

An approach to neutrosophic ideals

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Abstract

In this paper, we aim to introduce an approach to single-valued neutrosophic ideals over a given classical ring and over a given neutrosophic subring, respectively, as a continuation of our researches on algebraic structures over single-valued neutrosophic sets. We first propose the two types of neutrosophic ideals and then present their elementary properties.

1. Introduction

In many practical situations and in many complex systems like biological, behavioral and chemical etc., different types of uncertainties are encountered. Since the classical set is invalid to handle the described uncertainties, Zadeh [17] first gave the definition of a fuzzy set. According to this definition, a fuzzy set is a function described by a membership value takes degrees in the real unit interval. But, later it has been seen that this definition is inadequate by consideration not only the degree of membership but also the degree of nonmembership. So, Atanassov [2] described a set which is called an intuitionistic fuzzy set to handle mentioned ambiguity. Since this set have some problems in applications, Smarandache [15] introduced neutrosophy to deal with the problems involves indeterminate and inconsistent information. "It is a branch of philosophy which studies the origin, nature and scope of neutralities, as well as their interactions with different ideational spectra"[15]. Neutrosophic set is a generalization of the fuzzy set and intuitionistic fuzzy set, where the truth-membership, indeterminacy-membership, and falsity-membership are represented independently. Wang et al.[16] specified the definition of a neutrosophic set, named as a single valued neutrosophic set to make more applicable the theory to real life problems. The single valued neutrosophic set is a generalization of a classical set, fuzzy set, intuitionistic fuzzy set and paraconsistent set etc. Vasantha Kandasamy and Florentin Smarandache [9] studied the concept of neutrosophic algebraic structures.

In addition, single valued neutrosophic set is applied to algebraic and topological directions (see [1, 3, 4, 11, 13, 14]). Liu [10] defined the concept of a fuzzy ring and fuzzy ideal. Later, Martinez [12] and Dixit et al.[6] studied on fuzzy ring and obtain certain ring theoretical analogous. Hur et al.[7] proposed the notion of an intuitionistic fuzzy subring. Vasantha Kandasamy and Florentin Smarandache [8] studied the neutrosophic rings. In this work, in a different direction from [8], we give an approach to a single valued neutrosophic ideal of a classical ring as a continuation of neutrosophic algebraic structures discussed in [4, 5]. We define neutrosophic ideal and study some properties of this structure. Moreover, we examine homomorphic image and preimage of a neutrosophic ideal. By this way, we obtain the generalized form of the fuzzy ideal and intuitionistic fuzzy ideal of a classical ring.

2. Preliminaries

In this chapter, we recall the concepts of a neutrosophic set and a single valued neutrosophic set. Throughout this section, X denotes the universal set which is nonempty.

Definition 2.1. [15] A neutrosophic set N on X is defined by : $N = \{ \langle x, t_N(x), i_N(x), f_N(x) \rangle, x \in X \}$ where $t_N, i_N, f_N : X \rightarrow]^{-0}, 1^{+}[$ are functions satisfy the inequality $^{-0} \leq t_N(x) + i_N(x) + f_N(x) \leq 3^{+}$.

From philosophical point of view, the neutrosophic set takes the value from real standard or non standard subsets of $]^{-0}, 1^{+}[$. But it is hard to consider the degree which belongs to a real standard or a non-standard subset of $]^{-0}, 1^{+}[$, in real world applications, especially in medical,

engineering and statistical problems etc. Hence throughout this work, we deal with the following specified definition of a neutrosophic set which is called a single valued neutrosophic set.

Definition 2.2. [16] A single valued neutrosophic set (SVNS) N on X is characterized by the truth-membership function t_N , the indeterminacy-membership function i_N and the falsity-membership function f_N . For each point x in X , the values $t_N(x), i_N(x), f_N(x)$ take place in the real unit interval $[0, 1]$.

A neutrosophic set N can be written as

$$N = \sum_{i=1}^n \langle t_N(x_i), i_N(x_i), f_N(x_i) \rangle / x_i, x_i \in X.$$

Since the membership functions t_N, i_N, f_N are defined from the universal set X into the unit interval $[0, 1]$ as $t_N, i_N, f_N : X \rightarrow [0, 1]$, a (single valued) neutrosophic set N will be denoted by a mapping described by $N : X \rightarrow [0, 1] \times [0, 1] \times [0, 1]$ and where, $N(x) = (t_N(x), i_N(x), f_N(x))$, for simplicity. The family of all single-valued neutrosophic sets on X is denoted by $SNS(X)$.

