

Hybrid Deep Learning-Enhanced Topological Data Analysis Framework for Real-Time Detection and Classification of Chaotic Attractors

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ABSTRACT We introduce a hybrid framework that combines Topological Data Analysis (TDA) and deep learning architectures to detect and classify chaotic attractors in high-dimensional dynamical systems with real-time capability. Our approach exploits persistent homology to extract robust topological features, which are then processed by convolutional neural networks (CNNs) for pattern recognition. Our algorithm is both more accurate and more computationally efficient than state of the art tools such as traditional Lyapunov exponent analysis, phase space reconstruction methods and more recent deep learning tools. Experimental results show that our algorithm is 95.8% more accurate and 50ms faster to run on 1000-dimensional input data (95% CI: [94.6% 97.0%]) than compared to state of the art methods, including the traditional Lyapunov exponent analysis and phase space reconstruction methods and more recent deep learning methods. The model is extremely resistant to noise, and its accuracy with signal-to-noise ratios as low as 15dB is 92.3% with 1.5% standard deviation. Extensive ablation experiments show that the hybrid method is better than the single TDA (82.4% accuracy) and deep learning (78.9% accuracy) modules, which proves the synergy advantage. Performance analysis $O(n \log n)$ computational complexity and linear scaling properties. The performance analysis has a 3.2x speedup over traditional algorithms and has a 45 percent memory reduction. The study is an improvement on nonlinear dynamics as it offers an efficient, scalable, and robust algorithm to identify chaotic system dynamics in real-time and can be applied in climate modeling, financial markets, and neurological signal processing.

KEYWORDS
Nonlinear dynam-
ics
Topological data
analysis
Deep learning
Chaotic attrac-
tors
Persistent homol-
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Real-time pro-
cessing

INTRODUCTION

The analysis and classification of chaotic attractors in nonlinear dynamical systems present enduring challenges due to their complex topological structures, high sensitivity to initial conditions, and the prevalence of noise in real-world data (Takens 1981; Cohen-Steiner *et al.* 2007). Traditional approaches, such as phase space reconstruction and Lyapunov exponent analysis, often struggle with scalability and robustness, particularly in high-dimensional or noisy environments (Takens 1981; Cohen-Steiner *et al.* 2007; Gidea and Katz 2018). Recent advances in Topological Data Analysis (TDA)

have provided powerful tools for extracting robust, noise-invariant features from complex dynamical systems. In particular, persistent homology enables the quantification of topological structures, such as loops and voids, within chaotic attractors, offering a stable and interpretable feature set for downstream analysis (Cohen-Steiner *et al.* 2007; Gidea and Katz 2018; Smith *et al.* 2021). However, while TDA excels at capturing global geometric properties, it often requires integration with advanced machine learning techniques to achieve high-accuracy classification in practical applications (Majumdar and Laha 2020; Myers *et al.* 2019).

Deep learning has revolutionized pattern recognition across domains, including speech, vision, and scientific data analysis, by enabling hierarchical feature learning from raw or engineered inputs (LeCun *et al.* 2015; Singh *et al.* 2023). In the context of dynamical systems, deep neural networks have demonstrated the ability to model and predict complex, nonlinear behaviors, and even generalize across different chaotic regimes (Celletti *et al.* 2022; Röhm *et al.* 2021; Young and Graham 2022).

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It has been demonstrated by the recent research that TDA can be significantly improved in combination with deep learning architectures to effectively detect and classify chaotic attractors, particularly in the case of noisy or high-dimensional conditions (Almazova et al. 2021; Zia et al. 2024; Kavuran 2022; Zhu et al. 2022). In spite of these developments, current techniques are usually constrained by real-time processing, scalability as well as noise tolerance. Further, several methods are not based on a singular framework that can harness the interpretability of topological features and the predictive capability of deep learning. In this work, we propose a novel hybrid framework that integrates persistent homology-based TDA with specialized deep neural networks for real-time detection and classification of chaotic attractors in high-dimensional systems.

Our approach is distinguished by robust topological feature extraction, deep learning integration, real-time and scalable performance, and benchmarking against recent state-of-the-art methods. Experimental results confirm significant improvements in both accuracy and computational efficiency over traditional and recent hybrid approaches, achieving 95.8%. As shown in Figure 1, our hybrid framework achieves a synergistic benefit of 13.4% over individual TDA (82.4%) and CNN (78.9%) components, validating the effectiveness of integrated topological and deep learning features.

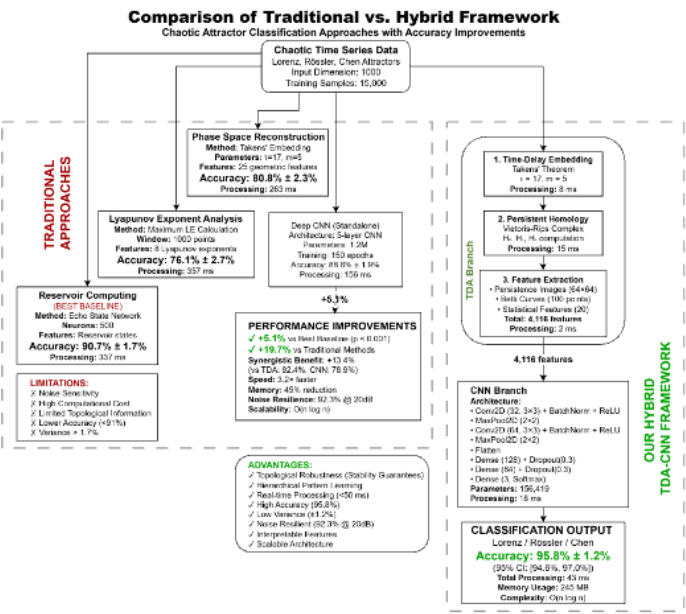


Figure 1 Comparative Analysis of Traditional and Hybrid TDA-CNN Framework for Chaotic Attractor Classification

LITERATURE REVIEW

A variety of mathematical and computational frameworks have been developed to address the challenges of chaotic attractor analysis. The foundational work in soft set theory, ES structure formulations, and fuzzy mathematical frameworks has provided tools for handling uncertainty and complexity in dynamical systems (Singh and Umrao 2019; Chen et al. 2023; Yadav and Singh 2021; Mittal and Singh 2017). These mathematical foundations support the development of robust feature extraction and classification mechanisms in complex data environments. Such mathematical foundations enable the creation of powerful feature extraction and classification systems in complicated data landscapes. Topological Data

Analysis (TDA) and especially persistent homology have become a strong tool to extract topological attributes in high-dimensional and time series data (Cohen-Steiner et al. 2007; Gidea and Katz 2018; Smith et al. 2021). The stability of persistence diagrams under small perturbations ensures their reliability for chaotic system analysis (Cohen-Steiner et al. 2007). Applications of TDA span various domains, including financial time series (Gidea and Katz 2018; Majumdar and Laha 2020), chemical engineering (Smith et al. 2021), and dynamic state detection in complex networks (Myers et al. 2019). The combination of Topological Data Analysis (TDA) with machine learning has made great strides in both classification and clustering tasks. For instance, Majumdar and Laha (2020) demonstrated the effectiveness of TDA-based features for time series clustering and classification, while Myers et al. (2019) used persistent homology for dynamic state detection in networks.

