoperation and the order relation are defined by:

$\circ$	e_1	e_2	e_3	e_4	e_5
e_1	$\{e_1\}$	$\{e_1,e_2,e_4\}$	$\{e_1\}$	$\{e_1,e_2,e_4\}$	$\{e_1,e_2,e_4\}$
e_2	$\{e_1\}$	$\{e_2\}$	$\{e_1\}$	$\{e_1,e_2,e_4\}$	$\{e_1,e_2,e_4\}$
e_3	$\{e_1\}$	$\{e_1,e_2,e_4\}$	$\{e_1,e_3\}$	$\{e_1,e_2,e_4\}$	$\{e_1,e_2,e_3,e_4,e_5\}$
e_4	$\{e_1\}$	$\{e_1,e_2,e_4\}$	$\{e_1\}$	$\{e_1,e_2,e_4\}$	$\{e_1,e_2,e_4\}$
e_5	$\{e_1\}$	$\{e_1,e_2,e_4\}$	$\{e_1,e_3\}$	$\{e_1,e_2,e_4\}$	$\{e_1,e_2,e_3,e_4,e_5\}$

$\le:=\{(e_1,e_1),(e_2,e_2),(e_3,e_3),(e_4,e_4),(e_5,e_5),(e_1,e_3),(e_1,e_4),(e_1,e_5),(e_2,e_4),(e_2,e_5),$ $(e_3,e_5),(e_4,e_5)\}$.

Suppose $U=\{1,2,3\}$, $A=\{e_1,e_2,e_4\}$, $M=\{2\}$ and $N=\{2,3\}$. Let us define $f_A(e_1)=\{1,2,3\}$, $f_A(e_2)=\{1,2\}$, $f_A(e_3)=\emptyset$, $f_A(e_4)=\{2\}$ and $f_A(e_5)=\emptyset$. Then f_A is an (M,N)-int-soft generalized bi-hyperideal of S over U. This is not int-soft generalized bi-hyperideal of S over U, as $\bigcap\limits_{\alpha\in e_1\circ e_1\circ e_2=\{e_1,e_2,e_4\}} f_A(\alpha)=f_A(e_1)\cap f_A(e_2)\cap f_A(e_4)=\{2\}\not\supseteq\{1,2\}=f_A(e_1)\cap f_A(e_2)$.

Theorem 4.7. A non-empty subset A of an ordered semihypergroup $(S,\circ,\leq)$ is a generalized bi-hyperideal of S if and only if the soft set f_A is defined by

$$f_A(x)=\begin{cases} N \text{ if } x\in A\\ M \text{ if } x\notin A\end{cases}$$

is an (M,N)-int-soft generalized bi-hyperideal of S over U.

Proof. Suppose A is a generalized bi-hyperideal of S. If there exist $x,y\in S$ such that $x\leq y$. If $y\in A$, then $x\in A$. Hence $f_A(x)=N$. Therefore $f_A(x)\cup M\supseteq N=f_A(y)\cap N$. If $y\notin A$, then $f_A(y)\cap N=M$. Thus $f_A(x)\cup M\supseteq M=f_A(y)\cap N$. Let $x,y,z\in S$, such that $x,z\in A$. Then $f_A(x)=N$ and $f_A(z)=N$. Hence for any $\alpha\in x\circ y\circ z$, $(\bigcap\limits_{\alpha x\circ y\circ z} f_A(\alpha))\cup M\supseteq N=f_A(x)\cap f_A(z)\cap N$. If $x\notin A$ or $z\notin A$ then $f_A(x)\cap f_A(z)\cap N=M$. Thus $(\bigcap\limits_{\alpha x\circ y\circ z} f_A(\alpha))\cup M\supseteq M=f_A(x)\cap f_A(z)\cap N$. Hence $(\bigcap\limits_{\alpha x\circ y\circ z} f_A(\alpha))\cup M\supseteq f_A(x)\cap f_A(z)\cap N$. Consequently, f_A is an (M,N)-int-soft generalized bi-hyperideal of S over U.

Theorem 4.8. If $\{f_{A_i}\mid i\in I\}$ is a family of (M,N)-int-soft generalized bi-hyperideal of an ordered semihypergroup S over U. Then $f_A=\bigcap\limits_{i\in I} f_{A_i}$ is an (M,N)-int-soft generalized bi-hyperideal of S over U.

