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RESEARCH ARTICLE

USING CIRCULAR INTUITIONISTIC FUZZY TOPSIS TO ELIMINATE THE HESITANCY IN THE FIELD OF AGRICULTURAL BIOLOGY

Zeynep GÖKKUŞ [1,*], Sevil ŞENTÜRK [2], Fırat ALATÜRK [3] , Baboo ALİ [4]

[1] Department of Computer Technology, Vocational School, Kastamonu University, Kastamonu, Türkiye
z.gokkus@gmail.com - 0000-0003-2767-8420

[2] Department of Statistics, Faculty of Science, Eskişehir Technical University, Eskişehir, Türkiye
sdeligoz@eskisehir.edu.tr - 0000-0002-9503-7388

[3] Department of Field Crops, Faculty of Agriculture, Çanakkale Onsekiz Mart University, Çanakkale, Türkiye
alaturkf@comu.edu.tr - 0000-0003-3394-5855

[4] Department of Field Crops, Faculty of Agriculture, Çanakkale Onsekiz Mart University, Çanakkale, Türkiye
babooali@gmail.com - 0000-0002-6989-7018

Abstract

Circular intuitionistic fuzzy sets (CIFSs) were introduced by Atanassov (2020) as a new extension of intuitionistic fuzzy sets. C-IFSs are represented by a circle of each element that is characterized by degrees of membership and non-membership. In a decision-making process based on experimental data, although uncertainty is low, hesitation can be high. In such cases, the decision-making process is affected by the decision-makers as well as the criteria. Therefore, there is a need to evaluate the expertise of decision-makers within the decision-making process. In Circular Intuitionistic Fuzzy Sets, where hesitation is represented, an approach is proposed for calculating decision-maker weights with the Technique for Order of Preferences by Similarity to Ideal Solution (TOPSIS), which is a multi-criteria decision-making method to eliminate hesitation. In this study, circular intuitionistic fuzzy sets were implemented into the TOPSIS method. The problem was handled from two different perspectives while creating the decision matrix. Sensitivity analyses were performed for both applications. These sensitivity analyses were carried out to examine the change in the ranking of the alternatives when the optimistic or pessimistic approaches of the decision makers, criterion weights, and the decision maker importance weights changed, respectively. In addition, intuitionistic and Pythagorean fuzzy TOPSIS methods were applied and presented as comparative analyses. According to the results obtained, the proposed approaches were satisfactory in eliminating the hesitation.

Keywords

Circular intuitionistic fuzzy sets,
TOPSIS,
Experimental decision-making problem,
Weights of decision-makers,
Hesitation

Time Scale of Article

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1. INTRODUCTION

Zadeh introduced ordinary fuzzy sets in which each element has the sum of membership and non-membership degrees equal to 1 [1]. Later, fuzzy sets expanded into new extensions with the approaches of different researchers. Then, Zadeh introduced Type-2 fuzzy sets as an extension of ordinary fuzzy sets to handle the uncertainty in membership functions [2]. Atanassov introduced intuitionistic fuzzy

*Corresponding Author: z.gokkus@gmail.com

sets (IFS's), which consist of a membership and a non-membership degree whose sum is ≤ 1 [3]. Here, the purpose of IFS is to consider the hesitations of experts. Smarandache studied Neutrosophic sets, which have degrees of accuracy, uncertainty, and inaccuracy for each element in the universe, and the largest sum of these three can be 3 [4]. Torra proposed Hesitant fuzzy sets (HFSs) to work with the set of potential membership degrees of an element in a fuzzy set [5]. On the other hand, Atanassov developed intuitionistic type-2 fuzzy sets (IFS2) and Yager developed Pythagorean fuzzy sets (PFS's), which are specified with a wider area for membership and non-membership degrees [6, 7]. Yager proposed q-rung orthopair fuzzy sets (Q-ROFSs) as a general class of IFSs and PFSs [8]. Moreover, Picture fuzzy sets (PFSs) [9], spherical fuzzy sets (SFSs) [10], and circular intuitionistic fuzzy sets (C-IFSs) [11] which are direct extensions of IFSs and where each element is characterized by membership, non-membership, and hesitation degrees were introduced. Extensions of fuzzy sets are represented in Figure 1.

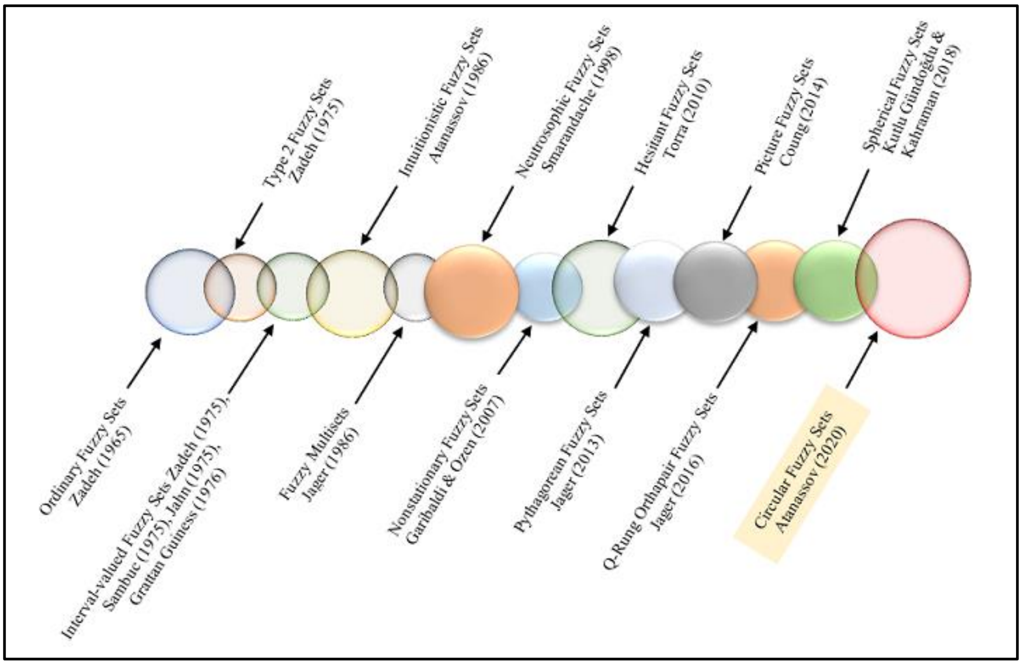

Figure 1. Extentions of Fuzzy Sets [12]

Atanassov developed C-IFSs, the newest version of fuzzy sets [11]. An element of a C-IFS is represented by a circle with center $\mu_A(x), \vartheta_A(x,)$, and radius r, which is the difference of C-IFSs from IFSs. These $\mu_A(x), \vartheta_A(x,)$ are degree of membership non-membership, respectively and where the sum of them within this circle is primarily equal to 1. This indicates that the fuzziness of membership functions is handled more flexibly. For this reason, CIFSs can be effectively used in MCDM methods, considering that their properties contribute significantly to more accurate results for all MCDM methods [12].

Multi-criteria decision-making (MCDM) methods have become the main research area in solving complex decision-making problems. The methods followed in determining the best alternative among a set of alternatives according to several criteria or to rank the alternatives according to criteria in order to reach a certain goal are generally called MCDM methods [13]. Several MCDM methods have been developed for solving MCDM problems with various conflicting criteria under uncertainty such as Analytic Hierarchy Process (AHP) [14], Analytic Network Process (ANP) [15], Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) [16], VIseKriterijumska Optimizacija I Kompromisno Resenje (VIKOR) [17], Weighted Aggregated Sum Product Assessment (WASPAS) [18]. Classical MCDM methods require precise numerical values and are therefore insufficient to represent uncertainty in linguistic evaluations. To represent uncertainty, MCDM methods have been extended using for all types of fuzzy sets [13].

Technique for Order of Preferences by Similarity to Ideal Solution (TOPSIS) is a multi-criteria decision-making method developed by Hwang and Yoon in 1981 to select the alternative closest to the positive ideal solution and the farthest alternative to the negative ideal solution [16]. Its main idea is to rank the alternatives according to the distance between the positive and negative ideal solutions. On the other hand, Chen presented a practical approach for decision making processes with the TOPSIS method for ordinary fuzzy numbers and for uncertainty and hesitancy situations [19]. In this way, the hesitations of the decision makers on the ranking can be eliminated. In the fuzzy TOPSIS method, linguistically expressed evaluations can be represented by fuzzy numbers and situations of uncertainty and hesitancy can be overcome more easily.

The TOPSIS method is one of the most common and practical MCDM methods in the literature in terms of eliminating hesitation in determining the ideal solution. In this study, C-IFS is more flexible and the latest fuzzy sets for eliminating hesitation, and the TOPSIS method is emphasized in order to eradicate hesitation and obtain the most suitable rank for the ideal solution.

Several TOPSIS methods for various fuzzy sets have been proposed and applied in many scientific fields (for instance see [10, 12, 20-41]). Readers may research for details from the referred publications.

However, in the literature, the applications of fuzzy MCDM methods for the field of agricultural biology, except for the selection of suitable agricultural areas, are limited [42]. The topic of fuzzy MCDM methods are highly prevalent in the literature. Over 6,000 articles with the term "Fuzzy MCDM" in the abstract and only approximately 8 (4.1%) of them related to “agricultural biology” can be found in the Scopus database. Fig. 2 and 3 show the distributions of these papers by year and subject area, respectively that were gathered from SCOPUS (URL (06.09.2024) https://www.scopus.com/results/results.uri?sort=plf-f&src=s&sid=4098c9f6059ee490eec4f6ceee97ad2d&sot=a&sdt=cl&cluster=scosubjabbr%2C%22AGRI%22%2Ct&sl=77&s=%28TITLE-ABS-KEY%28fuzzy%29+AND+TITLE-ABS-KEY%28mcdm%29+AND+TITLE-ABS-KEY%28agriculture%29%29&origin=resultsAnalyzer&zone=subjectArea&editSaveSearch=&txGid=0a6d992549603e84faca49f1270f21c2&sessionSearchId=4098c9f6059ee490eec4f6ceee97ad2d&limit=10).

