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Original Research Article

Formulation of Word Problems in Geometry by Gifted Pupils

ABSTRACT: This study investigated the ability level of talented and gifted pupils to define selected geometry terms and formulate a word problem for each of them. In order to perform this task correctly, pupils should be acquainted with the geometry term. Moreover, they must have at last experience in solving word problems. The research population consisted of 58 pupils from the 4th-6th grades who learn mathematics in a course which is adjusted to their high ability level. The research findings illustrate a medium level of mastery of the term definition knowledge. The formulated word problems were mainly taken from the pupils' previous experience and they are at the first level according to van Hiele. Only few pupils demonstrated creativity and write problems which were not similar to the ones they knew from the textbooks.

Key words: Ability of gifted, geometry definitions, geometry words problems, gifted education.

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INTRODUCTION

Geometry is one of the subjects learnt at elementary and high school. It is one of the difficult subjects both for learning and for teaching. Definitions of geometry terms play a major role in learning and understanding the subject. The definitions serve as a communication tool in addition to being a classification tool as well as a corner stone for proofs and problem solution. The criteria for the term definitions constitute a foundation for the deductive and logical structure of the subject.

Pupils who study geometry build their knowledge on the basis of terms which are known and familiar to them and by establishing contacts between them. Hence, it is recommended starting the inculcation of a new term in an informal way and by examples. From a didactic point of view, the teaching of a new term should include concepts known to the learners and should clearly present the relation to their near world of terms (Patkin & Levenberg, 2002; Yelin & Zaslavski, 2000). Only at a later stage, do pupils start learning the formal definitions of the terms and the problem solution process.

Studies indicate the important relation between development of learners' mathematical language and the ability to solve word problems (Hershkovitz & Nesher, 2003; Murray, 2012; Usiskin, 1996). Consequently, while teaching gifted or talented pupils, the need for developing the verbal ability together with the mathematical-logical thinking are attributed greater importance. Moreover, the need to offer an opportunity for expressing creativity and originality is essential (English, 1998).

The community exploring geometry learning is strongly founded in the theory of van Hiele (Patkin, 2014; van Hiele, 1987) as a means of examining the pupils' level in accordance with the level of questions they are capable of solving.

This study examined this issue in a different way. It did not check the pupils' level in geometry but rather the level of the verbal problem formulated by the pupils while using the given term. In fact, this study examines two aspects of geometry studies: gifted pupils' ability to define geometry terms and their ability to build word problems.

Theoretical background

Definitions of mathematical terms and their role in teaching the subject

The definition of a term has a central role in the study of mathematics in general and of geometry in particular. The word 'definition' originates from the word 'Finis' which means a border, a fence. The practical meaning of 'fence' around a term is to facilitate a clear and conclusive distinction between what complies with the definition of the term and what does not belong to this term (Mounwietten & Vinner, 1995). When a definition is concerned, it is important to point out that every term (except for Basic geometric terms– which are usually accepted without a definition) has a definition which determines its meaning by means of previous terms. Vinner (1991) specifies that in mathematics a term cannot be defined by its image as can be done in other areas. Mathematics recognizes, identifies and classifies concepts only through their formal definition.

The criteria of mathematical definition are well clarified. Some of them are necessary and some depend on the case or on the defining person's preference. (Alcock & Simpson, 2002; Shir, 2004; Yelin & Zaslavski, 2000). The five essential criteria as: consistency, conclusiveness, independence of representation, presence and equivalence. There are also some criteria which are not essential and they depend as mentioned on the situation and the defining person's preference. For example: the hierarchy criterion, minimalism or elegance.

Researchers who investigated the issue of mathematical definitions (Karni, 2004, Karni & Zaslavski,, 2004; Vinner, 1991) stipulate the functions that they have in the teaching of this subject. The definition constitutes a tool for building the mathematical term, serves as the cornerstone for structuring mathematical theories, and is a basis for establishing proofs, problem solution and a tool for mathematical communication. Moreover, all these together enrich the learners' language. In fact, definitions are the foundation for learning the subject of geometry and without them, pupils cannot advance to the higher levels and to the problem solution stage.

Teaching of word problems

The teaching of word problems is considered as one of the difficult and complex tasks with which mathematics teachers have to cope (Hershkovitz & Nesher, 2003). In order that pupils understand how to solve a problem rather than just recite facts and perform calculation operations, a suitable teaching method which promotes mathematical thinking and understanding should be adopted.

Skemp (1997) distinguishes between two interpretations of the concept "understanding": "instrumental understanding" and "relational understanding". He highlights the difference between knowing the rules and being able to apply them and knowing to do and why. This approach is supported by Hiebert (1986) who differentiates between procedural knowledge and conceptual knowledge. That is, being able to perform algorithmic procedures is not enough. It is necessary to form a relational understanding during the teaching process, while emphasizing the logical relation between the terms.

Researchers (Levenberg & Ophir, 2006; Morgan, 1996; Silver, 1996) attribute importance to the formulation of word problems by the learners. According to them, this is essential not merely for understanding the learnt subject but also as a tool which serves teachers for comprehending the pupils' way of thinking. Furthermore, this is the way by which teachers could develop learners' competences of mathematical writing and encourage creative writing

This theoretical framework leads up to the practical part of teaching word problems. Mathematics as a language is used for deriving information and understanding it, organizing the thinking and demonstrating generalizations when speaking and writing. Mathematical language has a unique linguistic structure and teaching word problem constitutes a bridge between pure mathematical situations while using mathematical language and our daily life (Patkin & Levenberg, 2012).

