Optimum Thickness of Outside Sheathing Insulation for Exterior Walls of Building Envelopes

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Highlights
• An optimization analysis is performed using P_1-P_2 method with the degree day method.
• The formulae for savings and payback time of sheathing insulation for exterior walls are introduced.
• The optimization formulations are verified for three test cities.

Abstract

Due to the high heat losses in the buildings, using thermal insulation for external walls is a very effective method to decrease the building energy demand and their influences on environment. The goal of this work is to determine optimal exterior sheathing thickness to make energy consumption in the buildings more efficient. The P_1-P_2 method combined with the degree day method is used to predict the optimum sheathing thickness for the United States test cities. The rockwool and insulating board are considered for the insulation materials. The sheathing thickness must be chosen near the optimum point to augment energy savings in the buildings. It is seen that the rockwool material having low thermal conductivity leads to a reduction in the payback period and insulation thickness with high net savings. The optimal insulation thicknesses of 0.064 m and 0.12 m with the payback period of 4.4 and 5.9 years were determined for the rockwool and insulating board, respectively, in New York City.

NOMENCLATURE

A	Area of wall (m²)
B	Constant defined in Equation (10)
C	Constant defined in Equation (10)
C_E	Cost of energy for heating, ($/W.hr)
C_v	Cost of insulation material per unit volume, ($/m³)
CDD	Cooling degree day, (K.days)
COP	Coefficient of performance
D	Constant defined in Equation (10)
d	Market discount rate in fraction
E	Constant defined in Equation (13)
F	Constant defined in Equation (13)
G	Constant defined in Equation (13)
h_i	Convective heat transfer coefficient for internal surface, (W/m²K)
h_o	Convective heat transfer coefficient for external surface, (W/m²K)
H	Constant defined in Equation (13)
HDD	Heating degree day, (K.days)
i	Energy price escalation rate in fraction
k_{ins}	Thermal conductivity of insulating material, (W/mK)
k_n	Thermal conductivity of nth layer of wall, (W/mK)
x	Sheathing insulation thickness, (m)

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x_n	Thickness of nth layer of wall, (m)
x_{opt}	Optimum insulation thickness, (m)
N	Technical life of the sheathing insulation for exterior walls, (year)
N_P	Payback time, (year)
P_1	Ratio of the life cycle energy cost savings to the first year energy cost savings, (year)
P_2	Ratio of the life cycle expenditures incurred because of the additional capital investment to the initial investment
R_{ins}	Internal resistance of insulating material, (m^{2}K/W)
R_w	Thermal resistance of an uninsulated wall, (m^{2}K/W)
S	Net overall life cycle savings of the sheathing insulation for exterior walls, ($)
Q_W	Instantaneous wall heat gain load, (W)
T_i	Inside temperature, (K)
T_o	Outside temperature, (K)
TDD	Total degree day, (K.days)
U	Overall heat transfer coefficient, (W/m^{2}K)
U_{ins}	Overall heat transfer coefficient of insulated wall, (W/m^{2}K)
U_{un}	Overall heat transfer coefficient of uninsulated wall, (W/m^{2}K)
Y_1	Constant defined in Equation (15)
Y_2	Constant defined in Equation (16)
Y_3	Constant defined in Equation (17)
Z	Constant defined in Equations (15), (16) and (17)

1. INTRODUCTION

A building envelope is a physical separator that differentiates the conditioned interior from the unconditioned outdoor. Exterior walls of the building enclosure are accountable for nearly all heat gains and losses [1]. They also influence fossil fuel consumption and hence emissions due burning fuels.

Therefore, the thermal performance enhancement of the insulation systems for buildings has been an increasing concern in recent years [2, 3]. Choosing appropriate wall insulation thickness is vital for maintaining a comfortable indoor climate while promoting energy efficiency and environmental sustainability [4, 5]. Various works have been conducted to obtain the optimal exterior insulation thickness for distinct climate conditions [6-19]. Malka et al. [13] used the RETScreen Expert model to calculate the optimum insulation thickness for various materials such as glass wool, graphite, expanded polystyrene, and rockwool with energy sources (fossil fuels, electricity and biomass) using Mathcad. It was observed that minimum optimal thickness was achieved by employing biomass with the corrected optimal insulation thickness in the range of 0.03 m - 0.11 m. Shahid et al. [18] examined impacts of temperature and cooling degree day CDD on insulation thickness for the hot weather in India. The findings revealed that the best suitable insulation materials are fiberglass rigid, urethane rigid and fiberglass urethane with optimum thickness of 0.0209-0.0270 m at 24 ºC. Las-Heras-Casas et al. [19] conducted life cycle cost analysis to select the optimal insulation thicknesses of the exterior walls for the buildings in Spain. They determined optimum thicknesses as 0.062 m - 0.114 m and 0.059 m - 0.105 m for new and renewed buildings, respectively.