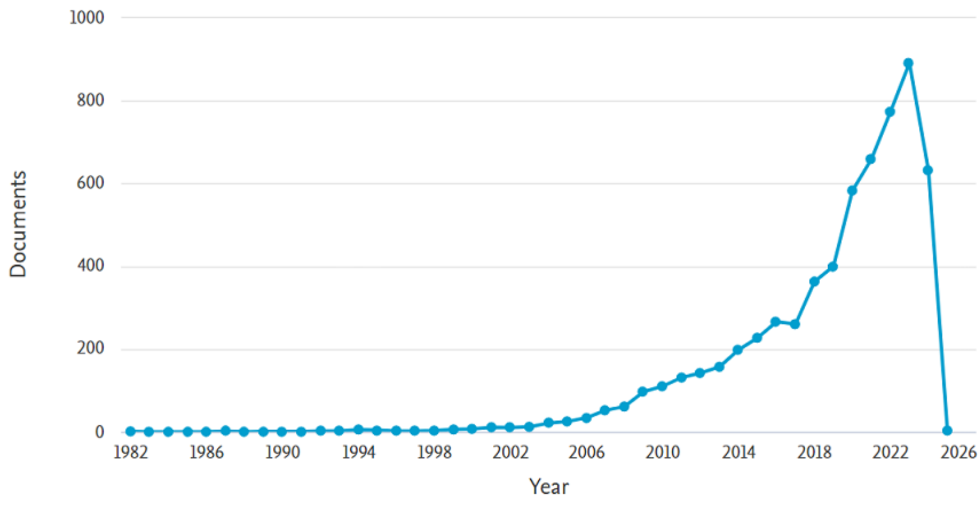

Figure 2. Documents by year

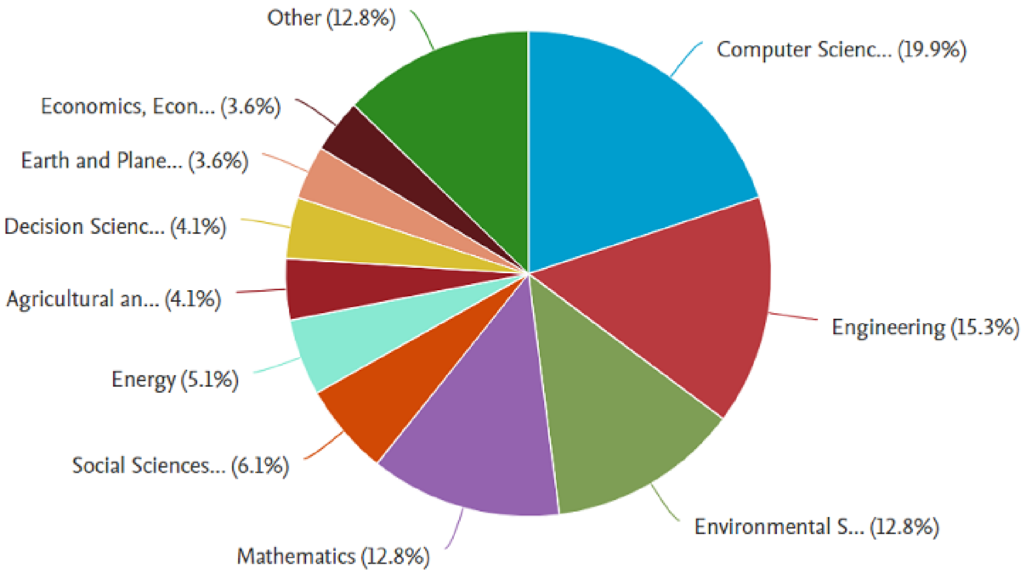

Figure 3. Documents by area

Among these publications, Manos et al proposed a mathematical program [43], and G wa Mbũgwa et al used fuzzy MCDM methods to identify the most suitable agricultural area for growing the Tiflon plant [44]. Mir and Padma evaluated and prioritized rice production practices and constraints under temperate climate conditions using the Fuzzy Analytic Hierarchy Process (FAHP) [45], while Jamil et al conducted a Crop Suitability Analysis in the Bijnor Region, UP, using geospatial tools and the fuzzy AHP method [46]. Hezam et al developed an effective decision-making model to evaluate irrigation systems under uncertainty [47]. In addition, Aslan applied AHP and fuzzy-AHP methods to determine the groundwater potential of the Van Basin [48], and Elleuch et al applied hybrid fuzzy multi-criteria decision-making to solve the irrigation water allocation problem in Tunisia [49]. Alaoui et al worked on 'Type 2 Fuzzy TOPSIS for Agricultural MCDM Problems [50].

The most crucial aim of this study is to show the applicability of fuzzy MCDM methods in experimental research. MCDM methods, which evaluate both expert opinions and experimental data together, combined with the flexibility of fuzzy logic, will provide advantages in ranking alternatives and/or determining the best alternative. The second aim is to develop a decision procedure suitable for the structure of the experimental data. This procedure incorporates expert judgment at every stage, and expert and criterion weights are calculated using a formula developed in accordance with circular intuitionistic fuzzy numbers. Thus, agricultural biology data was studied with the circular intuitionistic fuzzy TOPSIS (C-IF TOPSIS) method.

In this study, firstly, fuzzy numbers IFS, PFS, and C-IFS were defined briefly. A C-IF TOPSIS procedure was proposed for experimental data obtained from the field of agricultural biology, in which decision-maker (DM) importance weights were calculated effectively, and the mentioned data was explained. In Chapter 3, the proposed procedure was applied to the mentioned data from two different perspectives. Sensitivity analyses were performed for the proposed procedure from both perspectives. One of the sensitivity analyses were carried out to monitor the effect of the change in DM weights on the ranks. This was for observing how the ranks were affected by evaluations of decision-makers, while their rates of expertise change. In addition, comparative analyses were performed with IF-TOPSIS and PF-TOPSIS. Chapter 4 contains conclusions and further research.

2. MATERIAL AND METHODS

In this section, Circular Intuitionistic Fuzzy (C-IF) numbers introduced by [11] was briefly explained. Then, a C-IF TOPSIS procedure was proposed, which was taken into account studies performed in the field of agricultural biology. Here, the evaluations of the alternatives for criteria were based on quantitative measurements and the expertise weights of the decision makers (DMs) were effective, and DM weights were also included in the calculation. Finally, research carried out in the field of agricultural biology, in which the evaluations of the criteria are based on quantitative measurements, were described.

2.1. Preliminaries for IFS, PFS and C-IF

There are many extensions of Intuitionistic fuzzy sets (IFS). This study focused on IFS, Pythagorean fuzzy set (PFS), and C-IFS, and in the follow-up, the definitions of these clusters are given, respectively, and briefly.
In a finite set X, the intuitionistic fuzzy set (IFS) A is defined as $A = \{\langle x, \mu_A(x), \vartheta_A(x)\rangle | x \in X\}$ [3, 21]. Here, $\mu_A(x), \vartheta_A(x): X \rightarrow [0, 1]$ are membership and non-membership functions, respectively and $0 \leq \mu_A(x) + \vartheta_A(x) \leq 1$ dir. A third parameter, which is the index of whether x belongs to A or not and indicates the hesitation degree of $\pi_A(x) = 1 - \mu_A(x) - \vartheta_A(x)$ and $0 \leq \pi_A(x) \leq 1$. If π_A s small, the information about x is more precise, if large, more uncertain, and if zero, IFS turns into the ordinary fuzzy set [51].

Again in a finite set X, the Pythagorean fuzzy set (PFS) is defined as $P = \{\langle x, \mu_P(x), \vartheta_P(x)\rangle | x \in X\}$ [7]. Here, $\mu_P(x), \vartheta_P(x): X \rightarrow [0, 1]$ are membership and non-membership functions, respectively, and $0 \leq \left(\mu_P(x)\right)^2 + \left(\vartheta_P(x)\right)^2 \leq 1$. Similarly, a third parameter, which is the index of whether x belongs to A or not indicates the hesitation degree of $\pi_P(x) = \sqrt{1 - \left(\mu_P(x)\right)^2 - \left(\vartheta_P(x)\right)^2}$ and $0 \leq \left(\pi_P(x)\right)^2 \leq 1$.

Likewise, in a finite set X, the circular intuitionistic fuzzy set (C-IFS) is defined as $C = \{\langle x, \mu_C(x), \vartheta_C(x); r\rangle | x \in X\}$ [11]. Here, $\mu_A(x), \vartheta_A(x): X \rightarrow [0, 1]$ are membership and non-membership functions, respectively, $r: X \rightarrow [0, 1]$ is the radius of the circle around each element, $0 \leq \mu_C(x) + \vartheta_C(x) \leq 1$ and $r \in [0,1]$ for $\forall x \in X$. x is a third parameter, which is the index of whether x belongs to C or not and indicates the hesitation degree of C-IFS, is $\pi_C(x) = 1 - \mu_C(x) - \vartheta_C(x)$ and $0 \leq \pi_C(x) \leq 1$. However, as in other IFSs, each element is not represented by a triangular form, but by a circle with center $\langle \mu_C(x), \vartheta_C(x,)\rangle$ and radius r [11]. If $r = 0$ then CIFS turns into IFS [52]. The geometric representation of C-IFS is given below.

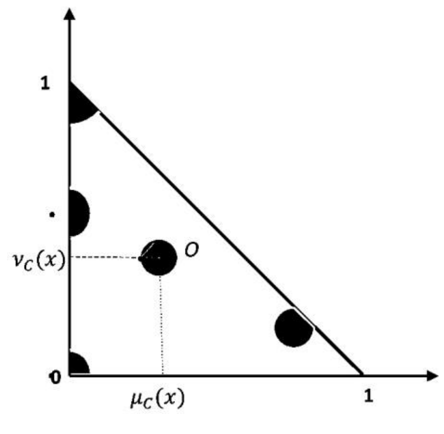

Figure 4. Geometric representation of C-IFS [11].

Readers may examine the arithmetic operations for all three fuzzy sets from mentioned publications.

2.2. C-IF TOPSIS

Fuzzy TOPSIS is an extension of the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) method, integrating fuzzy logic to handle uncertainty and imprecision in decision-making processes. It is commonly used in multi-criteria decision-making (MCDM) scenarios where the available data may be uncertain or vague. By incorporating fuzzy sets, Fuzzy TOPSIS evaluates alternatives based on their distance from the ideal and negative ideal solutions, allowing for more robust decision-making in complex, real-world problems, particularly when relying on expert judgments.

In this section, an approach that can be evaluated from two different perspectives is proposed for the solution of a fuzzy MCDM problem based on the data obtained directly from the field of agricultural biology with the C-IF TOPSIS method.

When DM weights are different, and evaluations of alternatives for criteria is based on measurements C-IF TOPSIS

When the empirical evaluation of alternatives according to criteria is based on measurements, in the M9 approach (see Subsection 3.1), DMs can specify the experimental results in the decision matrix according to their own opinions. In the M12 approach (see Subsection 3.2), DMs only evaluate the criteria, and the decision matrix is created by keeping the measurement values constant for each DM in the linguistic scale (like a single DM opinion). Because in such a decision-making problem, although the uncertainty is low (relative uncertainty based on experimental data), hesitation is at the forefront. Since this hesitation is directly related to DM opinions. So other C-IF TOPSIS approaches in the literature [12, 41, 52] have not been deemed appropriate, for include decision matrices obtained with equal DM weights and very different DM opinions. The proposed C-IF TOPSIS approach for both the M9 and M12 approaches is explained in the following steps and Figure 5.

