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The Differential Transform Method For Solving One-Dimensional Burger's Equation and K(m,p,1) Equation

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Abstract. In this paper, a differential transform method (DTM) has been applied to solve one-dimensional Burger's and K(m,p,1) equations with initial conditions and exact solutions have been obtained as same as [1-5]. The results show that DTM has got many merits and much more advantages and it is also a powerful mathematical tool for solving partial differential equations having wide applications in engineering and physics.

Key words: Two-dimensional differential transform method, one-dimensional Burger's equation, K(2,2,1) equation, K(3,3,1) equation.

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1. Introduction

Most scientific problems in physics and other fields such as biology, chemistry, mechanics, etc., are modeled by nonlinear partial differential equations. Except a limited number of these problems, most of them do not have any analytical solution. Some of them are solved by using numerical techniques and some by the analytical perturbation method. In recent years, some researchers used many powerful methods for obtaining exact solutions of nonlinear partial differential equations, such as inverse scattering method [6], Hirota's bilinear method [7], Backlaund transformation [8], Painleve expansion [9], sine-cosine method [10], homogenous balance method [11], homotopy perturbation method [12], variational method [13], asymptotic methods [14], nonperturbative methods [15], Adomian Pade approximation [16], improved tanh function method [17], Jacobi elliptic function expansion method [18], F- expansion method [19], Weierstrass semi-rational expansion method [20] and so on.

The differential transform method (namely DTM) was first introduced by Pukhov et al. [21] who solved linear and nonlinear initial value problems in electric circuit analysis. It is a semi-numerical and semi-analytic technique that formulizes Taylor series in totally different manner. With this technique, the given differential equation and its related initial conditions are transformed into a recurrence equation that finally leads to the solution of a system of algebraic equations as coefficients of a power series solution. This method is useful to obtain the exact and approximate solutions of linear and nonlinear differential equations. No need to linearization

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or discretization, large computational work and round-off errors are avoided [22]. Recently, this methods has been successfully applied to solve many types of nonlinear problems in science and engineering by many authors ([23]-[30]).

The aim of this paper is to extend the differential transform method to obtain exact solutions to the nonlinear dispersive one-dimensional Burger's and K(m,p,1) equation equation with initial condition, respectively:

$$u_t + u_x = vu_{xx} \tag{1.1}$$

$$u_t + (u^m)_x - (u^p)_{xxx} + u_{5x} = 0, \quad m > 1, \quad 1 \leq p \leq 3 \tag{1.2}$$

$$u(x,0) = f(x)$$

where $v > 0$ is the coefficient of kinematics.

In this letter, the basic idea of the DTM is introduced and then its applications in one-dimensional Burger's, K(2,2,1) and K(3,3,1) equations are studied for initial conditions. Closed form solutions are obtained as same as ([1]-[5]).

2. Two-Dimensional Differential Transform Method

The basic definitions and fundamental operations of the two-dimensional differential transform are defined in ([27]-[33]) as follows:

If function $u(x,y)$ is analytic and differentiated continuously with respect to x and y in the domain of interest, then let

$$U(k,h) = \frac{1}{k!h!}\left[\frac{\partial^{k+h}u(x,y)}{\partial x^k \partial y^h}\right]_{\substack{x=0\\y=0}} \tag{2.1}$$

where the spectrum $U(k,h)$ is the transformed function,which is also called T-function in brief. In this paper, the lowercase $u(x,y)$ represent the original function while the uppercase $U(k,h)$ stand for the transformed function (T-function).

The differential inverse transform of $U(k,h)$ is defined as follow:

$$u(x,y) = \sum_{k=0}^{\infty}\sum_{h=0}^{\infty} U(k,h)x^k y^h \tag{2.2}$$

Combining (2.1) and (2.2), it can be obtained that

$$u(x,y) = \sum_{k=0}^{\infty}\sum_{h=0}^{\infty} \frac{1}{k!h!}\left[\frac{\partial^{k+h}u(x,y)}{\partial x^k \partial y^h}\right]_{\substack{x=0\\y=0}} x^k y^h \tag{2.3}$$

In real applications, the function $u(x,y)$ is estimated by a finite number of terms of Eq. (2.2) or Eq. (2.3). Hence, Eq. (2.2) can be written as follow:

$$u(x,y) = \sum_{k=0}^{n}\sum_{h=0}^{m} U(k,h)x^k y^h. \tag{2.4}$$

Usually, the values of n and m are decided by convergency of the series coefficients.

The fundamental operations of two-dimensional differential transform method are listed in Table 1.