Based on the recommendations of the National Council of Teachers of Mathematics (NCTM, 2000), we find extensive reference to the issue of solving word problems while emphasizing the relevance of the problems presented to the pupils. The word problems presented to the pupils should be meaningful and close to their world so that mathematics is not perceived as a subject which is disconnected from reality. The objective is to make pupils aware of the relevance of the studied mathematical contents and their relation to their day-by-day life.

Greer (1993) who investigated the pupils' formulation of problems pointed out the pupils' lack of reference to the realistic content of the problem. That is, formulations which have no mathematical meaning and the use of numerical sizes which are incompatible with reality.

The main source from which pupils learn the issue of word problems in general and geometry in particular is the textbook. Most of the books have maintained for years a unique formulation of problems and only recently one can detect the changes which were generated as a result of the introduction of technology. Furthermore, the direct impact which text books have on the pupils' formulation is well known. Consequently, independent mathematical writing of the learners should be viewed as a type of creation which will sometimes appear as original and sometimes as an imitation of what is written in the books.

Teaching of gifted pupils

The characterization of gifted and talented pupils has various definitions (Tannenbaum, 1983; Renzuli, 1978; 2006). The steering committee of the Ministry of Education determines that "gifted" are the upper percentile of every year's population in each of the "giftedness" areas, provided they comply also with the criteria of motivation and creativity. Nevertheless, for unique programmes, the Ministry of Education identifies every year a wide variety of competent pupils, not necessarily those who meet all the giftedness criteria.

In order to teach mathematics to talented pupils, teachers should have a special training. Moreover, teaching geometry to this population requires acquisition of teaching methods and contents adjusted to them.

In their study, Shore & Kanevsky (1993) focused on thinking processes of these pupils while solving problems. The researchers enumerate the following characteristics: a. effective use of memory; b. effective use of thinking about thinking; c. speed of thinking; d. ability to classify and analyze problems; e. ability to look for and process information in an organized manner; f. flexibility in using solving strategies; g. preference of challenge problems.

Gifted pupils do not always think differently than regular pupils (Hershkovitz, Peled & Littler, 2009) However, the instructing teachers should know that their thinking is faster and they are capable of processing information much more rapidly, even when abstract or complex terms are concerned. These pupils tend to approach problems in special way, sometimes because they see a new way of completing the task or a relation to another process which is not immediate or algorithmically familiar. Moreover, gifted pupils need opportunities for creation and independence and mainly for additional challenging work of another type.

The van Hiele theory as a tool for determining levels of thinking and as a source of activities

The van Hiele theory (1987) engages in the development of geometrical thinking and it has been prevalent for more than five decades among mathematical education researchers. In 1959 the five levels of thinking of the theory were published in a hierarchical order. However today, it is customary to relate to four levels only: recognition or visualization, analysis or description, order or information deduction and formal deduction and rigor. Each of these stages has clear characteristics and even different-level competences. The competences are: visual, verbal, drawing, logical and applied. In recent years the van Hiele theory has been adopted also in other branches of mathematics, e.g. algebra or space geometry.

The clear hierarchical structure of the theory enables identification of the level at which pupils are, according to the level of question presented to them.

Along the lines of the theory, one can design learning activities which are adjusted to the learner population and in particular to gifted pupils. Alternately, as things have been structured in this study, the theory serves as a tool for identifying the level of the question composed by the pupils themselves.

METHODOLOGY

The aim of the study

The aim of the study was to investigate the ability of gifted pupils to define geometry terms, to write a word problems. To find the impact of word problems in geometry books on the formulation of the problems.

Research questions

- To what extent are gifted pupils capable of defining selected geometry terms?
- To what extent can gifted pupils formulate a word problem for a given geometry term?
- At what level (according to van Hiele) are the problems formulated by the pupils?

Research population

The research population comprised 58 "Gordon Club" (Gordon Center" – a centre for gifted 4th -6th graders at Gordon Academic College of Education, Haifa) pupils learning at the 4th-6th grades, 32 boys and 26 girls. Studies at "Gordon Club" are conducted once a week in the afternoon in a variety of extra-curricular courses. The pupils who responded to the questions have chosen to learn in the "Developing Mathematical Thinking" course. At that course they are exposed to a different, non-conventional way of seeing mathematics. Activities are based upon research methods and drawing conclusions, reveals the beauty of math and the connection between math and the daily world. The questions were circulated towards the end of the school year of studies in the course.

Research tools

In the first part of the questionnaire, the pupils were asked to define eight mathematical terms. All the terms are familiar to the 4th-6th graders from their geometry studies at school and at the "GordonClub" course.

The selected terms were: parallel lines, angle, circumference, rhombus, circle, tangent, diagonal and cube. In the second part of the questionnaire, the pupils were requested to formulate a word problem for each of the terms.

Data analysis method

Table 1: Example of the quantitative data analysis method for defining terms and formulating problems

	Term definition			Word problem		
The geometry term	No. of pupils who defined **correctly**	No. of pupils who defined **in correctly**	No. of pupils who **did not define at all**	No. of pupils who **formulated** a problem for the term	No. of pupils who **did not formulate** a problem for the term	No. of pupils who formulated **an unsuitable problem**
Angle						

Table 2: Classification of problems according to level

	No. of problems for the term according to level		
The geometry term	**Problem at level 1**	**Problem at level 2**	**Problem at level 3**
Angle			

Level 1: Recognition or visualization: Identifying geometric shapes and distinguishes between them. Each of the concepts or the shapes is perceived as a whole, in the way it is seen.