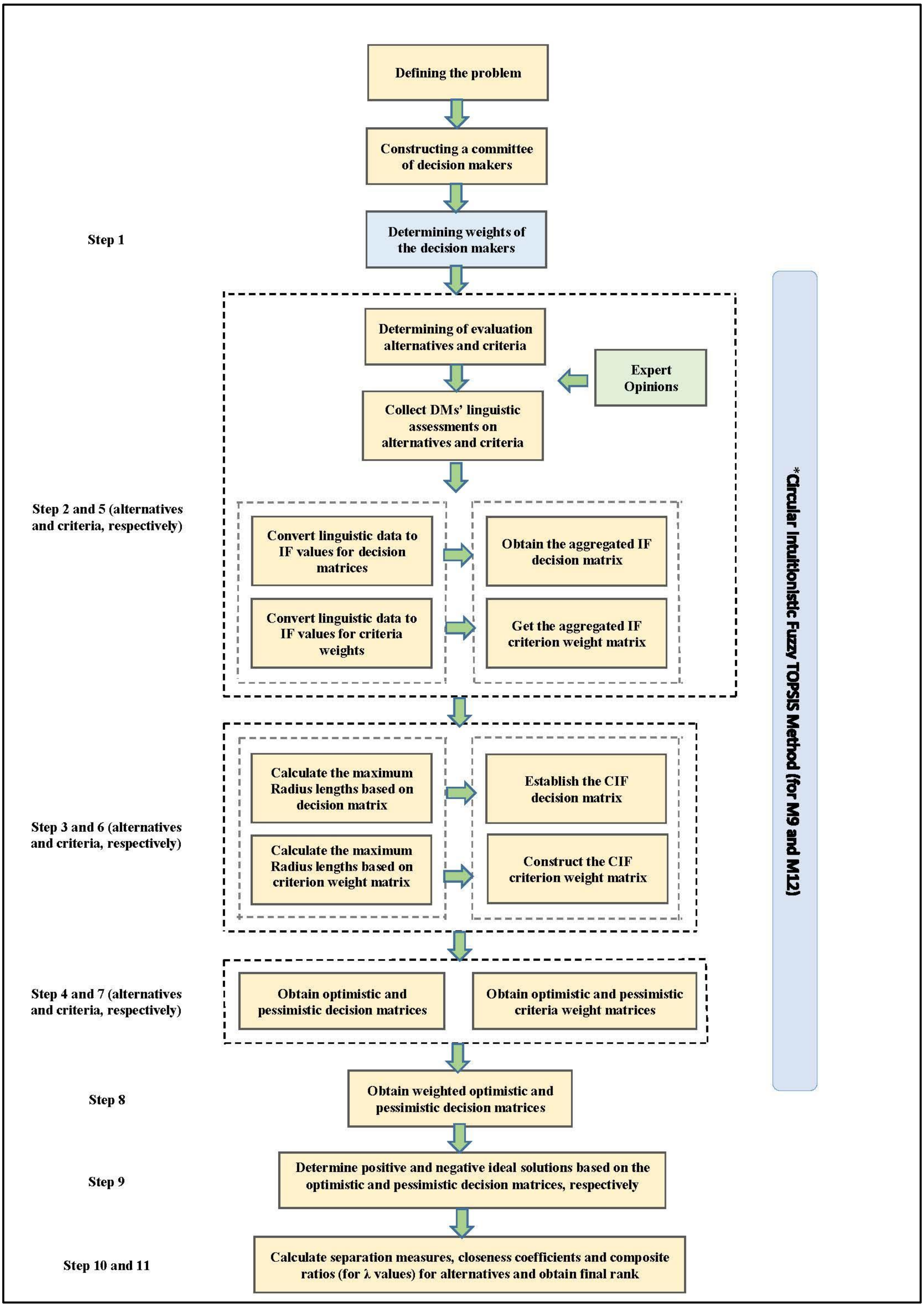

Figure 5. Framework of proposed approaches (both M9 and M12)

There are examples in the literature for both cases for IF TOPSIS and PF TOPSIS [21, 53], but there is no publication in which DM weights are different for C-IF TOPSIS. For this reason, a DM weight calculation formula, with Equation 1, suitable for the structure of the C-IF TOPSIS method was proposed for the first time. Step 1 is very important because it is suitable for the structure of C-IFs and is used in all decision steps of the TOPSIS method.

Step 1. The importance of l DMs in the study group is considered as linguistic terms (LTs) expressed with IFNs according to Table 1. Let the relative importance degree of the kth DM be $D_k = [\mu_k, \vartheta_k]$. The weight of the kth DM is then calculated as Equation 1 for C-IFNs as follows.

$$\sigma_k = \frac{(\mu_k - \vartheta_k)^2}{\sum_{k=1}^{l}(\mu_k - \vartheta_k)^2} . \tag{1}$$

Table 1. Linguistic variables for the relative importance ratings of decision makers [21]

Linguistic variables	Intuitionistic Fuzzy Numbers (μ, ϑ)
Very important	(0.90, 0.10)
Important	(0.75, 0.20)
Medium	(0.50, 0.45)
Unimportant	(0.35, 0.60)
Very unimportant	(0.10, 0.90)

Step 2. In this step, each DM evaluates the alternatives according to Table 2 and creates an individual decision matrix for M9, consistent with Table 16. For M12, the measurement values of the alternatives according to the criteria are adapted according to Table 2 and a decision matrix (Table 16) is created to be constant for each DM.

Table 2. Linguistic scale for ratings of alternatives and criteria [12, 41]

Linguistic variables	Intuitionistic Fuzzy Numbers (μ, ϑ)
Certainly High Importance (CHI)	(0.9, 0.10)
Very High Importance (VHI)	(0.8, 0,15)
High Importance (HI)	(0.7, 0.25)
Above Average Importance (AAI)	(0.6, 0.35)
Average Importance (AI)	(0.5, 0.45)
Under Average Importance (UAI)	(0.4, 0.55)
Low Importance (LI)	(0.3, 0.65)
Very Low Importance (VLI)	(0.2, 0.75)
Certainly Low Importance (CLI)	(0.1, 0.90)

Let, $S^k = \left(s_{ij}^k\right)_{axb}$ be the intuitionistic fuzzy decision matrix of each DM, with a and b representing the alternative and criterion numbers, respectively. With $\sigma = \{\sigma_1, \sigma_2, \dots, \sigma_l\}$ weight of each DM and $\sum_{k=1}^{l}\sigma_k = 1$, an aggregation intuitionistic fuzzy decision matrix is created to fuse all DM views to form a group view. For this, [54]'s IFWA (Intuitionistic Fuzzy Weighted Arithmetic) operator, represented in Equation 2, is used [21]. When $s_{ij} = \left(\mu_{C_i}(x_j), \vartheta_{C_i}(x_j)\right)$ $(i = 1, \dots, a; j = 1, \dots, b)$ and the aggregation intuitionistic fuzzy decision matrix is $S = \left(s_{ij}\right)_{axb}$, where

$$s_{ij} = IFWA_\sigma\left(s_{ij}^{(1)}, s_{ij}^{(2)}, \dots, s_{ij}^{(l)}\right) = \sigma_1 s_{ij}^{(1)} \oplus \sigma_2 s_{ij}^{(2)} \oplus \dots \oplus \sigma_l s_{ij}^{(l)} = \left[1 - \prod_{k=1}^{l}\left(1 - \mu_{ij}^{(k)}\right)^{\sigma_k}, \prod_{k=1}^{l}\left(\vartheta_{ij}^{(k)}\right)^{\sigma_k}\right]. \tag{2}$$

Step 3. Using the S^k and S matrices, Equation 3 calculates the maximum radius lengths $R^D = \left(r_{ij}^d\right)_{axb}$.

$$r_{ij} = \max_{1 \le j \le k_i} \sqrt{\left(\mu(C_i) - \mu_{ij}\right)^2 + \left(\vartheta(C_i) - \vartheta_{ij}\right)^2}. \tag{3}$$

Then circular intuitionistic fuzzy decision matrix $\widetilde{D} = \left(\tilde{d}_{ij}\right)_{axb}$ is obtained. Here, $\tilde{d}_{ij} = \left(\left(\mu_{ij}, \vartheta_{ij}\right); r_{ij}\right)$ is used to indicate the circular intuitionistic fuzzy number of i^{th} alternative with respect to j^{th} criterion [12, 41].

Step 4. Since the membership and non-membership values assigned by DMs are handled as a circle with radius r, here in a sample space with radius r, optimistic and pessimistic attitudes of DMs are represented in the area with radius r. Two different decision matrices are formed according to the optimistic and pessimistic attitudes of the DMs. Calculate the optimistic decision matrix $\widetilde{Q}_{ij}^{O_d} = \left(\tilde{q}_{ij}^{O_d}\right)_{axb}$, where $\tilde{q}_{ij}^{O_d} = \langle\left(\mu_{ij} + r_{ij}, \vartheta_{ij} - r_{ij}\right)\rangle$, and pessimistic decision matrix $\widetilde{Q}_{ij}^{P_d} = \left(\tilde{q}_{ij}^{P_d}\right)_{axb}$, where $\tilde{q}_{ij}^{P_d} = \langle\left(\mu_{ij} - r_{ij}, \vartheta_{ij} + r_{ij}\right)\rangle$ [12, 41].

Step 5. $\widehat{W}_k = \left(w_{jk}\right)_{1xb}$ matrix is obtained by evaluating the criteria of each DM according to Table 2, where w_{jk} indicates intuitionistic fuzzy pairs of k^{th} DM with respect to the j^{th} criterion. Then, $\widehat{W} = \left(\widehat{w}_j\right)_{1xb}$ aggregated intuitionistic fuzzy weighted matrix is created with Equation 2 in order to fuse all DM views to get a group view.

Step 6. Using the $\widehat{W}_k$ and $\widehat{W}$ matrices, the maximum radius lengths $R^D = \left(r_j^d\right)_{1xb}$ are calculated for the criteria, with Equation 4.

$$r_j = \max_{1 \le j \le k_i} \sqrt{\left(\mu(C_i) - \mu_j\right)^2 + \left(\vartheta(C_i) - \vartheta_j\right)^2}. \tag{4}$$

Here, with Step 5, a circular intuitionistic fuzzy weight matrix is obtained and represented by $\widehat{w}_j = \left(\left(\mu_j, \vartheta_j\right); r_j\right)$ [12, 41].