Original Function	Transformed Function
$w(x,y)=u(x,y)\pm v(x,y)$	$W(k,h)=U(k,h)\pm V(k,h)$.
$w(x,y)=\lambda u(x,y)$	$W(k,h)=\lambda U(k,h)$,(λ is a constant).
$w(x,y)=\frac{\partial u(x,y)}{\partial x}$	$W(k,h)=(k+1)U(k+1,h)$
$w(x,y)=\frac{\partial u(x,y)}{\partial y}$,	$W(k,h)=(h+1)U(k,h+1)$.
$w(x,y)=\frac{\partial^{r+s}u(x,y)}{\partial x^r \partial y^s}$	$W(k,h)=(k+1)(k+2)...(k+r)$ $(h+1)(h+2)...(h+s)U(k+r,h+s)$.
$w(x,y)=x^m y^n$	$W(k,h)=\delta(k-m,h-n)=\delta(k-m)\delta(h-n)$ $\delta(k-m,h-n)=\begin{cases}1, \text{ k=m and h=n}\\ 0, \text{ otherwise}\end{cases}$
$w(x,y)=u(x,y)v(x,y)$	$W(k,h)=\sum_{r=0}^{k}\sum_{s=0}^{h}U(r,h-s)V(k-r,s)$
$w(x,y)=u(x,y)v(x,y)\frac{\partial c^2(x,y)}{\partial x^2}$	$W(k,h)=\sum_{r=0}^{k}\sum_{t=0}^{k-r}\sum_{s=0}^{h}\sum_{p=0}^{h-s}(k-r-t+2)(k-r-t+1)$ $U(r,h-s-p)V(t,s)C(k-r-t+2,p)$
$w(x,y)=\frac{\partial u(x,y)}{\partial x}\frac{\partial v(x,y)}{\partial x}$	$W(k,h)=\sum_{r=0}^{k}\sum_{s=0}^{h}(r+1)(k-r+1)U(r+1,h-s)V(k-r+1,s)$
$w(x,y)=u(x,y)\frac{\partial v^2(x,y)}{\partial x^2}$	$W(k,h)=\sum_{r=0}^{k}\sum_{s=0}^{h}(k-r+2)(k-r+1)U(r,h-s)V(k-r+2,s)$.
$w(x,y)=u(x,y)v(x,y)q(x,y)$	$W(k,h)=\sum_{r=0}^{k}\sum_{t=0}^{k-r}\sum_{s=0}^{h}\sum_{p=0}^{h-s}U(r,h-s-p)V(t,s)Q(k-r-t,p)$

Table 1: The operations for the two-dimensional differential transform method

3. Differential Transform Method for One-Dimensional Burger's Equation

3.1. Here we consider the solution of Eq. (1.1) with the initial condition and boundary conditions as follows,

$$u(x,0)=u_0(x)=u_0\tan\frac{\pi}{l}x \tag{3.1}$$

$$u(0,t)=u(l,t)=0. \tag{3.2}$$

where $v>0$ is the coefficient of kinematic viscosity [1].

Taking two-dimensional transform of Eq. (1.1) by using the related definition Table 1, we have

$$(h+1)U(k,h+1)+\sum_{r=0}^{k}\sum_{s=0}^{h}(k-r+1)U(r,h-s)U(k-r+1,s)=$$
$$v(k+1)(k+2)U(k+2,h) \tag{3.3}$$

By using the series form of Eq. (3.1), some of the initial transformation coefficients $U(k,0)$ are listed in Table 2.

$U(1,0) = \frac{\left(\frac{2u_0\pi^2}{l^2}-\frac{u_0^2\pi}{l}\right)^2\pi}{2l}$	$U(3,0) = \frac{7\left(\frac{2u_0\pi^2}{l^2}-\frac{u_0^2\pi}{l}\right)^2\pi^3}{6l^3}$
$U(5,0) = \frac{47\left(\frac{2u_0\pi^2}{l^2}-\frac{u_0^2\pi}{l}\right)^2\pi^5}{30l^5}$	$U(7,0) = \frac{502\left(\frac{2u_0\pi^2}{l^2}-\frac{u_0^2\pi}{l}\right)^2\pi^7}{315l^7}$
$U(9,0) = \frac{553\left(\frac{2u_0\pi^2}{l^2}-\frac{u_0^2\pi}{l}\right)^2\pi^9}{405l^9}$	$U(11,0) = \frac{324419\left(\frac{2u_0\pi^2}{l^2}-\frac{u_0^2\pi}{l}\right)^2\pi^{11}}{31150l^{11}}$
$U(k,0) = 0$ if $k = 0,2,4,...$	$U(11,0) = \frac{324419\left(\frac{2u_0\pi^2}{l^2}-\frac{u_0^2\pi}{l}\right)^2\pi^{11}}{31150l^{11}}$

Table 2: Some values of U(k,0) of Ex 2.1.

By substituting $U(k,0)$ values in Table 2 into Eq. (3.3), we obtain some values of $U(k,h)$ in Table 3.

