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RESEARCH PAPER

Population projection of Southeast Asia with a time-delay logistic model

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Abstract

The rise in population is a vital problem in Southeast Asian countries. It has an immediate and long-term impact on basic human needs such as housing, food, clothes, and medical care. Population forecasting can be crucial to deciding which socioeconomic and population development initiatives should be implemented in a country. The unpredictable increase in population has several drawbacks. This study compares the accuracy of the process and the proximity of two mathematical models: the time-delay (i.e., maturation time) logistic population model and the classical logistic population model. The resources and ecosystems of Bangladesh, Pakistan, India, Nepal, Bhutan, and Afghanistan are under threat due to the increase in human population, as the population of these countries has increased sharply. Hence, countries should solve this problem creatively and efficiently. We determine and compare the population forecasting of the countries using the logistic population model and the time delay population model under different ecological circumstances. The classical logistic and time-delay logistic population model predicts a future population of the countries in Southeast Asia, where the present and past population projections have quite significant agreement with the existing dataset.

Keywords: Logistic population model; time-delay; population dataset; population projection; Southeast Asia

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1 Introduction

Researchers of mathematical ecology, epidemiology, disease dynamics, etc., should be concerned about population dynamics. They try to understand the fluctuations of population for a short

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period and the prediction of a country's total population for long periods. Mathematical models represent the relationships between birth, mortality, and other factors affecting population increase, which are crucial for comprehending population dynamics. The logistic population model, which offers a straightforward but effective framework for explaining population growth and carrying capacity, is a cornerstone among these models [1]. After being first published by Pierre François Verhulst in the 1800s, the logistic model has been extensively used to analyze population dynamics in everything from bacteria to mammals [2]. As resources become scarce, population growth is described as initially exponential, slowing down to attain a stable equilibrium known as the carrying capacity. Temporal delays in population responses to changes in the environment or the availability of resources are not considered by the logistic model, even though it has been useful in explaining population behaviors under steady-state settings.

On the other hand, George Evelyn Hutchinson investigated a time delay in the Verhulst equation governing population's self-regulatory mechanisms in 1948 [3]. He observed that these oscillations are caused by time lag, which explains some natural occurrences in animal populations, such as Daphnia [4]. The temporal delays added by the time-delay logistic population model expand the logistic framework and recognize that population reactions to shifting environmental circumstances or resource availability might not occur instantly [5]. Population stability and oscillatory dynamics have been demonstrated to be significantly impacted by temporal delays, which can result from various causes, e.g., generation time, migration patterns, or physiological processes. Considering the lag between birth and its effect on population increase, the logistic population model with time-delay provides a more accurate depiction of population dynamics. This maturation phase represents the biological truth that newly born individuals may not immediately contribute to population expansion because of infancy, juvenile development, or reproductive maturity [6]. Time delays are incorporated into the classical logistic population model to capture this crucial component of population dynamics and to give a more accurate picture of how populations react over time to changes in the availability of resources or the environment. As a result, the temporal delay logistic population model provides critical new understandings of biological population dynamics and resilience to environmental disturbances [7].

To clarify different behaviors and ecological implications, we consider the classical logistic population model and the time-delay logistic population model for predicting future populations of the countries in Southeast Asia, as presented in Section 2. In Section 3, we also investigate how the inclusion of temporal delays modifies the dynamics of population growth, stability, and resilience by integrating numerical simulations with mathematical analysis. Moreover, it includes a precise protocol for estimating the parameters of the model under the real dataset of populations of Southeast Asia. By using both models, a future prediction of populations is performed for all the countries of Southeast Asia. We also emphasize the possible implications for conservation and management techniques and discuss the ecological relevance of adding temporal delays to population models. Section 4 provides a summing up of the outcomes of this research.

2 Methodology

A nationally representative dataset from the Bangladesh Bureau of Statistics (BBS) population census conducted in Bangladesh between 1974 and 2022 [8], census population of India [9], census population of Pakistan [10], population of Nepal [11], population of Bhutan [12], and population of Afghanistan [13] are used in this study. In the beginning, we attempt to relate the population from 1974 to 2022, 1951 to 2011, and 1951 to 2011 census populations of Bangladesh, Pakistan, and India respectively; also 1960 to 2023 population of Nepal, Bhutan, and Afghanistan respectively for both the logistic population model and the time-delay logistic population model. Afterward, using these models, we aimed to forecast these six countries' populations until 2101. Many techniques, including the method of characteristics, Bellman's method of steps, the method of Laplace, and the continuous extension of the Runge-Kutta method, can be used to solve delay differential equations [14, 15].

Logistic population model

The logistic equation was introduced to describe population growth with a self-limiting term that serves as a controller for the unrestricted growth of the Malthusian model. It is commonly applied in studies of human, plant, animal, and microbial populations and is also used to predict technological and economic growth. The logistic equation is a non-linear equation of the form.

$$\frac{dN(t)}{dt} = rN(t)\left(1 - \frac{N(t)}{K}\right), \tag{1}$$

where r is the actual growth rate, $N(t)$ denotes the population density at time t, K is the environmental carrying capacity, and N_0 is the population density at the time $t = 0$.
The solution to the logistic equation (1) is

$$N(t) = \frac{N_0 K}{Ke^{-rt} + N_0(1 - e^{-rt})}, \tag{2}$$

where the carrying capacity, K (constant) is not realistic, the value of K may vary if the environment changes. When the carrying capacity K and growth rate a are continuous, equation (1) is regarded as an autonomous system, e.g., food availability can change after a change in the environment. This change may create a positive or negative effect on the population of a species [13, 16–18].
Instead of the constant carrying capacity, time-dependent carrying capacity $K(t)$ is used in many studies for various applications of population dynamics. Using a time-varying transport capacity makes the logistic equation nonautonomous due to the explicit time dependence of $K(t)$ [19]. As a result, the logistic equation (1) modified to

$$\frac{dN(t)}{dt} = rN(t)\left(1 - \frac{N(t)}{K(t)}\right); \quad N(0) = N_0, \tag{3}$$

where the carrying capacity

$$K(t) = K_s(1 - e^{-ct}). \tag{4}$$

Here, K_s is the bacterial fertilization level, c is the fertilization constant, $b = \left(1 - \frac{K_0}{K_s}\right)$, with $K(0) = K_0 (0 < b < 1)$ [20].

Time delay logistic population model

The time-delay logistic population model is a mathematical representation used to describe the growth of a population over time, incorporating a time delay in the effect of population density on the growth rate. This model extends the classical logistic growth model by introducing a delay term that accounts for the time it takes for individuals to respond to changes in population density.