Step 7. The area with radius r is represented for the optimistic and pessimistic attitudes of DMs. Two different criteria weight matrices are formed according to the DMs' attitude of being optimistic and pessimistic. Calculate the optimistic criterion weight matrix $\widetilde{Q}^{O_w} = \left(\tilde{q}_j^{O_w}\right)_{1xb}$, where $\tilde{q}_j^{O_w} = \langle\left(\mu_j + r_j, \vartheta_j - r_j\right)\rangle$, and pessimistic criterion weight matrix $\widetilde{Q}^{P_w} = \left(\tilde{q}_j^{P_w}\right)_{1xb}$, where $\tilde{q}_{ij}^{P_d} = \langle\left(\mu_j - r_j, \vartheta_j + r_j\right)\rangle$ [12, 41].

Step 8. With the matrices obtained in steps 4 and 7, the weighted optimistic decision matrix $\psi_{ij}^O = q_j^{O_w} \otimes q_{ij}^{O_d}$ and the weighted pessimistic decision matrix $\psi_{ij}^P = q_j^{P_w} \otimes q_{ij}^{P_d}$ are obtained.

Step 9. Positive and negative ideal solutions obtained with optimistic and pessimistic matrices are calculated with Equations 5 and 6, respectively [12, 41].

$$\left.\begin{array}{l} X^{O^*} = \left\{\langle\left(\max\limits_i \psi_{ij}^O \mid j \in J_1\right), \left(\min\limits_i \psi_{ij}^O \mid j \in J_2\right)\rangle \mid j = 1, 2, \ldots, b\right\}^T = \left\{\psi_1^{O^+}, \psi_2^{O^+}, \ldots, \psi_b^{O^+}\right\}^T \\ X^{O^-} = \left\{\langle\left(\min\limits_i \psi_{ij}^O \mid j \in J_1\right), \left(\max\limits_i \psi_{ij}^O \mid j \in J_2\right)\rangle \mid j = 1, 2, \ldots, b\right\}^T = \left\{\psi_1^{O^-}, \psi_2^{O^-}, \ldots, \psi_b^{O^-}\right\}^T \end{array}\right\}. \quad (5)$$

$$\left.\begin{array}{l} X^{P^*} = \left\{\langle\left(\max\limits_i \psi_{ij}^P \mid j \in J_1\right), \left(\min\limits_i \psi_{ij}^P \mid j \in J_2\right)\rangle \mid j = 1, 2, \ldots, b\right\}^T = \left\{\psi_1^{P^+}, \psi_2^{P^+}, \ldots, \psi_b^{P^+}\right\}^T \\ X^{P^-} = \left\{\langle\left(\min\limits_i \psi_{ij}^P \mid j \in J_1\right), \left(\max\limits_i \psi_{ij}^P \mid j \in J_2\right)\rangle \mid j = 1, 2, \ldots, b\right\}^T = \left\{\psi_1^{P^-}, \psi_2^{P^-}, \ldots, \psi_b^{P^-}\right\}^T \end{array}\right\}. \quad (6)$$

Step 10. According to the positive and negative ideal solutions obtained with Equation 5 and 6, separation measures are calculated with Equation 7 and 8, respectively [12, 41].

$$\left.\begin{array}{l} D_i^{O^*} = \left(\frac{1}{2b}\sum_{j=1}^{b}\left(\left|\mu_{ij} - \mu_j^{O^+}\right|^2 + \left|\vartheta_{ij} - \vartheta_j^{O^+}\right|^2\right)\right)^{1/2} \\ D_i^{O^-} = \left(\frac{1}{2b}\sum_{j=1}^{b}\left(\left|\mu_{ij} - \mu_j^{O^-}\right|^2 + \left|\vartheta_{ij} - \vartheta_j^{O^-}\right|^2\right)\right)^{1/2} \end{array}\right\}. \quad (7)$$

$$\left.\begin{array}{l} D_i^{P^*} = \left(\frac{1}{2b}\sum_{j=1}^{b}\left(\left|\mu_{ij} - \mu_j^{P^+}\right|^2 + \left|\vartheta_{ij} - \vartheta_j^{P^+}\right|^2\right)\right)^{1/2} \\ D_i^{P^-} = \left(\frac{1}{2b}\sum_{j=1}^{b}\left(\left|\mu_{ij} - \mu_j^{P^-}\right|^2 + \left|\vartheta_{ij} - \vartheta_j^{P^-}\right|^2\right)\right)^{1/2} \end{array}\right\}. \quad (8)$$

Step 11. According to Equation 7 and 8, the closeness coefficients (based on the optimistic and pessimistic matrices) are obtained with Equation 9 and 10, respectively. Finally, via Equation 9 and 10, the composite ratio (CR) scores which determine the final ranking of the alternatives, is calculate with Equation 11 [12, 41].

$$CC_i^O = \frac{D_i^{O^-}}{D_i^{O^+} + D_i^{O^-}}. \quad (9)$$

$$CC_i^P = \frac{D_i^{P^-}}{D_i^{P^+} + D_i^{P^-}}. \quad (10)$$

$$CC_i^{CR} = \lambda \times CC_i^O + (1 - \lambda) \times CC_i^P. \quad (11)$$

Here, λ and $(1 - \lambda)$ are the weight of optimistic and pessimistic attitudes of DMs, respectively.

2.3. Fuzzy Multi-Criteria Decision Making (MCDM) Problem for An Experimental Study in the Field of Agricultural Biology

In this study, the data obtained from the TÜBITAK-1001 Project numbered 120-O-527, supported by the Scientific and Technological Research Council of Turkey (TÜBITAK), was used. Therefore, we thank TÜBİTAK.

An agricultural research was carried out on the cultivation of four different sorghum varieties, and the harmful insects detected in the plants were counted, taking into account the different harvesting heights of the plants. It was aimed to rank sorghum varieties in terms of damage from these harmful insects and to determine the most damaged sorghum variety. Therefore, sorghum varieties were alternatives. The maturity period of the detected harmful insects, the number of species in which the insects were included, and the harvesting height of the sorghum varieties affected the damage to the plants. For this reason, the harvesting height of the plants, the maturity period of the harmful insects, and the number of species including the insects were determined as criteria. The harmful insects detected in each sorghum variety were counted and categorized according to these criteria and Table 16 was obtained.

Although the number of insects was determined precisely, researchers expressed their hesitations about which sorghum variety was most damaged and/or how to rank sorghum varieties in terms of damage. Therefore, it was deemed appropriate to consider this problem as a Fuzzy Multi-Criteria Decision Making (MCDM) problem which was explained as follows.

Goal: Identify the most damaged sorghum variety and/or rank four different sorghum varieties in terms of damage from harmful insects.

Criteria: Harvesting height of plants (30-120 cm), harvesting height of plants (150 cm and above), maturity stage of insects (larvae), maturity stage of insects (nymph), maturity stage of insects (adult), and number of insect species (NIS).

Alternatives: Nutri Honey, Nutrima, M81E ve Topper76 (sorghum varieties).

The structure of the fuzzy MCDM problem explained is represented in Figure 6.

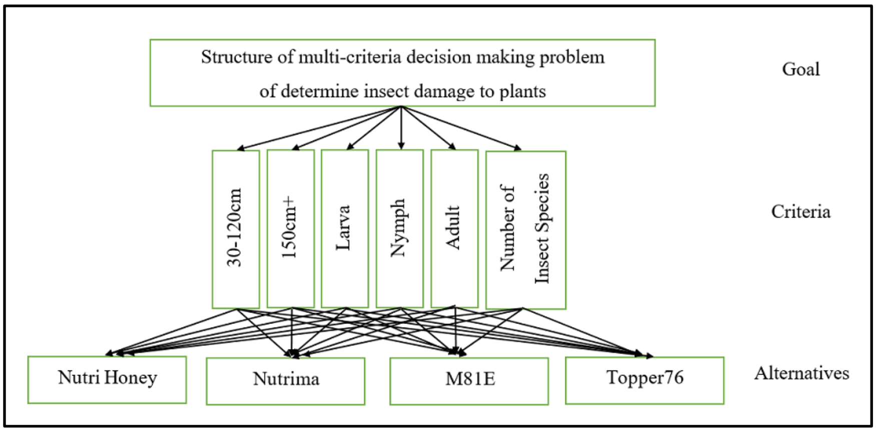

Figure 6. Hierarchical structure of the fuzzy MCDM problem

Here, 30-120cm and 150cm+ criteria represent the harvest heights of the alternatives in which the insects were detected, and the Larva, Nymph and Adult criteria represent the maturity periods of the insects were detected in the alternatives. A Number of species of insects (NIS) criterion was obtained by grouping insects according to the species they belong to. The plants were harvested after their heights exceeded 150cm. Insects are immobile in the larval stage and cannot move away from the plant. However, although they cause much damage in the nymph and adult stages, they may move away from the plant because they are mobile. Finally, the more different insect species a plant attracts, the more damage it suffers.

Due to the hesitancy of DMs in solving this problem, although there was no uncertainty, it was deemed appropriate to evaluate with both intuitionistic, pythagorean and circular intuitionistic fuzzy TOPSIS methods. In such studies, evaluation was made using fuzzy MCDM methods for the first time. In this study, we transformed the experimental data to into fuzzy numbers because this was a real-world problem and in real-world problems, decisions are usually made based on this point of view [55].

3. RESULTS

There are solutions for the above-mentioned two perspectives where the DM weights are different as follows respectively. The first (from Table 26; M9) was applied with the decision matrix for individual

DM assessments, and the second (from Table 26; M12) was applied with the decision matrix for fixed DM assessments.

3.1. Application Results

C-IF TOPSIS when DMs were created individual decision matrices (M9)

The problem described in subsection 2.3 was solved with the steps given below.

Step 1. In this study, a DM group of four, including entomologists and agronomists, was formed. Relative expertise weights of DMs were calculated with Equation 1 and were given in Table 3.

Table 3. Relative importance levels and weights of DMs

Decision Makers	DM1	DM2	DM3	DM4
Linguistic variables	Very important	Very important	Important	Medium
IFNs	(0.9, 0.1)	(0.9, 0.1)	(0.75, 0.2)	(0.5, 0.45)
Weights (σ_k)	0,40379	0,40379	0,19085	0,00158

Step 2. In order to compare four alternative sorghum varieties in terms of damage from insects, measurements were made according to 6 different criteria and these measurements are represented by linguistic variables according to Table 2. Each DM was evaluated the measurement results given in Table 16 and Table 4 was obtained.