$U(1,1) = \left(\frac{2u_0\pi^3t}{l^3} - \frac{u_0\pi^3t}{l^2}\right)$	$U(3,1) = \left(\frac{8u_0\pi^5t}{3l^5} - \frac{4u_0^2\pi^4t}{3l^4}\right)$
$U(5,1) = \left(\frac{34u_0\pi^7t}{15l^7} - \frac{17u_0^2\pi^6t}{15l^6}\right)$	$U(7,1) = \left(\frac{496u_0\pi^9t}{315l^9} - \frac{248u_0^2\pi^8t}{315l^8}\right)$
$U(9,1) = \left(\frac{276u_0\pi^{11}t}{2835l^{11}} - \frac{1382u_0^2\pi^{10}t}{2835l^{10}}\right)$	$U(k,1) = 0$ if $k = 0,2,4,...$

Table 3: Some values of U(k,1) of Ex 2.1.

Hence, substituting sufficient number of computed $U(k,h)$ values into Eq. (2.2), we have series solution as follow:

$$
\begin{aligned}
u(x,t) = \{ & \frac{\left(\frac{2u_0\pi^2}{l^2}-\frac{u_0^2\pi}{l}\right)^2\pi}{2l}x + \frac{7\left(\frac{2u_0\pi^2}{l^2}-\frac{u_0^2\pi}{l}\right)^2\pi^3}{6l^3}x^3 \\
& + \frac{47\left(\frac{2u_0\pi^2}{l^2}-\frac{u_0^2\pi}{l}\right)^2\pi^5}{30l^5}x^5 + \frac{502\left(\frac{2u_0\pi^2}{l^2}-\frac{u_0^2\pi}{l}\right)^2\pi^7}{315l^7}x^7 \\
& + \frac{553\left(\frac{2u_0\pi^2}{l^2}-\frac{u_0^2\pi}{l}\right)^2\pi^9}{405l^9}x^9 + \frac{324419\left(\frac{2u_0\pi^2}{l^2}-\frac{u_0^2\pi}{l}\right)^2\pi^{11}}{31150l^{11}}x^{11} + ...\} \\
& + \{\left(\frac{2u_0\pi^3t}{l^3} - \frac{u_0\pi^3t}{l^2}\right)xt + \left(\frac{8u_0\pi^5t}{3l^5} - \frac{4u_0^2\pi^4t}{3l^4}\right)x^3t \\
& + \left(\frac{34u_0\pi^7t}{15l^7} - \frac{17u_0^2\pi^6t}{15l^6}\right)x^5t + \left(\frac{496u_0\pi^9t}{315l^9} - \frac{248u_0^2\pi^8t}{315l^8}\right)x^7t \\
& + \left(\frac{276u_0\pi^{11}t}{2835l^{11}} - \frac{1382u_0^2\pi^{10}t}{2835l^{10}}\right)x^9t + ...\} + ...
\end{aligned} \tag{3.4}
$$

The closed form of the first curly bracket is $u_0 \tan \frac{\pi}{l}x$, the closed form of the second curly bracket is $(\frac{2u_0\pi^2}{l^2} - \frac{u_0^2\pi}{l})\sec^2(\frac{\pi}{l}x)\tan(\frac{\pi}{l}x)t$, and so on.

Letting $(\frac{2u_0\pi^2}{l^2} - \frac{u_0^2\pi}{l}) = C$ then Eq. (3.4) can be written as

$$\begin{aligned} u(x,t) &= u_0 \tan\frac{\pi}{l}x + C\sec^2(\frac{\pi}{l}x)\tan(\frac{\pi}{l}x)t + C\sec^4(\frac{\pi}{l}x)\tan(\frac{\pi}{l}x)\frac{t^2}{2} + ... \\ &= u_0 \tan\frac{\pi}{l}x\left\{1 + C\sec^2(\frac{\pi}{l}x)t + C\sec^4(\frac{\pi}{l}x)\frac{t^2}{2} + ...\right\} \end{aligned} \tag{3.5}$$

This is also the same result with obtained by decomposition method and in a closed form solution is given by Gorguis [1].

$$u(x,t) = u_0 \tan\frac{\pi}{l}x \exp(C\sec^2(\frac{\pi}{l}x)t) \tag{3.6}$$

3.2 Consider the solution of Eq. (1.1) the initial condition as in [1]

$$u(x,0) = 2x. \tag{3.7}$$

From the initial condition Eq. (3.7), we can write

$$\begin{aligned} &U(k,0) = 0 \text{ if } k = 0,2,3,4,5,6,..., \\ &U(1,0) = 2. \end{aligned} \tag{3.8}$$

For each k, substituting Eq. (3.8) into Eq. (3.3), and by recursive method, the result are listed as follows:

$$\begin{aligned} &U(k,1) = 0 \text{ if } k = 0,2,3,4,..., U(1,1) = -4, \\ &U(k,2) = 0 \text{ if } k = 0,2,3,4,..., U(1,2) = 8, \\ &U(k,3) = 0 \text{ if } k = 0,2,3,4,..., U(1,3) = -16. \end{aligned} \tag{3.9}$$

The rest of the terms of the series have been calculated using Maple. Substituting all $U(k,h)$ into Eq. (2.2), we have series solution as follow:

$$u(x,t) = 2x - 4xt + 8xt^2 - 16xt^3 + 32xt^4 - 64xt^5 + ... \tag{3.10}$$

The exact analytical solution of $u(x,t)$ is given

$$u(x,t) = \frac{2x}{1+2t} \tag{3.11}$$

which is exactly the same as those obtained by the Adomian decomposition method [1].