Wall sheathing is a layer or panel of material connected to the exterior wall to provide additional strength, insulation and protection from external factors. Exterior wall sheathing is a board or panel that may be made of several different materials such as plywood, OSB (oriented strand board), structural fiberboard, foam board, gypsum, etc. It performs as a protective layer that gives structural support and insulation to the walls and represents a barrier against outdoor elements such as wind and moisture. By providing these essential characteristics, insulating sheathing to exterior wall contributes to the overall energy efficiency of the building.

The current work is aimed to optimize sheathing thickness for exterior walls in buildings. The correlations are derived to predict an optimal sheathing insulation thickness. The cities from three different climate regions of the United States are selected to determine optimum insulation thickness for the worst-case scenario. The application of sheathing is vitally necessary for concrete exterior walls of uninsulated or insufficiently insulated building structures.

2. NUMERICAL PROCEDURE

The methodology adopted in the current study is described in this section. Wall heat gain load through the building enclosure is evaluated by means of following equation:

$$Q_w = UA(T_o - T_i) \tag{1}$$

where U is overall heat transfer coefficient and A is the area of wall. T_o and T_i are outside and inside temperatures. The overall heat transfer coefficient of uninsulated wall (U_{un}) is calculated as

$$U_{un} = \frac{1}{R_w} \tag{2}$$

The thermal resistance of an uninsulated wall (R_w) equals the sum of resistances of the inside and outside surfaces against heat convection and all layers of the wall against the heat conduction

$$R_w = \frac{1}{h_i} + \frac{x_1}{k_1} + \frac{x_2}{k_2} + ... + \frac{x_n}{k_n} + \frac{1}{h_o} \tag{3}$$

where h_i and $\underline{h_o}$ are convective heat transfer coefficient for internal and external surfaces, respectively. k_n is thermal conductivity of nth layer of wall. The cross-section of the wall employed in this work is illustrated in Figure 1.
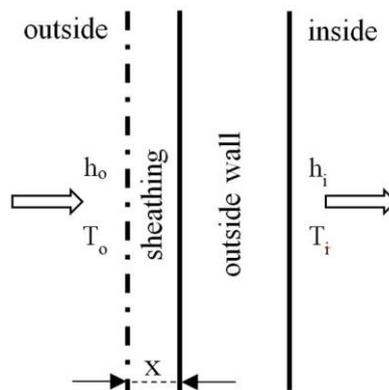
Figure 1. Cross section of wall with sheathing

The overall heat transfer coefficient of an insulated external wall (U_{ins}) is calculated as

$$U_{ins} = \frac{1}{R_{ins} + R_w} \tag{4}$$

$$\Delta U = U_{un} - U_{ins} = \frac{1}{R_w} - \frac{1}{R_w + \frac{x}{k_{ins}}} \quad . \tag{5}$$

ΔU in Equation (5) is the difference in the overall heat transfer coefficients between uninsulated and insulated walls. k_{ins} is thermal conductivity of insulating material. The derivative of Equation (5) with the sheathing thickness, *x*, is

$$\frac{\partial(\Delta U)}{\partial x} = \frac{k_{ins}}{\left(R_w k_{ins} + x\right)^2} \quad . \tag{6}$$

In this study, the life-cycle cost of exterior wall system including the insulation is calculated using a quick, convenient, and useful method called the P_1 - P_2 method [20] combined with the degree day method [21].

$$S = P_1 C_E \left(CDD + HDD\right) 24 \Delta U A - P_2 C_V A x \quad . \tag{7}$$

C_E is energy cost for heating. It is assumed to equal to cost for cooling because of *COP* effect of cooling system when using electricity. *HDD* and *CDD* are heating and cooling degree day, respectively. C_v is the initial cost per unit volume containing material, painting and labor costs. *CDD* and *HDD* have been commonly employed as indicators of cooling and heating load occurring due to outdoor temperature.

CDD and *HDD* values can be determined as the difference between the daily mean temperature and base temperature [22]. Total degree day (*TDD*) is calculated by Equation (8)

$$TDD = CDD + HDD \quad . \tag{8}$$

CDD and *HDD* values of several cities are given in Table 1.

Table 1. *CDD and HDD for the United States test cities [23]*

Cities	CDD (K.days)	HDD (K.days)
New York	734	2800
Lake Charles	1500	1000
Phoenix	2000	1000

CDD and *HDD* values in Table 1 are obtained approximately using Courtesy of SERI, Solar Radiation Energy Resource Atlas of the United States [23]. As seen in Table 1, three test cities including New York, Lake Charles and Phoenix are selected to investigate the effect of different climate regions on the optimum sheathing thickness. The first city is selected for an extremely cold climate. The second city has a mild climate. The third city is in a hot tropical zone. *HDD* value is higher than *CDD* for the cold climates, whereas *CDD* value is higher than *HDD* for the hot tropical climates. The combined effect of the *HDD* and *CDD* on the exterior wall sheathing thickness is examined in this study.