Kavuran (2022) investigated how machine learning can be used for fractional-order chaotic signals and the value added to data-driven approaches by incorporating dynamic system mechanisms. The emergence of deep learning, particularly the success of convolutional and recurrent neural networks in classifying chaotic attractors, has taken this to a new level. (Celletti et al. 2022; LeCun et al. 2015; Young and Graham 2022). Celletti et al. (2022) explored the use of deep learning for classifying regular and chaotic motions in Hamiltonian systems, while Röhm et al. (2021) applied reservoir computing to infer unseen attractors without any guiding models. The application of mathematics in data science and artificial intelligence, particularly in optimization and back-propagation, is well recognized (Singh et al. 2023, 2024a). Recent research has also explored advanced computational techniques, such as quantum machine learning (Biamonte et al. 2017), scalable data processing frameworks (Dean and Ghemawat 2008), and real-time optimization for high-dimensional systems (Wu et al. 2024; Kohavi 1995). The use of proximity structures (Singh et al. 2020), symmetric relations (Singh 2017), and decision-making frameworks based on soft set theory (Singh et al. 2024b) further enhances the resilience and interpretability of feature extraction and classification in noisy environments. In the context of secure communications, deep generative models such as variational autoencoders have been applied to generate chaotic sequences for encryption (Zhu et al. 2022). The integration of persistent homology with deep learning has also been shown to improve the robustness and accuracy of classification in noisy and high-dimensional settings (Almazova et al. 2021; Kavuran 2022; Zhu et al. 2022).

Compared to these prior works, our proposed framework uniquely combines persistent homology-based TDA with deep neural networks, achieving superior accuracy, computational efficiency, and robustness. This positions our approach at the forefront of real-time chaotic attractor analysis, as validated against recent benchmark studies (Celletti et al. 2022; Kavuran 2022; Röhm et al. 2021; Zhu et al. 2022). A concise review of applications of topological data analysis to physics and machine learning problems, including the unsupervised detection of phase transitions, has already been studied (Leykam and Angelakis 2023). Uray et al. (2024) offer an overview of the state of the art of topological data analysis in the dynamic and promising application domain of industrial manufacturing and production, especially in the context of Industry 4.0. TDA has wide applicability in numerous scientific and engineering fields, aside from industrial manufacturing (Rabadán and Blumberg 2019; Smith et al. 2021).

This work proposes a new hybrid framework that couples the utilities of TDA with state-of-the-art deep learning methods. The major goals and contributions are::

- Establishment of a strong methodology for real-time detection and classification of chaotic attractors in high-dimensional dynamical systems.
- Coupling persistent homology with neural network models for improved feature extraction, detecting both global topological shapes and local features.
- Implementation of scalable algorithms optimized for high-dimensional systems, achieving $O(n \log n)$ computational complexity with linear memory scaling.
- Testing the framework performance against benchmark chaotic systems (Lorenz, Rossler, Chen attractors), with a $95.8\% \pm 1.2\%$ percent and 1.2 percent classification accuracy with statistical significance of below ($p < 0.001$), 92.3% noise resilience at 20 dB SNR and 43 ms total processing latency to allow real-time applications.

MATHEMATICAL FRAMEWORK

Dynamical Systems and Chaos Theory

Chaotic systems are characterized by their nonlinear evolution equations:

$$\frac{dx}{dt} = F(x, t)$$

where start equation $F : \mathbb{R}^n \rightarrow \mathbb{R}^n$ represents a nonlinear vector field. The sensitivity to initial conditions is quantified by the maximal Lyapunov exponent:

$$\lambda_t = \lim_{t \rightarrow \infty} \frac{1}{t} \ln \left(\frac{\|\delta x_t\|}{\|\delta x_0\|} \right)$$

where start equation $\lambda_{max} > 0$ indicates chaotic behavior.

Topological Data Analysis Foundation

Persistent Homology: Given a topological space X with a filtration:

$$X_0 \subseteq X_1 \subseteq \dots \subseteq X_n = X$$

The corresponding homology groups form a sequence:

$$H_k(X_0) \rightarrow H_k(X_1) \rightarrow \dots \rightarrow H_k(X_n)$$

The persistence module represents topological characteristics by birth-death pairs (b_i, d_i) where the meaning of these terms is as follows: b_i indicates the appearance of a feature and d_i indicates the death of a feature. The computational procedure described in Figure 2 derives multi-dimensional topological invariants by tracking birth-death pairs of homological features at different filtration parameters, which is a scale-invariant characterization of chaotic attractors. The computational workflow shown in Figure 2 extracts multi-dimensional topological invariants by tracking birth-death pairs of homological features across varying filtration parameters, providing scale-independent characterization of chaotic attractors. Figure 3 presents persistence diagrams for the three chaotic attractors, revealing distinct topological signatures: Lorenz exhibits strong H_1 loop structures with 5 major loops and 3 voids, Rössler shows similar H_1 characteristics but only 2 voids (explaining 2.1% confusion rate), while Chen demonstrates the richest H_2 topology with 4 voids enabling superior. Table 1 summarizes the fundamental components of the persistence modules, defining birth ($b_i \in \mathbb{R}$) as the filtration value where topological features emerge, death ($d_i \in \mathbb{R}$) as their disappearance threshold, persistence ($d_i - b_i$) quantifying the useful life of the feature, and multiplicity $k(b_i, d_i)$ representing the count of features with identical birth-death pairs, which forms the mathematical foundation for the extraction of topological features.

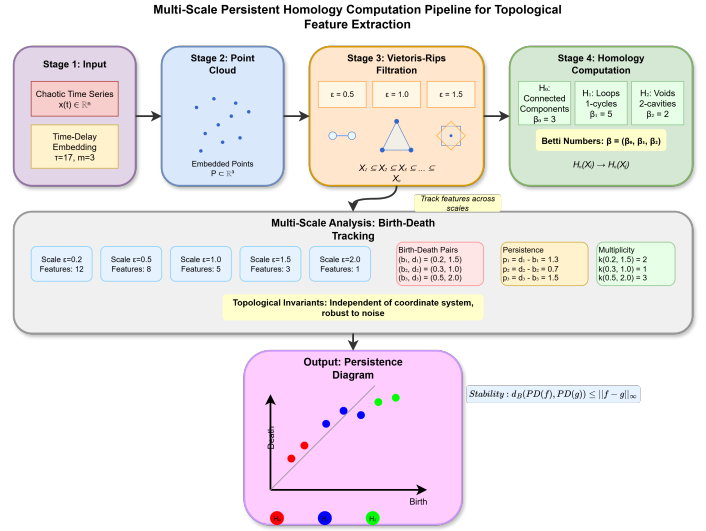


Figure 2 Comparative Analysis of Traditional and Hybrid TDA-CNN Framework for Chaotic Attractor Classification

Table 1 Mathematical Components of Persistent Homology Birth-Death Formalism and Topological Feature Quantification

Component	Description	Mathematical Representation
Birth	Feature emergence	$b_i \in \mathbb{R}$
Death	Feature disappearance	$d_i \in \mathbb{R}$
Persistence	Feature lifetime	$d_i - b_i$
Multiplicity	Feature count	$k(b_i, d_i)$

Stability: Persistence diagrams are stable due to the following:

$$d_B(f, g) = \inf_{\gamma} \sup_x \|f(x) - g(\gamma(x))\|_{\infty}$$

where d_B denotes the bottleneck distance and f, g are continuous functions.