The time-delay logistic population model is represented as follows:

$$\frac{dN(t)}{dt} = rN(t)\left(1 - \frac{N(t-\tau)}{K}\right), \tag{5}$$

where $N(t)$ represents the population at time t, r is the intrinsic growth rate, K is the carrying capacity, and τ is the time delay.

The population dynamics are represented by this equation, in which the logistic term $\left(1 - \frac{N(t-\tau)}{K}\right)$ modulates the population change rate, which is proportional to the current population size $N(t)$. With a time-delay of τ units, this term depicts the impact of population density on the growth rate. As the population gets closer to the carrying capacity K, the logistic term makes sure that the growth rate slows down to represent resource limitations and environmental constraints. This approach applies especially to biological and ecological environments where individual responses to population density changes are slow [3, 21–24].

3 Results and discussions

In this section, we first describe the estimation technique of the parameters for both the classical logistic population model and the logistic population model with a time delay. The parameters involved in the models: r (growth rate), K (carrying capacity), and τ (time-delay) are estimated in the following manners:

Estimation of growth rate

To calculate the growth rate of these six countries, we use the following formula. But here, we only show the growth rate calculation of Bangladesh. We know that a constant growth rate of the population for a long period is [25, 26]

$$r = -1 + \left(\frac{\text{Present population}}{\text{Starting Population}}\right)^{\frac{1}{\text{Time Period}}}.$$

Therefore, population growth during this period is

$$r = -1 + \left(\frac{171,186,372}{72,947,807}\right)^{\frac{1}{47}} = 0.018315 = 1.8315\%.$$

Similarly, we estimate the growth rate of Pakistan, India, Nepal, Bhutan, and Afghanistan, and the growth rates are 2.6259%, 2.03614%, 1.78%, 2.046%, and 2.55%, respectively.

Computation of carrying capacity

To estimate the carrying capacity of Bangladesh using the following formula [27],

$$CarryingCapacity = \frac{Total\_Resources}{Resource\_Consumption\_per\_Person}.$$

For this calculation, we follow the following steps:

Food production

Bangladesh has a diverse agricultural sector. In recent years, annual rice production has been around 35 million tons [28]. We can convert this to calories as $1kg$ of rice provides roughly $3,500$

calories. Therefore, 35 million tons of rice provide $122,500,000$ million calories. Also, for other food supplies, including fish, vegetables, and pulses, their production is 30.2 million tons, which provides $105,700,000$ million calories. Hence, the total food, including rice and other food sources, is $228,200,000$ million calories [29].

Water resources

We consider the annual renewable water resources. Bangladesh has about $1,200$ cubic kilometers of renewable freshwater. It becomes a significant factor if we estimate the per capita water need (drinking, sanitation, agriculture, etc.) to be around $2,000$ liters per person per day [30].

Resource consumption per person

The average caloric requirement for a person is about 2500 calories per day. Hence, the average caloric requirement for a person in a year is $9,12,500$ calories/year [31, 32]. For simplicity, we use the food as the primary resource and estimate the carrying capacity under the following assumptions.

- The estimation of the carrying capacity is mainly based on food production, while water availability, quality, and other factors, e.g., land for habitation and infrastructure, are also critical.
- It assumes that the current agricultural practices remain constant and do not degrade resources over time.

$$CarryingCapacity = \frac{228,200,000 million\_calories}{912,500 calories\_per\_person/year} = 250,082,191 \approx 250\ millions.$$

Likewise, we estimate the carrying capacity for Pakistan, India, Nepal, Bhutan, and Afghanistan, and hence the carrying capacities are 430,000,000, 2,000,000,000, 43,814,550, 1,500,000, and 400,000,000, respectively.

Estimation of time delay

We analyze the estimations of the Demographic and Health Survey (DHS) [33], World Bank Group [30], UN Population Division [34], Bangladesh Bureau of Statistics (BBS) [35], Fertility and Reproductive Health in Bangladesh [36], Pakistan Bureau of Statistics (PBS) [37], Ministry of Health and Family Welfare- India [38], Population Reference Bureau (PRB India) [39], Nepal Central Bureau of Statistics (CBS Nepal) [40], National Statistical Bureau- Bhutan (NSB Bhutan) [41], and Central Statistics Organization-Afghanistan (CSO Afghanistan) [42] for the estimation of time delay parameter (τ) of the countries in Southeast Asia and consider the following time delay periods:

Table 1. Estimated time delay period for different countries based on the demographic data [30, 33–42]

Country	Bangladesh	Pakistan	India	Nepal	Bhutan	Afghanistan
Time delay (τ)	17 years	19 years	18 years	19 years	20 years	18 years

In the case of Bangladesh, the logistic population model considers a growth rate of 1.8315% (See Table 1) and a carrying capacity of 250 million. On the other hand, the time delay logistic population model has a carrying capacity of 250,000,000 and a maturation period of 17 years over the time from 1974 to 2101.

Furthermore, we consider a carrying capacity of 430,000,000 and a growth rate of 2.6259Pakistan in the logistic population model. In contrast, the time delay logistic population model has a carrying capacity 430,000,000 and a maturation period of 19 years from 1974 to 2101.

In the logistic population model, we consider India's growth rate of 2.03614of 2,000 million people. In addition, for the time delay logistic population model, we determine that the maturity phase for the years 1951 to 2101 is 18 years. In the logistic population model, we take a carrying capacity of 43,814,550 people and a growth rate of 1.78% for Nepal. It is considered a maturation period of 19 years from 1960 to 2101 to calculate the time delay logistic population model with a growth rate of 1.78%. In the case of Bhutan, the logistic population model scenario, we take a growth rate of 2.046% and a carrying capacity of 1,500,000 people. Using the time delay logistic population model, we compute the population, fixing a growth rate of 2.046% and a maturation period of 20 for the period 1960 to 2101. Finally, we consider a growth rate of 2.55% and a carrying capacity of 400,000,000 people for Afghanistan in the case of the logistic population model. To estimate the population using a time delay logistic population model with 2.55% as the growth rate and 19 years for the maturation period from 1960 to 2101.

In the ensuing subsections, we attempt to describe the current populations of Bangladesh, Pakistan, India, Nepal, Bhutan, and Afghanistan, with the populations estimated from the logistic and time delay logistic population models, as well as with the probable future populations of all these countries, using tabular data and graphical representations.