Table 4. Linguistic decision matrix for each DM

		Alternatives			
Criteria	**Decision makers**	**Nutri Honey**	**Nutrima**	**M81E**	**Topper76**
30-120cm	DM1	HI	AAI	AI	VHI
	DM2	AAI	AI	UAI	HI
	DM3	HI	AAI	AI	VHI
	DM4	HI	AAI	AI	VHI
150cm+	DM1	UAI	AI	HI	AAI
	DM2	UAI	UAI	AAI	AI
	DM3	UAI	AI	VHI	AAI
	DM4	AI	AAI	VHI	HI
Larva	DM1	AAI	AI	AAI	UAI
	DM2	AAI	AI	AAI	UAI
	DM3	AI	UAI	AI	LI
	DM4	AAI	UAI	AI	LI
Nymph	DM1	UAI	AAI	HI	AAI
	DM2	AI	AAI	HI	AAI
	DM3	UAI	AI	AAI	AI
	DM4	LI	AI	AAI	UAI
Adult	DM1	AAI	AAI	HI	VHI
	DM2	HI	HI	VHI	VHI
	DM3	AAI	AI	VHI	CHI
	DM4	HI	AAI	VHI	CHI
NIS	DM1	UAI	AI	AAI	AI
	DM2	AI	AAI	HI	AAI
	DM3	AI	AAI	HI	AAI
	DM4	AI	AAI	VHI	HI

After the C-IFN equivalents of the LTs given in Table 4 were written in their place according to Table 2, they were combined with the DM weights in Table 3 and aggregated intuitionistic fuzzy decision matrix (μC_i, ϑC_i) was obtained with Equation 2.

Step 3. Radius lengths were calculated using Table 4 and Aggregated intuitionistic fuzzy decision matrix, and maximum Radius lengths were obtained with Equation 3. Circular Intuitionistic fuzzy decision matrix $((\mu C_i, \vartheta C_i); r_{ij})$ was presented with Table 5.

Table 5. Circular Intuitionistic fuzzy decision matrix $((\mu C_i, \vartheta C_i); r_{ij})$

Criteria	Alternatives			
	Nutri Honey	Nutrima	M81E	Topper76
30-120cm	(0.663, 0,286); 0.090	(0.562, 0.387); 0.088	(0.462, 0.488); 0.088	(0.764, 0.184); 0.092
150cm+	(0.400, 0.550); 0.141	(0.462, 0.488); 0.195	(0.688, 0.260); 0.156	(0.562, 0.387); 0.194
Larva	(0.583, 0.367); 0.117	(0.482, 0.468); 0.116	(0.582, 0.367); 0.117	(0.382, 0.568); 0.116
Nymph	(0.442, 0.507); 0.202	(0.582, 0.367); 0.117	(0.683, 0.267); 0.118	(0.582, 0.367); 0.258
Adult	(0.644, 0.305); 0.079	(0.628, 0.321); 0.182	(0.764, 0.184); 0.092	(0.825, 0.139); 0.084
NIS	(0.462, 0.488); 0.088	(0.562, 0.387); 0.088	(0.663, 0.286); 0.193	(0.562, 0.387); 0.194

Step 4. With the results obtained in Table 5, optimistic and pessimistic decision matrices were obtained, respectively, and given in Tables 6 and 7, respectively.

Table 6. Optimistic decision matrix $(\mu_{ij}+r_{ij}, \vartheta_{ij}-r_{ij})$

Criteria	Alternatives			
	Nutri Honey	Nutrima	M81E	Topper76
30-120cm	(0.753, 0.197)	(0.651, 0.299)	(0.549, 0.400)	(0.856, 0.092)
150cm+	(0.541, 0.409)	(0.657, 0.293)	(0.845, 0.103)	(0.757, 0.193)
Larva	(0.700, 0.250)	(0.598, 0.351)	(0.699, 0.251)	(0.498, 0.452)
Nymph	(0.644, 0.306)	(0.699, 0.251)	(0.800, 0.149)	(0.840, 0.109)
Adult	(0.723, 0.227)	(0.811, 0.138)	(0.856, 0.092)	(0.909, 0.054)
NIS	(0.549, 0.400)	(0.651, 0.299)	(0.856, 0.093)	(0.757, 0.193)

Table 7. Pessimistic decision matrix $(\mu_{ij}-r_{ij}, \vartheta_{ij}+r_{ij})$

Criteria	Alternatives			
	Nutri Honey	Nutrima	M81E	Topper76
30-120cm	(0.573, 0.376)	(0.474, 0.476)	(0.374, 0.576)	(0.672, 0.479)
150cm+	(0.259, 0.691)	(0.267, 0.683)	(0.532, 0.416)	(0.368, 0.682)
Larva	(0.466, 0.484)	(0.366, 0.584)	(0.466, 0.484)	(0.266, 0.584)
Nymph	(0.241, 0.709)	(0.466, 0.484)	(0.565, 0.384)	(0.324, 0.625)
Adult	(0.565, 0.384)	(0.446, 0.503)	(0.672, 0.276)	(0.741, 0.405)
NIS	(0.374, 0.576)	(0.474, 0.476)	(0.470, 0.479)	(0.368, 0.582)

Step 5. DMs evaluated the criteria with the LT given in Table 2 and criteria matrix was created with the corresponding C-IFNs. By using the DM weights given in Table 3, aggregated criterion weights were obtained via Equation 2. Criterion types, DM evaluations and aggregated criterion weights were listed in Table 8.

Table 8. Linguistic evaluations of criteria for each DM and aggregated criterion weights

Criteria	DM1	DM2	DM3	DM4	Aggregated Weights (μ_j, ϑ_j)	Type	
						Cost	Benefit
30-120cm	AAI	VHI	HI	HI	(0.714, 0.233)		X
150cm+	HI	AAI	VHI	VHI	(0.688, 0.260)		X
Larva	AI	AAI	HI	AI	(0.586, 0.363)		X
Nymph	HI	VHI	AAI	AI	(0.731, 0.217)		X
Adult	HI	HI	AAI	VHI	(0.683, 0.266)		X
NIS	AI	AI	HI	VHI	(0.547, 0.402)		X

All insects included in this study are pests and the aim of the study is to compare four sorghum varieties in terms of damage from insects. Therefore (although all criteria are cost), all criteria were evaluated as benefit type.

Step 6. Using Equation 4, radius lengths were calculated for each DM and maximum radius lengths were found and the C-IF criteria weight matrix given in Table 9 was obtained.

Table 9. Circular intuitionistic fuzzy criteria weight matrix $((\mu_j, \vartheta_j); r_j)$

Criteria	C-IF criteria weight matrix $((\mu_j, \vartheta_j); r_j)$
150cm+	(0.688, 0.260); 0.156
Larva	(0.586, 0.363); 0.161
Nymph	(0.731, 0.217); 0.328
Adult	(0.683, 0.266); 0.165
NIS	(0.547, 0.402); 0.357

Step 7. By using aggregated criteria weights given in Tables 8 and 9 and maximum radius lengths given in Table 9, optimistic and pessimistic criteria weight matrices were obtained as given in Table 10.

Table 10. Optimistic and pessimistic criteria weight matrices

Criteria	Optimistic criteria weights $(\mu_j+r_j, \vartheta_j-r_j)$	Pessimistic criteria weights $(\mu_j-r_j, \vartheta_j+r_j)$
30-120cm	(0.877, 0.070)	(0.551, 0.396)
150cm+	(0.845, 0.103)	(0.532, 0.416)
Larva	(0.747, 0.202)	(0.424, 0.525)
Nymph	(0.954, 0.008)	(0.403, 0.545)
Adult	(0.848, 0.102)	(0.518, 0.431)
NIS	(0.904, 0.045)	(0.190, 0.758)

Step 8. The weighted optimistic decision matrix was calculated using the values in Tables 6 and 10, and the pessimistic decision matrix was calculated using the values in Tables 7 and 10 were shown in Tables 11 and 12, respectively.

Table 11. Weighted optimistic decision matrix

	Alternatives			
Criteria	Nutri Honey	Nutrima	M81E	Topper76
30-120cm	(0.660, 0.253)	(0.571, 0.348)	(0.482, 0.442)	(0.751, 0.156)
150cm+	(0.457, 0.470)	(0.555, 0.366)	(0.714, 0.196)	(0.639, 0.276)
Larva	(0.522, 0.402)	(0.447, 0.483)	(0.522, 0.402)	(0.372, 0.563)
Nymph	(0.682, 0.229)	(0.740, 0.168)	(0.847, 0.055)	(0.890, 0.011)
Adult	(0.613, 0.305)	(0.688, 0.226)	(0.726, 0.185)	(0.771, 0.150)
NIS	(0.497, 0.427)	(0.588, 0.330)	(0.774, 0.134)	(0.684, 0.229)

Table 12. Weighted pessimistic decision matrix

	Alternatives			
Criteria	Nutri Honey	Nutrima	M81E	Topper76
30-120cm	(0.316, 0.623)	(0.261, 0.683)	(0.206, 0.744)	(0.370, 0.686)
150cm+	(0.138, 0.820)	(0.142, 0.815)	(0.283, 0.659)	(0.196, 0.814)
Larva	(0.198, 0.755)	(0.155, 0.802)	(0.198, 0.755)	(0.113, 0.802)
Nymph	(0.097, 0.868)	(0.188, 0.765)	(0.228, 0.720)	(0.131, 0.830)
Adult	(0.293, 0.650)	(0.231, 0.717)	(0.349, 0.588)	(0.384, 0.662)
NIS	(0.071, 0.897)	(0.090, 0.873)	(0.090, 0.874)	(0.070, 0.899)

Step 9. Positive and negative ideal solutions based on weighted optimistic decision matrix given in Table 11 were calculated with Equation 5, and positive and negative ideal solutions based on the weighted pessimistic decision matrix given in Table 12 were calculated with Equation 6 and presented with Table 13.

Table 13. Positive and negative ideal solutions based on the optimistic and pessimistic decision matrices

	Optimistic matrix		Pessimistic matrix	
Criteria	**PIS**	**NIS**	**PIS**	**NIS**
30-120cm	(0.751, 0.156)	(0.482, 0.442)	(0.370, 0.623)	(0.206, 0.744)
150cm+	(0.714, 0.196)	(0.457, 0.470)	(0.283, 0.659)	(0.138, 0.820)
Larva	(0.522, 0.402)	(0.372, 0.563)	(0.198, 0.755)	(0.113, 0.802)
Nymph	(0.890, 0.011)	(0.682, 0.229)	(0.228, 0.720)	(0.097, 0.868)
Adult	(0.771, 0.150)	(0.613, 0.305)	(0.384, 0.588)	(0.231, 0.717)
NIS	(0.774, 0.134)	(0.497, 0.427)	(0.090, 0.873)	(0.070, 0.899)

Step 10. Separation measures of alternatives were calculated separately for optimistic and pessimistic matrices. Separation measures of alternatives were obtained using Equation 7 for the optimistic matrix and Equation 8 for the pessimistic matrix were given in Table 14.