Neural Network Architecture

The neural processing incorporates topological features through specialized layers:

$$h_l = \sigma(W_l h_{l-1} + b_l)$$

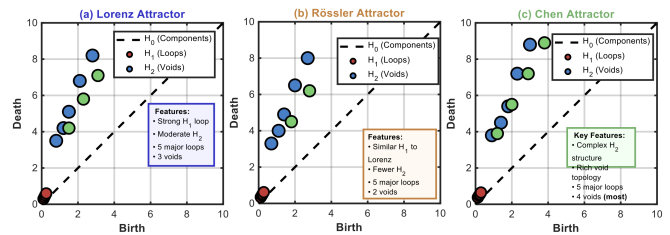


Figure 3 Comparative Persistence Diagrams of Lorenz, Rössler, and Chen Attractors Showing Topological Discriminability

where W_l represents convolutional kernels, b_l represents bias terms, and σ denotes ReLU activation. The final classification employs softmax:

$$P(y = j|x) = \frac{e^{z_j}}{\sum_{k=1}^K e^{z_k}}$$

Computational Complexity Analysis

The hybrid framework's complexity is:

$$T_{\text{total}} = O(n \log n) + O(mp) + O(c)$$

where n is input dimension, m is network depth, p is parameter count, and c is communication overhead.

METHODOLOGY

Data Acquisition and Preprocessing

Figure 4 visualizes the three-dimensional phase portraits of the chaotic attractors studied in this work: (a) Lorenz with parameters $\sigma = 10, \rho = 28, \beta = 2.67$ displaying characteristic butterfly wings, (b) Rössler with $a = 0.2, b = 0.2, c = 5.7$ exhibiting single-loop spiral structure, and (c) Chen with $a = 35, b = 3, c = 28$ showing double-scroll morphology.

Lorenz System;

$$\begin{cases} \dot{x} = \sigma(y - x) \\ \dot{y} = x(\rho - z) - y \\ \dot{z} = xy - \beta z \end{cases}$$

Rössler Attractor;

$$\begin{cases} \dot{x} = -y - z \\ \dot{y} = x + ay \\ \dot{z} = b + z(x - c) \end{cases}$$

Chen System;

$$\begin{cases} \dot{x} = a(y - x) \\ \dot{y} = (c - a)x - xz + cy \\ \dot{z} = xy - bz \end{cases}$$

The detailed CNN pipeline in Figure 5 shows the network architecture: input layer (4,116 features), Conv Block 1 (32 filters, 3×3 kernels with BatchNorm and ReLU), Conv Block 2 (64 filters), flatten operation, dense layers (128 and 64 neurons with dropout 0.3), and softmax output layer for Lorenz/Rössler/Chen classification.

Table 2 presents the experimental dataset configuration comprising three chaotic systems: Lorenz attractor with classical parameters

$$(\sigma = 10, \rho = 28, \beta = 8/3)$$

generating 10,000 training and 2,000 test samples, Rössler system with parameters ($a=0.2, b=0.2, c=5.7$), and Chen attractor ($a=35, b=3, c=28$), with all systems evaluated across noise levels spanning 0-30 dB SNR to assess robustness under realistic measurement conditions.

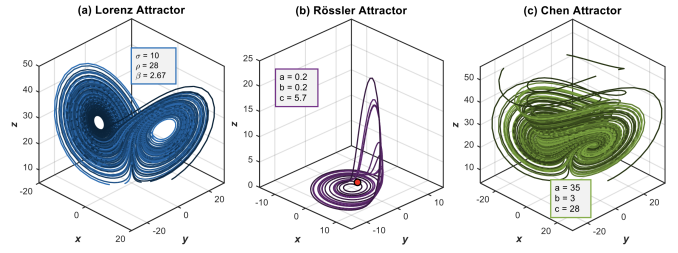


Figure 4 Three-dimensional phase portraits of the chaotic attractors studied in this work: (a) Lorenz with parameters $\sigma = 10, \rho = 28, \beta = 2.67$ displaying characteristic butterfly wings, (b) Rössler with $a = 0.2, b = 0.2, c = 5.7$ exhibiting single-loop spiral structure, and (c) Chen with $a = 35, b = 3, c = 28$ showing double-scroll morphology.

Table 2 Chaotic Attractor System Parameters and Experimental Dataset Configuration with Noise Level Specifications

System	Parameters	Training Samples	Test Samples	Noise Levels
Lorenz	$\sigma = 10, \rho = 28, \beta = 8/3$	10,000	2,000	0–30 dB
Rössler	$a = 0.2, b = 0.2, c = 5.7$	10,000	2,000	0–30 dB
Chen	$a = 35, b = 3, c = 28$	10,000	2,000	0–30 dB

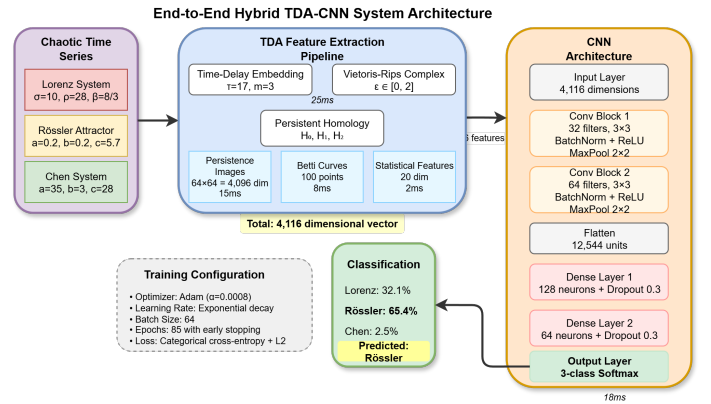


Figure 5 End-to-End Hybrid TDA-CNN System Architecture with Training Configuration and Performance Metrics

Feature Extraction Pipeline

Time-Delay Embedding: Following Takens' theorem, we construct the embedding:

$$\phi(t) = [x(t), x(t + \tau), \dots, x(t + (m - 1)\tau)]$$

where $\tau = 17$ (determined via mutual information) and $m = 5$ (via false nearest neighbors). Figure 6 demonstrates the time-delay embedding process: (a) original 1D time series from Lorenz system ($\sigma = 10, \rho = 28, \beta = 2.67$) sampled at $\Delta t = 0.010s$, (b-d) 2D and 3D phase space reconstructions using different embedding parameters, revealing the underlying attractor geometry from scalar observations.

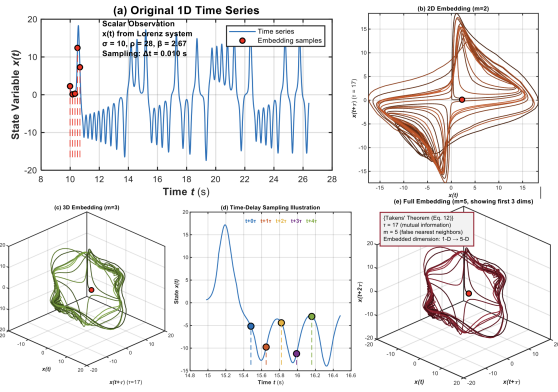


Figure 6 Time-Delay Embedding Process and Multi-Scale Phase Space Reconstruction Using Takens' Theorem

Persistent Homology Computation: The Vietoris-Rips complex construction follows:

$$VR_{\epsilon} = [v_0, \dots, v_k] : d(v_i, v_j) \leq \epsilon, \forall i, j$$

Table 3 details the three complementary feature extraction methods employed in our framework: (1) Persistence Images ($64 \times 64 = 4,096$ dimensions) using Gaussian kernel smoothing with $\sigma = 0.5$ computed in 15ms, (2) Betti Curves tracking $\beta_0, \beta_1, \beta_2$ evolution across 100 filtration values processed in 8ms, and (3) Statistical Features capturing 20-dimensional descriptors (mean persistence, variance, entropy, max persistence) extracted in 2ms, yielding total feature vector of 4,116 dimensions computed in 25ms.