Projection of existing census populations

Table 2. Comparison table of census population with the logistic population model and the time delay population model for Bangladesh

Year	Census population	Population according to the logistic population model (Growth rate 1.8315%)	Population according to the time delay population model (Growth rate 1.8315%, time delay 17 years)
1974	72,947,807	72,947,807	72,947,807
1981	86,154,836	79,743,257	79,881,011
1991	109,242,834	80,741,176	90,942,954
2001	131,670,484	92,121,618	103,163,846
2011	150,211,005	115,387,742	116,106,175
2022	171,186,372	150,491566	129,527,139

It is easy to deduce from Table 2 and Figure 1 above that the logistic population model's calculation appears more suitable for Bangladesh's population in 2022. Since 1,717,814,220 people were counted in the Bangladeshi census 2022, the projected population from the logistic population model was 150.491566, and from the time delay, the model's figure was 129,527,139. Moreover, the estimated populations using the time delay logistic population model in 1991, 2001, and 2011 are 90,942,954,103,163,846, and 116,106,175, respectively, which aligned with the census population of these years for Bangladesh.

In 2023, according to Table 3 and Figure 2, the population of Nepal was 30,896,590. However, 37,372,714 and 22,510,462 are estimated from the logistic population model and the time delay logistic population model, respectively. On top of that, in the years 1970, 1980, and 1990, populations calculated from these two models were nearly the same. However, the difference in the computed populations of these two models increased as time passed for the population of Nepal.

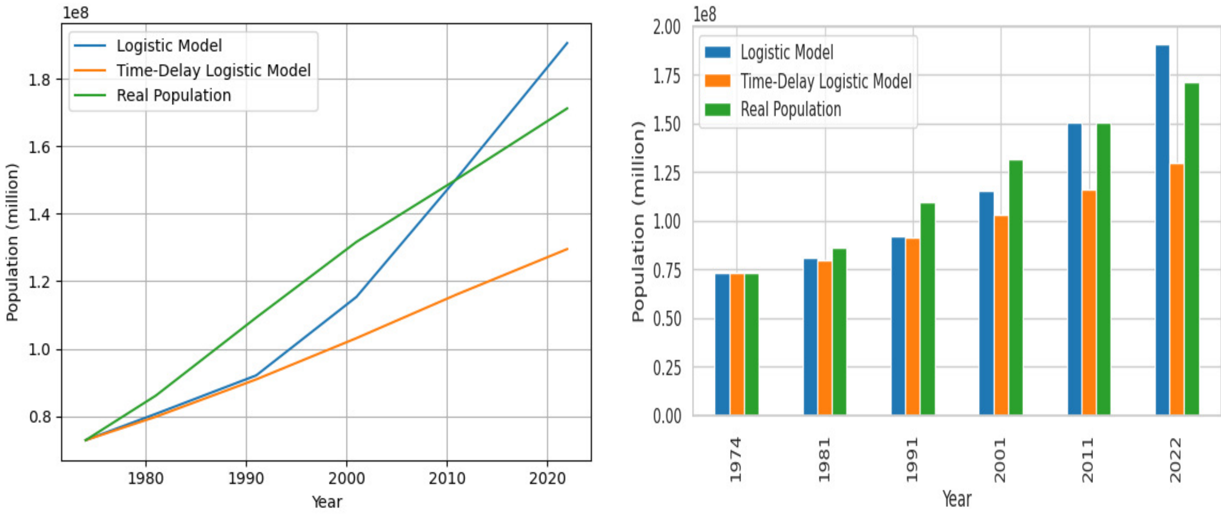


Figure 1. Comparison of the census population, the logistic population and the time delay population model for Bangladesh

Table 3. Comparison table of census population with the logistic population model and the time delay population model for Nepal

Year	Census population	Population according to the logistic population model (Growth rate 1.78%)	Population according to the time delay population model (Growth rate 1.78%, time delay 19 years)
1960	10,167,941	10,167,941	10,167,941
1970	12,501,285	11,321,920	11,657,135
1980	15,600,442	13,041,776	13,363,886
1990	19,616,530	16,699,888	15,266,749
2000	24,559,500	22,642,288	17,326,359
2010	27,161,567	30,202,107	19,518,228
2023	30,896,590	37,372,714	22,510,462

Table 4. Comparison table of census population with the logistic population model and the time delay population model for Pakistan

Year	Census population	Population according to the logistic population model (Growth rate 2.6259%)	Population according to the time delay population model (Growth rate 2.6259%, time delay 19 years)
1951	41,933,000	41,933,000	41,933,000
1961	50,840,000	50,582,438	53,144,439
1971	71,478,000	69,419,078	67,351,112
1981	87,120,000	101,479,308	85,016,728
1991	106,315,000	176,638,099	106,471,718
2001	124,355,000	288,832,812	132,025,150
2011	198,602,738	381,179,154	161,736,407

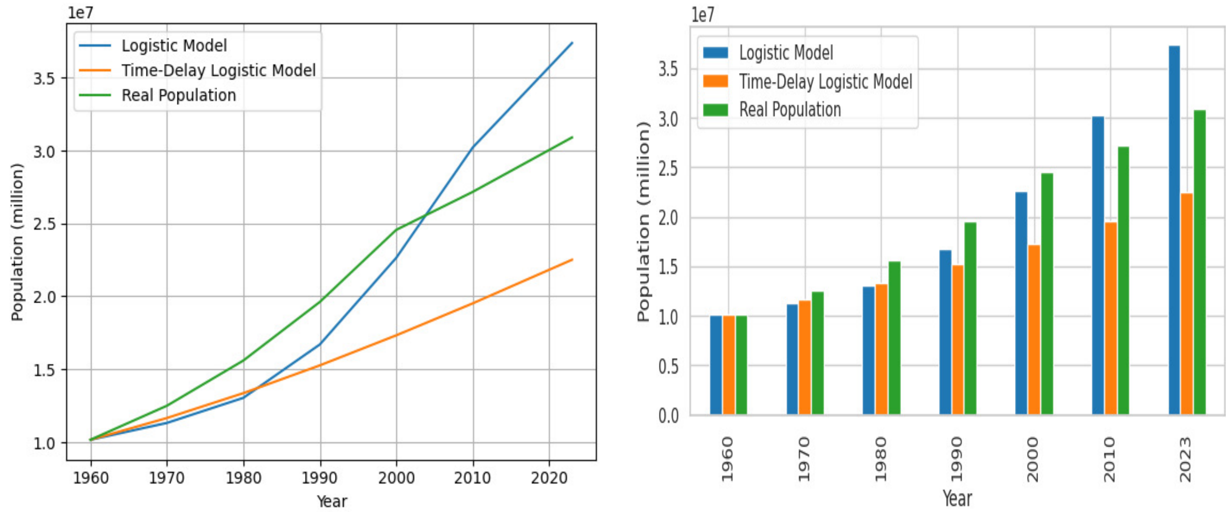


Figure 2. Comparison of the census population, the logistic population and the time delay population model for Nepal