Level 2: Analysis or description: Analyzing properties of shapes but not to attribute properties of a particular item to the properties of the group to which it belongs

Level 3: Order or informal deduction: Identifying a hierarchical order of connection between groups of different shapes according to their properties and definitions. But not proving claims related to the properties of the geometric shapes (Patkin, 2014).

FINDINGS

Ability to conceive a definition

The first part of the research findings presents the pupils' extent of ability to conceive a definition for each of the eight terms, namely, how many correct definitions were obtained for all the eight terms.

The 58 questionnaires should have yielded 468 definitions (8x58). The findings illustrate that only 40% of the total conceived definitions were correct. 31% of the definitions were incorrect and 29% of the terms for which no definitions were conceived at all.

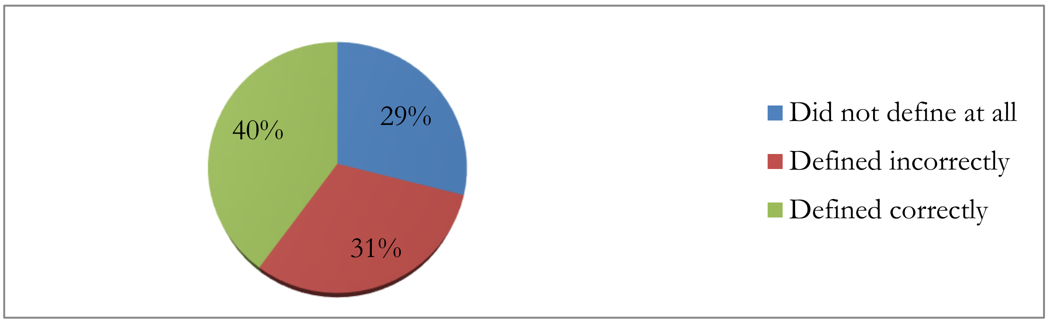

Figure 1: Classification of the definition (in percentage) for the entirety of the terms.

Specification of the ability to conceive a definition for each of the eight terms is presented in Figure 2.

Among the findings, there is one prominent term which most of the pupils encountered difficulties to define it (93.3%), i.e. the tangent. More than 50% of the pupils gave correct definitions for four terms: parallel lines, angle, circumference and rhombus. Very familiar terms such as diagonal and cube were unexpectedly not included in the previous group and were not defined by a larger number of pupils. These two terms (diagonal and cube) were defined correctly only by approximately 30% of the pupils.

Half of the pupils defined incorrectly two terms which are well known at elementary school, namely circle and diagonal. About 30% of the pupils gave incorrect definitions to the other terms.

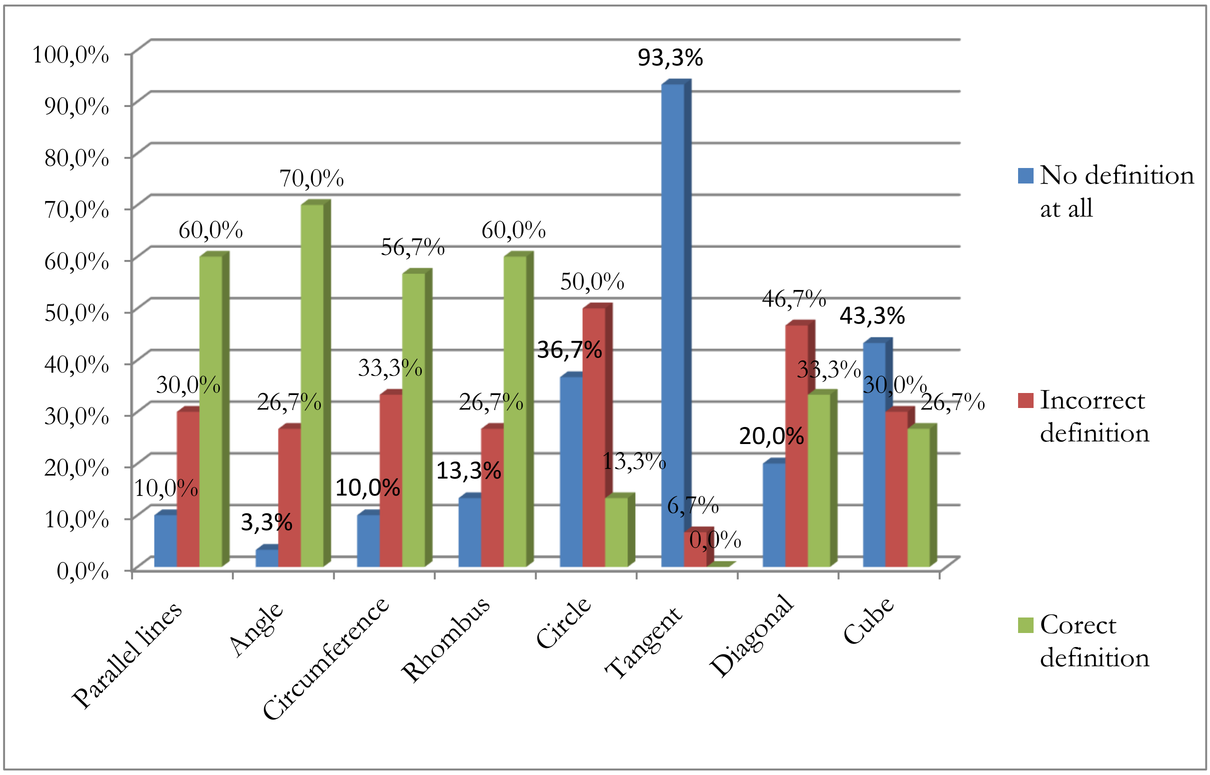

Figure 2 – Classification of the pupils' definition (in percentage) for each of the terms