3.3. We consider Eq. (1.1) with initial condition

$$u(x,0) = \frac{\alpha + \beta + (\beta - \alpha)\exp(\gamma)}{1 + \exp(\gamma)},\ t \geqslant 0 \tag{3.12}$$

where $\gamma = (\alpha/\upsilon)(x - \lambda)$ and the parameters α, β and υ are arbitrary constants.

$U(0,0)=\frac{\alpha+\beta+(\beta-\alpha)\exp(\mu)}{1+\exp(\mu)}$
$U(1,0)=-2\frac{\exp(\mu)\alpha^2}{[1+\exp(\mu)]^2\upsilon}$
$U(2,0)=\frac{\exp(\mu)\alpha^3(-1+\exp(\mu))}{[1+\exp(\mu)]^3\upsilon^2}$
$U(3,0)=\frac{1}{3}\frac{\exp(\mu)\alpha^4(-1+4\exp(\mu)-\exp(2\mu))}{[1+\exp(\mu)]^3\upsilon^2}$
$U(4,0)=\frac{1}{12}\frac{\exp(\mu)\alpha^5(-1+\exp(3\mu)-11\exp(2\mu)+11\exp(\mu))}{[1+\exp(\mu)]^5\upsilon^4}$
$U(5,0)=\frac{1}{60}\frac{\exp(\mu)\alpha^6(-1+26\exp(3\mu)+26\exp(2\mu)-\exp(4\mu)-66\exp(2\mu))}{[1+\exp(\mu)]^6\upsilon^5}$

Table 4: Some values of U(k,0) of Ex 2.3. ($\mu=-\frac{\alpha\lambda}{\upsilon}$)

From the initial condition Eq. (3.12), some of the initial transformation coefficients $U(k,0)$ are listed in Table 4.

Hence, substituting $U(k,0)$ values in Table 4 into Eq. (3.3), and by recursive method, some of the results are listed in Table 5. Substituting all $U(k,h)$ into Eq. (2.2), we have series solution as follow:

$$
\begin{aligned}
u(x,t) = {} & \{\frac{\alpha+\beta+(\beta-\alpha)\exp(\mu)}{1+\exp(\mu)} - 2\frac{\exp(\mu)\alpha^2}{[1+\exp(\mu)]^2\upsilon} + \frac{\exp(\mu)\alpha^3(-1+\exp(\mu))}{[1+\exp(\mu)]^3\upsilon^2}x^2 + ...\} \\
& \{2\frac{\exp(\mu)\alpha^2\beta}{[1+\exp(\mu)]^2\upsilon} - 2\frac{\exp(\mu)\alpha^3\beta(-1+\exp(\mu))}{[1+\exp(\mu)]^3\upsilon^2}xt \\
& -2\frac{\exp(\mu)\alpha^4\beta(1+4\exp(\mu)+\exp(2\mu))}{[1+\exp(\mu)]^4\upsilon^3}x^2t \\
& +\frac{1}{3}\frac{\exp(\mu)\alpha^5\beta(1-\exp(3\mu)+11\exp(2\mu)-11\exp(\mu))}{[1+\exp(\mu)]^5\upsilon^4}x^3t + ...\} \\
& +\{2\frac{\exp(\mu)\alpha^3\beta^2(-1+\exp(\mu))}{[1+\exp(\mu)]^3\upsilon^2}t^2 + \frac{\exp(\mu)\alpha^4\beta^2(-1-4\exp(\mu)+\exp(2\mu))}{[1+\exp(\mu)]^4\upsilon^3}xt^2 \\
& +\frac{\exp(\mu)\alpha^5\beta^2(-1+\exp(3\mu)-11\exp(2\mu)+11\exp(\mu))}{[1+\exp(\mu)]^5\upsilon\upsilon^4}x^2t^2 \\
& +\frac{1}{6}\frac{\exp(\mu)\alpha^6\beta^2(26\exp(\mu)-66\exp(2\mu)+26\exp(3\mu)-\exp(4\mu)-1)}{[1+\exp(\mu)]^6\upsilon^5}x^3t^2 + ...\} \\
& +\{\frac{1}{3}\frac{\exp(\mu)\alpha^4\beta^3(1-4\exp(\mu)+\exp(2\mu))}{[1+\exp(\mu)]^4\upsilon^3}t^3 \\
& +\frac{1}{3}\frac{\exp(\mu)\alpha^5\beta^3(1-\exp(3\mu)+11\exp(2\mu)-11\exp(k))}{[1+\exp(\mu)]^5\upsilon^4}xt^3 \\
& +\frac{1}{6}\frac{\exp(\mu)\alpha^6\beta^3(1-26\exp(2\mu)+66\exp(2\mu)-26\exp(3k)+\exp(4k))}{[1+\exp(\mu)]^6\upsilon^5}x^2t^3 \\
& +\frac{1}{18}\frac{\exp(\mu)\alpha^7\beta^3(1+302\exp(2\mu)-57\exp(\mu)+57\exp(4\mu)-\exp(5\mu)-302\exp(3k))}{[1+\exp(\mu)]^7\upsilon^6} \\
& \quad x^3t^3 + ...\} + ... \qquad (3.13)
\end{aligned}
$$