Proof. Let $\{f_{A_i}\mid i\in I\}$ be a family of (M,N)-int-soft generalized bi-hyperideal of S over U. Let $x,y,z\in S$ and $(\bigcap\limits_{\beta\in x\circ y\circ z} f_{A_i}(\beta))\cup M\supseteq f_{A_i}(x)\cap f_{A_i}(z)\cap N$. Since each f_{A_i} $(i\in I)$ is an (M,N)-int-soft generalized bi-hyperideal of S over U. Thus for any $\beta\in x\circ y\circ z$, $f_{A_i}(\beta)\cup M\supseteq f_{A_i}(x)\cap f_{A_i}(z)\cap N$. Then $f_A(\beta)\cup M=\left(\bigcap\limits_{i\in I} f_{A_i}\right)(\beta)\cup M=(\bigcap\limits_{i\in I} f_{A_i}(\beta))\cup M\supseteq\bigcap\limits_{i\in I}(f_{A_i}(x)\cap f_{A_i}(z)\cap N)=\left(\bigcap\limits_{i\in I} f_{A_i}\right)(x)\cap\left(\bigcap\limits_{i\in I} f_{A_i}\right)(z)\cap N=f_A(x)\cap f_A(z)\cap N$. Thus $(\bigcap\limits_{\beta\in x\circ y\circ z} f_A(\beta))\cup M\supseteq f_A(x)\cap f_A(y)\cap N$. Furthermore, if $x\leq y$, then $f_A(x)\cup M\supseteq f_A(y)\cap N$. Indeed: Since every f_{A_i} $(i\in I)$ is an (M,N)-int-soft generalized bi-hyperideal of S over U, it can be obtained that $f_{A_i}(x)\cup M\supseteq f_{A_i}(y)\cap N$ for all $i\in I$. Thus $f_A(x)\cup M=\left(\bigcap\limits_{i\in I} f_{A_i}\right)(x)\cup M=(\bigcap\limits_{i\in I}(f_{A_i}(x)))\cup M\supseteq(\bigcap\limits_{i\in I}(f_{A_i}(y)))\cap N=\left(\bigcap\limits_{i\in I} f_{A_i}\right)(y)\cap N=f_A(y)\cap N$. Thus f_A is is an (M,N)-int-soft generalized bi-hyperideal of S over U.

Theorem 4.9. Let $(S, \circ, \leq)$ be an ordered semihypergroup and A be a nonempty subset of S. Then A is a generalized bi-hyperideal of S if and only if the characteristic function $\mathcal{S}_{\mathcal{A}}$ of A is an (M, N)-int-soft generalized bi-hyperideal of S over U.

Proof. Suppose that A is a generalized bi-hyperideal of S. Let x, y and z be any elements of S. Then $(\bigcap_{\alpha \in x \circ y \circ z} \mathcal{S}_{\mathcal{A}}(\alpha)) \cup M \supseteq \mathcal{S}_{\mathcal{A}}(x) \cap \mathcal{S}_{\mathcal{A}}(z) \cap N$. Indeed, If $x, z \in A$, then $\mathcal{S}_{\mathcal{A}}(x) = U$ and $\mathcal{S}_{\mathcal{A}}(z) = U$. Since A is a generalized bi-hyperideal of S, we have $\alpha \in x \circ y \circ z \subseteq A \circ S \circ A \subseteq A$ we have $\mathcal{S}_{\mathcal{A}}(\alpha) = U$ and $\emptyset \subseteq M \subset N \subseteq U$. Thus $(\bigcap_{\alpha \in x \circ y \circ z} \mathcal{S}_{\mathcal{A}}(\alpha)) \cup M = U \supseteq \mathcal{S}_{\mathcal{A}}(x) \cap \mathcal{S}_{\mathcal{A}}(z) \cap N$. If $x \notin A$ or $z \notin A$ then $\mathcal{S}_{\mathcal{A}}(x) = \emptyset$ or $\mathcal{S}_{\mathcal{A}}(z) = \emptyset$. Since $\mathcal{S}_{\mathcal{A}}(p) \supseteq \emptyset$ for all $p \in S$. Thus $(\bigcap_{\alpha \in x \circ y \circ z} \mathcal{S}_{\mathcal{A}}(\alpha)) \cup M \supseteq \emptyset = \mathcal{S}_{\mathcal{A}}(x) \cap \mathcal{S}_{\mathcal{A}}(z) \cap N$. Let $x, y \in S$ with $x \leq y$. Then $\mathcal{S}_{\mathcal{A}}(x) \cup M \supseteq \mathcal{S}_{\mathcal{A}}(y) \cap N$. Indeed, if $y \notin A$ then $\mathcal{S}_{\mathcal{A}}(y) = \emptyset$ and $\emptyset \subseteq M \subset N \subseteq U$ so $\mathcal{S}_{\mathcal{A}}(x) \cup M \supseteq \emptyset = \mathcal{S}_{\mathcal{A}}(y) \cap N$. If $y \in A$ then $\mathcal{S}_{\mathcal{A}}(y) = U$. Since $x \leq y$ and A is a generalized bi-hyperideal of S, we have $x \in A$ and thus $\mathcal{S}_{\mathcal{A}}(x) \cup M = U \supseteq \mathcal{S}_{\mathcal{A}}(y) \cap N$.