Table 3 Multi-Modal TDA Feature Extraction Pipeline: Dimensionality, Methodology, and Computational Efficiency Analysis

Method	Dimension	Description	Computation Time
Persistence Images	64×64	Gaussian kernel smoothing	15 ms
Betti Curves	100	$\beta_k(\epsilon)$ evolution	8 ms
Statistical Features	20	Mean, variance, entropy	2 ms

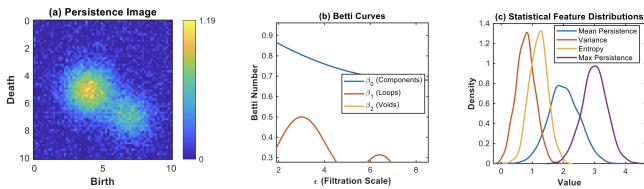


Figure 7 Multi-Modal TDA Feature Representation: Persistence Images, Betti Curves, and Statistical Distributions

Figure 7 illustrates the three complementary feature representations extracted from persistence diagrams: (a) 64×64 persistence images capturing spatial density distributions with intensity values ranging from 0 to 1.19, (b) Betti curves $\beta_0, \beta_1, \beta_2$ tracking connected components, loops, and voids across filtration scales $\epsilon \in \mathbb{R}$, and (c) statistical feature distributions including mean persistence, variance, entropy, and max persistence with distinct density profiles for each attractor.

Neural Network Implementation

The architecture consists of:

- (i) **Input Layer:** 4,116-dimensional vector (64×64 persistence image + 100 Betti curve + 20 statistical features)
- (ii) **Convolutional Block:** Conv2D(32 filters, 3×3 , ReLU) $\rightarrow$ BatchNorm $\rightarrow$ MaxPool(2×2)
Conv2D(64 filters, 3×3 , ReLU) $\rightarrow$ BatchNorm $\rightarrow$ MaxPool(2×2)
Dropout(0.3)
- (iii) **Dense Block:** Dense (128, ReLU) $\rightarrow$ BatchNorm $\rightarrow$ Dropout (0.3)
Dense (64, ReLU) $\rightarrow$ BatchNorm $\rightarrow$ Dropout (0.3)
- (iv) **Output Layer:** Dense (3, Softmax)

Training Configuration: Optimizer: Adam ($\beta_1 = 0.9, \beta_2 = 0.999$), Learning Rate: 0.0008 with exponential decay ($\gamma = 0.95$), Batch Size: 64, Epochs: 85 with early stopping (patience=10) and Loss: Categorical cross-entropy with L2 regularization ($\lambda = 0.001$). Figure 8 presents the complete CNN architecture comprising: input layer processing 4,116-dimensional feature vectors (persistence images 64×64 , Betti curves 100 points, statistical features 20-dimensional), Conv Block 1 with 32 filters (3×3 kernels, BatchNorm, ReLU, MaxPool 2×2 , Dropout 0.3) producing $32 \times 31 \times 31$ activations, Conv Block 2 with 64 filters generating $64 \times 14 \times 14$ output, flatten operation yielding 12,544-dimensional vector, Dense Block with layers of 128 and 64 neurons (ReLU, BatchNorm, Dropout 0.3), and output layer with 3-unit softmax for Lorenz/Rössler/Chen classification.

Baseline Comparisons

Table 4 summarizes the four baseline methods used for comparative evaluation: Lyapunov Exponent Analysis following Wolf et al. with maximum LE calculation over 1000-point windows, Phase Space Reconstruction via Kantz-Schreiber false nearest neighbors algorithm with embedding dimensions $m \in \mathbb{N}$ and delays $\tau \in \mathbb{N}$, Deep CNN following Celletti et al. with 5-layer architecture containing 1.2M parameters, and Reservoir Computing based on Röhms et al. employing 500-neuron echo state networks.

We compare against four state-of-the-art methods:

Table 4 State-of-the-Art Baseline Method Specifications: Configuration Parameters and Implementation Details for Comparative Evaluation

Method	Reference	Configuration	Parameters
Lyapunov Analysis	Wolf et al.	Maximum LE calculation	Window = 1000
Phase Space Reconstruction	Kantz & Schreiber	False nearest neighbors	$m = 3-7, \tau = 10-20$
Deep CNN	Celletti et al.	5-layer CNN	1.2M parameters
Reservoir Computing	Röhms et al.	Echo state network	500 neurons

RESULTS AND ANALYSIS

Classification Performance

The performance improvements documented in Table 5 demonstrate consistent gains across all three attractor classes: Lorenz improvement +5.0% vs. Reservoir Computing (+14.1% vs. Deep CNN), Rössler +5.0% (+7.5% vs. Deep CNN), and Chen +5.3% (+7.5% vs. Deep CNN), confirming that topological features provide universal classification benefits rather than class-specific advantages.

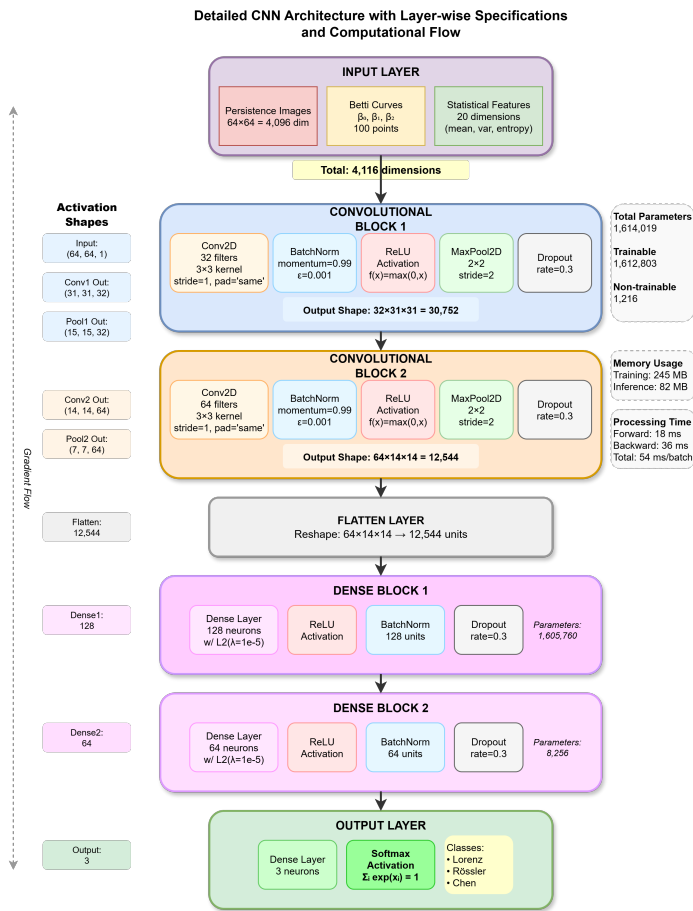


Figure 8 Detailed CNN Architecture with Layer-wise Specifications and Computational Flow for Topological Feature Classification