The closed form of the first curly bracket is $\frac{\alpha+\beta+(\beta-\alpha)\exp(\gamma)}{1+\exp(\gamma)}$, the closed form of

$U(0,1) = 2\frac{\exp(\mu)\alpha^2\beta}{(1+\exp(\mu))^2 v}$
$U(1,1) = -2\frac{\exp(\mu)\alpha^3\beta(-1+\exp(\mu))}{[1+\exp(\mu)]^3 v^2}$
$U(2,1) = -2\frac{\exp(\mu)\alpha^4\beta(1+4\exp(\mu)+\exp(2\mu))}{[1+\exp(\mu)]^4 v^3}$
$U(3,1) = \frac{1}{3}\frac{\exp(\mu)\alpha^5\beta(1-\exp(3\mu)+11\exp(2\mu)-11\exp(\mu))}{[1+\exp(\mu)]^5 v^4}$
$U(4,1) = \frac{1}{12}\frac{\exp(\mu)\alpha^6\beta(1-\exp(4\mu)-26\exp(3\mu)+66\exp(2\mu)-26\exp(\mu))}{(1+\exp(\mu))^6 v^5}$
$U(5,1) = \frac{1}{60}\frac{\exp(\mu)\alpha^7\beta(302\exp(3\mu)+\exp(5\mu)-57\exp(4\mu)+57\exp(\mu)-302\exp(2\mu)-1)}{[1+\exp(\mu)]^7 v^6}$
$U(0,2) = 2\frac{\exp(\mu)\alpha^3\beta^2(-1+\exp(\mu))}{[1+\exp(\mu)]^3 v^2}$
$U(1,2) = \frac{\exp(\mu)\alpha^4\beta^2(-1-4\exp(\mu)+\exp(2\mu))}{(1+\exp(\mu))^4 v^3}$
$U(2,2) = \frac{\exp(\mu)\alpha^5\beta^2(-1+\exp(3\mu)-11\exp(2\mu)+11\exp(\mu))}{[1+\exp(\mu)]^5 v^4}$
$U(3,2) = \frac{1}{6}\frac{\exp(\mu)\alpha^6\beta^2(26\exp(\mu)-66\exp(2\mu)+26\exp(3\mu)-\exp(4\mu)-1)}{[1+\exp(\mu)]^6 v^5}$
$U(4,2) = \frac{1}{24}\frac{\exp(\mu)\alpha^7\beta^2(302\exp(3\mu)+\exp(5\mu)-57\exp(4\mu)+57\exp(\mu)-302\exp(2\mu)-1)}{[1+\exp(\mu)]^7 v^6}$
$U(5,2) = \frac{1}{120}\frac{\exp(\mu)\alpha^8\beta^2(-120\exp(\mu)-2416\exp(3\mu)+1191\exp(2\mu)-120\exp(5\mu)+1191\exp(4\mu)+\exp(6\mu)+1)}{[1+\exp(\mu)]^8 v^7}$
$U(0,3) = \frac{1}{3}\frac{\exp(\mu)\alpha^4\beta^3(1-4\exp(\mu)+\exp(2\mu))}{[1+\exp(\mu)]^4 v^3}$
$U(1,3) = \frac{1}{3}\frac{\exp(\mu)\alpha^5\beta^3(1-\exp(3\mu)+11\exp(2\mu)-11\exp(k))}{[1+\exp(\mu)]^5 v^4}$
$U(2,3) = \frac{1}{6}\frac{\exp(\mu)\alpha^6\beta^3(1-26\exp(2\mu)+66\exp(2\mu)-26\exp(3k)+\exp(4k))}{[1+\exp(\mu)]^6 v^5}$
$U(3,3) = \frac{1}{18}\frac{\exp(\mu)\alpha^7\beta^3(1+302\exp(2\mu)-57\exp(\mu)+57\exp(4\mu)-\exp(5\mu)-302\exp(3k))}{[1+\exp(\mu)]^7 v^6}$

Table 5: Some values of $U(k,h)$ of Ex 3.3.