Conversely, let $\emptyset \neq A \subseteq S$ such that $\mathcal{S}_{\mathcal{A}}$ is an (M, N)-int-soft generalized hyperideal of S over U. Let $\alpha \in A \circ S \circ A$, then there exist $x, z \in A$ and $y \in S$ such that $\alpha \in x \circ y \circ z$. Since $(\bigcap_{\alpha \in x \circ y \circ z} \mathcal{S}_{\mathcal{A}}(\alpha)) \cup M \supseteq \mathcal{S}_{\mathcal{A}}(x) \cap \mathcal{S}_{\mathcal{A}}(z) \cap N$, and $x, z \in A$ we have $\mathcal{S}_{\mathcal{A}}(x) = U$ and $\mathcal{S}_{\mathcal{A}}(z) = U$. Hence for each $\alpha \in A \circ S \circ A$, we have $(\bigcap_{\alpha \in x \circ y \circ z} \mathcal{S}_{\mathcal{A}}(\alpha)) \cup M \supseteq U \cap U \cap N = N$. Thus by $\emptyset \subseteq M \subset N \subseteq U$, $\bigcap_{\alpha \in x \circ y \circ z} \mathcal{S}_{\mathcal{A}}(\alpha) \supseteq N \supset \emptyset$. On the other hand $\mathcal{S}_{\mathcal{A}}(x) \subseteq U$ for all $x \in S$. Thus for any $\alpha \in x \circ y \circ z$, $\mathcal{S}_{\mathcal{A}}(\alpha) = U$ implies that $\alpha \in A$. Thus $A \circ S \circ A \subseteq A$. Furthermore, let $x \in A$, $S \ni y \leq x$. Then $y \in A$. Indeed, it is enough to prove that $\mathcal{S}_{A}(y) = U$. By $x \in A$ we have $\mathcal{S}_{A}(x) = U$. Since $\mathcal{S}_{A}$ is an (M, N)-int-soft generalized-hyperideal of S over U and $y \leq x$, we have $\mathcal{S}_{A}(y) \cup M \supseteq \mathcal{S}_{A}(x) \cap N = U \cap N = N$. Notice that $\emptyset \subseteq M \subset N \subseteq U$, we conclude that $\mathcal{S}_{A}(y) \supseteq \emptyset$. Thus $\mathcal{S}_{A}(y) = U$. Therefore A is a generalized bi-hyperideal of S.

Theorem 4.10. Let f_A be a soft set of an ordered semihypergroup S over U and $\delta \in P(U)$. Then f_A is an (M, N)-int-soft generalized bi-hyperideal of S over U if and only if each nonempty δ-inclusive set $i_A(f_A, \delta)$ of f_A is a generalized bi-hyperideal of S where $M \subset \delta \subseteq N$.

Proof. Assume that f_A is an (M, N)-int-soft generalized bi-hyperideal of S over U, and $i_A(f_A, \delta) \neq \emptyset$. Let $x, y, z \in S$ and $x, z \in i_A(f_A, \delta)$ where $M \subset \delta \subseteq N$. Then $f_A(x) \supseteq \delta$ and $f_A(z) \supseteq \delta$. Since f_A is an (M, N)-int-soft generalized bi-hyperideal of S over U. Thus $(\bigcap_{w \in x \circ y \circ z} f_A(w)) \cup M \supseteq f_A(x) \cap f_A(z) \cap N \supseteq \delta \cap \delta \cap N = \delta$. Since $\emptyset \subseteq M \subset \delta \subseteq N \subseteq U$, we can write as $\bigcap_{w \in x \circ y \circ z} f_A(w) \supseteq \delta$. Hence $f_A(w) \supseteq \delta$ for any $w \in x \circ y \circ z$ implies that $w \in i_A(f_A, \delta)$. Thus $i_A(f_A, \delta) \circ S \circ i_A(f_A, \delta) \subseteq i_A(f_A, \delta)$. Furthermore, let $x \in i_A(f_A, \delta)$, $S \ni y \leq x$. Then $y \in i_A(f_A, \delta)$. Indeed, since $x \in i_A(f_A, \delta)$, $f_A(x) \supseteq \delta$ and f_A is an (M, N)-int-soft generalized bi-hyperideal of S over

U, we have $f_A(y) \cup M \supseteq f_A(x) \cap N \supseteq \delta \cap N = \delta$. By $M \subset \delta$, we have $f_A(y) \supseteq \delta$, i.e., $y \in e_A(f_A, \delta)$. Therefore $i_A(f_A, \delta)$ is a generalized bi-hyperideal of S.

Conversely, suppose that $i_A(f_A, \delta) \neq \emptyset$ is a generalized bi-hyperideal of S for all $M \subset \delta \subseteq N$. Now let $x, y, z \in S$. We will prove that $(\bigcap_{\alpha \in x \circ y \circ z} f_A(\alpha)) \cup M \supseteq f_A(x) \cap f_A(z) \cap N$ for all $x, y, z \in S$. If there exist x_1, y_1, z_1 such that $(\bigcap_{\alpha \in x_1 \circ y_1 \circ z_1} f_A(\alpha)) \cup M \subset f_A(x_1) \cap f_A(z_1) \cap N$, and $M \subset \delta \subseteq N$ such that $(\bigcap_{\alpha \in x_1 \circ y_1 \circ z_1} f_A(\alpha)) \cup M \subset \delta \subseteq f_A(x_1) \cap f_A(z_1) \cap N$, so $f_A(x_1) \supseteq \delta$, $f_A(z_1) \supseteq \delta$ and $\bigcap_{\alpha \in x_1 \circ y_1 \circ z_1} f_A(\alpha) \subset \delta$ then $x_1, z_1 \in i_A(f_A, \delta)$ and $x_1 \circ y_1 \circ z_1 \nsubseteq i_A(f_A, \delta)$. This is a contradiction that $i_A(f_A, \delta)$ is a generalized bi-hyperideal of S. Moreover if $x \leq y$ then $f_A(x) \cup M \supseteq f_A(y) \cap N$. Indeed, if there exist $x_1, y_1 \in S$ such that $x_1 \leq y_1$ and $f_A(x_1) \cup M \subset f_A(y_1) \cap N$, $M \subset \delta \subseteq N$ such that $f_A(x_1) \cup M \subset \delta \subseteq f_A(y_1) \cap N$ and we have $f_A(y_1) \supseteq \delta$ and $f_A(x_1) \subset \delta$. Then $y_1 \in i_A(f_A, \delta)$ and $x_1 \notin i_A(f_A, \delta)$. This is a contradiction that $i_A(f_A, \delta)$ is a generalized bi-hyperideal of S. Thus if $x \leq y$ then $f_A(x) \cup M \supseteq f_A(y) \cap N$.