The savings in cooling and heating strongly depend on thermal insulation cost. Optimum insulation thickness should be determined to make balance between the investment in thermal insulation and savings in heating and cooling.

Substituting Equation (5) and Equation (8) into Equation (7) yields

$$\frac{S}{A} = P_1 C_E \left(TDD\right) 24 \left(\frac{1}{R_w} - \frac{1}{R_w + \dfrac{x}{k_{ins}}}\right) - P_2 C_V x \quad . \tag{9}$$

Equation (9) can be written in the following form

$$\frac{S}{A} = Bx + \frac{1}{C + x} + D \tag{10}$$

where B, C and D are constants. The derivative of Equation (7) with the sheathing insulation thickness (x) is

$$\frac{\partial S}{\partial x} = P_1 C_E (TDD) 24 \frac{\partial(\Delta U)}{\partial x} A - P_2 C_V A. \tag{11}$$

Substituting Equation (6) into Equation (11) yields

$$\frac{\partial S}{\partial x} = P_1 C_E (TDD) 24 \frac{k_{ins}}{\left(R_w k_{ins} + x\right)^2} A - P_2 C_V A. \tag{12}$$

Equation (12) can be written in the following form

$$\frac{\partial S}{\partial x} = \frac{E}{\left(F + G.x\right)^2} - H \tag{13}$$

where E, F, G and H are constants. The optimum value of x (x_{opt}) is found by $\partial S/\partial x$ in Equation (12) equal to zero:

$$x_{opt} = \sqrt{\frac{P_1 C_E (TDD) 24 k_{ins}}{P_2 C_V}} - R_w k_{ins}. \tag{14}$$

Effects of TDD, C_V/k_{ins} and C_E on x_{opt} are examined following by arranging Equation (14) in the following forms

$$x_{opt} = Y_1 \sqrt{(TDD)} + Z \tag{15}$$

$$x_{opt} = \frac{Y_2}{\sqrt{C_V}} + Z \tag{16}$$

$$x_{opt} = Y_3 \sqrt{C_E} + Z \tag{17}$$

where Y_1, Y_2 and Y_3 are constants.

The second derivative of S always equals negative value. This certainly indicates a local maximum saving

$$\frac{\partial^2 S}{\partial x^2} < 0. \tag{18}$$

For $i = d$, the payback time, N_P is evaluated by specifying $S/A = 0$ in Equation (9) for x_{opt}

$$N_P = \frac{P_2 C_V x_{opt} (1+i)}{C_E (TDD) 24 \left(\dfrac{1}{R_w} - \dfrac{1}{R_w + \dfrac{x_{opt}}{k}} \right)}. \tag{19}$$

For $i \neq d$, P_1 and N_P are evaluated by following equations:

$$P_1 = \frac{1}{(d-i)}\left\{1-\left[\frac{1+i}{1+d}\right]^N\right\} \tag{20}$$

$$N_P = \ln\left[1+\frac{P_2 C_V x(d-i)}{C_E(TDD)24\left(\frac{1}{R_w}-\frac{1}{R_w+\frac{x_{opt}}{k}}\right)}\right] \Bigg/ \ln\left[\frac{1+i}{1+d}\right] . \tag{21}$$

3. THE RESEARCH FINDINGS AND DISCUSSION

Values of parameters used in this study are as follows: $k_{ins} = 0.035$ W/mK and $C_V = 750$ \$/m³ for rockwool insulation, $k_{ins} = 0.1$ W/mK and $C_V = 500$ \$/m³ for insulating board, $N = 30$ years, $i = 0.1$, $P_2 = 1$, $C_E = 5.10^{-5}$ \$/W.hr, $HDD = 2800$ K days and $CDD = 734$ K days for New York City [23].

$R_w = 0.3$ m²K/W. The value of R_w is determined for the worst-case scenario. The wall structure consists of 1 cm exterior gypsum board, 15 cm briquette and 1 cm interior gypsum board in Figure 2. Thermal conduction resistance values for the exterior gypsum board, briquette and interior gypsum board (R_{g1}, R_w and R_{g2}) are 0.01, 0.1 and 0.011 m²K/W. k_{g1}, k_w and k_{g2} are 1, 1.43 and 0.87 W/mK.
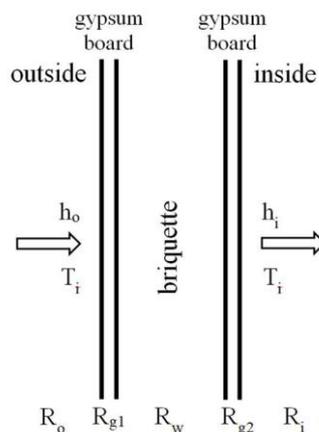
Figure 2. Schematic of the wall structure from outside to inside

Two layers of gypsum boards are fixed to the inside and outside of the wall [24, 25]. The cavity between the boards is filled with insulation layers such as the rockwool and insulating board. Each board includes a non-combustible gypsum core with paper-laminated surfaces. Gypsum is composed of a small amount of free water and chemically bound water. This plays a vital role in the performance of the assembly at high temperatures. During a fire, when gypsum board is heated up to about 100°C, a great amount of heat is absorbed to drive off the water [24, 25].