Ability to formulate a word problem for a given term

The research findings related to the ability to formulate a word problem indicate that 52% of the entire pupil population could formulate a suitable word problem for each of the terms. This concerns a word problem which is formulated so it can be examined according to van Hiele levels of thinking. Nevertheless, the other half of the participants did not formulate suitable word problems or failed to formulate at all as shown in Figure 3.

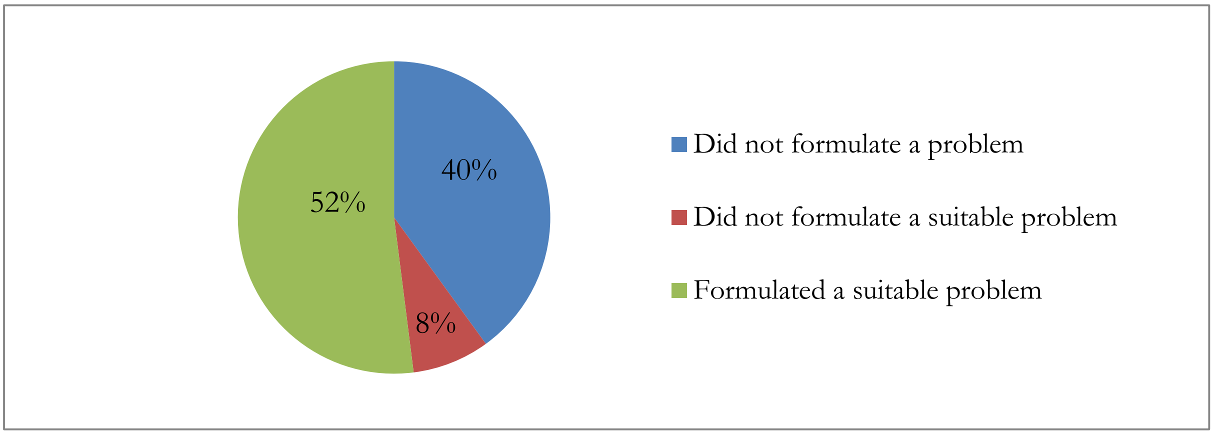

Figure 3. The Ability (in percentage) to formulate a word problem

A thorough examination of the ability to formulate a word problem for each of the terms shows that the term tangent was prominent once more. Similarly to the difficulty to conceive a definition for a term, here too the pupils found it difficult to formulate a word problem for this term.

In the case of the four terms – parallel lines, angle, circumference and rhombus – which were correctly defined by a large number of pupils, the findings show that a suitable word problem was also formulated for these terms.

About half of the pupils encountered a difficulty in formulating a word problem for the cube, diagonal and circle. This finding illustrates that some pupils could formulate geometric problems even if they were unable to conceive a definition for the term. For example, for the term diagonal, 43% of the pupils formulated a word problem but only 33% defined correctly the term. For the term circle, 33% of the pupils formulated suitable word problems but only 13% of them conceived a correct definition.