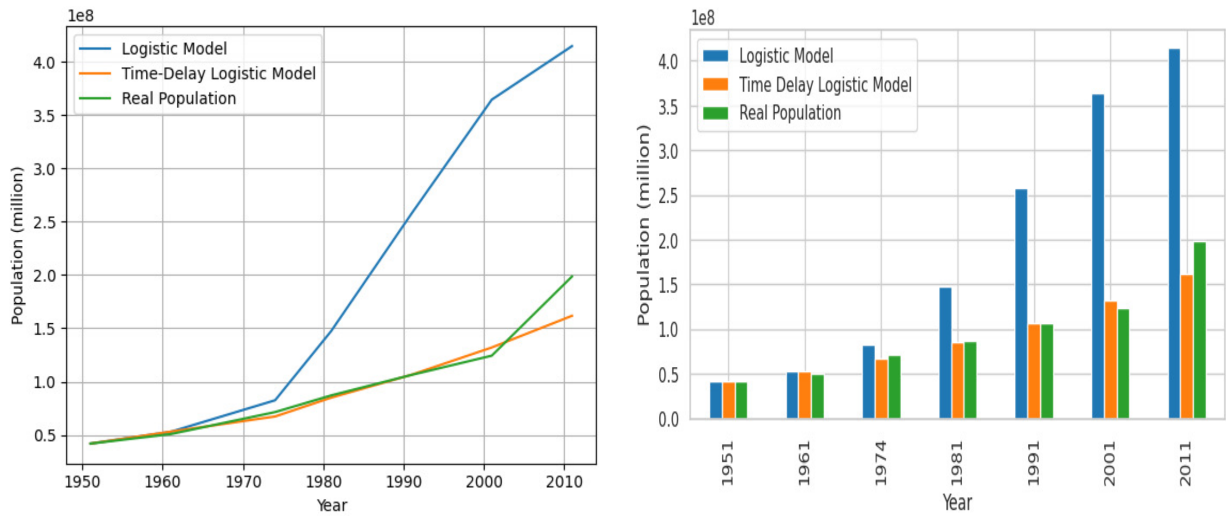


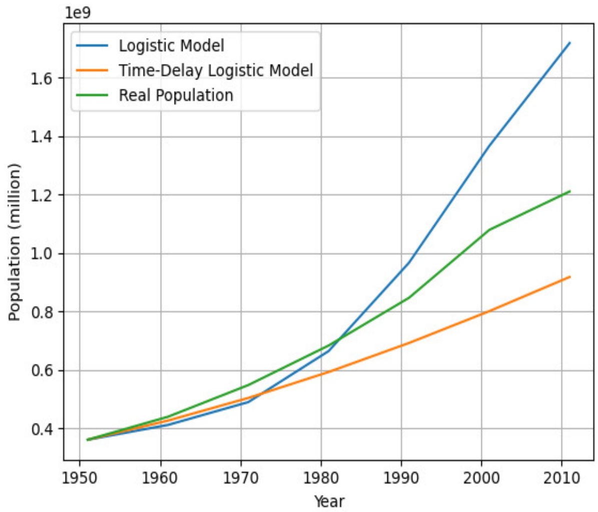
Figure 3. Comparison of the census population, the logistic population and the time-delay population model for Pakistan

Table 4 and Figure 3 illustrate that in 2011, the population of Pakistan was 198,602,738 in actuality, while the population of Pakistan was 381,179,154 according to the logistic population model. The population we obtained from the time delay logistic population model is 161,736,407. In other years, despite the logistic models, differences from the census population are increasing over time, and the results from the time delay logistic population model are similar to the census population of Pakistan.

Table 5 and Figure 5 portray that the population of India, as reported by the 2011 census, was 1,210,193,477, while the population of India is 1,717,814,220 according to the logistic population model and the population we obtain from the time delay logistic population model is 917,717,030. The data of the time delay model is more appropriate with the actual population of India. After 1980, the results of these two models deprived the census population of India.

Table 5. Comparison table of census population with the logistic population model and the time delay population model for India

Year	Census population	Population according to the logistic population model (Growth rate 2.03614%)	Population according to the time delay population model (Growth rate 2.03614%, time delay 18 years)
1951	361,088,090	361,088,090	361,088,090
1961	439,734,771	411,814,420	426,638,451
1971	548,159,657	489,874,321	504,021,333
1981	683,379,097	664,418,477	592,800,427
1991	846,471,039	966,509,286	692,006,336
2001	1,078,737,436	1,366,094,320	800,815,931
2011	1,210,193,477	1,717,814,220	917,717,030



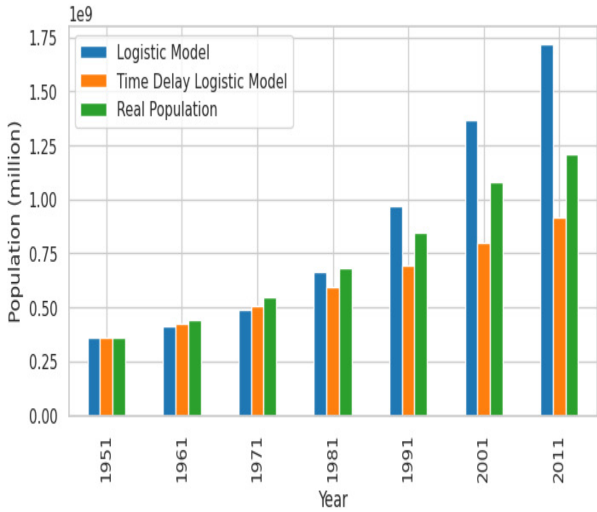


Figure 4. Comparison of the census population, the logistic population and the time delay population model for India

Table 6. Comparison table of census population with the logistic population model and the time delay population model for Bhutan

Year	Census population	Population according to the logistic population model (Growth rate 2.046%)	Population according to the time delay population model (Growth rate 2.046%, time delay 20 years)
1960	221,266	221,266	221,266
1970	298,894	253,953	263,426
1980	415,257	305,016	313,618
1990	558,442	422,605	372,330
2000	587,207	637,749	439,267
2010	705,516	946,762	514,401
2023	787,424	1,256,730	623,472

