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5E and CRA integration: Effects on geometric concept images and connection skills

5E ve CRA entegrasyonu: Geometrik kavram imajları ve ilişkilendirme becerileri üzerindeki etkisi

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Abstract: This study examined the effects of integrating the 5E Instructional Model with the Concrete-Representational-Abstract (CRA) approach on 10th-grade students' concept images of parallelograms and rhombuses and their mathematical connection skills. The study employed a quasi-experimental design with pretest-posttest control groups, involving 61 students (experimental group n=31, control group n=30). The experimental group received instruction using the integrated 5E-CRA model, while the control group received traditional instruction. Data were collected using the "Parallelogram and Rhombus Concept Assessment Tool" developed by the researcher. Results revealed that the integrated approach had very large positive effects on parallelogram concept image (d=2.78), rhombus concept image (d=2.42), and mathematical connection skills (d=2.36). The findings demonstrate that 5E-CRA integration offers an effective alternative to traditional methods in geometric concept instruction, highlighting the critical role of systematic progression from concrete to abstract in developing conceptual understanding and connection skills. The research findings provide guidance for mathematics teachers in using manipulatives and progressive instructional strategies

Keywords: *5E instructional model, concrete-representational-abstract approach, concept image, mathematical connections, geometry education, quadrilaterals.*

Özet: Bu araştırma, 5E Öğretim Modeli ile Somut-Yarı Somut-Soyut (CRA) yaklaşımının entegrasyonunun etkisini incelemiştir. Çalışmada 10. sınıf öğrencilerinin paralelkenar ve eşkenar dörtgen kavram imajları ile matematiksel ilişkilendirme becerileri üzerindeki etkiler araştırılmıştır. Çalışmada ön test-son test kontrol gruplu yarı deneysel desen kullanılmış olup, 61 öğrenci (deney grubu n=31, kontrol grubu n=30) araştırmaya dahil edilmiştir. Deney grubunda entegre 5E-CRA modeli kullanılarak öğretim gerçekleştirilirken, kontrol grubunda geleneksel öğretim uygulanmıştır. Veriler, araştırmacı tarafından geliştirilen "Paralelkenar ve Eşkenar Dörtgen Kavram Değerlendirme Aracı" kullanılarak toplanmıştır. Bulgular, entegre yaklaşımın paralelkenar kavram imajı (d=2.78), eşkenar dörtgen kavram imajı (d=2.42) ve matematiksel ilişkilendirme becerileri (d=2.36) üzerinde çok büyük pozitif etkiler yarattığını ortaya koymuştur. Elde edilen sonuçlar, 5E-CRA entegrasyonunun geometrik kavram öğretiminde geleneksel yöntemlere etkili bir alternatif sunduğunu göstermekte ve somuttan soyuta sistematik geçişin kavramsal anlayış ve ilişkilendirme becerilerinin gelişimindeki kritik rolünü vurgulamaktadır. Araştırma bulguları, matematik öğretmenlerine manipülatif kullanımı ve kademeli öğretim stratejileri konusunda rehberlik sağlamaktadır.

Anahtar Kelimeler: *5E öğretim modeli, somut-yarı somut-soyut yaklaşımı, kavram imajı, matematiksel ilişkilendirme, geometri eğitimi, dörtgenler*

1. Introduction

Geometry plays a critical role in developing students' spatial thinking, mathematical reasoning, and problem-solving skills as a fundamental area of mathematics. The challenges encountered in teaching this domain direct researchers and educators toward developing effective instructional approaches. In particular, the difficulties students experience in making sense of abstract geometric concepts necessitate the investigation of alternative instructional models and their integration.

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One of the most significant problems encountered in geometry instruction is that students struggle to understand relationships between geometric concepts and to connect these concepts with one another (Duval, 2006; Jones, 2002). The topic of quadrilaterals particularly stands out as an area where students experience difficulty in understanding the hierarchical structure among concepts (Fujita & Jones, 2007). Research shows that students struggle to grasp relationships among special quadrilaterals and to connect the properties of these quadrilaterals (Balgalmış & Işık-Ceyhan, 2019; Şimşek, 2019). When concepts such as parallelograms and rhombuses that are related to each other are concerned, students' concept images are generally incomplete or erroneous (Monaghan, 2000). For example, many students perceive a parallelogram only as a "slanted quadrilateral," cannot fully know the properties of rhombuses, and cannot grasp the hierarchical relationships among special quadrilaterals (Fujita, 2012).

The foundation of these problems lies in several key issues within traditional geometry instruction. These include the use of prototype representations of shapes, insufficient emphasis on relationships among concepts, and the adoption of a definition-formula-oriented approach (Hershkowitz, 1990; Van Hiele, 1986). Traditional instructional methods do not adequately support students in achieving conceptual understanding and developing connection skills (Battista, 2007). The difficulties experienced by Turkish students in the geometry domain in international assessments such as PISA and TIMSS (MEB, 2019; MEB, 2020) further increase the importance of developing and implementing effective instructional approaches.

The development of concept images and connection skills in geometry instruction is an important research and application area not only in Turkey but also at the international level. Studies conducted worldwide show that students experience similar difficulties in making sense of geometric concepts and establishing relationships among them (Battista, 2007; Jones & Tzekaki, 2016; Sinclair et al., 2016). International comparative studies reveal that students' geometry performance is lower compared to other mathematics domains in many countries worldwide. TIMSS 2019 results indicate that students worldwide show lower performance in geometry and measurement domains (Mullis et al., 2020). Similarly, in PISA assessments, students have been observed to struggle with questions requiring spatial thinking and geometric reasoning (OECD, 2019).

Both in the United States (National Council of Teachers of Mathematics [NCTM], 2000) and in European Union countries (Đokić et al., 2021; Tessema et al., 2024), developing conceptual understanding and connection skills in geometry instruction is among the priority goals of mathematics education reforms. Particularly, the hierarchical classification and understanding of properties of quadrilaterals has been identified as a common area where students struggle in many countries (Fujita & Jones, 2007; Usiskin et al., 2008). This situation reveals the need to investigate alternative instructional approaches and their integration for deep understanding of geometric concepts and ensuring lasting learning.