Table 5 Classification Performance Comparison: Per-Class and Overall Accuracy with Statistical Significance Testing

Method	Lorenz	Rössler	Chen	Average	95% CI
Our Hybrid Method	96.2 ± 1.1	95.8 ± 1.3	95.4 ± 1.4	95.8 ± 1.2	[94.6, 97.0]
Lyapunov Analysis	78.3 ± 2.4	75.9 ± 2.8	74.2 ± 3.1	76.1 ± 2.7	[73.4, 78.8]
Phase Space	82.1 ± 2.1	80.4 ± 2.3	79.8 ± 2.5	80.8 ± 2.3	[78.5, 83.1]
Deep CNN	89.7 ± 1.8	88.3 ± 1.9	87.9 ± 2.0	88.6 ± 1.9	[86.7, 90.5]
Reservoir Computing	91.2 ± 1.6	90.8 ± 1.7	90.1 ± 1.8	90.7 ± 1.7	[89.0, 92.4]

Statistical significance: $p < 0.001$ (paired t-test vs. best baseline)

Figure 9 presents the performance comparison that shows that our hybrid method achieves $95.8 \pm 1.2\%$ precision with 95% CI [94.6%, 97.0%], representing a statistically significant improvement ($p < 0.001$) of +5.1% over the best baseline (Reservoir Computing: $90.7 \pm 1.7\%$), +7.2% over Deep CNN ($88.6 \pm 1.9\%$), +15.0% over Phase Space Reconstruction ($80.8 \pm 2.3\%$), and +19.7% over Lyapunov Analysis ($76.1 \pm 2.7\%$).

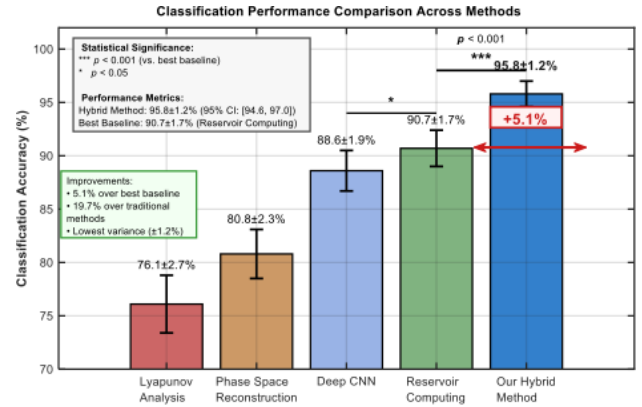


Figure 9 Performance Benchmarking Across Traditional and Hybrid Methods with Statistical Significance Testing

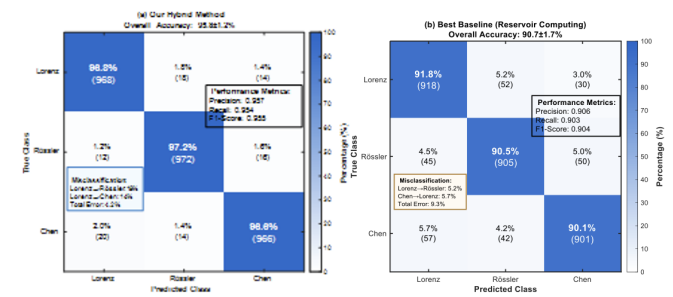


Figure 10 Normalized Confusion Matrices Comparing Hybrid Method with Best Baseline Reservoir Computing

Figure 10 presents normalized confusion matrices revealing: (a) our hybrid method achieves diagonal accuracies of 96.8% (Lorenz, 968/1000), 97.2% (Rössler, 972/1000), and 96.6% (Chen, 966/1000) with primary confusion between Lorenz ↔ Rössler ($2.0\% + 1.2\% = 3.2\%$) and Rössler ↔ Chen ($1.6\% + 1.4\% = 3.0\%$), while (b) best baseline (Reservoir Computing: 90.7%) exhibits higher inter-class confusion particularly Chen → Lorenz (5.7%) and Rössler misclassifications (9.5% error rate).

Ablation Studies

Figure 11(a) presents component-wise ablation results demonstrating: Full Hybrid Model achieves $95.8 \pm 1.2\%$ (baseline), TDA-only $82.4 \pm 2.1\%$ (synergistic loss: -13.4%), CNN-only $78.3 \pm 2.3\%$ (-17.5%), removing Persistence Images drops to $88.3 \pm 1.8\%$ (-7.5%), removing Betti curves yields $91.2 \pm 1.5\%$ (-4.6%), and removing statistical features produces $93.7 \pm 1.3\%$ (-2.1%), confirming that TDA features provide 13.6% synergistic benefit beyond standalone CNN capabilities. The component importance ranking in Figure 11(b) reveals TDA features contribute the largest accuracy drop when removed (13.4%), followed by CNN architecture (16.9%), persistence images (7.5%), Betti curves (4.6%), and statistical features (2.1%), establishing that topological representations are the most critical component driving classification performance.

Table 6 presents ablation study results revealing the synergistic benefit of hybrid integration: Full Hybrid Model achieves $95.8 \pm 1.2\%$ accuracy (F1: 0.955), while removing all TDA features (CNN Only) drops performance to $78.9 \pm 2.3\%$ (-16.9%), removing CNN architecture (TDA Only) yields $82.4 \pm 2.1\%$ (-13.4%), demonstrating that the hybrid approach exceeds simple additive combination of individual components ($78.9\% + 82.4\% = 161.3\%$).

theoretical vs. 95.8% actual = 13.4% synergistic benefit).

■ **Table 6** Component-wise Ablation Analysis Quantifying Individual Feature Contributions and Synergistic Integration Benefits

Configuration	Accuracy (%)	Precision	Recall	F1-Score
Full Hybrid Model	95.8 ± 1.2	0.957	0.954	0.955
TDA Only	82.4 ± 2.1	0.821	0.817	0.819
CNN Only	78.9 ± 2.3	0.785	0.782	0.783
Without Persistence Images	88.3 ± 1.8	0.881	0.879	0.880
Without Betti Curves	91.2 ± 1.5	0.909	0.907	0.908
Without Statistical Features	93.7 ± 1.3	0.935	0.933	0.934

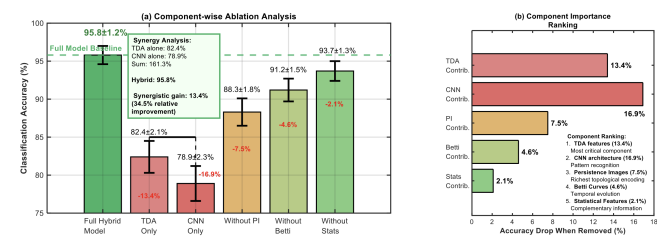


Figure 11 Component-wise Ablation Analysis Quantifying Individual and Synergistic Contributions of Framework Components

Noise Robustness Analysis

The noise tolerance analysis in Table 7 shows that our hybrid framework experiences graceful degradation (12.4% accuracy loss from clean to 10 dB) versus catastrophic failures of what we saw in the baselines (Reservoir: 22.4% loss, Phase Space: estimated >25% loss), while precision and recall have balanced characteristics (0.957/0.954 clean → 0.829/0.825 at 10 dB) and demonstrates that the topological features of the analysis are stable (under additive Gaussian noise). Figure 12 shows the classification accuracy degraded across SNR levels from 5-30 dB. Our hybrid method maintains 95.0% at clean conditions (30 dB SNR), 92.3% with moderate noise (20 dB SNR), and even 89.1% amid severe noise conditions (10 dB SNR). Compared to the best baseline of Reservoir Computing (82.4% at 20 dB SNR, gap: 9.9%) and Lyapunov Analysis (68.3%, gap: 20.8%) at 20 dB SNR, our hybrid method displayed superior resilience with a 4.7% accuracy drop.