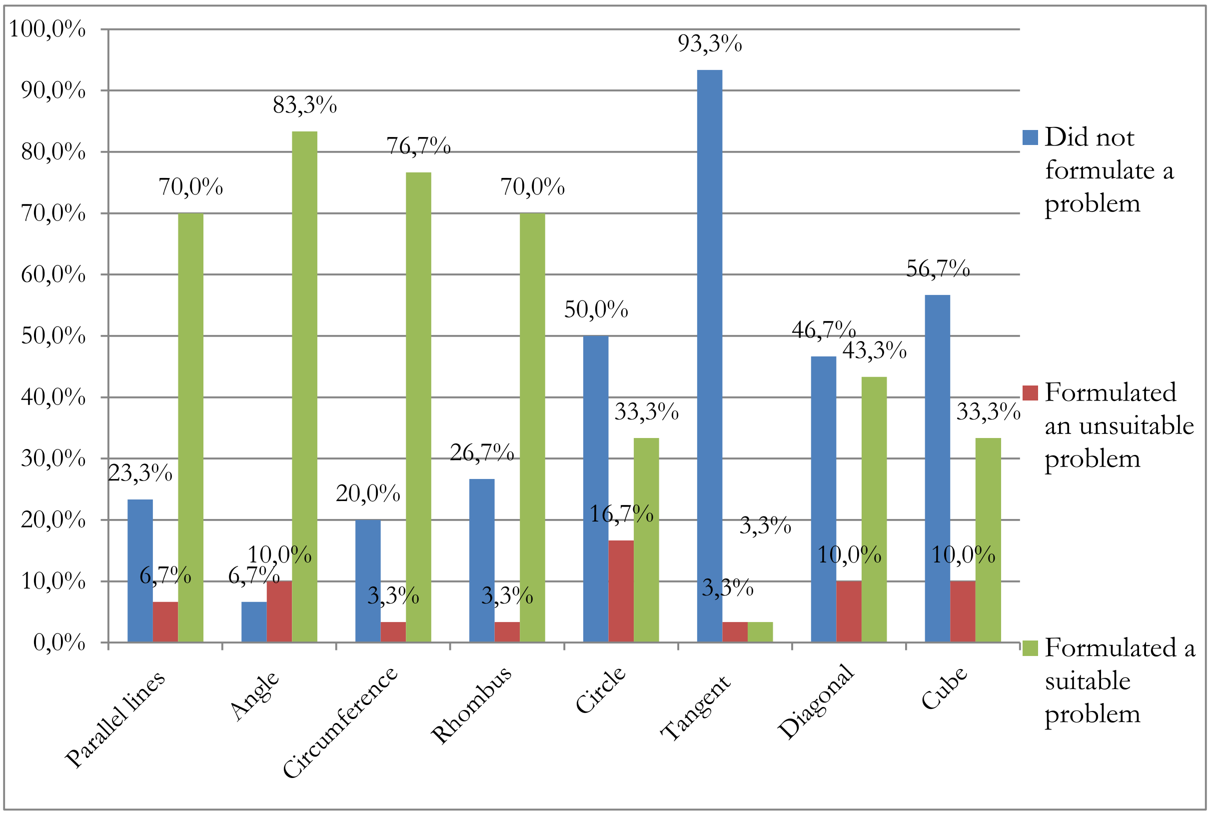

Figure 4 – Classification of word problems (in percentage) according to the given term

Analysis of the level of questions formulated by the pupils

Out of the total number of word problems formulated for the eight terms, the findings indicate 236 suitable problems which can be classified according to van Hiele levels, as presented by Figure 5.

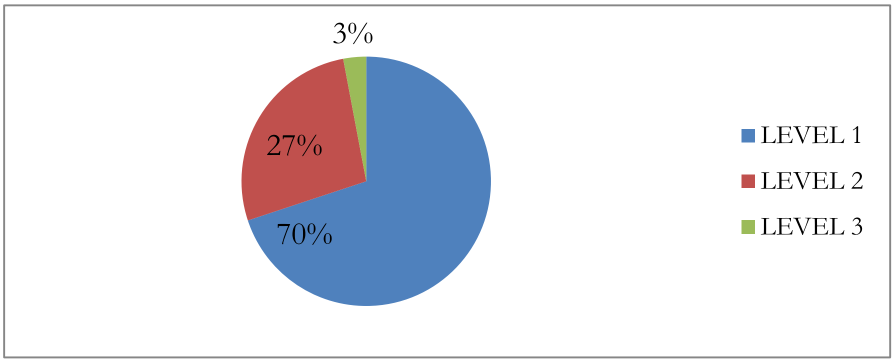

Figure 5: Classification of word problems (in percentage) according to van Hiele levels

Examination of the question level according to van Hiele levels indicated that most of the questions were formulated at the first level (70%). The findings show that 27% of the problems were at the second level and only 3% of the total formulated problems were at the third level. As could be anticipated in light of the pupils' age and their knowledge of geometry, no word problems were found at the fourth level.

Terms which the pupils were able to define, also received the maximum number of problems. However, most of them were at the first level according to van Hiele levels. The term circumference was the most prominent, getting more word problems at the third level. Although the pupils were well acquainted with most of the terms and they even solved in class word problems which included these terms, the problems they formulated were at the first level.

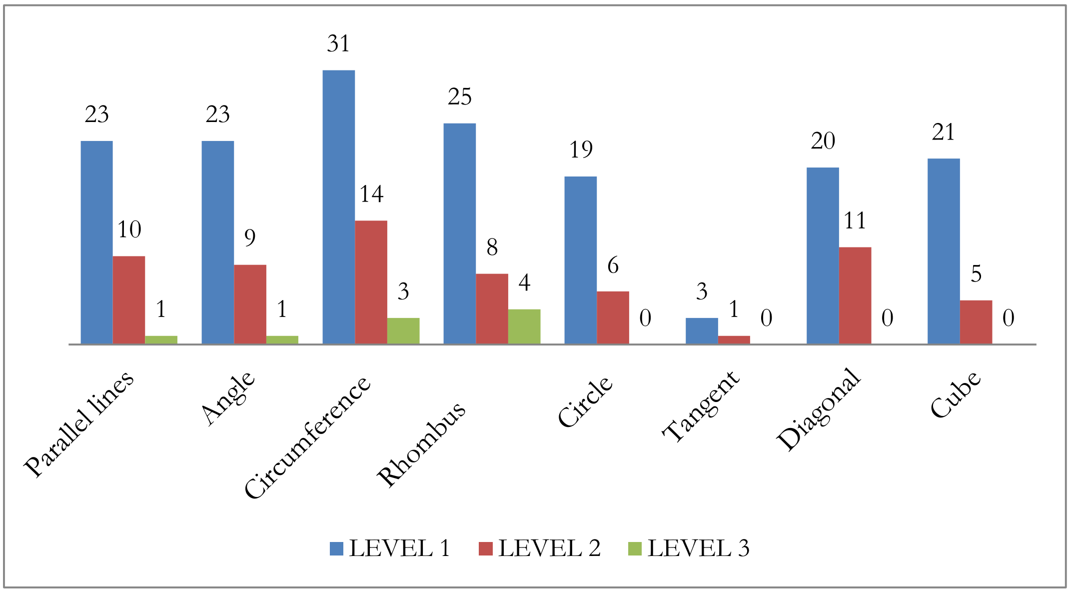

Figure 6: Number of formulated problems for each term according to van Hiele levels

Examples of problems according to Van Hiele levels of thinking, as formulated by the pupils

Example 1: Problem for the term diagonal – at the first level

A word problem in which the term diagonal is included in the problem content. Indicate the diagonals.