In this context, the integration of the 5E Instructional Model based on constructivist learning theory and the Concrete-Representational-Abstract (CRA) approach that provides systematic transition from concrete to abstract is considered an important alternative that can contribute to overcoming the challenges encountered in geometry instruction. This research is based on four fundamental theoretical frameworks: Concept Image theory (Tall & Vinner, 1981), Van Hiele's Geometric Thinking Levels theory (Van Hiele, 1986), the 5E Instructional Model (Bybee et al., 2006), and the Concrete-Representational-Abstract approach (Bruner, 1966; Witzel, 2005).

The Concept Image theory developed by Tall and Vinner (1981) encompasses the entire cognitive structure formed in students' minds during the process of learning mathematical concepts. Concept image includes all mental pictures, properties, processes, and experiences related to the concept. Concept definition, on the other hand, is the formal, mathematical definition of the concept. The relationship between concept image and concept definition is of critical importance in understanding mathematical concepts. Ideally, students' concept images should be compatible and consistent with the concept definition. However, in geometry instruction, students' concept images are generally limited to prototype examples, and this situation leads to concept misconceptions (Hershkowitz, 1990; Vinner & Hershkowitz, 1980).

The Geometric Thinking Levels theory developed by Van Hiele (1986) suggests that students' geometric thinking skills develop at five levels: Visual Level (Level 1), Analysis Level (Level 2), Relational Level (Level 3), Deduction Level (Level 4), and Higher Level (Level 5). At the visual level, students recognize shapes only by their appearance. At the analysis level, they begin to identify and analyze the properties of shapes. At the relational level, they understand relationships among shapes and can make hierarchical classifications. At the deduction level, they can make geometric proofs and develop theorems. At the higher level, they can compare and evaluate different geometry systems.

Mathematical connections, as defined by the National Council of Teachers of Mathematics (NCTM), refer to the ability to "recognize and use connections among mathematical ideas; understand how mathematical ideas interconnect and build on one another to produce a coherent whole; recognize and apply mathematics in contexts outside of mathematics" (NCTM, 2000, as cited in Mathframework, 2024). Mathematical connections are broadly categorized into two main types: intra-mathematical connections and extra-mathematical connections (De Gamboa et al., 2023; Rodríguez-Nieto et al., 2022). Intra-mathematical connections occur within the domain of mathematics and involve relationships between mathematical concepts, procedures, representations, and ideas (Hatisaru et al., 2024). These connections can be further subdivided into several specific types: (a) part-whole connections, which include inclusion relationships (when one concept is contained within another) and generalization relationships (when a concept is a generalization of a particular case); (b) feature/property connections, which occur when characteristics or properties of mathematical concepts are related; (c) representation connections, which involve linking different representations of the same mathematical concept; and (d) procedural connections, which relate different mathematical procedures or algorithms (Businskas, 2008; Rodríguez-Nieto et al., 2022). These connections align with Van Hiele's Level 2 (Analysis) and Level 3 (Abstraction-Deduction) geometric thinking levels, where students can identify properties of shapes and understand relationships among different geometric concepts (Van Hiele, 1986). Extra-mathematical connections involve relationships between mathematics and other disciplines, real-world contexts, or everyday life experiences (Caviedes et al., 2024). These connections help students understand the relevance and applicability of mathematics beyond the classroom setting. In the context of this study, the focus is specifically on intra-mathematical connections related to geometric concepts, particularly: (1) hierarchical connections between parallelogram and rhombus concepts (understanding that every rhombus is also a parallelogram), and (2) transformation connections (understanding how one geometric shape can be transformed into another through systematic modifications of its properties). These connections align with Van Hiele's Level 2 (Analysis) and Level 3 (Abstraction-Deduction) geometric thinking levels, where students can identify properties of shapes and understand relationships among different geometric concepts (Van Hiele, 1986).

The 5E Instructional Model is an instructional model developed based on constructivist learning theory and emphasizing active participation of students (Bybee et al., 2006). The model suggests that the learning process consists of five phases: Engage, Explore, Explain, Elaborate, and Evaluate. In the engagement phase, students' interest is captured and their prior knowledge is activated. This phase ensures that students become curious about the topic and motivated to learn. In the exploration phase, students discover the concept through their own experiences. In this phase, students are provided with opportunities to investigate their questions, collect data, and develop hypotheses. In the explanation phase, the formal definition and properties of the concept are systematized under teacher guidance. Students share their discoveries, and the teacher provides necessary terminology and explanations. In the elaboration phase, the concept is applied and extended in different contexts. Students have the opportunity to use newly learned concepts in different situations. In the evaluation phase, students' conceptual understanding and skills are assessed (Bybee, 2009).

The Concrete-Representational-Abstract (CRA) approach is a three-stage instructional approach developed inspired by Jerome Bruner's information processing theory (Witzel, 2005). In this approach, the learning process occurs in three stages: direct interaction with concrete objects, schema and visual-based representations, and abstract mathematical symbols and formulas. In the concrete stage, students work with real objects and manipulatives. This stage ensures physical experience of concepts and formation of concrete meaning. In the representational stage, students represent their concrete experiences with drawings, schemas, and graphics. This stage serves as a bridge between concrete and abstract thinking. In the abstract stage, students work with mathematical symbols, formulas, and proofs. This stage ensures concepts are fully expressed mathematically (Flores, 2010).

In this research, an instructional process based on the integration of the 5E Instructional Model and the CRA approach was designed. There is a natural harmony between the phases of the 5E model (Engage, Explore, Explain, Elaborate, Evaluate) and the phases of the CRA approach (Concrete, Representational, Abstract). This harmony was integrated as follows: capturing students' interest in the engagement phase using concrete manipulatives and real-life examples; discovering concept properties with concrete materials and transitioning to representational representations in the exploration phase; visualizing concepts using drawings, schemas, and diagrams in the explanation phase; applying concepts in different contexts and using mathematical formulas in problem-solving activities in the elaboration phase; and evaluating students' conceptual understanding through abstract mathematical expressions and problems in the evaluation phase.

The 5E Instructional Model and CRA approach are recognized as effective instructional strategies at the international level. The 5E Model was developed by the Biological Sciences Curriculum Study (BSCS) in the United States and has been applied in many fields including mathematics instruction over time (Bybee et al., 2006). It is widely used in countries with advanced education systems such as Australia, Canada, Singapore, and Finland (Eisenkraft, 2003). The CRA approach particularly forms the foundation of Singapore mathematics teaching methodology. This approach, also known as the "Singapore Model," is considered one of the important factors in Singapore's success in mathematics education, which ranks high in PISA and TIMSS assessments (Leong et al., 2015).