the second curly bracket is $\frac{2\alpha\beta\exp(\gamma)}{[1+\exp(\gamma)]^2 v}$, the closed form of the third curly bracket is $\frac{\alpha^3\beta^2\exp(\gamma)(-1+\exp(\gamma))}{[1+\exp(\gamma)]^3 v^2}$ and so on. Eq. (3.13) can be written as

$$\begin{aligned} u(x,t) = &\frac{\alpha+\beta+(\beta-\alpha)\exp(\gamma)}{1+\exp(\gamma)} + \frac{2\alpha^2\beta\exp(\gamma)}{[1+\exp(\gamma)]^2 v}t \\ &+\frac{\alpha^3\beta^2\exp(\gamma)(-1+\exp(\gamma))}{[1+\exp(\gamma)]^3 v^2}t^2 + \frac{\alpha^4\beta^3\exp(\gamma)[1-4\exp(\gamma)+\exp(\gamma)^2)}{3[1+\exp(\gamma)]^4 v^3}t^3 + ... \end{aligned} \tag{3.14}$$

and so on, in the same manner the rest of components of the iteration formula were obtained using the Maple Package. The solution of $u(x,t)$ in closed form is

$$u(x,t) = \frac{\alpha+\beta+(\beta-\alpha)\exp(\zeta)}{1+\exp(\zeta)}$$

where $\zeta = (\alpha/v)(x-\beta t-\lambda)$, which are exactly the same as obtained by Adomian decomposition method [3] and variation iteration method [2]. The behavior of the solutions obtained by the differential transform method is shown for different values of times in Fig.1.

4. Differential Transform Method for K(2,2,1) Equation

4.1. Let we take $m = 2, p = 2$ in Eq. (1.2), hence we have

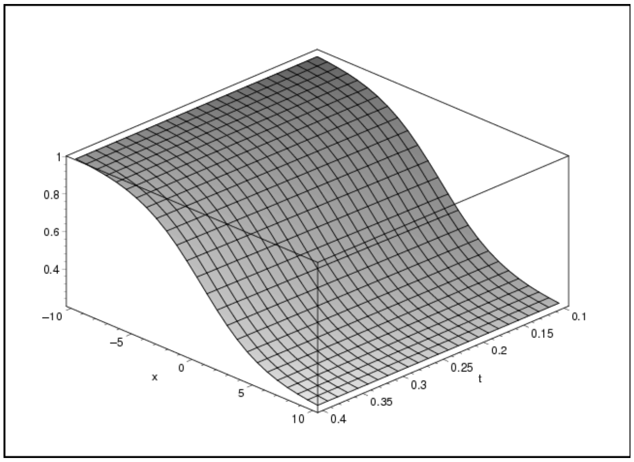

Fig.1. The behavior of $u(x,t)$ evaluates by the differential transform method versus x for different values of time with fixed values $\alpha = 0.4, \beta = 0.6, v = 1, \lambda = 0.125$.

$$u_t + (u^2)_x - (u^2)_{xxx} + u_{5x} = 0 \tag{4.1}$$

Here we consider the solution of Eq.(4.1) with the initial condition as follow,

$$u(x,0) = \frac{16c-1}{12}\cosh^2(\frac{x}{4}) \tag{4.2}$$

where c is an arbitrary constant.

Taking the two-dimensional transform of Eq.(4.1) by using the related definitions in Table 1, we have

$$\begin{aligned}(h+1)U(k,h+1) = & -2\sum_{r=0}^{k}\sum_{s=0}^{h}(k+1-r)U(r,h-s)U(k+1-r,s) \\ & +6\sum_{r=0}^{k}\sum_{s=0}^{h}(r+1)(k+1-r)(k+2-r)U(r+1,h-s)U(k+2-r,s) \\ & +2\sum_{r=0}^{k}\sum_{s=0}^{h}(k+1-r)(k+2-r)(k+3-r)U(r,h-s)U(k+3-r,s) \\ & -(k+1)(k+2)(k+3)(k+4)(k+5)U(k+5,h) \qquad (4.3)\end{aligned}$$

By using the series form of Eq. (4.2), some of the initial transformation coefficients $U(k,0)$ can be written as bellows:

$$
\begin{aligned}
&U(k,0) = 0 \text{ if } \ k = 1, 3, 5, ...,\\
&U(0,0) = \frac{4c}{3} - \frac{1}{12}, U(2,0) = \frac{c}{12} - \frac{1}{92}, U(4,0) = \frac{c}{576} - \frac{1}{9216}\\
&U(6,0) = \frac{c}{69120} - \frac{1}{1105920}, U(8,0) = \frac{c}{15482880} - \frac{1}{247726080}, ...
\end{aligned}
\tag{4.4}
$$

By substituting Eq.(4.4) into Eq.(4.3), we obtain some values of $U(k,h)$ which are none zero and given in Table 6.