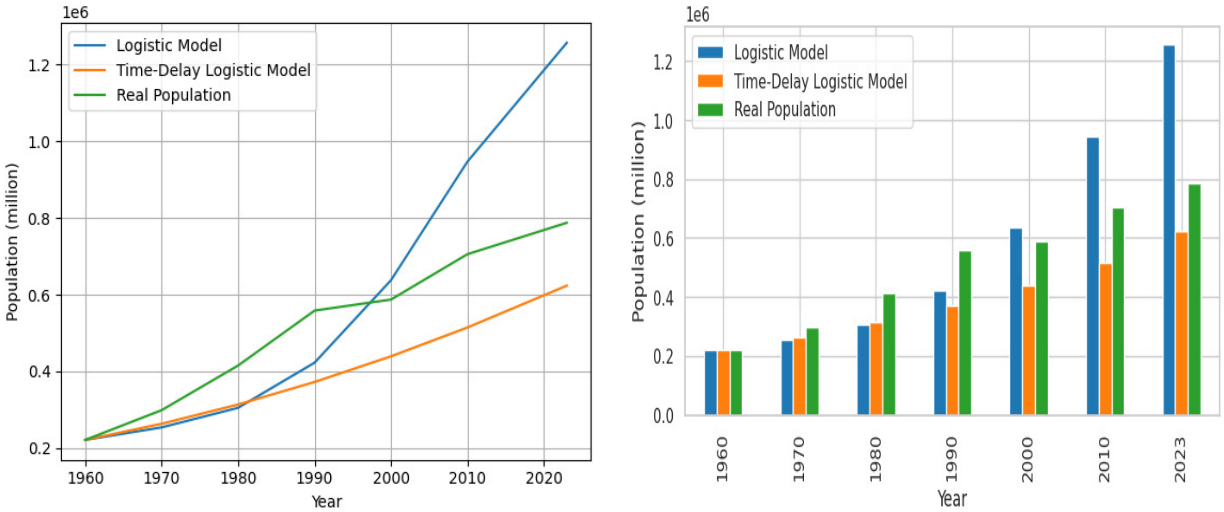


Figure 5. Comparison of the census population, the logistic population and the time delay population model for Bhutan

From Table 6 and Figure 5, the 2023 census of Bhutan reported 787,424 residents; the logistic population model puts that number at 1,256,730; the time delay logistic population model gives us 623,472 residents; the data from the time delay model is more consistent with the actual population of Bhutan for the year 2023. However, before the year 2000, the results from both models showed approximate results for the population projection of Bhutan.

Table 7. Comparison table of census population with the logistic population model and the time delay population model for Afghanistan

Year	Census population	Population according to the logistic population model (Growth rate 2.55%)	Population according to the time delay population model (Growth rate 2.55%, time delay 18 years)
1960	8,622,466	8,622,466	8,622,466
1970	10,752,971	10,522,384	11,065,491
1980	12,486,631	13,811,799	14,200,117
1990	10,694,796	23,031,440	18,203,072
2000	19,542,982	47,479,885	23,291,175
2010	28,189,672	110,807,224	29,730,869
2023	42,239,854	251,191,807	40,640,040

Based on Table 7 and Figure 6, the 2023 census of Afghanistan reported 42,239,854 residents. The logistic population model provides the number of residents in Afghanistan at 251,191,807, while the time delay logistic population model gives us 40,640,040 residents. Before 1990, these three populations were nearly equal, but after 1990, these estimated populations and census populations show fluctuations; mainly, the logistic models' projections depart from the time delay logistic population model's population and the census population for Afghanistan.

Based on the above-tabulated data, graphical representations, and descriptions, it is challenging to determine which of these two models is better or more suited based on this section. Therefore, we examine the future population projections for Bangladesh, Pakistan, India, Nepal, Bhutan, and

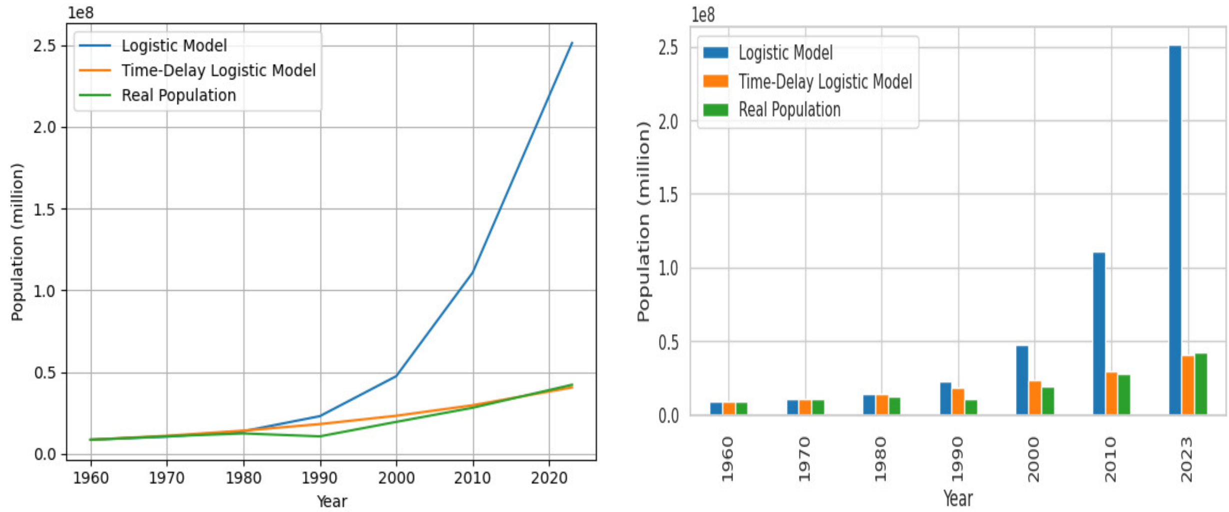


Figure 6. Comparison of the census population, the logistic population and the time delay population model for Afghanistan

Afghanistan using both the logistic and time-delay logistic models to determine which model is more suitable for forecasting population.

Prediction of populations using the logistic and the time delay logistic model

Table 8. Comparison table of the future predicted population of Bangladesh between the logistic population model and the time delay population model

Year	Population according to the logistic population model (Growth rate 1.8315%)	Population according to the time delay population model (Growth rate 1.8315%, time delay 17 years)
1974	72,947,807	72,947,807
1981	79,743,257	79,881,011
1991	80,741,176	90,942,954
2001	92,121,618	103,163,846
2011	115,387,742	116,106,175
2022	150,491566	129,527,139
2031	190,540,966	143,150,229
2041	222,692,843	156,667,918
2051	240,356,690	169,763,329
2061	247,296,991	182,135,958
2071	249,381,683	193,530,655
2081	249,882,980	203,759,843
2091	249,981,589	212,714,723
2101	249,997,589	220,366,474

At a glance, it is evident from the above-tabulated data (Table 8 - Table 13) and graphs (Figure 7 - Figure 12) of the logistic population model that, after some time, the projected populations of Bangladesh, Pakistan, India, Nepal, Bhutan, and Afghanistan approached their carrying capacity. More specifically, 249,381,683 people are expected to live in Bangladesh in 2071, roughly equivalent to the country's carrying capacity of 250 million people. In the same way, Pakistan's projected