International research shows that both the 5E Model (Huda et al., 2020; Ültay & Çalık, 2016) and the CRA approach (Bouck et al., 2017; Leong et al., 2015) are effective in mathematics instruction. Hu et al. (2017) state that among the important advantages of the 5E model are enhancing students' ability to use prior knowledge and facilitating their construction of new knowledge. In Putri et al.'s (2021) study, it was determined that the 5E learning cycle supported by

GeoGebra helps students understand the relationships among concepts and use these relationships effectively. Regarding the CRA approach, Satsangi and Bouck (2015) found that this approach is effective in helping students with special learning difficulties understand area and perimeter concepts. Witzel, Mercer, and Miller (2003) revealed that the CRA approach improves mathematical problem-solving skills and increases the permanence of learned information.

However, the integration of these two approaches is an original area that has not yet been sufficiently researched. This research aims to contribute originally to international literature by examining the integration of 5E and CRA approaches. This study combines internationally recognized theoretical frameworks such as concept image (Tall & Vinner, 1981) and Van Hiele's geometric thinking levels (Van Hiele, 1986) in geometry instruction. The research has the potential to contribute to efforts for improving geometry instruction on a global scale. Studies examining the effect of the integration of these two approaches on concept image and connection skills, particularly in the topic of quadrilaterals, are limited. This research aims to fill this gap in the literature and develop an effective instructional model in geometry instruction.

The research focuses on developing students' images of parallelogram and rhombus concepts and strengthening their connection skills between these concepts. Concept image and connection skills are of critical importance in conceptual understanding of mathematics and achieving lasting learning (Tall & Vinner, 1981). In this context, the purpose of this research is to examine the effect of the integration of the 5E Instructional Model and the Concrete-Representational-Abstract (CRA) approach on 10th-grade students' parallelogram and rhombus concept images and mathematical connection skills.

To achieve this purpose, the following research questions guide the study:

RQ1: *To what extent does the integration of the 5E Instructional Model and CRA approach affect 10th-grade students' concept images of parallelograms and rhombuses compared to traditional instruction?*

RQ2: *How does the integrated 5E-CRA approach influence students' mathematical connection skills between geometric concepts compared to traditional instruction?*

The following hypotheses were tested in connection with this main purpose:

H_1: *The parallelogram concept image scores of the experimental group that received the integration of the 5E Instructional Model and CRA approach are significantly higher than those of the control group that received traditional instruction.*

H_2: *The rhombus concept image scores of the experimental group that received the integration of the 5E Instructional Model and CRA approach are significantly higher than those of the control group that received traditional instruction.*

H_3: *The mathematical connection skill scores of the experimental group that received the integration of the 5E Instructional Model and CRA approach are significantly higher than those of the control group that received traditional instruction.*

2. Methodology

This research was conducted to examine the effect of integrating the 5E Instructional Model and the Concrete-Representational-Abstract (CRA) approach on 10th-grade students' geometric concept images and mathematical connection skills. The study employed a quasi-experimental design with pretest-posttest control groups (Campbell et al., 1963). This design was selected due to the practical constraints of working with intact classroom groups while maintaining experimental control through random assignment. The research included one experimental group and one control group, and the same measurement tool was applied to both groups before and after the implementation. The experimental group was instructed using an integrated 5E-CRA approach, while the control group received conventional instruction.

2.1. Study Group

This research was conducted with 10th-grade students attending a public high school located in the central district of Burdur during the spring semester of the 2024-2025 academic year. A convenience sampling method was employed due to practical constraints and accessibility considerations. The selection criteria included: (a) 10th-grade classes accessible to the researcher, (b) voluntary participation of both teachers and students, (c) classes that had not yet covered quadrilateral topics in their curriculum, and (d) willingness of school administration to participate in the research. Two intact classes meeting these criteria were selected from a public high school in Burdur, Turkey. To minimize selection bias, random assignment was used to designate one class as the experimental group (n=31) and the other as the control group (n=30). Pre-implementation equivalence testing confirmed that the groups were statistically comparable on all measured variables (see Table 8).

Random assignment placed one class in the experimental group (n=31) and the other in the control group (n=30). The demographic characteristics of participants are summarized in Table 1.

Table 1

Demographic Characteristics of the Study Group

Characteristic	Category	Experimental Group	Control Group	Total
Gender	Male	14 (45.2%)	13 (43.3%)	27 (44.3%)
	Female	17 (54.8%)	17 (56.7%)	34 (55.7%)
Total		31 (50.8%)	30 (49.2%)	61 (100%)

As seen in Table 1, the study group consists of 61 students in total. The experimental group includes 31 students (50.8%), and the control group includes 30 students (49.2%). In terms of gender distribution, there are 14 males (45.2%) and 17 females (54.8%) in the experimental group, and 13 males (43.3%) and 17 females (56.7%) in the control group.

2.2. Data Collection Tool and Data Collection Process

The "Parallelogram and Rhombus Concept Assessment Tool" developed by the researcher was used as the data collection tool in this research. In developing the tool, theoretical frameworks proposed by Battista (2007), Bruner (1966), Bybee et al. (2006), Tall and Vinner (1981), and Van Hiele (1986) were taken as the basis. In the development of the assessment tool, first the relevant literature was reviewed and existing measurement tools used in geometric concept instruction were examined (Clements & Battista, 1992; Gutierrez et al., 1991). For content validity of the tool, opinions of two mathematics education experts and one experienced mathematics teacher were obtained. Pilot implementation was conducted with 20 tenth-grade students not included in the research.

The tool was designed with four questions in accordance with the hierarchical structure in Van Hiele's (1986) geometric thinking levels. Details of the questions are presented in Table 2.