Cost per unit area of exterior walls (S/A) with outside sheathing thickness, x is plotted in Figure 3. S/A versus the life cycle period (N) of the rockwool and insulating board using exterior walls of the building is plotted in Figure 4.
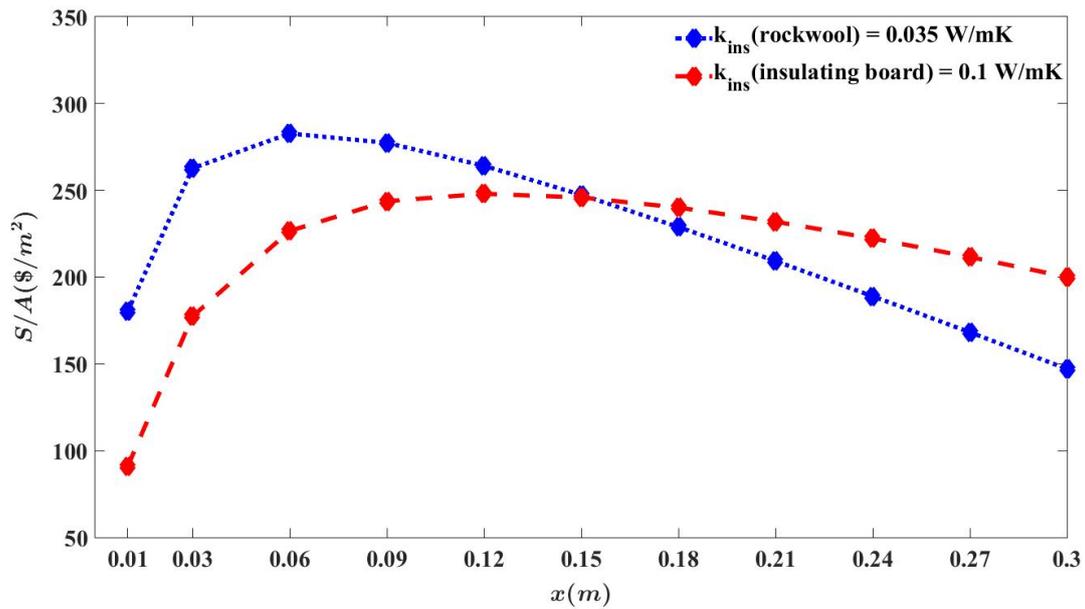
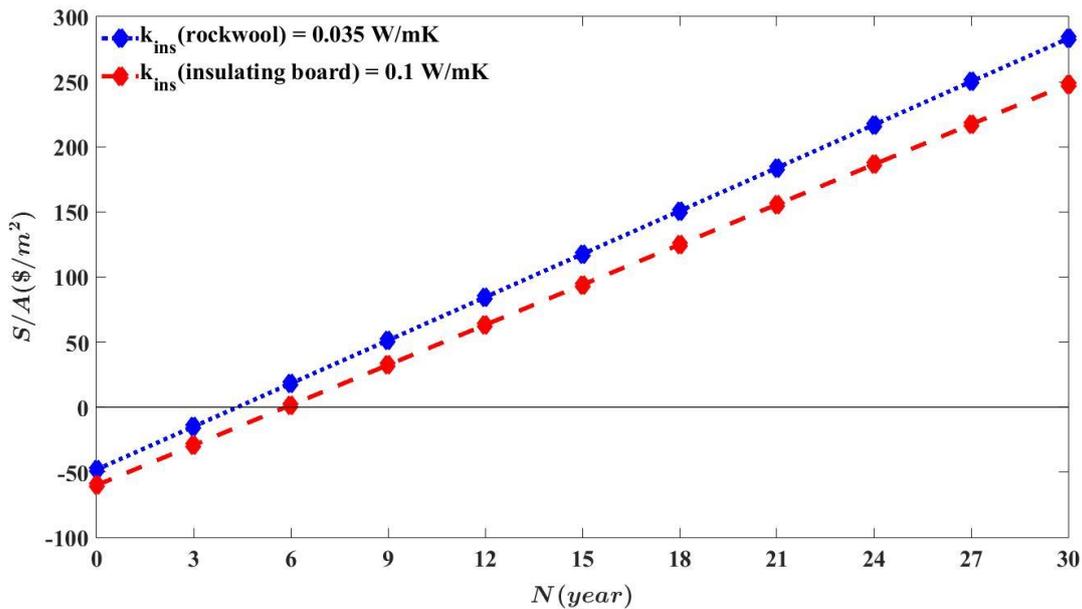**Figure 3.** *S/A versus x*