Table 14. Separation measures of alternatives based on optimistic and pessimistic decision matrices

	Optimistic decision matrix		Pessimistic decision matrix	
Alternatives	$\boldsymbol{D_i^{O^*}}$	$\boldsymbol{D_i^{O^-}}$	$\boldsymbol{D_i^{O^*}}$	$\boldsymbol{D_i^{O^-}}$
Nutri Honey	0.196	0.098	0.092	0.061
Nutrima	0.150	0.085	0.095	0.048
M81E	0.116	0.191	0.060	0.103
Topper76	0.081	0.191	0.078	0.072

Step 11. The closeness coefficients ($CC_i^O$and CC_i^P) of each alternative for the optimistic and pessimistic matrices were calculated by Equation 9 and 10, respectively. Then their composite ratio (CR_i) was calculated with Equation 11. In this study, optimistic and pessimistic attitudes of DMs were taken as equal (λ=0.5). Finally, the alternatives were listed according to their CR scores.

Table 15. Closeness coefficients of alternatives based on optimistic and pessimistic decision matrices ($CC_i^O$and CC_i^P, respectively) and composite ratio scores (CR_i) with ranks

Alternatives	$\mathbf{CC_i^O}$	Rank	$\mathbf{CC_i^P}$	Rank	$\mathbf{CR_i}$	Final Rank
Nutri Honey	0.334	4	0.398	3	0.366	3
Nutrima	0.361	3	0.339	4	0.350	4
M81E	0.622	1	0.633	1	0.627	1
Topper76	0.704	2	0.481	2	0.593	2

The last column of Table 15 is shown in the final rank and the sorghum variety most damaged by insects is M81E. The final rank is M81E>Topper76>Nutri Honey> Nutrima.

C-IF TOPSIS when a fixed decision matrix was created for each DM (M12)

The problem described in Subsection 2.3 was solved with the steps given below.

Step 1. Same as Subsection 3.1.
Step 2. Since the measurements were certain, it was treated as if there was only one DM view and a fixed assessment was made for all DMs (like a single DM). Table 16 includes the number of pests detected in the alternatives for each criterion and the corresponding linguistic variables.

Table 16. Evaluation of alternatives according to criteria and fixed linguistic decision matrix for all DMs

Alternatives	Criteria											
	30-120cm	LT	150cm+	LT	Larva	LT	Nymph	LT	Adult	LT	NIS	LT
Nutri Honey	665	CHV	163	VLV	248	LV	43	CLV	537	HV	118	HV
Nutrima	643	VHV	194	LV	215	LV	97	VLV	525	HV	126	HV
M81E	620	VHV	380	AV	246	LV	152	VLV	702	CHV	156	VHV
Topper76	691	CHV	289	UAV	164	VLV	90	VLV	726	CHV	128	VH

Here, the first five criteria were evaluated by adapting them to a nine-point scale in the range of 0-750 insect numbers. Since the NIS criterion was grouped according to the species to which the insects belong, the numerical distribution according to the alternatives was different from the other criteria and was adapted to the scale differently, therefore it was different from the other criteria in terms of the LTs. After substituting the IFN equivalents of the LTs given in Table 16, they were combined with Equation 2 using the DM weights in Table 3, and an Aggregated intuitionistic fuzzy decision matrix (μC_i, ϑC_i) was obtained.

Step 3. Radius lengths were calculated using Table 16 and Aggregated intuitionistic fuzzy decision matrix. Maximum Radius lengths were obtained with Equation 3. The circular Intuitionistic fuzzy decision matrix ((μC_i, ϑC_i); r_i) was presented in Table 17.

Table 17. Circular Intuitionistic fuzzy decision matrix ((μC_i, ϑC_i); r_{ij})

Criteria	Alternatives			
	Nutri Honey	Nutrima	M81E	Topper76
30-120cm	(0.9, 0.10); 2.78E^{-17}	(0.8, 0.15); 2.78E^{-17}	(0.8, 0.15); 2.78E^{-17}	(0.9, 0.10); 2.78E^{-17}
150cm+	(0.2, 0.75); 1.24E^{-16}	(0.3, 0.65); 1.24E^{-16}	(0.5, 0.45); 1.24E^{-16}	(0.4, 0.55); 1.11E^{-16}
Larva	(0.3, 0.65); 1.24E^{-16}	(0.3, 0.65); 1.24E^{-16}	(0.3, 0.65); 1.24E^{-16}	(0.2, 0.75); 1.24E^{-16}
Nymph	(0.1, 0.90); 2.78E^{-17}	(0.2, 0.75); 1.24E^{-16}	(0.2, 0.75); 1.24E^{-16}	(0.2, 0.75); 1.24E^{-16}
Adult	(0.7, 0.25); 1.11E^{-16}	(0.7, 0.25); 1.11E^{-16}	(0.9, 0.10); 2.78E^{-17}	(0.9, 0.10); 2.78E^{-17}
NIS	(0.7, 0.25); 1.11E^{-16}	(0.7, 0.25); 1.11E^{-16}	(0.8, 0.15); 2.78E^{-17}	(0.7, 0.25); 1.11E^{-16}

When Table 17 was examined, it was noteworthy that the max r_{ij} values were quite small. This situation represented the reconciliation of the DMs and was a desirable situation [52]. Since alternative evaluations of DMs were fixed in this study, r_{ij}s were obtained in this way.

Step 4. Since the max rij values were extremely small, the optimistic and pessimistic decision matrices were equal and presented in Table 18 (and they were also equal to the aggregated intuitionistic decision matrix).

Table 18. Optimistic decision matrix ($\mu_{ij}+r_{ij}$, $\vartheta_{ij}-r_{ij}$) and Pessimistic decision matrix ($\mu_{ij}-r_{ij}$, $\vartheta_{ij}+r_{ij}$)

Criteria	Alternatives			
	Nutri Honey	Nutrima	M81E	Topper76
30-120cm	(0.9, 0.10)	(0.8, 0.15)	(0.8, 0.15)	(0.9, 0.10)
150cm+	(0.2, 0.75)	(0.3, 0.65)	(0.5, 0.45)	(0.4, 0.55)
Larva	(0.3, 0.65)	(0.3, 0.65)	(0.3, 0.65)	(0.2, 0.75)
Nymph	(0.1, 0.90)	(0.2, 0.75)	(0.2, 0.75)	(0.2, 0.75)
Adult	(0.7, 0.25)	(0.7, 0.25)	(0.9, 0.10)	(0.9, 0.10)
NIS	(0.7, 0.25)	(0.7, 0.25)	(0.8, 0.15)	(0.7, 0.25)

Steps 5, 6, and 7 were the same as in Subsection 3.1.

Step 8. Using the values in Tables 17 and 18, weighted optimistic and pessimistic decision matrices were obtained as shown in Tables 19 and 20, respectively.

Table 19. Weighted optimistic decision matrix

	Alternatives			
Criteria	**Nutri Honey**	**Nutrima**	**M81E**	**Topper76**
30-120cm	(0.790, 0.163)	(0.702, 0.209)	(0.702, 0.209)	(0.790, 0.163)
150cm+	(0.169, 0.776)	(0.253, 0.686)	(0.422, 0.507)	(0.338, 0.596)
Larva	(0.224, 0.721)	(0.224, 0.721)	(0.224, 0.721)	(0.149, 0.801)
Nymph	(0.106, 0.899)	(0.212, 0.722)	(0.212, 0.722)	(0.212, 0.722)
Adult	(0.594, 0.326)	(0.594, 0.326)	(0.763, 0.191)	(0.763, 0.191)
NIS	(0.633, 0.284)	(0.633, 0.284)	(0.723, 0.188)	(0.633, 0.284)

Table 20. Weighted pessimistic decision matrix

	Alternatives			
Criteria	**Nutri Honey**	**Nutrima**	**M81E**	**Topper76**
30-120cm	(0.496, 0.457)	(0.441, 0.487)	(0.441, 0.487)	(0.496, 0.487)
150cm+	(0.106, 0.854)	(0.160, 0.796)	(0.266, 0.679)	(0.213, 0.796)
Larva	(0.127, 0.834)	(0.127, 0.834)	(0.127, 0.834)	(0.085, 0.834)
Nymph	(0.040, 0.954)	(0.081, 0.886)	(0.081, 0.886)	(0.081, 0.886)
Adult	(0.363, 0.573)	(0.363, 0.573)	(0.467, 0.488)	(0.467, 0.573)
NIS	(0.133, 0.819)	(0.133, 0.819)	(0.152, 0.795)	(0.133, 0.819)

Step 9. The positive and negative ideal solutions based on the weighted optimistic decision matrix given in Table 19 were calculated with Equation 5, respectively, and the positive and negative ideal solutions based on the weighted pessimistic decision matrix given in Table 20 were calculated with Equation 6, respectively, and presented with Table 21.

Table 21. Positive and negative ideal solutions based on the optimistic and pessimistic decision matrices

	Optimistic matrix		Pessimistic matrix	
Criteria	**PIS**	**NIS**	**PIS**	**NIS**
30-120cm	(0.790, 0.163)	(0.702, 0.209)	(0.496, 0.457)	(0.441, 0.487)
150cm+	(0.422, 0.507)	(0.169, 0.776)	(0.266, 0.679)	(0.106, 0.854)
Larva	(0.224, 0.721)	(0.149, 0.801)	(0.127, 0.834)	(0.085, 0.834)
Nymph	(0.212, 0.722)	(0.106, 0.899)	(0.081, 0.886)	(0.040, 0.954)
Adult	(0.763, 0.191)	(0.594, 0.326)	(0.467, 0.488)	(0.363, 0.573)
NIS	(0.723, 0.188)	(0.633, 0.284)	(0.152, 0.795)	(0.133, 0.819)

Step 10. Separation measures of alternatives obtained using Equation 7 for the optimistic matrix and Equation 8 for the pessimistic matrix were shown in Table 22.

Table 22. Separation measures of alternatives based on optimistic and pessimistic decision matrices

	Optimistic decision matrix		Pessimistic decision matrix	
Alternatives	$\mathbf{D_i^{O^*}}$	$\mathbf{D_i^{O^-}}$	$\mathbf{D_i^{O^*}}$	$\mathbf{D_i^{O^-}}$
Nutri Honey	0.141	0.043	0.082	0.022
Nutrima	0.106	0.074	0.063	0.035
M81E	0.029	0.145	0.018	0.083
Topper76	0.061	0.114	0.048	0.054

Step 11. The closeness coefficients ($CC_i^O$and CC_i^P) of each alternative for the optimistic and pessimistic matrices were calculated by Equations 9 and 10, respectively. Then their composite ratio (CR_i) was calculated with Equation 11. In this study, the optimistic and pessimistic attitudes of DMs were taken into account as equal (λ=0.5). Finally, the alternatives were ranked according to their CR scores.