Table 2

Assessment Tool Question Structure and Theoretical Foundation

Question No	Question Type	Theoretical Foundation	Measured Dimension	Van Hiele Level	Description
1	Comparative Table	Tall & Vinner (1981) Van Hiele (1986)	Concept Image Property Component	Level 1-2 (Analysis-Abstraction)	Evaluating 6 basic properties of parallelogram and rhombus in ✓/✗ format
2	Multiple Choice	Clements & Battista (1992) Van Hiele (1999)	Concept Image Visual Component	Level 0-2 (Visualization-Abstraction)	Classifying 4 different quadrilateral shapes into parallelogram/rhombus categories
3	True-False	Battista (2007) Van Hiele (1986)	Mathematical Connection Hierarchical Thinking	Level 2-3 (Abstraction-Deduction)	Evaluating and justifying relationships among concepts
4	Open-ended	Duval (2007) Van Hiele (1986)	Mathematical Connection Transformation Process	Level 2-3 (Abstraction-Deduction)	Explaining the process of transforming parallelogram to rhombus

Analytical rubrics based on Van Hiele's (1986) geometric thinking levels were developed for each question. The rubrics were designed as four-level in the 0-3 point range, and scoring criteria are detailed in Table 3.

Table 3

Assessment Tool Rubric System

Question	Measured Dimension	3 Points (Van Hiele Level 2)	2 Points (Van Hiele Level 1)	1 Point (Van Hiele Level 0)	0 Points (Below Level 0)
1. Property Identification	Concept Image - Property Component	Correctly marks all 6 properties for both concepts	Correctly marks 4-5 properties for both concepts	Correctly marks 2-3 properties for both concepts	Correctly marks 0-1 properties for both concepts
2. Shape Recognition	Concept Image - Visual Component	Correctly classifies all 4 shapes into categories	Correctly classifies 3 shapes into categories	Correctly classifies 1-2 shapes into categories	Cannot correctly classify any shape
3. Relationship Establishment	Mathematical Connection	Correct answer to both questions + complete mathematical justification	Correct answer to both questions + partial justification or 1 question correct + complete justification	Only correct/incorrect marking, no/insufficient justification	Wrong answers or blank
4. Transformation	Mathematical Connection	Conceptual transformation logic + detailed explanation	Basic approach + partial explanation	General change statement, no systematic explanation	Cannot transform or completely wrong

The implementation process of the research lasted a total of 4 weeks (12 class hours). The total process, including pre- and post-implementation data collection phases, was planned as 6 weeks. Ethical approval for this study was granted by the Non-Interventional Clinical Research Ethics Committee of Burdur Mehmet Akif Ersoy University prior to data collection (Ethics Approval Code: GO 2025/1335). Details of the implementation process are presented in Table 4. The implementation plan was carried out in line with the learning outcomes "*Explains the basic elements and properties of quadrilaterals and solves problems. Explains the angle, side, diagonal, and area properties of special quadrilaterals and solves problems*."

Table 4

Implementation Process and Time Schedule

Week	Topic	5E Phase	CRA Level	Main Activities	Duration
1	Pretest	-	-	Application of parallelogram and rhombus concept assessment tool	1 class hour
2-3	Parallelogram	Engage + Explore	Concrete → Representational	Discovery with manipulatives, paper folding, measurement studies	6 class hours
3	Parallelogram	Explain + Elaborate	Representational → Abstract	Formal definition, problem solving, proof activities	
4-5	Rhombus	Engage + Explore	Concrete → Representational	Modeling, diagonal discovery, symmetry studies	6 class hours
5	Rhombus	Explain + Elaborate	Representational → Abstract	Property systematization, connection	
6	Posttest	Evaluate	Abstract	Application of parallelogram and rhombus concept assessment tool	1 class hour

In the experimental group, lesson plans based on the systematic integration of the 5E Instructional Model (Bybee et al., 2006) and the CRA approach (Bruner, 1966) were implemented. As seen in Table 5, each 5E phase was paired with specific CRA levels in this integration process.

Table 5

5E-CRA Integration

5E Phase	CRA Level	Materials Used	Main Activities
Engage	Concrete	Straws, geoboard, daily life objects	Creating quadrilaterals with manipulatives, daily life examples
Explore	Concrete → Representational	Manipulatives, paper folding, measurement tools	Quadrilateral construction, property discovery, drawings
Explain	Representational	Systematic drawings, schemas, concept maps	Formal definition construction, property systematization
Elaborate	Representational → Abstract	Problem papers, proof schemas, real-life examples	Problem solving, proof activities, application
Evaluate	Abstract	Assessment questions, project materials	Individual assessment, conceptual test

To clarify the implementation differences between experimental and control groups, the basic characteristics of the instructional processes carried out in both groups are presented comparatively in Table 6.

Table 6

Experimental and Control Group Implementation Comparison

Dimension	Experimental Group	Control Group
Approach	Student-centered, discovery-focused	Teacher-centered, lecture-focused
Materials	Manipulatives, paper folding, discovery sheets	Textbook, board, traditional tools
Activities	Concrete discovery → representational → abstract transition	Definition explanation → example → practice
Student Role	Active explorer, problem solver	Passive listener, implementer

As seen in Table 6, while the instruction based on 5E and CRA integration in the experimental group supported students' active participation and developing conceptual understanding starting from concrete experiences, the traditional approach in the control group followed a process focused on direct information transmission and repetitive practice. That is, in this process, traditional instructional approach based on the current curriculum was applied in the control group. The same time period (12 class hours) was allocated to teaching parallelogram and rhombus concepts in the control group as in the experimental group.

2.3. Data Analysis

The study adopted a quantitative data analysis approach, utilizing descriptive statistics including means, standard deviations, ranges, and frequencies, as well as inferential statistics comprising paired-samples t-tests, independent-samples t-tests, and Chi-square tests. Prior to analyses, necessary assumptions for the applicability of parametric tests were tested.

Different analysis strategies were adopted for the three hypotheses of the research. For parallelogram and rhombus concept image hypotheses, pretest-posttest comparisons were made using scores from questions 1 and 2 respectively. For the mathematical connection hypothesis, scores from questions 3 and 4 were combined and analyzed. Cohen's d value was calculated for each hypothesis to determine effect size. Small effect $0.20 \le d < 0.50$, medium effect $0.50 \le d < 0.80$, and large effect $d \ge 0.80$ were interpreted according to Cohen's (1988) criteria. Data analysis according to hypotheses is presented in Table 7.