$U(1,1) = \frac{c}{96} - \frac{1}{6}c^2$	$U(3,1) = -\frac{1}{144}c^2 + \frac{1}{2304}c$
$U(5,1) = \frac{1}{11520}c^2 + \frac{1}{184320}c$	$U(7,1) = -\frac{1}{1935360}c^2 + \frac{1}{30965760}c$
$U(0,2) = -\frac{1}{192}c^2 + \frac{1}{12}c^3$	$U(2,2) = -\frac{1}{1536}c^2 + \frac{1}{96}c^3$
$U(4,2) = -\frac{1}{73728}c^2 + \frac{1}{4608}c^3$	$U(6,2) = -\frac{1}{8847360}c^2 + \frac{1}{552960}c^3$
$U(8,2) = -\frac{1}{1981808640}c^2 + \frac{1}{123863040}c^3$	$U(10,2) = -\frac{1}{713451110400}c^2 + \frac{1}{44590694400}c^3$
$U(1,3) = -\frac{1}{144}c^4 + \frac{1}{2304}c^3$	$U(3,3) = -\frac{1}{3456}c^4 + \frac{1}{55296}c^3$
$U(5,3) = -\frac{1}{276480}c^4 + \frac{1}{4423680}c^3$	$U(7,3) = -\frac{1}{46448640}c^4 + \frac{1}{743178240}c^3$
$U(9,3) = -\frac{1}{13377208320}c^4 + \frac{1}{214035333120}c^3$	$U(0,4) = \frac{1}{576}c^5 - \frac{1}{9216}c^4$
$U(2,4) = \frac{1}{576}c^5 - \frac{1}{9216}c^4$	$U(4,4) = \frac{1}{4608}c^5 - \frac{1}{73728}c^4$
$U(6,4) = \frac{1}{221184}c^5 - \frac{1}{3538944}c^4$	$U(8,4) = \frac{1}{26542080}c^5 - \frac{1}{424673280}c^4$

Table 6: Some values of $U(k,h)$ of Ex 3.1.

Hence, substituting sufficient number of computed $U(k,h)$ values into Eq. (2.4), we obtained three terms series solutions for each unknown functions as follow:

$$
\begin{aligned}
u(x,t) = &\{\frac{4}{3}c - \frac{1}{12} + (\frac{1}{12}c - \frac{1}{92})x^2 + (\frac{1}{576}c - \frac{1}{9216})x^4 + (\frac{1}{69120}c - \frac{1}{1105920})x^6 + ...\}\\
&+\{(-\frac{1}{6}c^2 + \frac{1}{96}c)xt + (-\frac{1}{144}c^2 + \frac{1}{2304}c)x^3t + (-\frac{1}{11520}c^2 + \frac{1}{184320}c)x^5t\\
&+(-\frac{1}{1935360}c^2 + \frac{1}{3096576}c)x^7t + ...\}\\
&+\{(-\frac{1}{192}c^2 + \frac{1}{12}c^3) + (-\frac{1}{1536}c^2 + \frac{1}{96}c^3)x^2t^2 + (-\frac{1}{73728}c^2 + \frac{1}{4608}c^3)x^4t^2\\
&+(-\frac{1}{8847360}c^2 + \frac{1}{552960}c^3)x^6t^2 + ...\} + ...
\end{aligned}
\tag{4.5}
$$

Approximating the series in Eq. (4.5) appropriately, $u(x,t)$ in closed form are given as follow:

$$u(x,t) = \frac{16c-1}{12}\cosh^2(\frac{ct-x}{4}) \tag{4.6}$$

which is like that obtained by Adomian decomposition method [4] and He's variational iteration method [5].

4.2. Now, we repeat the same procedure for obtaining DTM solution , but with other initial conditions in the form of [4],

$$u(x,0) = -\frac{16c-1}{12}\sinh^2(\frac{x}{4}) \tag{4.7}$$

$$u(x,t) = -\frac{16c-1}{12}\sinh^2(\frac{ct-x}{4}) \tag{4.8}$$

5. Differential Transform Method for K(3,3,1) Equation

5.1. Let we take $m = 3, p = 3$ in Eq. (1.2), hence we have

$$u_t + (u^3)_x - (u^3)_{xxx} + u_{5x} = 0 \tag{5.1}$$

Consider the solution of Eq. (5.1) with the initial conditions as in [4],

$$u(x,0) = \sqrt{\frac{81c-1}{54}}\cosh(\frac{x}{3}) \tag{5.2}$$

By taking the two-dimensional transform of Eq. (5.1) by using the related definitions from Table 1, we have

$$\begin{aligned}(h+1)U(k,h+1) = & -3\sum_{r=0}^{k}\sum_{t=0}^{k-r}\sum_{s=0}^{h}\sum_{p=0}^{h-s}(k-r-t+1)U(r,h-s-p)U(t,s)U(k-r-t+1,p) \\ & +6\sum_{r=0}^{k}\sum_{t=0}^{k-r}\sum_{s=0}^{h}\sum_{p=0}^{h-s}(r+1)(t+1)(k-r-t+1)U(r+1,h-s-p) \\ & U(t+1,s)U(k-r-t+1,p) \\ & +18\sum_{r=0}^{k}\sum_{t=0}^{k-r}\sum_{s=0}^{h}\sum_{p=0}^{h-s}(t+1)(k-r-t+1)(k-r-t+2)U(r,h-s-p) \\ & U(t+1,s)U(k-r-t+2,p) \\ & +3\sum_{r=0}^{k}\sum_{t=0}^{k-r}\sum_{s=0}^{h}\sum_{p=0}^{h-s}(k-r-t+1)(k-r-t+2)(k-r-t+3)U(r,h-s-p) \\ & U(t,s)U(k-r-t+3,p) \\ & -(k+1)(k+2)(k+3)(k+4)(k+5)U(k+5,t)\end{aligned} \tag{5.3}$$