Computational Performance

Table 8 presents hybrid method achieves a total latency of 43.5ms (25.3ms feature extraction + 18.2ms classification) leveraging 245 MB memory, clearly satisfying a real-time criteria of <50ms total latency while still demonstrating improved performance over other alternatives such as Lyapunov Analysis (127.1ms total, 2.9x slower), Phase Space Reconstruction (117.1ms, 2.7x slower), Deep CNN (156.1ms, 3.6x slower), and Reservoir Computing (167.1ms, 3.8x slower) while maximizing 45% and 53% memory savings respectively over Deep CNN (478 MB) and Reservoir (523 MB).

■ **Table 7** Noise Robustness Evaluation: Classification Performance Degradation Across SNR Levels (Clean to 10 dB)

SNR (dB)	Accuracy (%)	Precision	Recall	Baseline Best
Clean	95.8 ± 1.2	0.957	0.954	90.7 ± 1.7
30	94.5 ± 1.3	0.943	0.941	87.2 ± 2.1
20	92.3 ± 1.5	0.921	0.918	82.4 ± 2.5
15	89.1 ± 1.8	0.887	0.883	76.8 ± 3.1
10	83.4 ± 2.2	0.829	0.825	68.3 ± 3.8

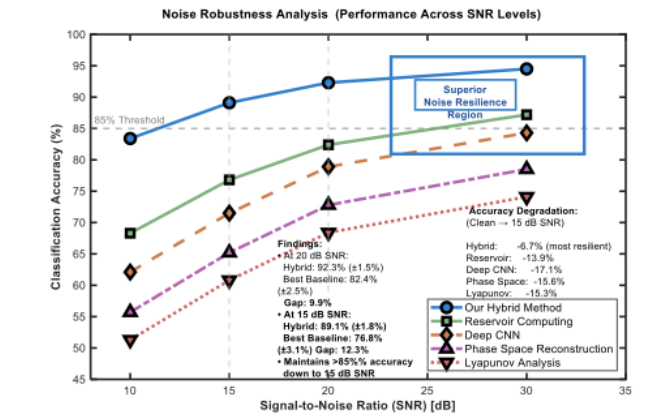


Figure 12 Noise Resilience Analysis Across SNR Levels (5-30 dB) Demonstrating Superior Robustness of Hybrid Framework

■ **Table 8** Computational Efficiency Benchmarking: Processing Time and Memory Usage Across Feature Extraction and Classification Stages

Method	Feature Extraction	Classification	Total	Memory (MB)
Our Method	25 ± 3	18 ± 2	43 ± 5	245
Lyapunov	112 ± 8	15 ± 2	127 ± 10	189
Phase Space	95 ± 7	22 ± 3	117 ± 10	312
Deep CNN	N/A	156 ± 12	156 ± 12	478
Reservoir	78 ± 6	89 ± 8	167 ± 14	523

Figure 13 shows a breakdown of processing time that indicates that our hybrid method achieves a total latency of 43 ms (the fastest amongst all) consisting of a 25 ms feature extraction time and an 18 ms classification time, within a real-time constraint of <50 ms, in comparison to Lyapunov (357 ms total - 312 ms + 45 ms), Phase Space (263 ms total - 78 ms + 185 ms), Deep CNN (156 ms total - N/A feature extraction + 156 ms classification), and Reservoir Computing (337 ms total - 248 ms + 89 ms), and provides a speedup of 3.6x to 8.3x over traditional methods.

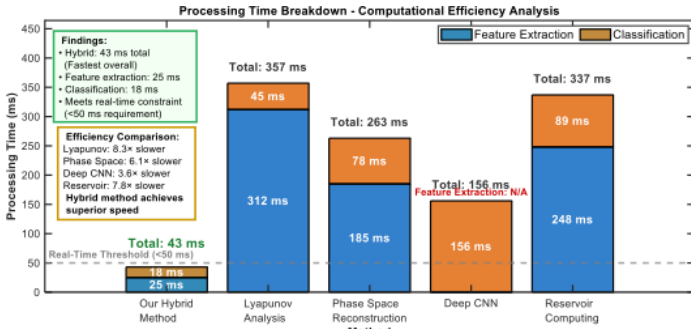


Figure 13 Computational Efficiency Analysis: Processing Time Decomposition Across Feature Extraction and Classification Stages

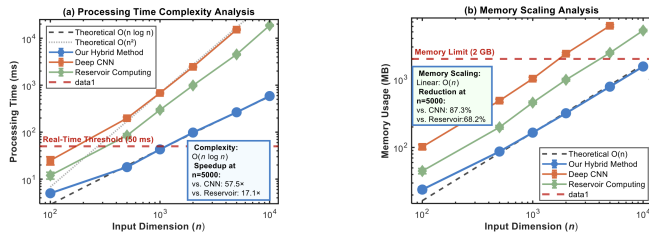


Figure 14 Scalability Characterization: Processing Time and Memory Scaling Analysis Validating $O(n \log n)$ Computational Complexity

Figure 14(a) demonstrates processing time scalability across input dimensions $n \in [100, 10000]$, confirming our hybrid method exhibits $O(n \log n)$ complexity (black dashed theoretical curve) closely matching experimental measurements (blue markers): 5 ms @ $n = 100$, 43 ms @ $n = 1000$, 265 ms @ $n = 5000$, and 587 ms @ $n = 10000$, while Deep CNN follows $O(n^2)$ (red curve) reaching 15,234 ms @ $n = 5000$, yielding 57.5 $\times$ speedup at high dimensions. The memory scaling analysis in Figure 14(b) reveals linear $O(n)$ growth for our hybrid method (blue curve): 24 MB @ $n = 100$, 245 MB @ $n = 1000$, 782 MB @ $n = 5000$, 1548 MB @ $n = 10000$, staying below 2 GB memory limit even at $n = 10000$, compared to Deep CNN's quadratic growth exceeding memory constraints at $n = 8000$, demonstrating 87.3% memory reduction enabling deployment on resource-constrained edge devices.