Table 7

Data Analysis According to RQ/Hypotheses

RQ/Hypothesis	Dependent Variable	Data Source	Statistical Analysis
RQ1/H_1	Parallelogram Concept Image	Question 1 parallelogram column (property identification, 0-3 points) Question 2 parallelogram category (shape recognition, 0-3 points)	Summary statistics, Within-group t-test, Between-groups t-test, Effect size calculation
RQ1/H_2	Rhombus Concept Image	Question 1 rhombus column (property identification, 0-3 points) Question 2 rhombus category (shape recognition, 0-3 points)	Summary statistics, Within-group t-test, Between-groups t-test, Effect size calculation
RQ2/H_3	Mathematical Connection Skills	Question 3 conceptual connection (relationship establishment, 0-3 points) Question 4 transformation process (transformation, 0-3 points)	Summary statistics, Within-group t-test, Between-groups t-test, Effect size calculation

To determine the pre-implementation equivalence of experimental and control groups, independent samples t-test was applied on pretest scores. The results presented in Table 8 show that there are no statistically significant differences between groups in terms of all measured variables ($p > 0.05$ for all comparisons). Specifically, the groups showed equivalent performance on parallelogram concept image ($t(59) = 0.18$, $p = 0.857$), rhombus concept image ($t(59) = 0.09$, $p = 0.925$), relationship establishment skills ($t(59) = 0.15$, $p = 0.878$), and transformation abilities ($t(59) = 0.11$, $p = 0.912$).

Table 8

Group Equivalence Test Results

Variable	Experimental Group			Control Group			*t*	*df*	*p*
	M	*SD*	*n*	*M*	*SD*	*n*			
Parallelogram Concept Image	0.65	0.49	31	0.67	0.48	30	0.18	59	0.857
Rhombus Concept Image	0.68	0.48	31	0.67	0.48	30	0.09	59	0.925
Relationship Establishment	0.52	0.51	31	0.50	0.51	30	0.15	59	0.878
Transformation	0.61	0.50	31	0.60	0.50	30	0.11	59	0.912

Additionally, Chi-square analysis revealed no significant differences in gender distribution between groups ($\chi^2 = 0.037$, $p = 0.847$), with the experimental group consisting of 14 males and 17 females, and the control group consisting of 13 males and 17 females. These results confirm that the two groups were statistically equivalent at baseline, ensuring that any post-intervention differences can be attributed to the experimental manipulation rather than pre-existing group differences. This equivalence provides a solid foundation for valid comparison of treatment effects between the integrated 5E-CRA approach and traditional instruction.

Whether difference scores showed normal distribution for the applicability of parametric tests was examined with Shapiro-Wilk test. For parallelogram concept image difference scores, $W = 0.961$ ($p = 0.312$) in the experimental group, $W = 0.958$ ($p = 0.278$) in the control group; for rhombus concept image difference scores, $W = 0.955$ ($p = 0.218$) in the experimental group, $W = 0.952$ ($p = 0.191$) in the control group; for mathematical connection difference scores, $W = 0.967$ ($p = 0.425$) in the experimental group, $W = 0.964$ ($p = 0.391$) in the control group were obtained. All $p > 0.05$ values show that the normality assumption is satisfied.

For analysis of development in students' geometric thinking levels, rubric scores were converted to Van Hiele levels (0 points=Below Level 0, 1 point=Level 0, 2 points=Level 1, 3 points=Level 2). Level transitions were analyzed with frequency tables and cross-tabulations. SPSS 28.0 program was used in data analysis.

2.4. Validity and Reliability of the Research

Validity and reliability studies of the Parallelogram and Rhombus Concept Assessment Tool were conducted. Content validity was examined within the scope of validity study, and internal consistency and inter-rater reliability were analyzed within the scope of reliability studies. Validity and reliability results are presented in Table 9.

Table 9.

Tool Validity and Reliability Results

Psychometric Property	Method/Dimension	Result	Value	Criterion
Content Validity	3 expert opinions	Satisfied	All questions	-
Internal Consistency	Total tool	High	$\alpha = 0.84$	$\alpha \geq 0.70$
Internal Consistency	Concept image dimension	High	$\alpha = 0.78$	$\alpha \geq 0.70$
Internal Consistency	Connection dimension	High	$\alpha = 0.81$	$\alpha \geq 0.70$
Inter-rater	Question 3	Very high	$r = 0.89$	$r \geq 0.80$
Inter-rater	Question 4	Very high	$r = 0.92$	$r \geq 0.80$

For content validity, opinions were obtained from two faculty members with doctoral degrees in mathematics education and one mathematics teacher. Experts were asked to evaluate the level of representation of each question for the conceptual dimensions intended to be measured, and content validity was provided for all questions. As seen in Table 9, within the scope of reliability analysis, Cronbach's Alpha coefficient for the entire tool was calculated as $\alpha = 0.84$. This value meets the $\alpha \geq 0.70$ criterion suggested by Nunnally (1978). For sub-dimensions, concept image dimension (questions 1 and 2) was found as $\alpha = 0.78$, mathematical connection dimension (questions 3 and 4) as $\alpha = 0.81$. For open-ended questions, agreement between two independent raters was calculated as $r = 0.89$ for question 3, $r = 0.92$ for question 4, and showed very high level agreement according to Cohen (1988) criteria.

3. Results

The findings obtained from the research are presented to answer the two research questions through testing three hypotheses. For RQ1 (concept image development), findings from H_1 and H_2 are presented. For RQ2 (mathematical connection skills), findings from H_3 are presented. For each hypothesis, first within-group development (pretest-posttest comparison) then between-group comparison results are given.

3.1. Findings for RQ1: Parallelogram Concept Image Development

Paired samples t-test was applied to test the hypothesis "The parallelogram concept image scores of the experimental group that received the integration of the 5E Instructional Model and CRA approach are significantly higher than those of the control group that received traditional instruction." Descriptive statistics and within-group comparison results for the H_1 hypothesis are presented in Table 10.

Table 10

Descriptive Statistics and Within-Group Comparison for H_1 Hypothesis

Group	Test	*n*	*M*	*SD*	*Min*	*Max*	*t*	*df*	*p*	*Cohen's d*
Experimental	Pretest	31	0.65	0.49	0	1	14.20	30	<0.001	2.55
	Posttest	31	2.55	0.51	2	3				
Control	Pretest	30	0.67	0.48	0	1	6.60	29	<0.001	1.20
	Posttest	30	1.27	0.45	1	2				

As seen in Table 10, a statistically significant difference was found between pretest scores (M = 0.65, SD = 0.49) and posttest scores (M = 2.55, SD = 0.51) in the experimental group [t(30) = 14.20, p < 0.001]. Cohen's d = 2.55 value indicates very large effect size. In the control group, a significant difference was detected between pretest scores (M = 0.67, SD = 0.48) and posttest scores (M = 1.27, SD = 0.45) [t(29) = 6.60, p < 0.001]. This group's Cohen's d = 1.20 value indicates large effect size.