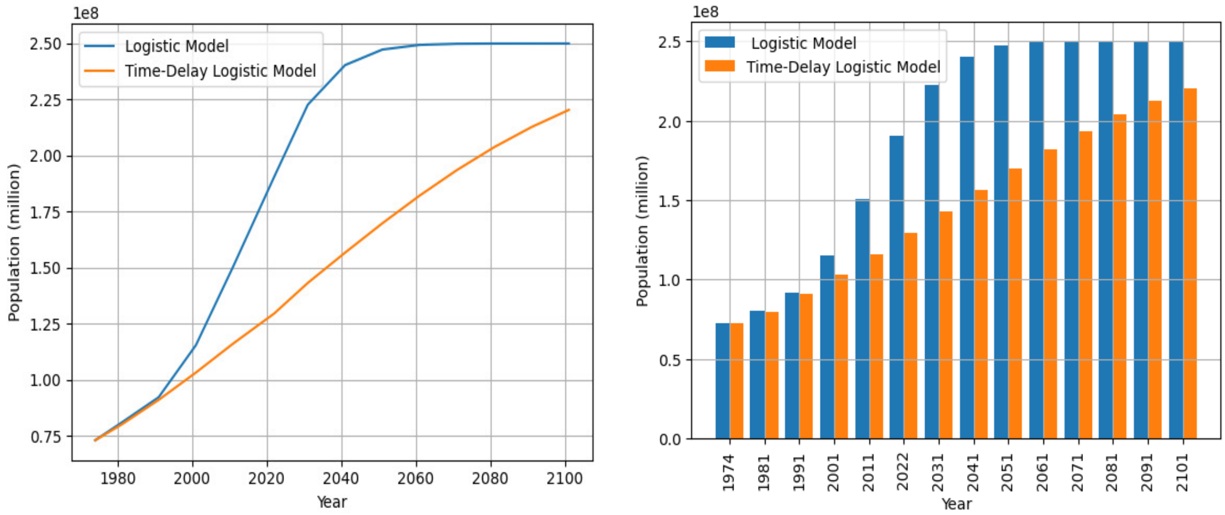


Figure 7. Comparison of the future predicted population of Bangladesh between the logistic population model and the time delay population model

Table 9. Comparison of the future population of Pakistan between the logistic population model and the time delay population model

Year	Population according to the logistic population model (Growth rate 2.6259%)	Population according to the time delay population model (Growth rate 2.6259%, time delay 19 years)
1951	41,933,000	41,933,000
1961	50,582,438	53,144,439
1971	69,419,078	67,351,112
1981	101,479,308	85,016,728
1991	176,638,099	106,471,718
2001	288,832,812	132,025,150
2011	381,179,154	161,736,407
2021	419,180,073	195,247,905
2031	428,286,578	231,722,380
2041	429,795,046	269,671,379
2051	429,981,203	307,139,583
2061	429,998,674	341,886,682
2071	429,999,928	371,886,061
2081	429,999,997	395,714,307
2091	429,999,999	412,915,201
2101	429,999,999	423,980,642

population of 430 million in 2041 exceeded that country's utmost capacity. Furthermore, the estimated population of India, Nepal, Bhutan, and Afghanistan in 2061, 2060, 2040, and 2050 will be roughly equivalent to these countries' carrying capacities. Therefore, following these years above, it is unlikely that the populations of these three countries will grow realistically or behave like a stable population, which is inconsistent with the dynamics of natural population growth. Conversely, the time delay logistic population model's tabular data and graphs show that, throughout the entire period, the population numbers, bars, and graphs are much below the carrying capacity of these six countries. Using the time delay population model, we can predict

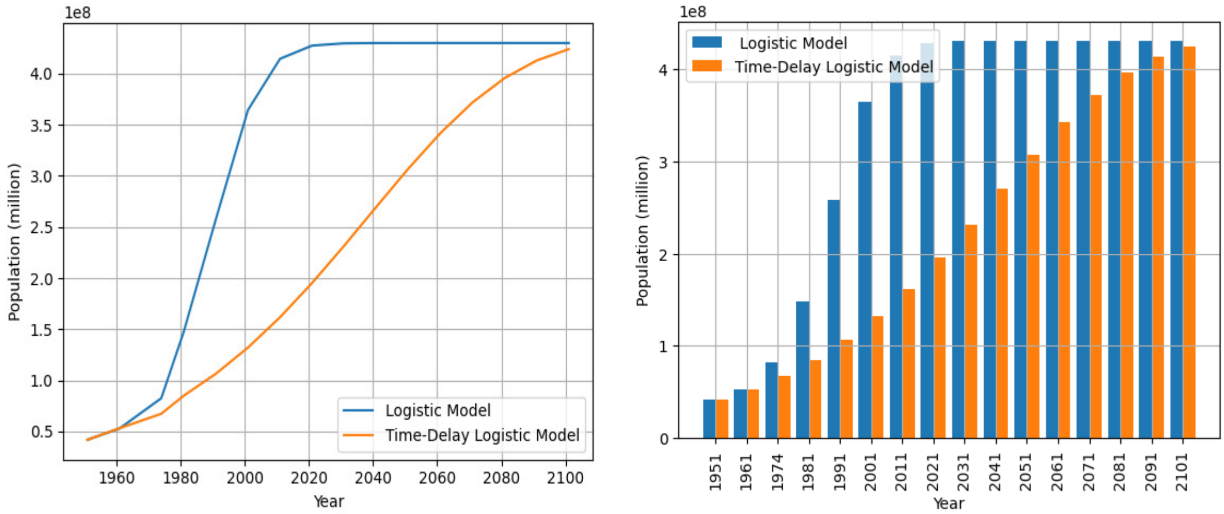
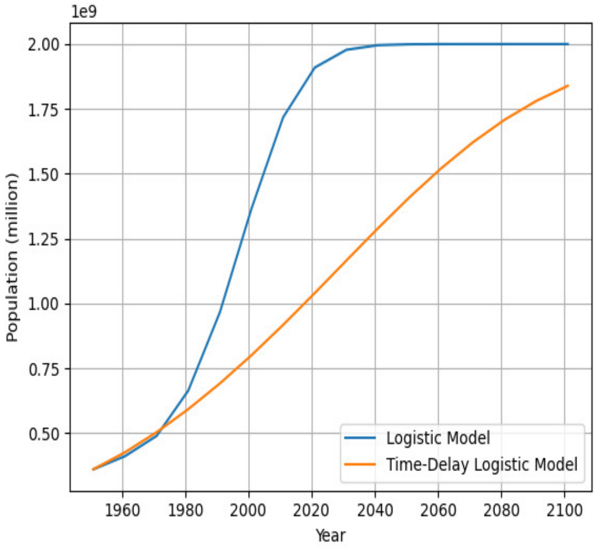