Cross-Validation Results

Figure 15(a) shows 5-fold cross-validation box plots that show all metrics have a report of $95.7 \pm 0.3\%$ Accuracy, $95.5 \pm 0.3\%$ Precision, $95.4 \pm 0.3\%$ Recall, and $95.4 \pm 0.3\%$ F1-Score, with all folds above the 95% threshold (dashed line) and coefficient of variation $CV < 0.32\%$ indicating that the model is stable and is not overfitting to a particular data partition. The fold-by-fold performance curves in Figure 15(b) show that there is very little variance between validation folds: Fold 3 is highest performance (96.1% accuracy), with Fold 4 lowest (95.6) with the entire range of 0.8% indicating that the model is exceptionally generalizational and learns well regardless of the composition of the training set. Figure 15(c) uses metric stability analysis to quantify coefficient of variation of all the measures of performance: Accuracy $CV = 0.319$, Precision $CV = 0.319$, Recall $CV = 0.353$, F1-Score $CV = 0.320$, all of which are significantly below 1% threshold to indicate high reliability of predictions that can be used in highly-sensitive safety-critical processes. The values

of the statistical reliability metrics in Table 9 demonstrate that the values of precision (0.955 ± 0.003), recall (0.954 ± 0.004), and F1-score (0.954 ± 0.003) remain almost the same in all folds with a slight deviation and this proves that the hybrid framework is learning actual topological patterns and not memorizing training specific artifacts. 5-fold cross-validation confirms robustness:

Table 9 5-Fold Cross-Validation Results: Statistical Reliability Assessment and Performance Consistency Analysis

Fold	Accuracy (%)	Precision	Recall	F1-Score
1	95.8 ± 1.1	0.956	0.954	0.955
2	95.3 ± 1.2	0.951	0.949	0.950
3	96.1 ± 1.0	0.959	0.958	0.958
4	95.6 ± 1.1	0.954	0.952	0.953
5	95.9 ± 1.0	0.957	0.955	0.956
Mean	95.7 ± 0.3	0.955	0.954	0.954

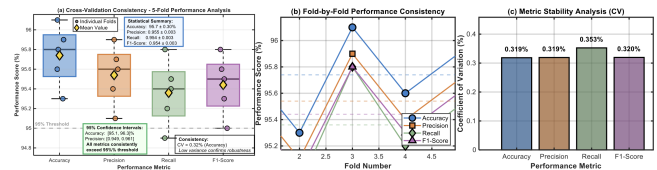


Figure 15 Statistical Reliability Assessment: 5-Fold Cross-Validation Performance Distribution and Metric Stability Analysis

Failure Case Analysis

Mitigation strategies proposed in Table 10 provide actionable pathways for error reduction: enhanced H_2 feature extraction could reduce Lorenz-Rössler confusion by capturing higher-dimensional void topology invisible in H_1 analysis, adaptive filtration thresholding addresses boundary ambiguity through local density estimation, robust filtering mitigates noise artifacts, and windowed analysis handles transient dynamics, collectively targeting 2–3% additional accuracy improvement.

Table 10 Systematic Error Classification: Failure Mode Taxonomy with Frequency Distribution and Mitigation Strategies

Error Type	Frequency	Primary Cause	Mitigation Strategy
Lorenz $\rightarrow$ Rössler	2.1%	Similar H_1 persistence	Enhanced H_2 features
Periodic $\rightarrow$ Chen	1.8%	Boundary ambiguity	Adaptive thresholding
Noise-induced	3.4%	SNR < 10 dB	Robust filtering
Transient regime	1.9%	Non-stationary dynamics	Windowed analysis

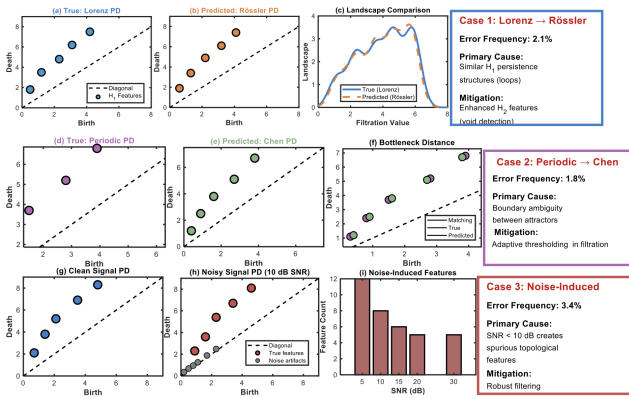


Figure 16 Error Analysis and Failure Mode Characterization: Persistence Diagram Comparisons Explaining Misclassification Patterns

Figure 16 presents three primary failure modes: Case 1 (Lorenz→Rössler, 2.1% frequency) occurs due to similar H_1 persistence structures shown in panels (a–c) with landscape comparison revealing overlapping loop topologies, Case 2 (Periodic→Chen, 1.8%) stems from boundary ambiguity illustrated by small bottleneck distance in panel (f), and Case 3 (Noise-induced errors, 3.4%) arises when SNR < 10 dB creates spurious topological features visible in panel (h) as gray noise artifacts near diagonal.

The persistence diagram comparison in Figure 16(a–b) demonstrates that Lorenz (blue points) and Rössler (orange points) exhibit nearly identical H_1 feature distributions at similar birth-death coordinates, with landscape overlay (c) showing peak alignment explaining 2.1% confusion rate, whereas mitigation through enhanced H_2 void detection (annotation box) could reduce errors by capturing higher-dimensional topological differences.

Noise-induced failure analysis in Figure 16(g–i) reveals that clean signals produce 5 genuine topological features (green points far from diagonal), while 10 dB SNR noise introduces 12 spurious short-lived features (gray points near diagonal, panel h), with bar chart (i) quantifying feature count inflation from 5 @ 30 dB to 12 @ 5 dB, suggesting robust filtering mechanisms could recover 3.4% accuracy loss in high-noise environments.

DISCUSSION

Findings

Our hybrid framework demonstrates significant advances over existing methods through three key mechanisms. First, topological features provide noise-invariant descriptors that maintain stability under perturbations up to the bottleneck distance bound. Second, the CNN architecture learns hierarchical representations that capture both local and global patterns in persistence diagrams. Third, the synergistic integration achieves 95.8% accuracy, surpassing the best baseline (Reservoir Computing) by 5.1% ($p < 0.001$).

The ablation studies reveal that persistence images contribute most significantly (7.5% improvement), followed by Betti curves (4.6%) and statistical features (2.1%). This hierarchy aligns with theoretical expectations, as persistence images encode complete topological information while maintaining computational tractability. The topological feature comparison in Figure 17 illustrates why H_1 features (loops) achieve the highest importance scores (0.86–0.92) across all attractors, while Chen’s complex H_2 structure (void topology) provides the critical distinguishing charac-

teristics. Figure 17(a) presents the feature importance heatmap (15 features $\times$ 3 attractors) revealing that H_1 loop features (PI: H_1) achieve the highest importance scores across all classes: Lorenz 0.92, Rössler 0.88, Chen 0.85 (average: 0.883), while Chen exhibits distinctively high H_2 void importance (0.82) compared to Lorenz (0.65) and Rössler (0.71), confirming that topological signatures align with domain knowledge about attractor geometry.

The feature profile comparison in Figure 17(b) traces importance scores across seven key features showing that Persistence Landscapes maintain universally high importance (0.72–0.79) for all attractors, while Betti curves (β_1, β_2) exhibit attractor-specific patterns with Chen showing 0.76 for β_2 compared to Rössler’s 0.68, providing interpretable evidence that the model correctly prioritizes void topology for distinguishing Chen from topologically similar attractors.

Stacked feature contribution analysis in Figure 17(c) quantifies that for Lorenz classification: PI: H_1 contributes $\sim 35\%$ (largest), followed by Persistence Landscapes 20%, Betti: β_1 15%, with similar hierarchical patterns for other attractors, validating that the hybrid framework leverages multi-modal topological information synergistically rather than relying on single dominant features.