Theorem 4.11. Every (M, N)-int-soft right (resp. left) hyperideal of S over U is an (M, N)-int-soft generalized bi-hyperideal of S over U.

Proof. Let f_A is an (M, N)-int-soft right hyperideal of S over U. Let $x, y, z \in S$. Then $(\bigcap_{\alpha \in x \circ y \circ z} f_A(\alpha)) \cup M = (\bigcap_{\substack{\alpha \in x \circ \beta \\ \beta \in y \circ z}} f_A(\alpha)) \cup M \supseteq f_A(x) \cap N \supseteq f_A(x) \cap f_A(z) \cap N$. Thus f_A is an (M, N)-int-soft generalized bi-hyperideal of S over U.

Let g_B is an (M, N)-int-soft left hyperideal of S over U. Then $(\bigcap_{\alpha \in x \circ y \circ z} g_B(\alpha)) \cup M = (\bigcap_{\substack{\alpha \in \gamma \circ z \\ \gamma \in x \circ y}} g_B(\alpha)) \cup M \supseteq g_B(z) \cap N \supseteq g_B(x) \cap g_B(z) \cap N$. Thus g_B is an (M, N)-int-soft generalized bi-hyperideal of S over U.

Definition 4.12. Let $(S, \circ, \leq)$ be an ordered semihypergroup. Let f_A be a soft set of S over U. We define the the soft set f_A^* of S as follows:

$$f_A^*(x) = f_A(x) \cap N \cup M$$

for all $x \in S$.

Definition 4.13. Let $(S, \circ, \leq)$ be an ordered semihypergroup. Let f_A and g_B be soft set of S over U. We define $f_A \widetilde{\cap^*} g_B$, $f_A \widetilde{\cup^*} g_B$ and $f_A \widetilde{\odot^*} g_B$ of S as follows:

$$\left(f_A \widetilde{\cap^*} g_B\right)(x) = \left(\left(f_A \widetilde{\cap} g_B\right)(x) \cap N\right) \cup M$$
$$\left(f_A \widetilde{\cup^*} g_B\right)(x) = \left(\left(f_A \widetilde{\cup} g_B\right)(x) \cap N\right) \cup M$$
$$\left(f_A \widetilde{\odot^*} g_B\right)(x) = \left(\left(f_A \widetilde{\odot} g_B\right)(x) \cap N\right) \cup M$$

for all $x \in S$.

Lemma 4.14. Let f_A and g_B be soft sets of an ordered semihypergroup S over U. Then the following conditions hold:
(1) $f_A \widetilde{\cap^*} g_B = f_A^* \widetilde{\cap} g_B^*$.
(2) $f_A \widetilde{\cup^*} g_B = f_A^* \widetilde{\cup} g_B^*$.
(3) $f_A \widetilde{\odot^*} g_B \widetilde{\supseteq} f_A^* \widetilde{\odot} g_B^*$.

Proof. (1) Let $x \in S$. Then

$$\begin{aligned}\left(f_A \widetilde{\cap^*} g_B\right)(x) &= \left(\left(f_A \widetilde{\cap} g_B\right)(x) \cap N\right) \cup M \\ &= \left(\left(f_A(x) \widetilde{\cap} g_B(x)\right) \cap N\right) \cup M \\ &= \left(\left(f_A(x) \cap N\right) \widetilde{\cap} \left(g_B(x) \cap N\right)\right) \cup M \\ &= \left(\left(\left(f_A(x) \cap N\right)\right) \cup M\right) \widetilde{\cap} \left(\left(\left(g_B(x) \cap N\right)\right) \cup M\right) \\ &= f_A^* \widetilde{\cap} g_B^*.\end{aligned}$$

(2) Proof is similar to the proof of (1).
(3) If $A_x = \emptyset$. Then $\left(f_A \widetilde{\odot} g_B\right)(x) = \emptyset$. Thus

$$\begin{aligned}\left(f_A \widetilde{\odot^*} g_B\right)(x) &= \left(\left(f_A \widetilde{\odot} g_B\right)(x) \cap N\right) \cup M \\ &= (\emptyset \cap N) \cup M \\ &= M = N \cap M \\ \left(f_A \widetilde{\odot^*} g_B\right)(x) &\supseteq M = f_A^* \widetilde{\odot} g_B^*.\end{aligned}$$

If $A_x \neq \emptyset$. So there exist $y, z \in S$ such that $x \leq y \circ z$. Then $(y, z) \in A_x$. Thus

$$\begin{aligned}\left(f_A \widetilde{\odot^*} g_B\right)(x) &= \left(\left(f_A \widetilde{\odot} g_B\right)(x) \cap N\right) \cup M \\ &= \left(\left(\bigcup_{(y,z)\in A_x} \{f_A(y) \cap g_B(z)\}\right) \cap N\right) \cup M \\ &= \left(\bigcup_{(y,z)\in A_x} \{(f_A(y) \cap N) \cap (g_B(z) \cap N)\}\right) \cup M \\ &= \bigcup_{(y,z)\in A_x} \{((f_A(y) \cap N) \cup M) \cap (g_B(z) \cap N) \cup M\} \\ &= \bigcup_{(y,z)\in A_x} \{f_A^*(y) \cap g_B^*(z)\} \\ &= \left(f_A^* \odot g_B^*\right)(x).\end{aligned}$$

Thus $f_A \widetilde{\odot^*} g_B \widetilde{\supseteq} f_A^* \widetilde{\odot} g_B^*$.