Figure 4. *S/A versus N*

As seen in Figure 3, the savings per unit area increase with the thicknesses of insulation materials up to local maximum points which equal optimal thickness (x_{opt}). The values of x_{opt} are found as 0.064 m and 0.12 m when using rockwool and insulating board, respectively, for New York City. On the other hand, x_{opt} values of the insulating board for Phoenix and Lake Charles in Table 1 are 0.11 m and 0.10 m, respectively. As illustrated in Equation (14), x_{opt} is highly dependent on TDD values. It is apparent in Figure 3 that further increasing thicknesses of insulation materials causes a reduction in the saving. It leads to the conclusion that the external wall insulation system is the most cost-efficient at optimum thickness. Higher lifetime leads to a linear increase in the energy recovery savings for both materials. The payback time (N_P) of 4.4 and 5.9 years is found by using Equation (19) in Figure 4.

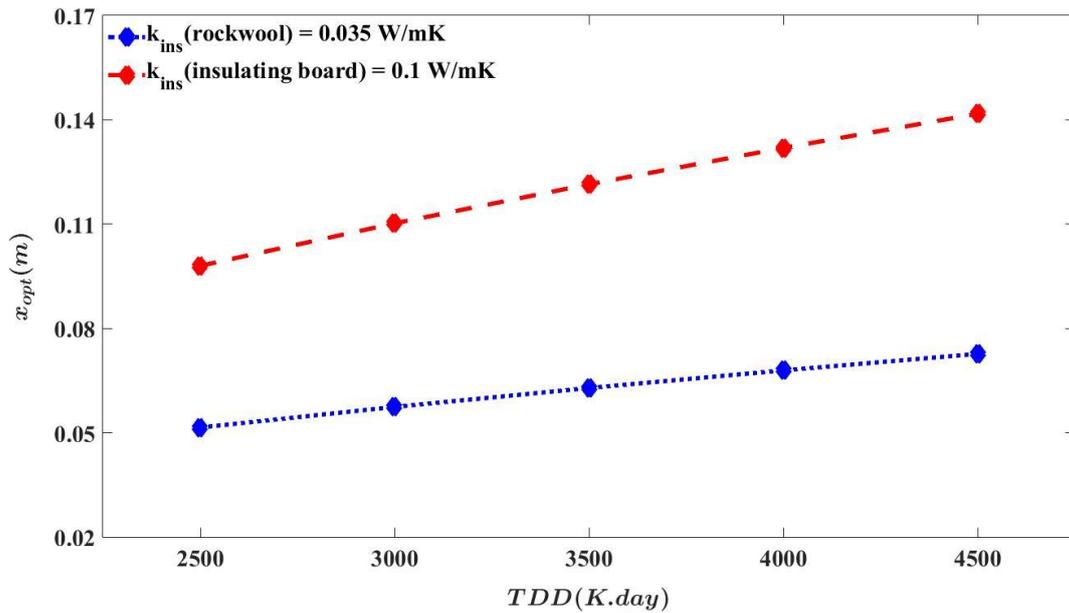Figure 5 illustrates the change of the optimum sheathing insulation thickness, x_{opt} with *TDD*.

Figure 5. *Effect of TDD on x_{opt}*

As expected, an increase in *TDD* leads to increasing x_{opt} and thus additional capital investment for the rockwool and insulating board. It is clear in Figure 5 that x_{opt} is strongly dependent on the total energy needs (cooling and heating). Lower x_{opt} is obtained with the rockwool since its thermal conductivity is smaller than the thermal conductivity of insulating board.

Figure 6 illustrates the variation of the optimum insulation thickness (x_{opt}) with the thermal conductivity cost (C_V/k_{ins}) for the rockwool and insulating board.
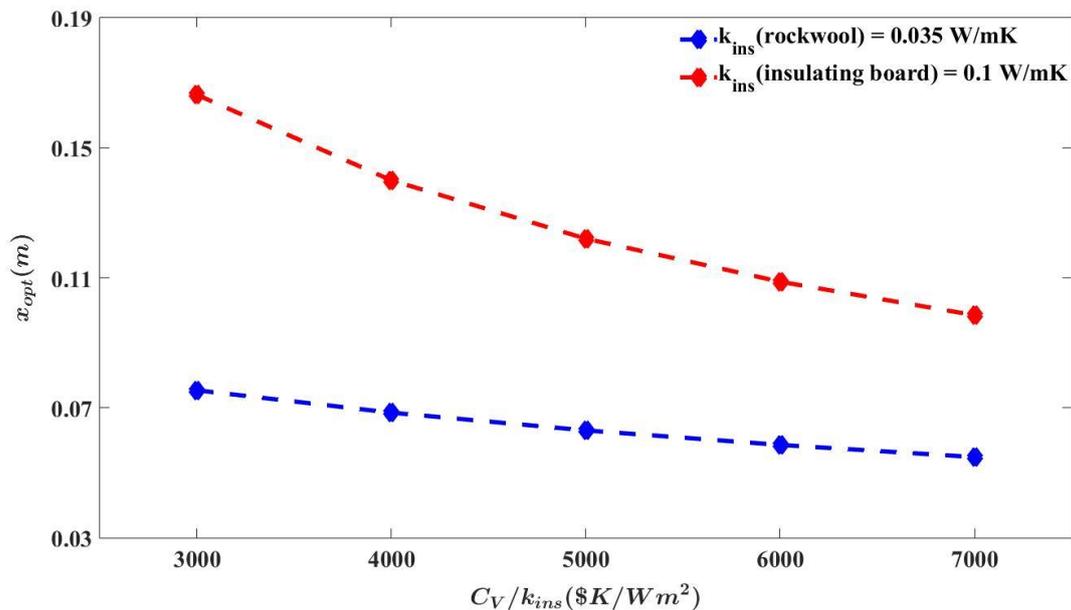