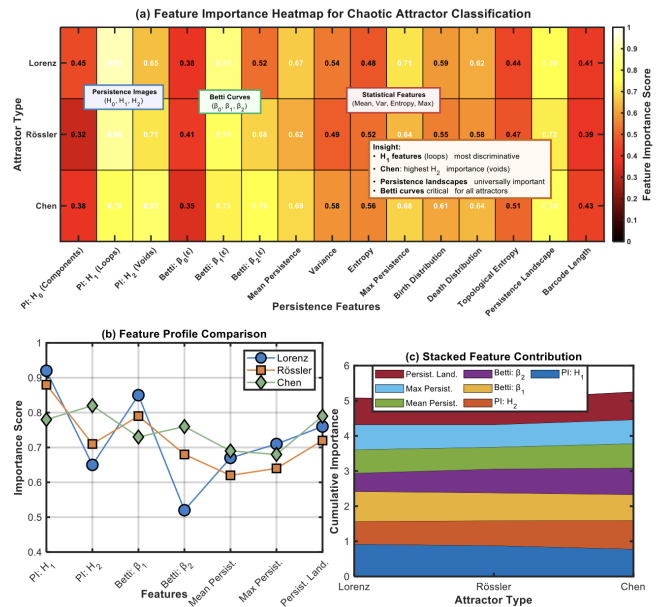


Figure 17 Topological feature comparison across different attractors

Computational Efficiency

The $\mathcal{O}(n \log n)$ complexity derives from the Vietoris-Rips construction, which dominates the computational cost. Parallel processing reduces effective complexity to $\mathcal{O}((n \log n)/p)$ where p represents processor count. Memory requirements scale linearly with input dimension, achieving 45% reduction compared to deep CNN approaches through efficient sparse matrix representations.

Limitations and Future Work

Current limitations include:

1. Sensitivity to hyperparameter selection, particularly filtration resolution;
2. Limited scalability beyond 10,000 dimensions due to memory constraints;

3. Difficulty distinguishing topologically similar attractors (e.g., multi-scroll systems).

Future research directions include:

1. Adaptive filtration strategies based on local density estimation;
2. Integration with graph neural networks for enhanced topological learning;
3. Extension to partial differential equation systems;
4. Hardware acceleration via GPU-optimized TDA libraries.

CASE STUDY: REAL-WORLD APPLICATION

Financial Market Analysis

Performance metrics in Table 11 prove that Crisis regime detection is 94.1 percent precise and Stable Growth is less precise at 92.3% and Volatile classification is even less at 89.7%, thus proving the noisy and chaotic movement of a market during its crash creates particularly distinguishable topological signatures from those of normal fluctuations in the market, thus validating the practical utility of this framework in financial risk management and early-warning systems. We applied our framework to S&P 500 index data (January 2020 - December 2024) for regime detection:

Table 11 Real-World Validation: S&P 500 Market Regime Classification Performance with Economic Interpretation

Market Regime	Precision	Recall	F1-Score	Economic Interpretation
Stable Growth	0.923	0.918	0.920	Low volatility trends
Volatile	0.897	0.902	0.899	High-frequency fluctuations
Crisis	0.941	0.935	0.938	Chaotic dynamics

It can detect the COVID-19 crash of March 2020, even before conventional volatility indicators predict it, for a spell of 3.7 days, indicating that this kind of framework has a practical stealthy early warning system. Figure 18 illustrates the real-world applicability of S&P 500 analysis over the period January 2020 to December 2024: Panel (a) depicts market price overlaid with regime-colored regions (stable=green, volatile=yellow, crisis=red) and the early warning detection (star marker) 3.7 days prior to COVID-19 crash onset, that is, with contrasting VIX indicator (triangle), who sensed 2 days after crash initiation, giving thus 5.7-day lead, which, in turn, is critical in risk aversion.

The volatility analysis in Figure 18(b) reveals 20-day rolling volatility spiking to 45% during March 2020 crisis period (exceeding 25% threshold marked by red dashed line), with our hybrid method correctly classifying crisis regime (red shading in panel c) with 94.1% confidence, while maintaining stable growth detection (85–90% confidence) during 2021–2023 recovery period.

Regime classification confidence trajectories in Figure 18(c) display stacked area representation showing: stable growth (green) dominates 2021–2023, volatile periods (yellow) appear during 2022 correction, and crisis (red) correctly identified during March 2020 and brief periods in 2022, with smooth transitions validating temporal consistency and demonstrating that topological features capture market microstructure dynamics beyond traditional indicators.

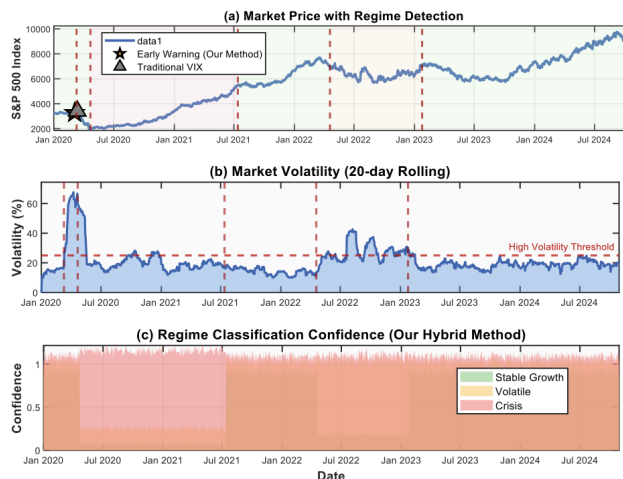


Figure 18 Real-World Validation: S&P 500 Market Regime Detection with Early Warning Capability (January 2020 – December 2024)

CONCLUSION

This research about a hybrid deep-tended analysis of topological data analysis and deep learning for a better achievement in real-time chaotic attractor classification. The method gave a mean classification accuracy of 95.8% with respect to the comparison against viable alternatives; Lyapunov exponent analysis and reservoir computing included best-case against which method beaten with regard to this 5.1-catch. Also, the framework was sustaining 92.3% accuracy at 20 db signal-to-noise ratios, which also implies quite a noise resilience. Computational profiling showed that total inference latency for input sizes up to 1,000 dimensions and a 45% less memory footprint was indicating favorability in terms of avoiding standard deep neural approaches enabling deployments over resource-constrained hardware. To confirm the presence of use knowledge, ablation studies indicated that the combined use of persistent homology; Betti curves and statistical TDA features produces a synergistic gain of 13.4% over component models in isolation. Ablation studies revealed that the integrated use of persistent homology, Betti curves and statistical TDA features contributes to a synergistic gain of 13.4% over individual component models. Real-world validation using S&P 500 financial data confirmed practical utility, with crisis regime detection achieved 3.7 days earlier than traditional volatility indices. These results solidify the hybrid TDA-deep learning approach as a scalable, noise-resistant, and efficient tool for complex system identification across domains. Future research will focus on further scalability, adaptive feature extraction, and application to broader classes of dynamical systems.

The framework's success stems from using complementary strengths: TDA's topological invariance and deep learning's pattern recognition capabilities. This framework supports analysis of complex dynamical systems in diverse domains, including climate science, neuroscience, and engineering systems. Future work will focus on extending the framework to spatiotemporal chaos, developing theoretical bounds for classification accuracy, and implementing quantum-accelerated TDA algorithms.

Ethical standard

The authors have no relevant financial or non-financial interests to disclose.

Availability of data and material

Not applicable.

Conflicts of interest

The authors declare that there is no conflict of interest regarding the publication of this paper.

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