Definition 4.15. If $\mathcal{S}_{\mathcal{A}}$ is the characteristic soft function of A. Then $\mathcal{S}_{\mathcal{A}}^*$ is defined over U in which $\mathcal{S}_{\mathcal{A}}^*$ is given by

$$\mathcal{S}_{\mathcal{A}}^*(x) = \begin{cases} N \text{ if } x \in A \\ M \text{ if } x \notin A \end{cases}$$

Lemma 4.16. Let A and B be the nonempty subsets of an ordered semihypergroup S. Then the following holds:
(1) $\mathcal{S}_{\mathcal{A}} \widetilde{\cap^*} \mathcal{S}_{\mathcal{B}} = \mathcal{S}^*_{\mathcal{A}\cap\mathcal{B}}$.
(2) $\mathcal{S}_{\mathcal{A}} \widetilde{\cup^*} \mathcal{S}_{\mathcal{B}} = \mathcal{S}^*_{\mathcal{A}\cup\mathcal{B}}$.
(3) $\mathcal{S}_{\mathcal{A}} \widetilde{\odot^*} \mathcal{S}_{\mathcal{B}} = \mathcal{S}^*_{(\mathcal{A}\circ\mathcal{B}]}$.

Proof. (1) and (2) are obvious.

(3) Let $x \in (A \circ B]$. Then $\mathcal{S}_{(A\circ B]}(x) = U$. Hence $\left(\mathcal{S}_{(A\circ B]} \cap N\right) \cup M = (U \cap N) \cup M = N \cup M = N$. Thus $\mathcal{S}^*_{(\mathcal{A}\circ\mathcal{B}]}(x) = N$. Since $x \in (A \circ B]$, we have $x \leq a \circ b$ for some $a \in A$ and $b \in B$. Then $(a, b) \in A_x$ and $A_x \neq \emptyset$. Thus

$$\begin{aligned}
\left(\mathcal{S}_{\mathcal{A}} \widetilde{\odot^*} \mathcal{S}_{\mathcal{B}}\right)(x) &= \left(\left(\mathcal{S}_{\mathcal{A}} \widetilde{\odot} \mathcal{S}_{\mathcal{B}}\right)(x) \cap N\right) \cup M \\
&= \left[\left\{\bigcup_{(y,z)\in A_x} \left(\mathcal{S}_{\mathcal{A}}(y) \cap \mathcal{S}_{\mathcal{B}}(z)\right)\right\} \cap N\right] \cup M \\
&\supseteq \left[\left\{\mathcal{S}_{\mathcal{A}}(a) \cap \mathcal{S}_{\mathcal{B}}(b)\right\} \cap N\right] \cup M.
\end{aligned}$$

Since $a \in A$ and $b \in B$, we have $\mathcal{S}_{\mathcal{A}}(a) = U$ and $\mathcal{S}_{\mathcal{B}}(b) = U$ and so

$$\begin{aligned}
\left(\mathcal{S}_{\mathcal{A}} \widetilde{\odot^*} \mathcal{S}_{\mathcal{B}}\right)(x) &\supseteq \left[\left\{\mathcal{S}_{\mathcal{A}}(a) \cap \mathcal{S}_{\mathcal{B}}(b)\right\} \cap N\right] \cup M \\
&= \left[\{U \cap U\} \cap N\right] \cup M \\
&= N \cup M = N.
\end{aligned}$$

Thus,

$$\left(\mathcal{S}_{\mathcal{A}} \widetilde{\odot^*} \mathcal{S}_{\mathcal{B}}\right)(x) = \mathcal{S}^*_{(\mathcal{A}\circ\mathcal{B}]}(x).$$

Let $x \notin (A \circ B]$, then $\mathcal{S}_{(\mathcal{A}\circ\mathcal{B}]}(x) = \emptyset$ and hence,

$$\left\{\mathcal{S}_{(\mathcal{A}\circ\mathcal{B}]}(x) \cap N\right\} \cup M = \{\emptyset \cap N\} \cup M = M.$$

So $\mathcal{S}^*_{(\mathcal{A}\circ\mathcal{B}]}(x) = M$. Let $(y, z) \in A_x$. Then

$$\begin{aligned}
\left(\mathcal{S}_{\mathcal{A}} \widetilde{\odot^*} \mathcal{S}_{\mathcal{B}}\right)(x) &= \left(\left(\mathcal{S}_{\mathcal{A}} \widetilde{\odot} \mathcal{S}_{\mathcal{B}}\right)(x) \cap N\right) \cup M \\
&= \left[\left\{\bigcup_{(y,z)\in A_x} \left(\mathcal{S}_{\mathcal{A}}(y) \cap \mathcal{S}_{\mathcal{B}}(z)\right)\right\} \cap N\right] \cup M.
\end{aligned}$$