Figure 6. *Effect of C_V/k_{ins} on x_{opt}*

It is clear in Figure 6 that x_{opt} decreases continuously with an increase in C_V/k_{ins} for both insulation materials. Besides, as shown in Figure 6, the optimum thickness of the insulation board is significantly higher than the rockwool insulation due to its high thermal conductivity.

Therefore, the rockwool is better thermal insulation material for reducing heat loss from the exterior wall because of its low thermal conductivity. The payback period also decreases by using rockwool insulation due to high net savings and low sheathing insulation thickness in Figure 3.

Figure 7 illustrates the variation of x_{opt} with cost of electricity (C_E) for rockwool and insulating board. It is apparent in Figure 7 that x_{opt} increases with the increasing C_E.
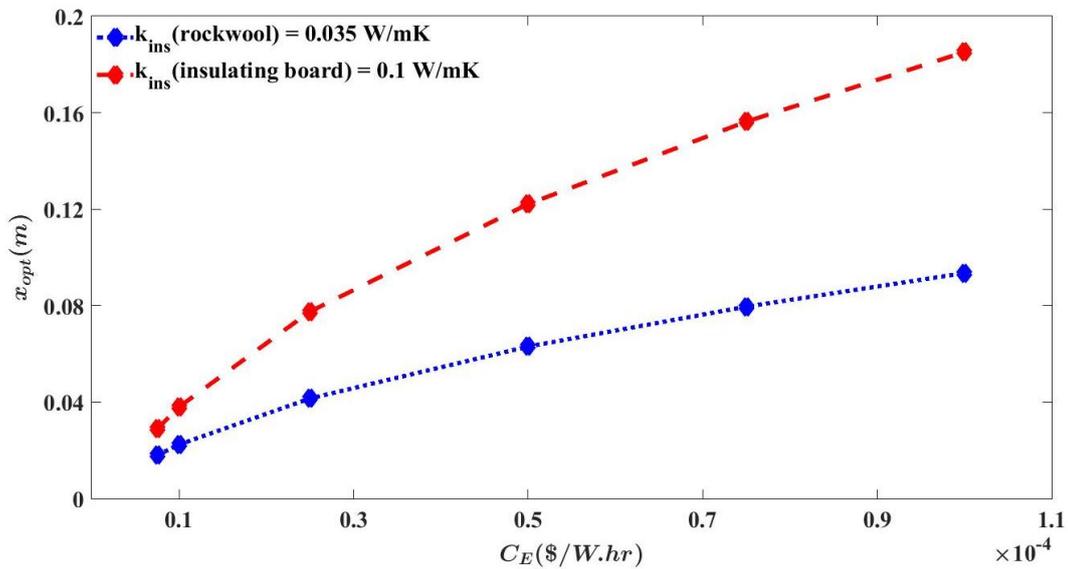
Figure 7. *Effect of C_e on x_{opt}*