Between-group comparison results for the H_1 hypothesis are presented in Table 11.

Table 11

Between-Group Comparison for H_1 Hypothesis

Group	*n*	*M*	*SD*	*Min*	*Max*	*t*	*df*	*p*	*Cohen's d*
Experimental	31	2.55	0.51	2	3	10.85	59	<0.001	2.78
Control	30	1.27	0.45	1	2				

As seen in Table 11, a statistically significant difference was found between experimental group posttest scores (M = 2.55, SD = 0.51) and control group posttest scores (M = 1.27, SD = 0.45) in favor of the experimental group [t(59) = 10.85, p < 0.001]. Cohen's d = 2.78 value indicates very large effect size and reveals that the H_1 hypothesis is accepted.

3.2. Findings for RQ1: Rhombus Concept Image Development

Descriptive statistics and within-group comparison results for the analyses conducted to test the hypothesis "The rhombus concept image scores of the experimental group that received the integration of the 5E Instructional Model and CRA approach are significantly higher than those of the control group that received traditional instruction" are presented in Table 12.

Table 12

Descriptive Statistics and Within-Group Comparison for H_2 Hypothesis

Group	Test	*n*	*M*	*SD*	*Min*	*Max*	*t*	*df*	*p*	*Cohen's d*
Experimental	Pretest	31	0.68	0.48	0	1	13.89	30	<0.001	2.49
	Posttest	31	2.52	0.51	2	3				
Control	Pretest	30	0.67	0.48	0	1	7.54	29	<0.001	1.38
	Posttest	30	1.33	0.48	1	2				

As seen in Table 12, a statistically significant difference was found between pretest scores (M = 0.68, SD = 0.48) and posttest scores (M = 2.52, SD = 0.51) in the experimental group [t(30) = 13.89, p < 0.001]. Cohen's d = 2.49 value indicates very large effect size. A significant difference was detected between pretest scores (M = 0.67, SD = 0.48) and posttest scores (M = 1.33, SD = 0.48) in the control group [t(29) = 7.54, p < 0.001]. This group's Cohen's d = 1.38 value indicates large effect size.

Between-group comparison results for the H_2 hypothesis are presented in Table 13.

Table 13

Between-Group Comparison for H_2 Hypothesis

Group	*n*	*M*	*SD*	*Min*	*Max*	*t*	*df*	*p*	*Cohen's d*
Experimental	31	2.52	0.51	2	3	9.47	59	<0.001	2.42
Control	30	1.33	0.48	1	2				

As seen in Table 13, a statistically significant difference was found between experimental group posttest scores (M = 2.52, SD = 0.51) and control group posttest scores (M = 1.33, SD = 0.48) in favor of the experimental group [t(59) = 9.47, p < 0.001]. Cohen's d = 2.42 value indicates very large effect size and reveals that the H_2 hypothesis is accepted.

3.3. Findings for RQ2: Mathematical Connection Skills Development

Analyses were conducted on total scores of the third and fourth questions to test the hypothesis "The mathematical connection skill scores of the experimental group that received the integration of the 5E Instructional Model and CRA approach are significantly higher than those of the control group that received traditional instruction." Descriptive statistics and within-group comparison results for the H_3 hypothesis are presented in Table 14.

Table 14

Descriptive Statistics and Within-Group Comparison for H_3 Hypothesis

Group	Test	*n*	*M*	*SD*	*Min*	*Max*	*t*	*df*	*p*	*Cohen's d*
Experimental	Pretest	31	1.13	0.81	0	2	18.75	30	<0.001	3.37
	Posttest	31	4.84	0.90	4	6				
Control	Pretest	30	1.10	0.80	0	2	8.94	29	<0.001	1.63
	Posttest	30	2.73	0.87	2	4				

As seen in Table 14, a statistically significant difference was found between pretest scores (M = 1.13, SD = 0.81) and posttest scores (M = 4.84, SD = 0.90) in the experimental group [t(30) = 18.75, p < 0.001]. Cohen's d = 3.37 value indicates very large effect size and the highest effect size among the three hypotheses. A significant difference was detected between pretest scores (M = 1.10, SD = 0.80) and posttest scores (M = 2.73, SD = 0.87) in the control group [t(29) = 8.94, $p < 0.001$]. This group's Cohen's d = 1.63 value indicates large effect size.

Between-group comparison results for the H_3 hypothesis are presented in Table 15.

Table 15

Between-Group Comparison for H_3 Hypothesis

Group	*n*	*M*	*SD*	*Min*	*Max*	*t*	*df*	*p*	*Cohen's d*
Experimental	31	4.84	0.90	4	6	9.21	59	<0.001	2.36
Control	30	2.73	0.87	2	4				

As seen in Table 15, a statistically significant difference was found between experimental group posttest scores (M = 4.84, SD = 0.90) and control group posttest scores (M = 2.73, SD = 0.87) in favor of the experimental group [t(59) = 9.21, $p < 0.001$]. Cohen's d = 2.36 value indicates very large effect size and reveals that the H_3 hypothesis is accepted.

When effect sizes obtained in all hypotheses of the research are examined, very large effects (Cohen's d > 2.0) were obtained in the experimental group, and large effects (Cohen's d > 1.0) were obtained in the control group. Between-group comparisons reveal very large effect sizes in favor of the experimental group. These results show that all H_1, H_2, and H_3 hypotheses are accepted and that the integration of the 5E Instructional Model and CRA approach is effective in developing both parallelogram and rhombus concept images and mathematical connection skills between these concepts.

4. Discussion

The findings of this research provide clear answers to both research questions posed in this study. Regarding RQ1 (the extent to which the integrated 5E-CRA approach affects students' concept images), the results demonstrate significant improvements in both parallelogram and rhombus concept image development. For RQ2 (how the approach influences mathematical connection skills), substantial enhancement in students' abilities to establish relationships between geometric concepts was observed.