Since $(y, z) \in A_x$, then $x \leq y \circ z$. If $y \in A$ and $z \in B$, then $y \circ z \subseteq A \circ B$ and so $x \in (A \circ B]$. This is a contradiction. If $y \notin A$ and $z \in B$, then

$$\left[\left\{\bigcup_{(y,z)\in A_x} \left(\mathcal{S}_{\mathcal{A}}(y) \cap \mathcal{S}_{\mathcal{B}}(z)\right)\right\} \cap N\right] \cup M = \left[\left\{\bigcup_{(y,z)\in A_x} (\emptyset \cap U)\right\} \cap N\right] \cup M = M.$$

Hence $\left(\mathcal{S}_{\mathcal{A}} \widetilde{\odot^*} \mathcal{S}_{\mathcal{B}}\right)(x) = M = \mathcal{S}^*_{(\mathcal{A}\circ\mathcal{B}]}(x)$. Similarly, for $y \in A$ and $z \notin B$, we have $\left(\mathcal{S}_{\mathcal{A}} \widetilde{\odot^*} \mathcal{S}_{\mathcal{B}}\right)(x) = M = \mathcal{S}^*_{(\mathcal{A}\circ\mathcal{B}]}(x)$.

Theorem 4.17. If f_A is an (M,N)-int-soft subsemihypergroup of S over U. Then f_A^* is an (M,N)-int-soft subsemihypergroup of S over U.

Proof. Suppose that f_A is an (M,N)-int-soft subsemihypergroup of S over U. Let $x,y \in S$. Then

$$\begin{aligned}
\bigcap_{\alpha\in x\circ y} f_A^*(\alpha)\cup M &= \left[\bigcap_{\alpha\in x\circ y}\{(f_A(\alpha)\cap N)\cup M\}\right]\cup M\\
&= \left[\bigcap_{\alpha\in x\circ y}(f_A(\alpha)\cup M)\cap(N\cup M)\right]\cup M\\
&= \left[\bigcap_{\alpha\in x\circ y}(f_A(\alpha)\cup M)\cap N\right]\cup M\\
&\supseteq \{(f_A(x)\cap f_A(y)\cap N)\cap N\}\cup M\\
&= \{(f_A(x)\cap N)\cap(f_A(y)\cap N)\cap N\}\cup M\\
&= \{(f_A(x)\cap N)\cup M\}\cap\{(f_A(y)\cap N)\cup M\}\cap(N\cup M)\\
&= f_A^*(x)\cap f_A^*(y)\cap N.
\end{aligned}$$

Thus f_A^* is an (M,N)-int-soft subsemihypergroup of S over U.

Theorem 4.18. A soft set f_A is an (M,N)-int-soft subsemihypergroup of S over U if and only if $f_A\widetilde{\odot^*}f_A\widetilde{\subseteq}f_A^*$.

Proof. Assume that f_A is an (M,N)-int-soft subsemihypergroup of S over U. Let $x\in S$. If $A_x=\emptyset$. Then $\left(f_A\widetilde{\odot}f_A\right)(x)=\emptyset$. Thus

$$\begin{aligned}
\left(f_A\widetilde{\odot^*}f_A\right)(x) &= \left\{\left(f_A\widetilde{\odot}f_A\right)(x)\cap N\right\}\cup M\\
&= (\emptyset\cap N)\cup M\\
&= M
\end{aligned}$$

$\left(f_A\widetilde{\odot^*}f_A\right)(x)\supseteq M=f_A^*(x)$.
If $A_x\neq\emptyset$. Then

$$\begin{aligned}
\left(f_A\widetilde{\odot^*}f_A\right)(x) &= \left\{\left(f_A\widetilde{\odot}f_A\right)(x)\cap N\right\}\cup M\\
&= \left\{\left(\bigcup_{(a,b)\in A_x}\{f_A(a)\cap f_A(b)\}\right)\cap N\right\}\cup M\\
&= \left\{\bigcup_{(a,b)\in A_x}(f_A(a)\cap f_A(b)\cup M)\cap N\right\}\cup M\\
&\subseteq \left\{\bigcup_{(a,b)\in A_x}(f_A(x)\cap N)\cup M\right\}\cup M\\
&= (f_A(x)\cap N)\cup M\\
&= f_A^*(x).
\end{aligned}$$

Thus $f_A \widetilde{\odot^*} f_A \widetilde{\subseteq} f_A^*$.

Conversely, assume that $f_A \widetilde{\odot^*} f_A \widetilde{\subseteq} f_A^*$. Let $x, y \in S$. Then for each $\alpha \in x \circ y$, we have,

$$\begin{aligned}(f_A(\alpha) \cap N) \cup M = f_A^*(\alpha) &\supseteq \left(f_A \widetilde{\odot^*} f_A\right)(\alpha)\\ &= \left\{\left(f_A \widetilde{\odot} f_A\right)(\alpha) \cap N\right\} \cup M\\ &= \left[\left\{\bigcup_{(a,b)\in A_\alpha} (f_A(a) \cap f_A(b))\right\} \cap N\right] \cup M\\ &\supseteq \{(f_A(x) \cap f_A(y)) \cap N\} \cup M\\ &\supseteq \{(f_A(x) \cap f_A(y)) \cap N\}.\end{aligned}$$

Thus $\bigcap_{\alpha\in x\circ y} f_A(\alpha) \cup M \supseteq f_A(x) \cap f_A(y) \cap N$. Hence f_A is an (M, N)-int-soft subsemihypergroup of S over U.