Figure 7. Example of problem for the term diagonal – at the first level

Example 2: Problem for the term angle – at the first level

A word problem in which the term angle is included in the problem content.
Indicate the diagonals.

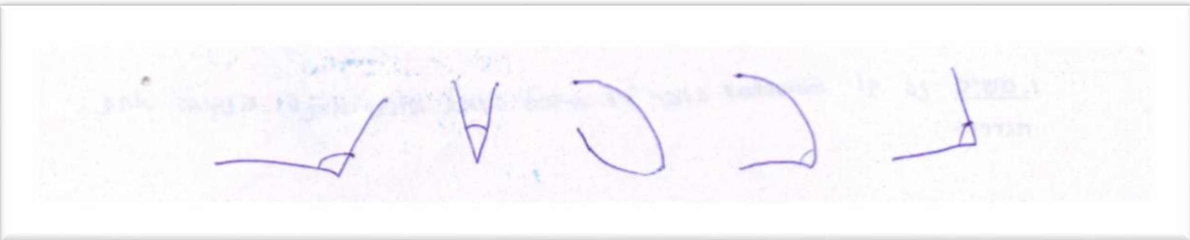

Figure 8. Example of problem for the term angle – at the first level

Example 3: Problem for the term circumference – at the second level
A word problem in which the term <u>circumference</u> is included in the problem content.

In a rectangle ABCD
AB = CD = 10
BC = 6
Calculate the circumference of the rectangle

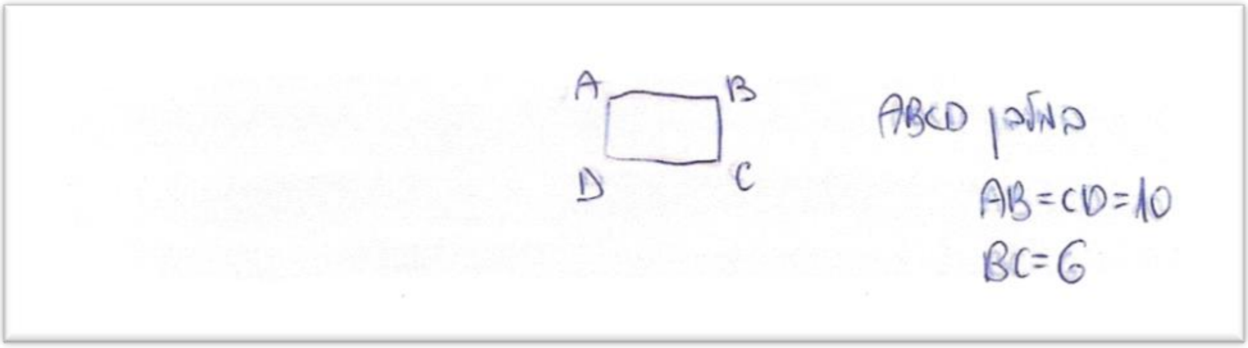

Figure 9. Example of problem for the term circumference – at the second l

Example 4: Problem for the term rhombus – at the third level
A word problem in which the term <u>rhombus</u> is included in the problem content.
Is a rhombus a square? Please explain why it is or why it is not.

Example 5: Problem for the term diagonal – at the second-third level (knowledge of the Pythagorean Theorem is required)
A word problem in which the term <u>diagonal</u> is included in the problem content.
A rectangle ABCD is given and its diagonal and side are known. Calculate the circumference of the rectangle.

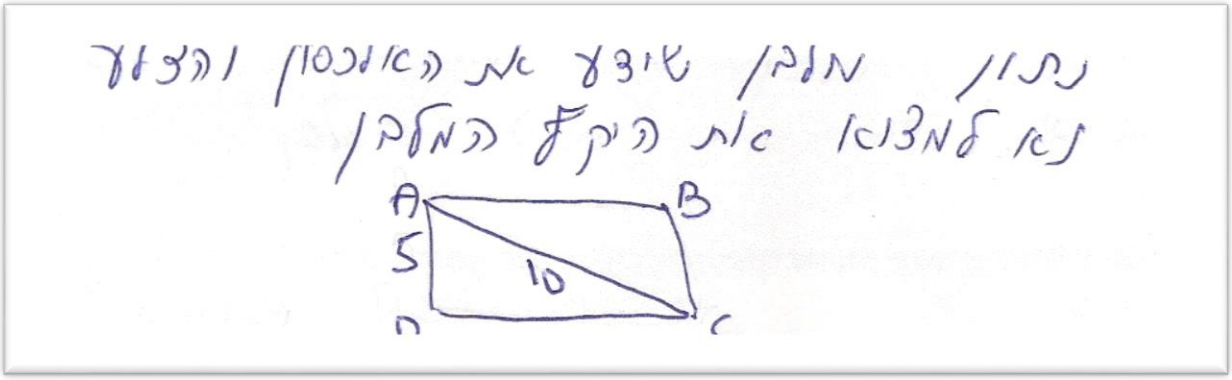

Figure 10. Example of problem for the term diagonal – at the second-third level

The mathematical language in word problems

Examination of the questions` formulation gave rise to another issue which initially we did not aim to explore, namely the use of exceptional formulations which differ from the familiar ones in the textbooks. In addition to faults in the mathematical language and formulation of problems with no direct relation to the term, the findings showed also problems or definitions of terms which presented a formulation and thinking ability which were not common