In this research, the effect of the integration of the 5E Instructional Model and CRA approach on 10th-grade students' parallelogram and rhombus concept images and mathematical connection skills was examined. The findings reveal that the integrated approach exhibits superior performance compared to traditional methods in geometric concept instruction. These results were evaluated in light of the four fundamental theoretical frameworks on which the research is based.

4.1. Parallelogram Concept Image Development

The improvement observed in parallelogram concept image development is important from the perspective of Tall and Vinner's (1981) Concept Image theory. This development shows that the integrated approach creates more consistent cognitive structures compatible with concept definition related to parallelogram concept in students' minds. The development in students' abilities to recognize parallelogram properties and apply these properties in different contexts reveals the effectiveness of the approach.

This development is also consistent with similar studies in international literature. The comprehensive meta-analysis study conducted by Polanin et al. (2024) revealed that the 5E model is significantly more effective than traditional instruction in STEM fields including mathematics and exhibits large effect sizes. Lin et al. (2014) also state that the positive effects of the 5E model in science learning contribute to students' deep understanding of concepts and enrich instructional processes.

When evaluated in terms of Van Hiele levels, the development in parallelogram concept image shows that it supports students' transitions from visual level (Level 1) to analysis level (Level 2). The process of discovering parallelogram properties with manipulatives in the concrete stage of the CRA approach, representing these properties with drawings in the representational stage, and expressing them with mathematical definitions in the abstract stage exhibits perfect harmony with Van Hiele's (1986) hierarchical thinking development theory.

The prototype image problem commonly seen in traditional instructional approaches (Hershkowitz, 1990; Vinner & Hershkowitz, 1980) was minimized in the integrated approach through systematic transition from concrete experiences to abstract thinking. Students became able to grasp parallelogram as a quadrilateral with opposite sides parallel rather than perceiving it only as a "slanted quadrilateral."

However, international literature also highlights important challenges associated with the CRA approach implementation. Research indicates that students do not always benefit equally from manipulative use, with some studies reporting that students may focus on the physical properties of manipulatives rather than the underlying mathematical concepts (Flores & Hinton, 2022). Additionally, one study found that although the CRA approach showed significant knowledge gains, most high school students did not believe it supported their learning, suggesting potential resistance to manipulative use in secondary mathematics classrooms (Prosser & Bismarck, 2023). Furthermore, effective CRA implementation requires substantial teacher preparation time and ongoing professional development to ensure meaningful connections between concrete experiences and abstract mathematical concepts (Bouck et al., 2017).

Overall, the findings demonstrate that CRA integration within the 5E framework provides a robust foundation for parallelogram concept development, though successful implementation requires careful attention to teacher preparation and student engagement strategies.

4.2. Rhombus Concept Image Development

The improvements observed in rhombus concept image development show that the integrated approach is effective not only in parallelogram concept but also in similar geometric structures. This result is particularly important in terms of concept image theory because rhombus concept is one of the concepts students frequently confuse and develop erroneous images about (Monaghan, 2000).

The success of the integrated approach stems from students experiencing rhombus properties with manipulatives in the exploration phase of the 5E model and understanding these properties at different representation levels through the systematic transition of the CRA approach. The meta-analysis study conducted by Ebner et al. (2024) revealed that the CRA approach consistently exhibits large positive effects in mathematics instruction. In this study, it was stated that the approach's providing systematic transition from manipulatives to abstract concepts particularly supports conceptual understanding.

The contribution of multiple representations emphasized by Flores and Hinton (2022) to conceptual understanding was clearly seen in rhombus concept in our research as well. Students discovered rhombus properties with manipulatives in the concrete stage, represented these properties with drawings in the representational stage, and were able to express them with mathematical definitions in the abstract stage.

From the Van Hiele levels perspective, the development in rhombus concept image shows that it improves students' abilities to identify and analyze shape properties at the analysis level (Level 2). At this level, students became able to understand the definition of rhombus as "a quadrilateral with all sides equal" and compare this property with other quadrilaterals.

Despite these positive outcomes, the 5E instructional model presents several implementation challenges that warrant consideration. Research has documented that the Engage and Explore phases, while pedagogically valuable, can be particularly time-consuming and may require extensive curriculum restructuring (Polanin et al., 2024). Teachers have reported initial student reluctance to engage in active learning activities, particularly when students are accustomed to more traditional, teacher-centered instructional approaches (Bybee, 2015). Moreover, the effectiveness of the 5E model shows considerable variation across different settings and implementations, suggesting that successful application requires careful adaptation to local contexts and student populations (Polanin et al., 2024). The model's inquiry-based nature also demands significant changes in teacher practice, potentially creating challenges for educators without adequate training or institutional support.

In summary, while the 5E instructional model proved highly effective for rhombus concept development in this study, educators must consider implementation challenges and provide adequate support systems to maximize its potential benefits.

4.3. Mathematical Connection Skills Development

The development observed in mathematical connection skills reveals that the integrated approach not only helps students learn individual concepts but also develops their abilities to establish relationships among concepts. This result reflects the relational level (Level 3) characteristics of Van Hiele theory. At this level, students can understand relationships among shapes and make hierarchical classifications.

The integrated approach's support for this skill stems from the 5E model's application of concepts in different contexts in the elaboration phase and the CRA approach's establishment of connections among multiple representations. Particularly, students showed improvement in understanding the hierarchical relationship between parallelogram and rhombus (every rhombus is also a parallelogram).

This situation becomes even more meaningful in light of Fujita and Jones's (2007) finding that hierarchical classification of quadrilaterals is a challenging area for students. Recent research continues to emphasize that intentional instruction and systematic progression through geometric thinking levels are essential for effective geometry education (Polanin et al., 2024).

The development in mathematical connection skills is also consistent with the goals of developing conceptual understanding and relationship establishment skills emphasized by the National Council of Teachers of Mathematics (NCTM, 2000). This situation shows that the integrated approach is consistent with international mathematics education standards.

While the integration of 5E and CRA approaches demonstrated substantial benefits in this study, the combination of these two instructional frameworks introduces additional complexity that may limit scalability. The systematic progression through both 5E phases and CRA stages requires considerable instructional time, potentially reducing curriculum coverage in traditional educational settings with rigid pacing guides (Ebner et al., 2024). Moreover, the integrated approach demands teachers who are proficient in both models, which may necessitate extensive professional development and ongoing support. Research also suggests that the effectiveness of such integrated approaches can vary significantly based on teacher expertise, student characteristics, and available resources (Flores et al., 2024). These practical constraints highlight the need for careful consideration of implementation logistics when scaling up such interventions beyond controlled research environments.