Figure 8 demonstrates the variation of x_{opt} with the heat transfer resistance of the uninsulated wall (R_w) for rockwool and insulating board.
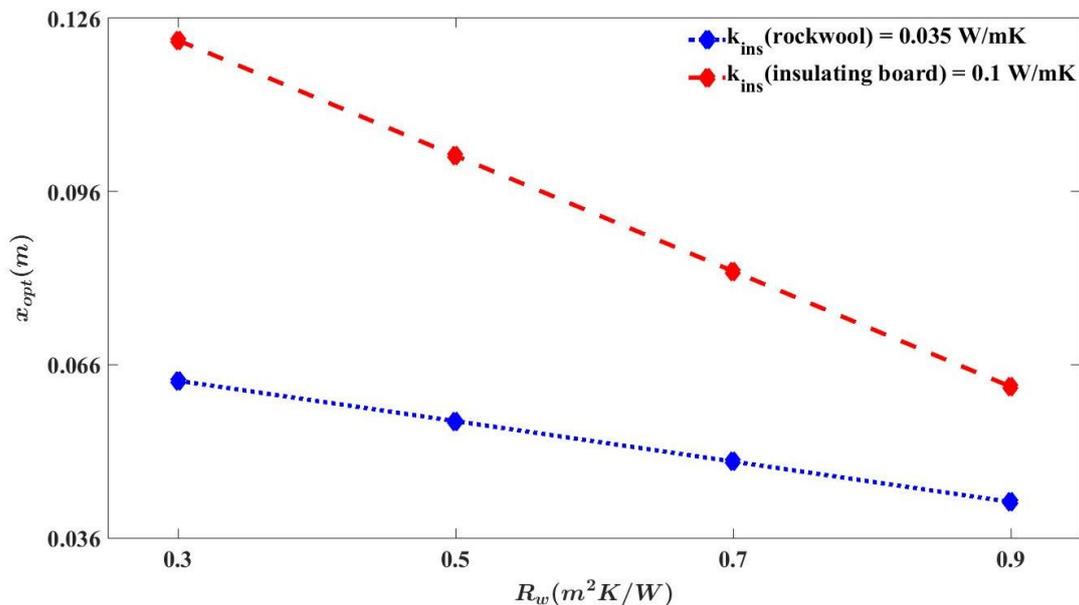
Figure 8. *Effect of R_w on x_{opt}*

It is clear in Figure 8 that an increase in R_w results in a decrease in x_{opt}. The elevated R_w indicates high levels of insulation. The higher R_w value leads to the greater the insulating effectiveness. It is more obvious for the rockwall insulation.

In summary, Equation (14) as a function of different parameters including C_E, C_V, TDD, R_w and k_{ins} can be useful to correctly evaluate the economic optimum insulation thickness for the buildings.

4. CONCLUSION

In the present work, optimum exterior sheathing insulation thickness are determined based on the P_1-P_2 method with the degree day method. It is seen that optimal insulation thickness depends on various properties including insulation and electricity costs, degree day and climate condition etc. The general correletaion (Equation (14)) is improved in this study takes account of economic characteristics, insulation materials and climatic conditions. Too much insulation results in a decrease in the energy cost at the expense of higher initial invesment. It is observed that the optimum sheathing thickness is in inverse proportion to the thermal conductivity cost. The optimum thicknesses of 0.064 m and 0.12 m and the payback time of 4.4 and 5.9 years are found by using rockwool and insulating board, respectively, for New York City.

CONFLICTS OF INTEREST

No conflict of interest was declared by the authors.

REFERENCES

[1] Lopes, G.C., Vicente, R., Azenha, M., and Ferreira, T.M., "A systematic review of Prefabricated Enclosure Wall Panel Systems: Focus on technology driven for performance requirements", Sustainable Cities and Society, 40: 688-703, (2018). DOI: https://doi.org/10.1016/j.scs.2017.12.027

[2] Vijayan, D.S., Sivasuriyan, A., Patchamuthu, P., and Jayaseelan, R., "Thermal performance of energy-efficient buildings for sustainable development", Environmental Science and Pollution Research, 29: 51130-51142, (2022). DOI: https://doi.org/10.1007/s11356-021-17602-3

[3] Chandhran, K.S.D., and Elavenil, S., "A comprehensive state-of-the-art review of sustainable thermal insulation system used in external walls for reduction in energy consumption in buildings", International Journal of Green Energy, 20(9): 895-913, (2023). DOI: https://doi.org/10.1080/15435075.2022.2120769

[4] Li, X., and Tingley, D.D., "A whole life, national approach to optimize the thickness of wall insulation", Renewable and Sustainable Energy Reviews, 174: 113137, (2023). DOI: https://doi.org/10.1016/j.rser.2022.113137

[5] Kaynakli, O., "A review of the economical and optimum thermal insulation thickness for building applications", Renewable and Sustainable Energy Reviews, 16(1): 415-425, (2012). DOI: https://doi.org/10.1016/j.rser.2011.08.006

[6] Wati, E., Meukam, P., and Nematchoua, M.K., "Influence of external shading on optimum insulation thickness of building walls in a tropical region", Applied Thermal Engineering, 90: 754-762, (2015). DOI: https://doi.org/10.1016/j.applthermaleng.2015.07.052

[7] Ashouri, M., Astaraei, F.R., Ghasempour, R, Ahmadi, M.H., and Feidt, M., "Optimum insulation thickness determination of a building wall using exergetic life cycle assessment", Applied Thermal Engineering, 106: 307-315, (2016). DOI: https://doi.org/10.1016/j.applthermaleng.2016.05.190

[8] Aktemur, C., Bılgın, F., and Tunçkol, S., "Optimisation on the thermal insulation layer thickness in buildings with environmental analysis: an updated comprehensive study for Turkey's all provinces", Journal of Thermal Engineering, 7: 1239-1256, (2021). DOI: https://doi.org/10.18186/thermal.978057