The research aim was not to investigate the mathematical language or to engage in mathematical literacy. However, there is room to present these two aspects through the "exceptional" problems.

Problems for the term parallel lines:

1. Danny drew a line and above it another line. At the middle between them, the distance was 7. On the left side 5 cms and on the right side 10 cms. Are the lines parallel?
2. A train is travelling from Haifa to Tel Aviv on track A. A second train is travelling from Tel Aviv to Haifa on track B. The two trains leave their starting point at the same time. A traffic accident will happen if the trains collide. Are the tracks parallel?

Problems for the term cube:

3. Can we build a cube of 5 identical squares and another big square?
4. You have at your disposal a cardboard, scissors and glue. Use them to build a cube whose dimensions are 2 cms.

Problem for the term tangent:

5. Prove that the tangent to a circle is parallel to the radius.

Problem for the term circumference:

6. Calculate the circumference of a hexagon if we know that its sides are 7 cms, 8 cms, 6 cms, 5 cms, 4 cms and 3 cms.

Problem for the term angle:

7. In an isosceles triangle the head angle is 30º and the two sides which are adjacent to the angle are 12 cms long. Calculate the basis angle.

Problem for the term rhombus:

8. A polygon with 4 sides is given. The diagonals are perpendicular to each other and all the sides are equal. Prove that the polygon is a rhombus.

Exceptional definition of the term tangent:

9. "Like wheels and train rails".

DISCUSSION and CONCLUSIONS

This study aimed to explore to what extent gifted pupils are capable of defining selected geometry terms and formulating a word problem for each of the terms. To date, there has been very little research on mathematics conceptual understanding and mathematical thinking for talented elementary pupils.

Regarding the first research question "To what extent are gifted pupils capable of defining selected geometry terms?", the research findings illustrate a medium level of ability to provide formal definitions of geometry terms. About half of the pupils encountered difficulties in conceiving definitions or wrote incorrect ones. Hence, the emphasis on giving formal definitions in the 4th – 6th grades is not full. Moreover, one can assume that the younger pupils, the 4th graders, had more difficulties than the older pupils in writing the definition because they have not yet studied it.

Elementary education does not require memorization of definitions in geometry lessons. Nevertheless, the research population consists of gifted pupils who are able not only to memorize definitions but also to understand them thoroughly.

Terms which the pupils did define correctly, such as the term angle, were those terms which are included in the textbooks and it is customary to revise their definition again and again.

Formulating word problems is not an activity which is prevalent in most geometry lessons. The questions are usually formulated by the teacher or presented in the textbook. This type of activity is highly important for developing both mathematical thinking and mathematical literacy.

Most of the word problems which were written were based on the pupils' previous experience. Moreover, some pupils opted not to formulate a word problem but to present a drawing with only a question as they had seen in the textbooks.

Many written problems were at the first and easy level according to van Hiele levels. Only about one third of the formulated problems were at the second level and few problems at the third level. Please note that only half of the written problems were suitable to and formulated in such a way which facilitated classification by levels. That is, for many terms no suitable problem was written at all or the problem formulated was incorrect.

Only a small number of participants demonstrated creativity and did not cite questions which were similar to those appearing in the textbooks. It was to be expected that gifted pupils would manifest more creativity and formulation ability at a higher level. However, this finding is not surprising since a considerable number of the pupils experienced formulation of geometry questions for the first time. So its time to adopt the way of teaching in project M^3 (Mentoring, Mathematical Minds) and to ask pupils to investigate "one problem per day" (Sheffield, 2009).

Based on the fact that we are dealing with a population of gifted and talented pupils who have above average knowledge and learning ability, the research findings do not reflect at all the population of regular school pupils. Consequently, geometry teaching at school must engage in much more than just presentation of geometric shapes or in measurements. Challenging in mathematics learning not only by via problem solving but with connecting tasks and selected problems with multiple solutions.(Taylor,2009).

Diversified, enriching and challenging activities, including development of literacy capability, have to be part of the learning. They should be presented to all the pupil population in general and to gifted and talented pupils in particular. Great importance should be attributed to the use of varied enrichment sources rather than just the familiar textbooks as an exclusive and standard model for the way of formulating the questions.

To sum up, for enhancing and developing geometry thinking of the gifted pupil population, diversified and exceptional teaching approaches are necessary. Consequently, recommendations of this study should impact the teaching methods of geometry teachers as well as the teacher education programmes for gifted and talented pupils.

REFERENCES

Alcock, L., & Simpson, A. (2002). Definitions: Dealing with Categories Mathematically. *For the Learning of Mathematics 22(2), 28-34.*

Cifarelli, V. (1999), Abductive inference: connections between problem posing and solving. *Proceedings of the 23th PME,* Haifa, 2, 217-224.