From the initial conditions Eq. (5.2), we can write

$$\begin{aligned}& U(k,0) = 0 \text{ if } k = 1,3,5,..., U(0,0) = \frac{\sqrt{486c-6}}{18}, U(2,0) = \frac{\sqrt{486c-6}}{324}, \\ & U(4,0) = \frac{\sqrt{486c-6}}{34992}, U(6,0) = \frac{\sqrt{486c-6}}{9447840}, U(8,0) = \frac{\sqrt{486c-6}}{4761711360}, ...\end{aligned} \tag{5.4}$$

Hence, substituting Eq.(5.4) into Eq.(5.3) , and by recursive method, some of the results are listed in Table 7.

$U(1,1) = -\frac{\sqrt{486c-6}c}{162}$	$U(3,1) = -\frac{\sqrt{486c-6}c}{8748}$	$U(5,1) = -\frac{\sqrt{486c-6}c}{1574640}$
$U(7,1) = -\frac{\sqrt{486c-6}c}{595213920}$	$U(0,2) = \frac{\sqrt{486c-6}c^2}{324}$	$U(2,2) = \frac{\sqrt{486c-6}c^2}{5832}$
$U(4,2) = \frac{\sqrt{486c-6}c^2}{629856}$	$U(6,2) = \frac{\sqrt{486c-6}c^2}{170061120}$	$U(8,2) = \frac{\sqrt{486c-6}c^2}{85710804480}$
$U(1,3) = -\frac{\sqrt{486c-6}c^3}{8748})$	$U(3,3) = -\frac{\sqrt{486c-6}c^3}{472392}$	$U(5,3) = -\frac{\sqrt{486c-6}c^3}{85030560}$
$U(7,3) = -\frac{\sqrt{486c-6}c^3}{32141551680}$	$U(9,3) = -\frac{\sqrt{486c-6}c^3}{20827725488640}$	$U(11,3) = -\frac{\sqrt{486c-6}c^3}{20619448233753600}$

Table 7: Some values of $U(k,h)$ of Ex 4.1.

Consequently, substituting sufficient number of $U(k,h)$ values into Eq.(2.2), we have series solution as follow:

$$\begin{aligned} u(x,t) = {} & \{\frac{\sqrt{486c-6}}{18} + \frac{\sqrt{486c-6}}{324}x^2 + \frac{\sqrt{486c-6}}{34992}x^4 + \frac{\sqrt{486c-6}}{9447840}x^6 + ...\} \\ & +\{-\frac{\sqrt{486c-6}c}{162}xt - \frac{\sqrt{486c-6}c}{8748}x^3t - \frac{\sqrt{486c-6}c}{1574640}x^5t \\ & -\frac{\sqrt{486c-6}c}{595213920}x^7t + ...\} \\ & +\{\frac{\sqrt{486c-6}c^2}{324}t^2 + \frac{\sqrt{486c-6}c^2}{5832}x^2t^2 + \frac{\sqrt{486c-6}c^2}{629856}x^4t^2 \\ & +\frac{\sqrt{486c-6}c^2}{170061120}x^6t^2 + ...\} + ... \end{aligned} \tag{5.5}$$

Approximating the series in Eq.(5.5) appropriately, $u(x,t)$ in closed form is given

$$u(x,t) = \sqrt{\frac{81c-1}{54}}\cosh(\frac{ct-x}{3}). \tag{5.6}$$

which is like that obtained by Adomian decomposition method [4] and He's variational iteration method [5].

5.2. As a conclusive work, now, we repeat the solution steps of the same problem, but with other initial conditions as in the form [4].

$$u(x,0) = -\sqrt{\frac{81c-1}{54}}\cosh(\frac{x}{3}) \tag{5.7}$$

$$u(x,t) = -\sqrt{\frac{81c-1}{54}}\cosh(\frac{ct-x}{3})$$

6. Conclusion

In this paper, the differential transform method has been successfully applied to finding the solution of a Burger's and $K(m,p,1)$ equations. The solution obtained by the differential transform method is an infinite power series for appropriate initial condition, which can, in turn, be expressed in a closed form, the exact solution. The results show that, the differential transform method is a powerful mathematical tool to solving Burger's and K(m,p,1) equations.

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