[9] Karamı, M., Anbarzadeh, E., and Delfanı, S., "Effect of HVAC system size on the optimum insulation thickness of the buildings in different climate zones", Journal of Thermal Engineering, 8: 249-267, (2021). DOI: https://doi.org/10.18186/thermal.1086236

[10] Benallel, A., Tilioua, A., Mellaikhafi, A., and Hamdi, M.A.A., "Thickness optimization of exterior wall insulation for different climatic regions in Morocco", Materials Today: Proceedings, 58: 1541-1548, (2022). DOI: https://doi.org/10.1016/j.matpr.2022.03.324

[11] Feng, Y.H, Yu, Z.T, Lu, J., and Xu, X., "Optimum insulation thickness of external walls by integrating indoor moisture buffering effect: a case study in the hot-summer-cold-winter zone of China", Journal of Zhejiang University-SCIENCE A, 23(12): 998-1012, (2022). DOI: https://doi.org/10.1631/2023.A2200158

[12] Kharbouch, Y., Boushaba, H., and Ameur, M., "Optimisation of the insulation thickness for the external walls in Morocco while considering the effect of climate change", Thermal Science and Engineering Progress, 35: 101455, (2022). DOI: https://doi.org/10.1016/j.tsep.2022.101455

[13] Malka, L., Kuriqi, A., and Haxhimusa, A., "Optimum insulation thickness design of exterior walls and overhauling cost to enhance the energy efficiency of Albanian's buildings stock", Journal of Cleaner Production, 381: 135160, (2022). DOI: https://doi.org/10.1016/j.jclepro.2022.135160

[14] Ozbek, K., Gelis, K., and Ozyurt, O., "Optimization of external wall insulation thickness in buildings using response surface methodology", International Journal of Energy and Environmental Engineering, 13: 1367-1381, (2022). DOI: https://doi.org/10.1007/s40095-022-00490-9

[15] Aşıkoğlu, A., "Cost analysis of insulation materials used to increase energy performance in buildings with Net Present Value method", Journal of Sustainable Construction Materials and Technologies, 8: 134-145, (2023). DOI: https://doi.org/10.47481/jscmt.1270831

[16] Terhan, M., and Abak, S.S., "Energy, exergy analysis and optimization of insulation thickness on buildings in a low-temperature district heating system", Journal of Thermal Engineering, 9: 161-178, (2023). DOI: https://doi.org/10.18186/thermal.1243502

[17] Fohagui, F.C.V., Koholé, Y.W., Alombah, N.H., Asoh, D.A., and Tchuen, G., "Optimum insulation thickness and exergy savings of building walls in Bamenda: a comparative analysis", Energy Storage and Saving, 3(1): 60-70, (2024). DOI: https://doi.org/10.1016/j.enss.2024.01.001

[18] Shahid, M., Karimi, M.N., and Mishra, A.K., "Optimum insulation thickness for external building walls for different climate zone in India", Journal of Thermal Engineering, 10(5): 1198-1211, (2024). DOI: https://doi.org/10.14744/thermal.0000861

[19] Las-Heras-Casas, J., López-Ochoa, L.M., Olasolo-Alonso, P., and López-González, L.M., "Adapting buildings to climate change by using the optimal thickness of thermal insulation for external walls: the case of Spain (2025–2055)", Journal of Cleaner Production, 490: 144715, (2025). DOI: https://doi.org/10.1016/j.jclepro.2025.144715

[20] Duffie, J.A., and Beckman, W.A., Solar Engineering of Thermal Processes, Fourth Edition, Jhon Wiley and Sons, New Jersey, 467-469, (2013). DOI: https://doi.org/10.1002/9781118671603

[21] ASHRAE, "2021 ASHRAE Handbook Fundamentals", ASHRAE, Peachtree Corners, 305-310, (2021).

[22] Bhatnagar, M., Mathur, J., and Garg, V., "Determining base temperature for heating and cooling degree-days for India", Journal of Building Engineering, 18: 270-280, (2018). DOI: https://doi.org/10.1016/j.jobe.2018.03.020

[23] Reddy T., Kreider, J.F., Curtiss, P.S., and Rabl, A., Heating and Cooling of Buildings: Design for Efficiency, Third Edition, CRC Press, Boca Raton, (2016). DOI: https://doi.org/10.1201/9781315374567

[24] Rahmanian, I., "Thermal and mechanical properties of gypsum boards and their influences on fire resistance of gypsum board based systems", Phd.Thesis, University of Manchester Faculty of Engineering and Physical Sciences, Manchester, 29-50, (2011).

[25] Kubba, S., Green Construction Project Management and Cost Oversight, First Editon, Butterworth-Heinemann, Oxford (2010). DOI: https://doi.org/10.1016/C2009-0-20060-2