Figure 8. Comparison of the future predicted population of Pakistan between the logistic population model and the time delay population model

Table 10. Comparison of the future population of India between the logistic population model and the time delay population model

Year	Population according to the logistic population model (Growth rate 2.03614%)	Population according to the time delay population model (Growth rate 2.03614%, time delay 18 years)
1951	361,088,090	361,088,090
1961	411,814,420	426,638,451
1971	489,874,321	504,021,333
1981	664,418,477	592,800,427
1991	966,509,286	692,006,336
2001	1,366,094,320	800,815,931
2011	1,717,814,220	917,717,030
2021	1,909,417,089	1,040,373,270
2031	1,977,894,180	1,165,764,748
2041	1,995,713,218	1,290,258,308
2051	1,999,326,685	1,410,029,649
2061	1,999,913,857	1,521,422,334
2071	1,999,991,011	1,621,447,052
2081	1,999,999,234	1,708,107,007
2091	1,999,999,946	1,780,506,839
2101	1,999,999,996	1,838,905,785

these six countries' far-future populations after this time frame (1951/1974–2101). In the year 2101, the populations of Bangladesh, Pakistan, India, Nepal, Bhutan, and Afghanistan are projected to be 220,366,474, 423,980,642, 1,838,905,785, 38,267,699, 1,306,052, and 205,662,268, respectively.



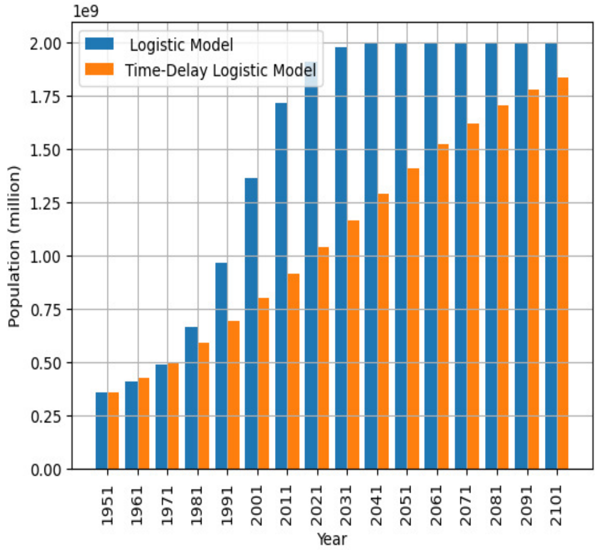


Figure 9. Comparison of the future predicted population of India between the logistic population model and the time delay population model

Table 11. Comparison of the future population of Nepal between the logistic population model and the time delay population model

Year	Population according to the logistic population model (Growth rate 1.78%)	Population according to the time delay population model (Growth rate 1.78%, time delay 19 years)
1960	10,167,941	10,167,941
1970	11,321,920	11,657,135
1980	13,041,776	13,363,886
1990	16,699,888	15,266,749
2000	22,642,288	17,326,359
2010	30,202,107	19,518,228
2020	37,073,732	21,809,627
2030	41,280,395	24,158,875
2040	43,067,452	26,517,286
2050	43,635,529	28,831,706
2060	43,778,998	31,049,270
2070	43,808,656	33,121,413
2080	43,813,732	35,008,891
2090	43,814,455	36,684,691
2101	43,814,540	38,267,699

4 Conclusions

The classical logistic population model has been transformed into the time-delay population model by incorporating the time-delay τ, which corresponds to the amount of time required for a newborn baby to mature before affecting the reproduction or growth of the population. We have estimated the parameters of both models based on the real population data set for all countries in Southeast Asia. This study has demonstrated the population projection for the period 1960-2023 employing both the classical logistic and the time-delay logistic model. It is observed that such a projection has a very good fit with the census of populations for Bangladesh, Pakistan, India,

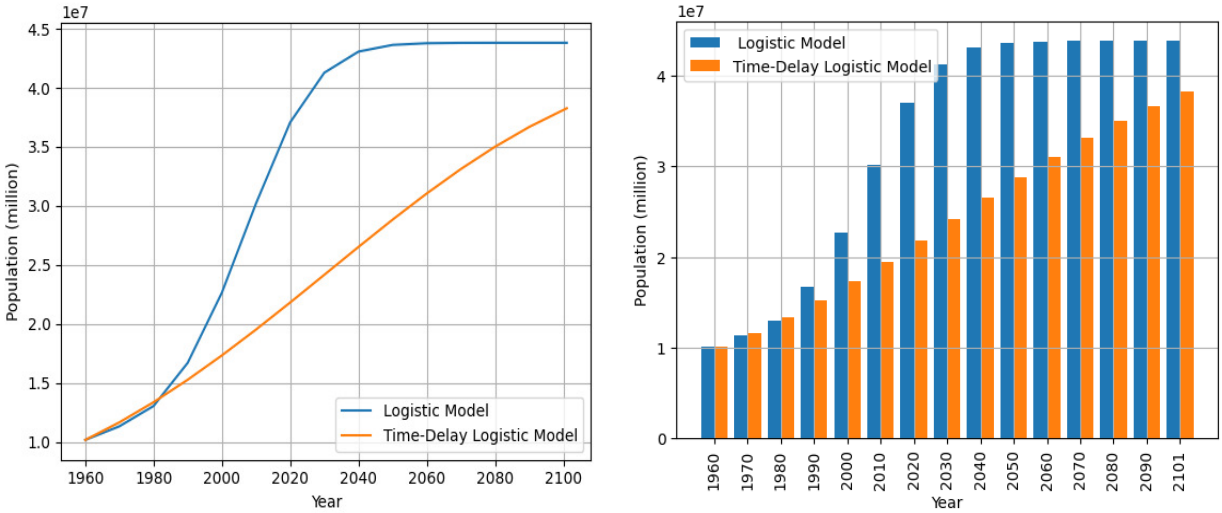


Figure 10. Comparison of the future predicted population of Nepal between the logistic population model and the time delay population model

Table 12. Comparison of the future population of Bhutan between the logistic population model and the time delay population model

Year	Population according to the logistic population model (Growth rate 2.046%)	Population according to the time delay population model (Growth rate 2.046%, time delay 20 years)
1960	221,266	221,266
1970	253,953	263,426
1980	305,016	313,618
1990	422,605	372,330
2000	637,749	439,267
2010	946,762	514,401
2020	1,243,959	597,251
2030	1,416,317	686,755
2040	1,479,548	781,206
2050	1,496,067	878,245
2060	1,499,385	975,015
2070	1,499,922	1,068,368
2080	1,499,991	1,155,253
2090	1,499,999	1,233,068
2101	1,499,999	1,306,052

Nepal, Bhutan, and Afghanistan, whilst for a short period, the logistic model fits well, and the time-delay model fits significantly over a large period. Furthermore, our research indicates that the logistic population model is not capable of providing a better approximation for the anticipated population over an extended period because, after a certain amount of time, it appears to be fairly stable and does not tend to increase realistically. However, this issue has been resolved by the logistic population model with time delay, as predicted in this study. Therefore, to predict the future population and look for solutions related to the enormous population, the concerned authorities should use the logistic population model with a delay time for a large period and the classical logistic population model for a short period.