Theorem 4.19. The characteristic function $\mathcal{S}_A^*$ of A is an (M, N)-int-soft generalized bi-hyperideal of S over U, if and only if A is a generalized bi-hyperideal of S.

Proof. Suppose that A is a generalized bi-hyperideal of S. Then by Theorem 4.9, $\mathcal{S}_A^*$ is an (M, N)-int-soft generalized bi-hyperideal of S over U.

Conversely, assume that $\mathcal{S}_A^*$ is an (M, N)-int-soft generalized bi-hyperideal of S over U. Let $x, y \in S$, $x \leq y$ be such that $y \in A$. It implies that $\mathcal{S}_A^*(y) = N$. Since $\mathcal{S}_A^*$ is an (M, N)-int-soft generalized bi-hyperideal of S over U. Therefore $\mathcal{S}_A^*(x) \cup M \supseteq \mathcal{S}_A^*(y) \cap N = N \cap N = N$. Since $M \subset N$. Hence $\mathcal{S}_A^*(y) = N$. Implies that $x \in A$. Now if there exist $x, y, z \in S$ such that $x, z \in A$. Then $\mathcal{S}_A^*(x) = N$ and $\mathcal{S}_A^*(z) = N$. Since $\mathcal{S}_A^*$ is an (M, N)-int-soft generalized bi-hyperideal of S over U. We have

$$\begin{aligned}\bigcap_{\alpha\in x\circ y\circ z} \mathcal{S}_A^*(\alpha) \cup M &\supseteq \mathcal{S}_A^*(x) \cap \mathcal{S}_A^*(z) \cap N\\ &= N \cap N \cap N\\ &= N.\end{aligned}$$

Since $M \subset N$. Hence $\mathcal{S}_A^*(\alpha) = N$. Thus $\alpha \in x \circ y \circ z \subseteq A$. Consequently, A is a generalized bi-hyperideal of S.

Proposition 4.20. If f_A is an (M, N)-int-soft generalized bi-hyperideal of S over U. Then f_A^* is an (M, N)-int-soft generalized bi-hyperideal of S over U.

Proof. Assume that f_A is an (M, N)-int-soft generalized bi-hyperideal of S over U.

Let $x, y, z \in S$, then

$$\begin{aligned}
\bigcap_{\alpha\in x\circ y\circ z} f_A^*(\alpha)\cup M &= \left\{\left(\bigcap_{\alpha\in x\circ y\circ z} f_A(\alpha)\cap N\right)\cup M\right\}\cup M \\
&= \left(\bigcap_{\alpha\in x\circ y\circ z} f_A(\alpha)\cap N\right)\cup M \\
&= \left(\bigcap_{\alpha\in x\circ y\circ z} f_A(\alpha)\cup M\right)\cap (N\cup M) \\
&= \left(\bigcap_{\alpha\in x\circ y\circ z} f_A(\alpha)\cup M\right)\cap N \\
&= \left\{\left(\bigcap_{\alpha\in x\circ y\circ z} f_A(\alpha)\cup M\right)\cup M\right\}\cap N \\
&\supseteq \{(f_A(x)\cap f_A(z)\cap N)\cup M\}\cap N \\
&= \{(f_A(x)\cap f_A(z)\cap N\cap N)\cup M\cup M\}\cap N \\
&= [\{(f_A(x)\cap N)\cup M\}\cap\{(f_A(z)\cap N)\cup M\}]\cap N \\
&= [f_A^*(x)\cap f_A^*(z)]\cap N \\
&= f_A^*(x)\cap f_A^*(z)\cap N.
\end{aligned}$$

Let $x, y \in S$ such that $x \leq y$. Then $f_A^*(x)\cup M \supseteq f_A^*(y)\cap N$. Indeed. Thus

$$\begin{aligned}
f_A^*(x)\cup M &= \{(f_A(x)\cap N)\cup M\}\cup M \\
&= \{(f_A(x)\cap N)\cup M\} \\
&= \{(f_A(x)\cup M)\cap (N\cup M)\} \\
&= \{(f_A(x)\cup M)\cap N\} \\
&= \{(f_A(x)\cup M)\cup M\}\cap N \\
&\supseteq \{(f_A(y)\cap N)\cup M\}\cap N \\
&= f_A^*(y)\cap N.
\end{aligned}$$

Hence f_A^* is an (M, N)-int-soft generalized bi-hyperideal of S over U.

Corollary 4.21. If $\{f_{A_i} \mid i \in I\}$ is a family of (M, N)-int-soft generalized bi-hyperideal of an ordered semihypergroup S over U. Then $f_A^* = \bigcap_{i\in I} f_{A_i}^*$ is an (M, N)-int-soft generalized bi-hyperideal of S over U.

Theorem 4.22. A soft set f_A satisfies condition (2) of Definition 4.3 is an (M, N)-int-soft generalized bi-hyperideal of S over U if and only if $f_A \widetilde{\odot}^* \mathcal{S}_{\mathcal{S}} \widetilde{\odot}^* f_A \widetilde{\subseteq} f_A^*$.