Table 23. Closeness coefficients of alternatives based on optimistic and pessimistic decision matrices ($CC_i^O$and CC_i^P, respectively) and composite ratio scores (CR_i) with ranks

Alternatives	CC_i^O	Rank	CC_i^P	Rank	CR_i	Final Rank
Nutri Honey	0.232	4	0.210	4	0.221	4
Nutrima	0.412	3	0.354	3	0.383	3
M81E	0.835	1	0.821	1	0.828	1
Topper76	0.652	2	0.530	2	0.591	2

The last column of Table 23 shows the final rank and the sorghum varieties most damaged by insects is M81E. The final rank is M81E>Topper76>Nutrima>Nutri Honey.

3.2. Sensitivity Analyses

Sensitivity analysis scenarios for optimistic and pessimistic attitudes of DMs

Here, the varying CR_i scores for λ=0, 0.1, …, 1 for optimistic and pessimistic attitudes of DMs were compared for both M9 and M12. The results were shown in Table 24 and Figure 7.

Table 24. Sensitivity analysis for λ value [52]

Methods	Score of $\widetilde{D}_{IFWA}$	CR_i scores of alternatives				Rank
		Nutri Honey	Nutrima	M81E	Topper76	
M9	λ=0	0.398	0.339	0.633	0.481	M81E>Topper76> Nutri Honey>Nutrima
	λ=0.1	0.391	0.341	0.631	0.503	M81E>Topper76> Nutri Honey>Nutrima
	λ=0.2	0.385	0.343	0.630	0.526	M81E>Topper76> Nutri Honey>Nutrima
	λ=0.3	0.379	0.346	0.629	0.548	M81E>Topper76> Nutri Honey>Nutrima
	λ=0.4	0.372	0.348	0.628	0.570	M81E>Topper76> Nutri Honey>Nutrima
	λ=0.5	0.366	0.350	0.627	0.593	M81E>Topper76> Nutri Honey>Nutrima
	λ=0.6	0.360	0.352	0.626	0.615	M81E>Topper76> Nutri Honey>Nutrima
	λ=0.7	0.353	0.355	0.625	0.637	Topper76> M81E> Nutrima>Nutri Honey
	λ=0.8	0.347	0.357	0.624	0.659	Topper76> M81E> Nutrima>Nutri Honey
	λ=0.9	0.341	0.359	0.623	0.682	Topper76> M81E> Nutrima>Nutri Honey
	λ=1	0.334	0.361	0.622	0.704	Topper76> M81E> Nutrima>Nutri Honey
M12	λ=0	0.210	0.354	0.821	0.530	M81E>Topper76>Nutrima>Nutri Honey
	λ=0.1	0.212	0.359	0.823	0.542	M81E>Topper76>Nutrima>Nutri Honey
	λ=0.2	0.214	0.365	0.824	0.554	M81E>Topper76>Nutrima>Nutri Honey
	λ=0.3	0.216	0.371	0.825	0.567	M81E>Topper76>Nutrima>Nutri Honey
	λ=0.4	0.219	0.377	0.827	0.579	M81E>Topper76>Nutrima>Nutri Honey
	λ=0.5	0.221	0.383	0.828	0.591	M81E>Topper76>Nutrima>Nutri Honey
	λ=0.6	0.223	0.389	0.829	0.603	M81E>Topper76>Nutrima>Nutri Honey
	λ=0.7	0.225	0.394	0.831	0.616	M81E>Topper76>Nutrima>Nutri Honey
	λ=0.8	0.227	0.400	0.832	0.628	M81E>Topper76>Nutrima>Nutri Honey
	λ=0.9	0.229	0.406	0.833	0.640	M81E>Topper76>Nutrima>Nutri Honey
	λ=1	0.232	0.412	0.835	0.652	M81E>Topper76>Nutrima>Nutri Honey

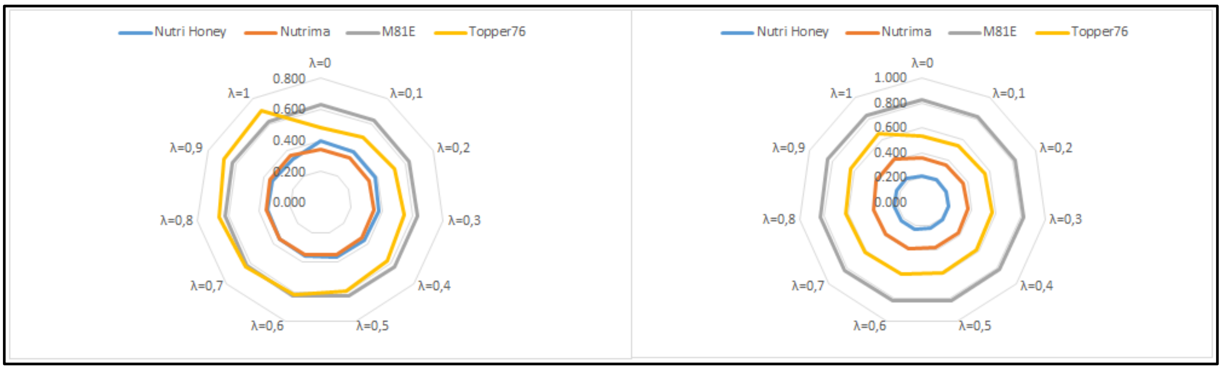

Figure 7. Sensitivity analysis for λ value based on DMs optimistic and pessimistic attitudes (M9 and M12, respectively)

In the M9 method, the optimistic (λ>0.5) or pessimistic (λ<0.5) approaches of the DMs had a great effect on the ranking. For M9, the rank for λ≥0.6 is Topper76> M81E> Nutrima>Nutri Honey, while for λ<0.6 the rank is M81E>Topper76> Nutri Honey> Nutrima. However, it is seen that the change in the λ value did not cause a significant change in the ranking of the alternatives for the M12 method. For the M12 method, the optimistic or pessimistic approaches of the DMs had little effect and the rank was unchanged and was M81E>Topper76>Nutrima> Nutri Honey.

Here, results showed that the optimistic approaches of DMs were completely changed the rank of alternatives in the M9 method. Topper76 when DMs were optimists and M81E when pessimists were the most damaged sorghum variety. On the contrary, optimistic or pessimistic approaches of DMs have not affected the rank in the M12 method, and the most damaged sorghum variety was M81E.

Sensitivity analysis scenarios for criteria weights

By changing the criterion weights, it was aimed to show that the proposed methods were robust and it was examined whether there was a change in the order of the alternatives. Here, the proposed C-IF TOPSIS methods were applied through scenarios where DMs evaluated the criteria in eight different ways. Related results were shown in Table 25 and Figure 8.

Table 25. Sensitivity analysis for different weights of criteria

		CR_i scores of alternatives				
Methods	**Scenarios**	**Nutri Honey**	**Nutrima**	**M81E**	**Topper76**	**Rank**
M9	S1	0.369	0.347	0.636	0.582	M81E>Topper76> Nutri Honey>Nutrima
	S2	0.370	0.379	0.620	0.589	M81E>Topper76>Nutrima>Nutri Honey
	S3	0.382	0.391	0.598	0.592	M81E>Topper76>Nutrima>Nutri Honey
	S4	0.364	0.370	0.647	0.552	M81E>Topper76>Nutrima>Nutri Honey
	S5	0.342	0.351	0.647	0.595	M81E>Topper76>Nutrima>Nutri Honey
	S6	0.413	0.319	0.665	0.548	M81E>Topper76> Nutri Honey>Nutrima
	S7	0.366	0.352	0.631	0.590	M81E>Topper76> Nutri Honey>Nutrima
	S8	0.367	0.345	0.624	0.597	M81E>Topper76> Nutri Honey>Nutrima
M12	S1	0.219	0.381	0.827	0.591	M81E>Topper76>Nutrima>Nutri Honey
	S2	0.193	0.381	0.852	0.608	M81E>Topper76>Nutrima>Nutri Honey
	S3	0.245	0.406	0.803	0.616	M81E>Topper76>Nutrima>Nutri Honey
	S4	0.222	0.375	0.842	0.567	M81E>Topper76>Nutrima>Nutri Honey
	S5	0.189	0.325	0.846	0.578	M81E>Topper76>Nutrima>Nutri Honey
	S6	0.257	0.370	0.844	0.513	M81E>Topper76>Nutrima>Nutri Honey
	S7	0.223	0.381	0.829	0.590	M81E>Topper76>Nutrima>Nutri Honey
	S8	0.219	0.381	0.827	0.591	M81E>Topper76>Nutrima>Nutri Honey

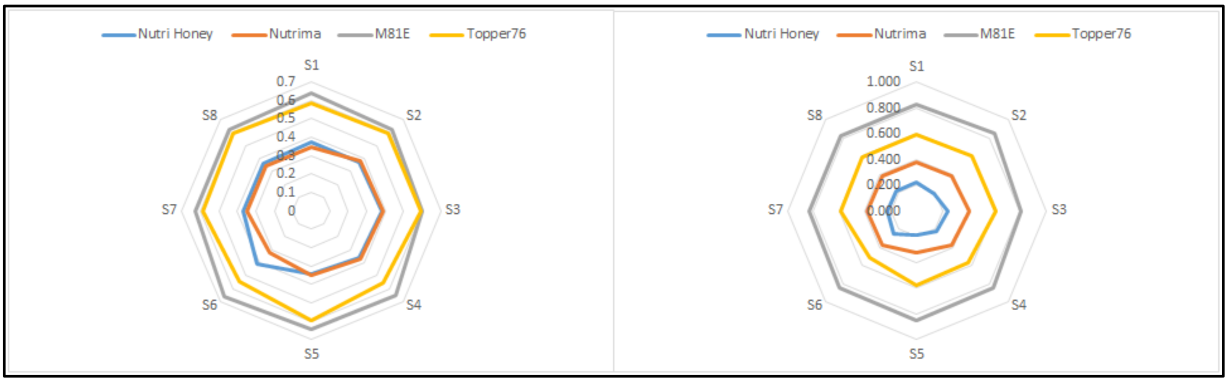

Figure 8. Sensitivity analysis based on the criteria weights (M9 and M12, respectively)

Here S1 represented the scenario with the current criterion weights. S2, S3, S4 and S5 were scenarios where the criteria evaluations of DM1, DM2, DM3 and DM4 were fixed, respectively. S6 was the scenario that occurred when a criterion evaluation was made in the opposite direction of each DM evaluation (for example, if the LT value of the current criterion assessment was VH, in S6 was also VL, etc.). When S7 evaluated the criteria of each DM with a higher value from its current evaluation (for example, if the LT value of the current criterion was AI, in S7 was also AAI, etc.), and when S8 evaluated the criteria of each DM with a lower value from its current evaluation (for example, if the LT value of current criterion was L, in S8 was also VL, etc.) was the resulting scenarios. For M9, the ranks in S2, S3, S4, and S5 were the same and M81E>Topper76>Nutrima>Nutri Honey, in other scenarios the ranks were same as M81E>Topper76>Nutri Honey > Nutrima. For M12, the ranks in all scenarios were same as M81E>Topper76>Nutrima>Nutri Honey.