English, L. N. (1998). Children's problem posing within formal and informal contexts. *Journal for Research in Mathematics Education, 29(1), 83-107.*

Greer, B. (1993). The mathematical modeling perspective on wor(l)d problems. *Journal of Mathematical Behavior, 12, 239-250).*

Hershkovitz, S., & Nesher, P. (2003).The role of schemes in solving word problems. *The Mathematics Educator, 7*(2), 1-24

Hershkovitz, S., Peled, I., & Littler, G.(2009).Mathematical Creativity and giftedness in elementary school :Task and teacher promoting creativity for all. In *: Creativity in mathematics education of gifted students*(255-271).Edited by: Leikin, Berman, Koichu. Sense publishers,

Hiebert, J. (1986). Conceptual and procedural knowledge: The Case on Mathematics. Hillsdale, NJ: Erlbaum.

Levenberg, I., & Ophir, S. (2006). Is it possible to tell about numbers? *Reflection and Research in Teacher education*, 10, 1-7. [Hebrew]

Morgan, C.(1996). Language and assessment issues in mathematics education. *Proceeding of the 20th conference of PME.* Valencia, Spain, 4, 19-25.

Mountwietten, M., & Vinner, S. (1995). Processes of acquiring mathematical terms by means of definitions and examples among elementary school pupils. *Theory and Practice in Studies Planning*, 10, 149-161. [Hebrew]

Murray, H.(2012). Problems with Word Problems in Mathematics. *Learning and Teaching Mathematics,* 13, 55-58.

National Council of Teachers of Mathematics (2000).*Principles and Standards for School Mathematics*. Reston, Va.: NCTM.

Patkin, D. (2014). Global van Hiele (GVH) Questionnaire as a tool for mapping knowledge and understanding of plane and solid geometry. *Journal of the Korean Society of Mathematical Education. Series D: Research in Mathematical Education,* 18(2), 103-128.

Patkin, D., & Levenberg, I. (2012) Geometry from the world around us. *Learning and Teaching Mathematics. A Journal of AMESA,* 13, p.14-18.

Renzulli, J. S. (1978). What makes giftedness? Reexamining a definition. *Phi Delta Kappan,* 60, 180-184, 261.

Renzulli, J. S., Koehler, J. & Fogarty, E. (2006)."Hound tooth operation intervention theory". *Gifted child today, 29,15-24.*

Sheffield, L. J. (2009).Developing mathematical creativity-questions may be the answer. In*: Creativity in mathematics education of gifted students*(72-85).Edited by: Leikin, Berman, Koichu. Sense publishers,

Shir, K., (2004). *Learning of a super-term in mathematics: The case of the term mathematical definition.* Ph.D. thesis. Haifa: Technion – Technological Institute of Israel.[Hebrew]

Shir, K., & Zaslavski, A. (2003).*The way pupils perceive the role of mathematical definitions.* A paper presented at the Hemi conference – the Organisation for the promotion of mathematical education in Israel, Department of Technology and Sciences Teaching, Technion – Technological Institute of Israel, Haifa, 20 May 2003.

Silver, E. (1996).An analysis of arithmetic problem posing by middle school students. *Journal for Research in Mathematics Education,* 27(5), 521–539.

Shore, B. M., & Kanevsky, L. (1993). Thinking processes: Being and becoming gifted. In K. A. Heller, F. J. Monks, & A. H. Passow (Eds.), *International Handbook for Research and Development on Giftedness and Talent* (pp. 133-148). London: Pergamon.

Skemp, R. (1997) Relational understanding and instrumental understanding. *Mathematics teaching,* 77,20-26.

Tannenbaum, A.J. (1983).Gifted children: *Psychological and emotional perspectives*. New York: Macmillan.

Taylor, P.(2009)Challenge in mathematics learning-where to from here? In *: Creativity in mathematics education of gifted students*(72-85).Edited by:Leikin, Berman, Koichu. Sense publishers,

Usiskin, Z.(1996**).**Mathematics as a Language .In *Communication in Mathematics, K–12 and Beyond, 1996 Yearbook of the National Council of Teachers of Mathematics* (NCTM).

Van Hiele, P.M.(1987. *Van-Hiele Levels, A Method to Facilitate the Finding of Levels of Thinking in Geometry* by *Using the Levels in Arithmetic*. Paper Presented at the Conference on Learning and Teaching Geometry: Issues for Research and Practice. Syracuse University.

Van Hiele, P. M. (1999). Developing geometric thinking through activities that begin with play. *Teaching Children Mathematic*, 5 (6), February 1999, 310-316.

Vinner, S. (1991).The role of definitions in teaching and learning of mathematics. *Advanced Mathematical Thinking*, 5, 65-81.

Yelin, K., &Zaslavski, A. (2000).*Perception of pre-service teachers of the concept mathematical definition*. A paper presented at IALA Conference – Israeli Association for the Research of Education, 12. Tel Aviv: Tel Aviv University, 25 October 2000. [Hebrew]

APPENDICES

Appendix No. 1: Example of a pupils' questionnaire

Assignments in geometry.

Below are several geometry terms.

a. Write a definition for each of the terms.
b. Compose a word problem (you can also attach a drawing) whereby the indicated term is included in the problem content.

1. Parallel lines

a. Definition –

b. A word problem whereby the term parallel lines is included in the problem content.