Definition 2.3. [13, 16] Let $N, M \in SNS(X)$. Then

(1) N is contained in M , denoted as $N \subseteq M$, if and only if $N(x) \leq M(x)$. This means that $t_N(x) \leq t_M(x), i_N(x) \leq i_M(x)$ and $f_N(x) \geq f_M(x)$. Two sets N, M are called equal, i.e., $N = M$ iff $N \subseteq M$ and $M \subseteq N$.

(2) the union $K = N \cup M$ is defined as $K(x) = N(x) \vee M(x)$ where $N(x) \vee M(x) = (t_N(x) \vee t_M(x), i_N(x) \vee i_M(x), f_N(x) \wedge f_M(x))$, for each $x \in X$. This means that $t_K(x) = \max\{t_N(x), t_M(x)\}, i_K(x) = \max\{i_N(x), i_M(x)\}$ and $f_K(x) = \min\{f_N(x), f_M(x)\}$.

(3) the intersection $K = N \cap M$ is defined as $K(x) = N(x) \wedge M(x)$ where $N(x) \wedge M(x) = (t_N(x) \wedge t_M(x), i_N(x) \wedge i_M(x), f_N(x) \vee f_M(x))$, for each $x \in X$. This means that $t_K(x) = \min\{t_N(x), t_M(x)\}, i_K(x) = \min\{i_N(x), i_M(x)\}$ and $f_K(x) = \max\{f_N(x), f_M(x)\}$.

(4) the complement of N is denoted by N^c and it is defined as $N^c(x) = (f_N(x), 1 - i_N(x), t_N(x))$, for each $x \in X$. Here $(N^c)^c = N$.

The details of the set theoretical operations can be found in [13, 16].

Definition 2.4. Let $g : X_1 \rightarrow X_2$ be a function and N, M be the neutrosophic sets of X_1 and X_2 , respectively. Then the image of N is a neutrosophic set of X_2 and it is defined as follows:

$$g(N)(y) = (t_{g(N)}(y), i_{g(N)}(y), f_{g(N)}(y)) = (g(t_N)(y), g(i_N)(y), g(f_N)(y)), \forall y \in X_2 \text{ where}$$

$$g(t_N)(y) = \begin{cases} \bigvee t_N(x), & \text{if } x \in g^{-1}(y); \\ 0, & \text{otherwise} \end{cases}, \quad g(i_N)(y) = \begin{cases} \bigvee i_N(x), & \text{if } x \in g^{-1}(y); \\ 0, & \text{otherwise} \end{cases},$$

$$g(f_N)(y) = \begin{cases} \bigwedge f_N(x), & \text{if } x \in g^{-1}(y); \\ 1, & \text{otherwise.} \end{cases}$$

And the preimage of M is a neutrosophic set of X_1 and it is defined as follows:

$$g^{-1}(M)(x) = (t_{g^{-1}(M)}(x), i_{g^{-1}(M)}(x), f_{g^{-1}(M)}(x)) = (t_M(g(x)), i_M(g(x)), f_M(g(x))) = M(g(x)), \forall x \in X_1.$$

Definition 2.5. [4] Let $N \in SNS(X)$ and $\beta \in [0, 1]$. Define the β -level sets of N as follows:

$$(t_N)_\beta = \{x \in X \mid t_N(x) \geq \beta\}, (i_N)_\beta = \{x \in X \mid i_N(x) \geq \beta\}, \text{ and } (f_N)^\beta = \{x \in X \mid f_N(x) \leq \beta\}.$$

Following properties are easily proved by using the definitions.

(1) If $N \subseteq M$ and $\beta \in [0, 1]$, then $(t_N)_\beta \subseteq (t_M)_\beta, (i_N)_\beta \subseteq (i_M)_\beta$, and $(f_N)^\beta \supseteq (f_M)^\beta$.

(2) $\beta \leq \gamma$ implies $(t_N)_\beta \supseteq (t_N)_\gamma, (i_N)_\beta \supseteq (i_N)_\gamma$, and $(f_N)^\beta \subseteq (f_N)^\gamma$.

Definition 2.6. [5] Let $R = (R, +, \cdot)$ be a classical ring and N be a neutrosophic set on R . Then N is called a neutrosophic subring of R if the following properties are satisfied: for each $r, s \in R$,

(R1) $N(r + s) \geq N(r) \wedge N(s)$.

(R2) $N(-r) \geq N(r)$.

(R3) $N(r \cdot s) \geq N(r) \wedge N(s)$.

From now on, R denotes a classical ring, unless otherwise specified.