These findings collectively suggest that the integrated 5E-CRA approach offers significant promise for developing mathematical connection skills, but its successful adoption requires strategic planning to address resource demands and implementation complexity.

4.4. Research Limitations

The developments observed in all dimensions in the research show that 5E-CRA integration offers an effective alternative in geometry instruction. These results reveal the robustness of the approach's theoretical foundations and its effectiveness in practical application. It also shows that the CRA approach (Leong et al., 2015), which is one of the fundamental factors of Singapore's success in mathematics education, can be integrated with different instructional models and contribute to improving geometry instruction on a global scale. The 5E model's consistent positive effects in various educational environments emphasized by Polanin et al. (2024) also supports the potential of this integration. However, this research also has some limitations. First, the study is limited to a specific school and grade level, and similar studies are needed in different school types and grade levels for generalizability of findings. Additionally, the implementation period is relatively short, and follow-up studies are needed to evaluate long-term effects. The research was conducted in a quantitative context, and there are no qualitative findings to determine experiences related to the implementation process. However, determining concept images was limited to recognizing concepts and knowing their properties, while connection skills were limited to transformation and establishing inter-conceptual connections.

5. Conclusion and Recommendations

This research addressed two main research questions regarding the effect of integrating the 5E Instructional Model and CRA approach on 10th-grade students' geometric concept images and mathematical connection skills. The research results reveal that the integrated approach exhibits superior performance compared to traditional methods in geometric concept instruction.

In terms of parallelogram concept image, 5E and CRA integration significantly improved students' conceptual understanding [$t(59)=10.85$, $p<0.001$, $d=2.78$]. The instructional process starting with concrete materials and transitioning to abstract thinking strongly supported students' abilities to understand and construct parallelogram concept in their minds. The very large effect size obtained shows that the practical significance of the implementation is quite high.

Similarly, the integrated approach provided significant superiority in rhombus concept as well [$t(59)=9.47$, $p<0.001$, $d=2.42$]. This result reveals that 5E and CRA integration is effective not only in a single concept but also in similar geometric structures. Students were able to better grasp rhombus properties through concrete experiences and strengthen their visual images.

In the context of mathematical connection skills, students showed remarkable development in establishing inter-conceptual connections and understanding transformation processes [$t(59)=9.21$, $p<0.001$, $d=2.36$]. This finding shows that the integrated approach not only helps students learn individual concepts but also supports higher-order mathematical thinking skills.

When effect sizes are compared, it is seen that the implementation had the greatest effect on parallelogram concept image development ($d=2.78$), followed by rhombus concept image ($d=2.42$) and mathematical connection ($d=2.36$) skills. This situation reveals that 5E and CRA integration is particularly strong in developing basic geometric concept images, while showing relatively less but still very large effects in connection skills. This difference may stem from concept image development being more directly supported by concrete experiences, while connection skills require more complex cognitive processes.

When evaluated in terms of Van Hiele geometric thinking levels, it was seen that students in the experimental group reached higher levels after implementation. Particularly, significant differences in favor of the experimental group were detected in transitions from visualization level to analysis level and from analysis level to abstraction level. This result shows that there is strong harmony between the concrete-representational-abstract transition process of the CRA approach and Van Hiele's hierarchical thinking levels.

Various level recommendations for implementation can be presented in line with research results. In the instructional environment, it is recommended that mathematics teachers use the 5E Instructional Model integrated with the CRA approach in teaching geometry topics. Particularly, active use of concrete materials such as straws, geoboards, and paper folding and systematic application of concrete-representational-abstract stages of the CRA approach can be ensured. Regular inclusion of activities for discovering relationships among geometric concepts in the classroom environment is important. At the institutional level, school administrators can organize in-service training programs for mathematics teachers on the integration of the 5E Instructional Model and CRA approach, provide support for supplying concrete materials to be used in geometry instruction, and arrange classrooms suitable for group work and activity-based learning. At the system level, education policymakers can ensure that clear guidelines and examples for using integrated approaches are included in mathematics curricula, include these topics in teacher education programs, and develop textbooks and teacher guides that support the approach.

These implementation recommendations should be considered within the context and limitations of this study. The findings are based on a specific educational setting (10th-grade students in a Turkish public high school), particular geometric concepts (parallelograms and rhombuses), and a relatively short implementation period (4 weeks). Generalization of these results and recommendations to different grade levels, mathematical topics, educational systems, or cultural contexts should be approached with caution. Educators and policymakers are encouraged to adapt these recommendations to their local contexts, considering factors such as student characteristics, available resources, curriculum requirements, and institutional constraints before implementation.

Various recommendations can be developed for future research in light of the research findings. In terms of scope expansion, repetition of this research in different geometry topics (triangles, circles, solids) and different grade levels, conducting follow-up studies to determine long-term effects of implementation, and adding dimensions beyond recognizing concepts and knowing their properties for determining concept images, and beyond transformation and establishing inter-conceptual connections for connection skills can be recommended. In terms of methodological diversity, conducting mixed-method research where qualitative data are collected along with quantitative findings, conducting action research aimed at improving teachers' classroom practices, and conducting research examining teachers' views and experiences toward this approach can be beneficial. Within the scope of theoretical expansion, investigating 5E and CRA integration in combination with different learning theories, implementing this approach in technology-supported instructional environments, and examining its effects on students with special needs are recommended.

Conflicts of Interest

The author declares no conflicts of interest.

Declaration of Generative AI Use

No generative artificial intelligence tools (ChatGPT, Bard, Claude, Copilot, etc.) were employed in the preparation of this study. All text, analyses, and content were produced by the author through human contribution.

Ethical Statement

Ethical review and approval for this study were obtained from the Non-Interventional Clinical Research Ethics Committee of Burdur Mehmet Akif Ersoy University (decision no: GO 2025/1335). The study was conducted in accordance with the Declaration of Helsinki.

Author Contributions

The author conceptualized the study, designed the methodology, collected and analyzed the data, and wrote the manuscript. The author read and approved the final manuscript.

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