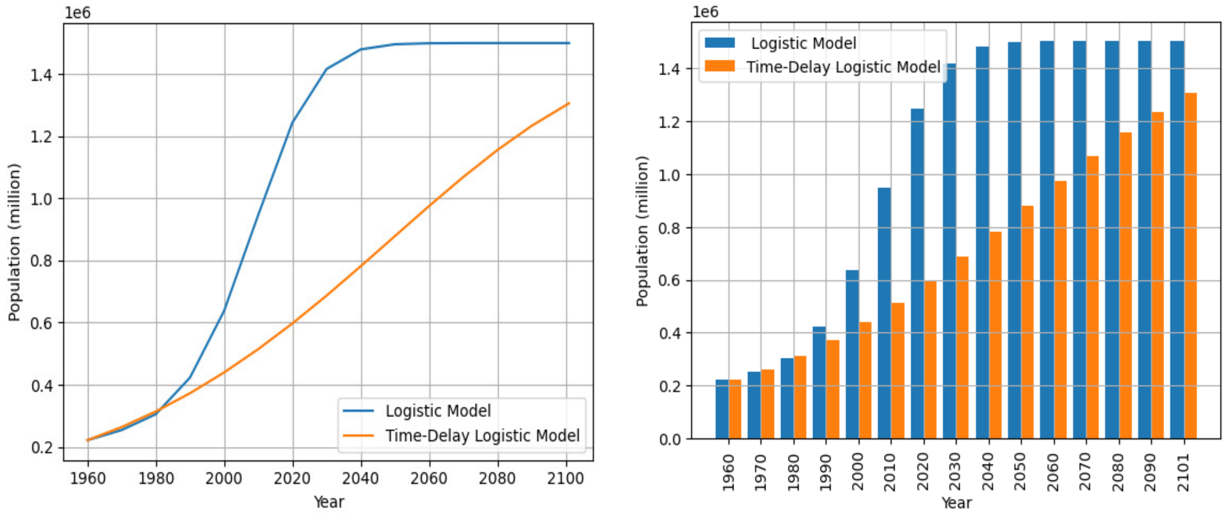


Figure 11. Comparison of the future predicted population of Bhutan between the logistic population model and the time delay population model

Table 13. Comparison table of the future predicted population of Afghanistan between the logistic population model and the time delay population model

Year	Population according to the logistic population model (Growth rate 2.55%)	Population according to the time delay population model (Growth rate 2.55%, time delay 18 years)
1960	8,622,466	8,622,466
1970	10,522,384	11,065,491
1980	13,811,799	14,200,117
1990	23,031,440	18,203,072
2000	47,479,885	23,291,175
2010	110,807,224	29,730,869
2020	233,792,646	37,833,447
2030	347,806,960	47,959,374
2040	390,416,754	60,498,367
2050	398,759,320	75,858,040
2060	399,877,793	94,412,919
2070	399,990,695	116,422,737
2080	399,999,451	141,970,066
2090	399,999,974	170,838,301
2101	399,999,999	205,662,268

Declarations

Use of AI tools

The authors declare that they have not used Artificial Intelligence (AI) tools in the creation of this article.

Data availability statement

All data generated or analyzed during this study are included in this article.

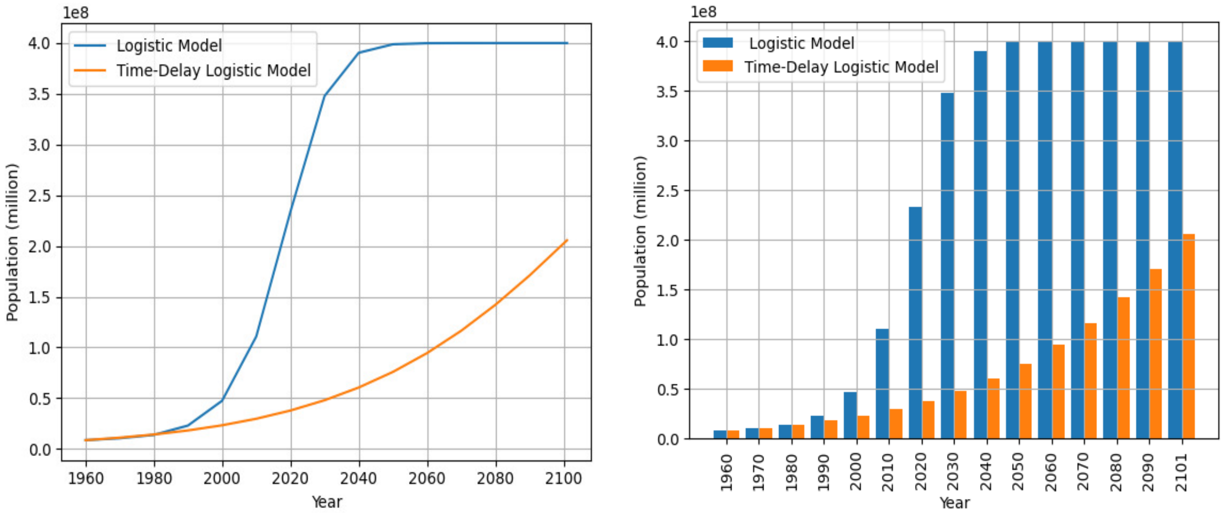


Figure 12. Comparison of the future predicted population of Afghanistan between the logistic population model and the time-delay population model

Ethical approval

The authors state that this research complies with ethical standards. This research does not involve either human participants or animals.

Consent for publication

Not applicable

Conflicts of interest

The authors declare that they have no conflict of interest.

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Author's contributions

M.S.A.: Methodology, Software, Validation, Visualization, Writing - Original Draft, M.M.H.: Formal Analysis, Investigation, Writing - Original Draft, Visualization, M.H.K.: Methodology, Validation, Supervision, Writing-Review & Editing, Project Administration, M.O.G.: Validation, Supervision, Project Administration. The authors have read and agreed to the published version of the manuscript.

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