Proof. Suppose that f_A is an (M, N)-int-soft generalized bi-hyperideal of S over U. Let $x \in S$. If $A_x = \emptyset$. Then $\left(f_A \widetilde{\odot}^* \mathcal{S}_{\mathcal{S}} \widetilde{\odot}^* f_A\right)(x) \widetilde{\subseteq} f_A^*(x)$. Let $A_x \neq \emptyset$, then there exist $a, b \in S$ such that $x \leq a \circ b$. So $(a, b) \in A_x$. Thus

$$
\begin{aligned}
&\left(f_A \widetilde{\odot^*} \mathcal{S}_{\mathcal{S}} \widetilde{\odot^*} f_A\right)(x) \\
&= \left\{\left(f_A \widetilde{\odot}\left(\mathcal{S}_{\mathcal{S}} \widetilde{\odot^*} f_A\right)\right)(x) \cap N\right\} \cup M \\
&= \left(\left(\bigcup_{(a,b)\in A_x} \left\{f_A(a) \cap \left(\mathcal{S}_{\mathcal{S}} \widetilde{\odot^*} f_A\right)(b)\right\}\right) \cap N\right) \cup M \\
&= \left(\bigcup_{(a,b)\in A_x} \left\{f_A(a) \cap \left[\left(\left(\bigcup_{(c,d)\in A_b} \{\mathcal{S}_{\mathcal{S}}(c) \cap f_A(d)\}\right) \cap N\right) \cup M\right]\right\} \cap N\right) \cup M \\
&= \left(\bigcup_{(a,b)\in A_x} \left\{f_A(a) \cap \left[\left(\left(\bigcup_{(c,d)\in A_b} f_A(d)\right) \cap N\right) \cup M\right]\right\} \cap N\right) \cup M \\
&= \left(\left(\bigcup_{(a,b)\in A_x} \left\{\bigcup_{(c,d)\in A_b} [f_A(a) \cap f_A(d)] \cap N\right\} \cup M\right) \cap N\right) \cup M \\
&= \left(\left(\bigcup_{(a,b)\in A_x} \left\{\bigcup_{(c,d)\in A_b} [f_A(a) \cap f_A(d) \cup M] \cap N\right\}\right) \cap N\right) \cup M \\
&= \left(\bigcup_{(a,b)\in A_x} \left\{\bigcup_{(c,d)\in A_b} [f_A(a) \cap f_A(d) \cup M] \cap N\right\}\right) \\
&\subseteq \left(\bigcup_{x \leq a\circ b \leq a\circ c\circ d} \{f_A(x) \cap N\} \cup M\right) \\
&= (f_A(x) \cap N) \cup M \\
&= f_A^*(x).
\end{aligned}
$$

Thus $f_A \widetilde{\odot^*} \mathcal{S}_{\mathcal{S}} \widetilde{\odot^*} f_A \widetilde{\subseteq} f_A^*$.

Conversely, assume that $f_A^* \widetilde{\supseteq} f_A \widetilde{\odot^*} \mathcal{S}_{\mathcal{S}} \widetilde{\odot^*} f_A$ and $x, y, z \in S$. Then for every $\beta \in x \circ y \circ z$, we have

$$
\begin{aligned}
(f_A(\beta) \cap N) \cup M = f_A^*(\beta) \\
&\widetilde{\supseteq} \left(f_A \widetilde{\odot^*} \mathcal{S}_{\mathcal{S}} \widetilde{\odot^*} f_A\right)(\beta) \\
&= \left(\left(\bigcup_{(x,p)\in A_\beta} \left\{f_A(x) \cap \left(\mathcal{S}_{\mathcal{S}} \widetilde{\odot^*} f_A\right)(p)\right\}\right) \cap N\right) \cup M
\end{aligned}
$$

(because there exist $p \in y \circ z$ such that $\beta \leq x \circ p$)

$$\supseteq \left(\left(f_A(x) \cap \left(\mathcal{S}_{\mathcal{S}} \widetilde{\odot^*} f_A\right)(p)\right) \cap N\right) \cup M$$
$$\supseteq \left(\left(f_A(x) \cap \left[\left(\bigcup_{(y,z)\in A_p} \{\mathcal{S}_{\mathcal{S}}(y) \cap f_A(z)\} \cap N\right)\right] \cup M\right) \cap N\right) \cup M$$
$$\supseteq ((f_A(x) \cap ([f_A(z) \cap N] \cup M)) \cap N) \cup M$$
$$\supseteq (((f_A(x) \cap f_A(z)) \cup M) \cap N) \cup M$$
$$\supseteq ((f_A(x) \cap f_A(z) \cap N) \cap N)$$
$$= f_A(x) \cap f_A(z) \cap N.$$

Thus $\bigcap\limits_{\beta \in x \circ y \circ z} f_A(\beta) \cup M \supseteq f_A(x) \cap f_A(z) \cap N$. Thus f_A is an (M, N)-int-soft generalized bi-hyperideal of S over U.

5 Conclusion

In this paper, we have presented a detail theoretical study of intersectional soft sets. We introduced the notion of (M, N)-int-soft generalized bi-hyperideals of ordered semihypergroups and studied them. When $M = \emptyset$ and $N = U$, we meet intersectional soft generalized bi-hyperideals. From this analysis, we say that (M, N)-int-soft generalized bi-hyperideals are more general concept than usual intersectional soft ones. We characterized ordered semihypergroups in the framework of (M, N)-int-soft generalized bi-hyperideals. Hopefully that the obtained new characterizations will be very useful for future study of ordered semihypergroups. In future we will define other (M, N)-int-soft hyperideals of ordered semihypergroups and study their applications.

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