According to these analyses, the changes in the criteria weights were caused a change in the ranking of the third and fourth alternatives for the M9 method but did not cause any change in the ranks of the alternatives for the M12 method. The reason for the partial changes in the M9 ranks was the individual evaluations of DMs. However, that's why the ranks changeless, especially in the first two, was that DMs' different expertise weights. This was shown that the proposed method was robust.

Sensitivity analysis scenarios for DM weights

By changing the DM importance weights, it was aimed to show that the proposed methods were robust and it was examined whether there was a change in the ranking of the alternatives in both M9 and M12. Here, scenarios were developed and implemented based on the evaluation of DM importance values for both perspectives in 39 different ways. Related results were shown in Figure 9.

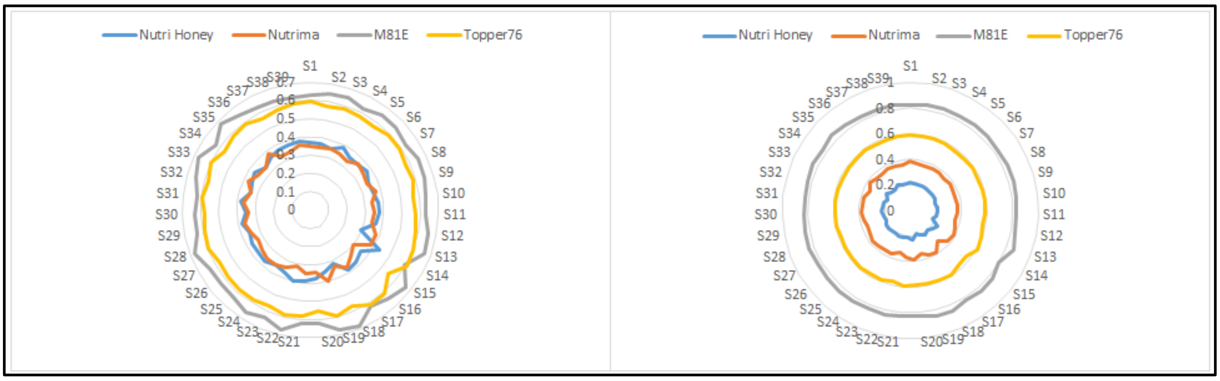

Figure 9. Sensitivity analysis based on the DM weights (M9 and M12, respectively)

Here, the $\mathrm{CR_i}$ scores of the alternatives were ranked over 39 different importance weights for both perspectives by changing the importance of each DM between M and VI as LTs. In one scenario (S14) for M9, the ranking Topper76>M81E>Nutri Honey>Nutrima was obtained. In other scenarios, the M81E alternative was in the first place, the Topper76 alternative was in the second place, and the third and fourth alternatives, although varying, were usually Nutri Honey and Nutrima, respectively. For M12, the ranks in all scenarios were the same as M81E>Topper76>Nutrima>Nutri Honey.

Here, if the DM weights were changed, it was checked whether the rank changed. The first and second alternatives in one of the 39 scenarios designed for the M9 method, and the third and fourth alternatives in seven of them were changed. In other words, Topper76 was determined as the most damaged sorghum variety in only one scenario and M81E in 38 scenarios. The reason for partial changes in the M9 ranks was the individual evaluations of DMs. A significant ranking change was observed in only one scenario. The reason for this was that in calculating the expertise weights of DMs, medium to very high values were used and the poor expertise of the DMs was not taken into account. Considering the overall scenarios, it can be seen that the ranks were almost identical and the proposed method was robustness. In the M12 method, it was observed that there was no change in ranks.

3.3. Comparative Analysis

IF TOPSIS, PF TOPSIS and C-IF TOPSIS methods are very similar to each other [12, 41, 52]. To solve the fuzzy MCDM problem explained above, IF TOPSIS, PF TOPSIS and C-IF TOPSIS methods were considered from two different perspectives. Each method was solved for cases where DM weights were equal and different, respectively.

Table 26. Results of comperative analyses

Methods			$\mathrm{CR_i}$ scores of alternatives and ranks							
			Nutri Honey	Rank	Nutrima	Rank	M81E	Rank	Topper 76	Rank
Equal DM weights and different alternative significant levels	IF TOPSIS	M1	0,299	4	0,328	3	0,722	1	0,591	2
	PF TOPSIS	M2	0,311	4	0,347	3	0,748	1	0,578	2
	C-IF TOPSIS	M3	0,373	3	0,349	4	0,636	1	0,575	2
Equal DM weights and fixed alternative significant levels	IF TOPSIS	M4	0,277	4	0,322	3	0,754	1	0,717	2
	PF TOPSIS	M5	0,224	4	0,419	3	0,826	1	0,631	2
	C-IF TOPSIS	M6	0,214	4	0,367	3	0,834	1	0,585	2
Different DM weights and different alternative significant levels	IF TOPSIS	M7	0,309	4	0,329	3	0,685	1	0,604	2
	PF TOPSIS	M8	0,315	4	0,349	3	0,732	1	0,581	2
	*C-IF TOPSIS	M9	0.366	3	0.350	4	0.627	1	0.593	2
Different DM weights and fixed alternative significant levels	IF TOPSIS	M10	0,282	4	0,322	3	0,750	1	0,720	2
	PF TOPSIS	M11	0,224	4	0,424	3	0,828	1	0,631	2
	*C-IF TOPSIS	M12	0.221	4	0.383	3	0.828	1	0.591	2

*Different DM weights for C-IF TOPSIS

When the Table 26 was examined, it was clear that there were only partial changes in the third and fourth ranks only for M3 and M9, while the ranks were same in other methods.

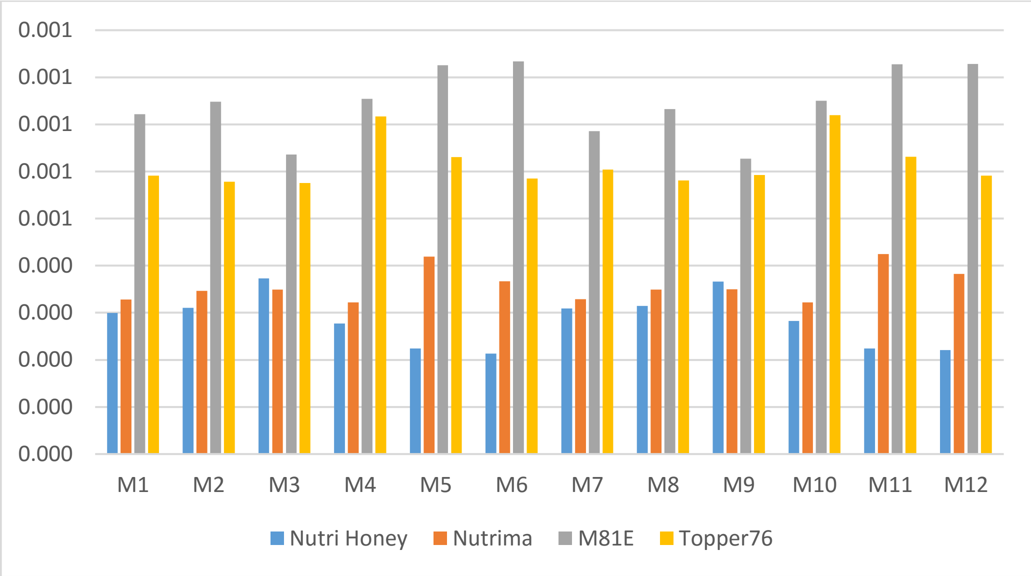

Figure 10. Result of comparative analyses

It was clear that the M81E was in the first place in all methods. In 11 of the 12 methods applied, the ranks were same as M81E>Topper76>Nutrima>Nutri Honey.

IF-TOPSIS and PF-TOPSIS methods were applied for equal and different DM weights (DM weights obtained with the formula for IF-TOPSIS and PF-TOPSIS methods proposed by [52, 53]) and also for fixed and different decision matrices. Moreover, for equal DM weights, the C-IF TOPSIS method was applied by creating fixed and different decision matrices.

We had two reasons for conducting the comparative analyses. First, to determine whether our approach was consistent with other Fuzzy TOPSIS results accepted in the literature, and second, to investigate the reasons for any discrepancies when different results were obtained. Upon reviewing the results, it was observed that only for M3 and M9 did the alternatives ranked third and fourth switch places. Therefore, our approach was consistent. The slight variations in the results for M3 and M9 were due to the decision-makers not reaching a consensus on the numerical outcomes. This highlighted the reliability of the M12 approach for similar studies.

4. CONCLUSION

In this study, Circular Intuitionistic Fuzzy Sets (C-IFSs) were applied to the TOPSIS method to address hesitation in multi-criteria decision-making problems, specifically within the field of agricultural biology. By incorporating decision-maker weights through the use of C-IFSs, the study successfully demonstrated the flexibility of this novel approach in handling uncertainty and hesitancy. Sensitivity analyses, conducted from various perspectives, provided valuable insights into how changes in decision-maker expertise and criteria importance affected the ranking of alternatives.

The comparative analysis between Intuitionistic Fuzzy TOPSIS (IF-TOPSIS), Pythagorean Fuzzy TOPSIS (PF-TOPSIS), and C-IF TOPSIS methods showed consistent results across different scenarios. Specifically, the M81E sorghum variety consistently ranked as the most damaged by pests, validating the robustness of the C-IF TOPSIS approach. While slight variations were observed in the ranking of third and fourth alternatives in specific cases (M3 and M9), these differences were attributed to minor disagreements among decision-makers regarding numerical evaluations. This underlines the importance of properly accounting for decision-maker expertise when handling hesitation in fuzzy multi-criteria decision-making problems.

Overall, the C-IF TOPSIS method proved to be a reliable and robust tool, particularly in scenarios involving decision-maker hesitation, making it a valuable contribution to the ongoing development of fuzzy decision-making methods. Further research can extend the application of this method to other domains, exploring its potential to enhance decision-making in more complex and uncertain environments.

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CONFLICT OF INTEREST

No other authors declared a potential conflict of interest regarding this article.

CRediT AUTHOR STATEMENT

Zeynep Gökkuş: Formal analysis, Writing - original draft, Visualization, Conceptualization, Supervision. **Sevil Şentürk:** Formal analysis, Writing - original draft, Visualization, Conceptualization, Supervision. **Fırat Alaltürk:** Obtaining experimental data. **Baboo Ali:** Obtaining experimental data.

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