Example 2.7. [5] Let us take into consideration the classical ring $R = \mathbb{Z}_4 = \{\bar{0}, \bar{1}, \bar{2}, \bar{3}\}$ with the operations \oplus and \odot defined as $\bar{x} \oplus \bar{y} = \overline{x + y}$ and $\bar{x} \odot \bar{y} = \overline{x \cdot y}$ for all $\bar{x}, \bar{y} \in \mathbb{Z}_4$, respectively. Define the neutrosophic set N on R as follows:

$$N = \{ \langle 0.8, 0.4, 0.1 \rangle / \bar{0} + \langle 0.5, 0.3, 0.5 \rangle / \bar{1} + \langle 0.7, 0.4, 0.3 \rangle / \bar{2} + \langle 0.5, 0.3, 0.5 \rangle / \bar{3} \}.$$

It is clear that the neutrosophic set N is a neutrosophic subring of R .

Theorem 2.8. [5] Let R be a classical ring and $N \in SNS(R)$. Then $N \in NSR(R)$ if and only if the following properties are satisfied for all $r, s \in R$;

(1) $N(r - s) \geq N(r) \wedge N(s)$.

(2) $N(r \cdot s) \geq N(r) \wedge N(s)$.

3. Neutrosophic ideals

In this section, we propose two definitions as neutrosophic ideal of a neutrosophic subring and a neutrosophic ideal of a classical ring. We investigate some properties and characterizations of a neutrosophic ideal of a given classical ring.

Definition 3.1. Let R be a classical ring and I be a neutrosophic set on R . Then I is called a neutrosophic left ideal over R if the followings are satisfied for each $r, s \in R$,

(LI1) $I(r - s) \geq I(r) \wedge I(s)$.

(LI2) $I(r \cdot s) \geq I(s)$.

Definition 3.2. Let R be a classical ring and I be a neutrosophic set on R . Then I is called a neutrosophic right ideal over R if the followings are satisfied for each $r, s \in R$,

$$(RI1) I(r-s) \geq I(r) \wedge I(s).$$

$$(RI2) I(r \cdot s) \geq I(r).$$

Definition 3.3. Let R be a classical ring and I be a neutrosophic set on R . Then I is called a neutrosophic ideal over R if the followings are satisfied for each $r, s \in R$,

$$(II) I(r-s) \geq I(r) \wedge I(s).$$

$$(I2) I(r \cdot s) \geq \max\{I(r), I(s)\}.$$

Remark 3.4. Each neutrosophic ideal over a classical ring R is a neutrosophic subring of R , but the converse is not true in general. For instance, let R be a ring and let $C = \{c \in R \mid cr = rc \text{ for all } r \in R\}$ denote the center of R . Define a neutrosophic set N on R as follows:

$$N(s) = \begin{cases} (1, 1, 0), & \text{if } s \in C \\ (0, 0, 1), & \text{otherwise} \end{cases}$$

It is clear that N is a neutrosophic subring of R , but may not be an ideal.

Theorem 3.5. Let I and J be two neutrosophic left (respectively, right) ideals of a classical ring R . Then the intersection $I \cap J$ is a neutrosophic left (respectively, right) ideal of R .

Proof. Let $r, s \in R$ be arbitrary and I, J be the left ideals of R . Let us show that

$(I \cap J)(r-s) \geq (I \cap J)(r) \wedge (I \cap J)(s)$, and $(I \cap J)(r \cdot s) \geq (I \cap J)(s)$. First consider the truth-membership degree of the intersection for the first condition,

$$\begin{aligned} t_{I \cap J}(r-s) &= t_I(r-s) \wedge t_J(r-s) \\ &\geq (t_I(r) \wedge t_I(s)) \wedge (t_J(r) \wedge t_J(s)) \\ &= (t_I(r) \wedge t_J(r)) \wedge (t_I(s) \wedge t_J(s)) = t_{I \cap J}(r) \wedge t_{I \cap J}(s). \end{aligned}$$

The other inequalities $i_{I \cap J}(r-s) \geq i_{I \cap J}(r) \wedge i_{I \cap J}(s)$ and $f_{I \cap J}(r-s) \leq f_{I \cap J}(r) \vee f_{I \cap J}(s)$ are similarly proved for each $r, s \in R$. For the second condition, let us consider the falsity degree of the intersection,

$$f_{I \cap J}(r \cdot s) = f_I(r \cdot s) \vee f_J(r \cdot s) \leq f_I(s) \vee f_J(s) = f_{I \cap J}(s).$$

The other inequalities $t_{I \cap J}(r \cdot s) \geq t_{I \cap J}(s)$ and $i_{I \cap J}(r \cdot s) \geq i_{I \cap J}(s)$ are similarly proved for each $r, s \in R$.

Consequently, $I \cap J$ is a neutrosophic ideal of R , as desired. \square

Theorem 3.6. Let R be a classical ring and I be a neutrosophic set on R . Then I is a neutrosophic (respectively, left, right) ideal over R if and only if for arbitrary $\beta \in [0, 1]$, if β -level sets of I are nonempty, then $(t_I)_\beta, (i_I)_\beta$ and $(f_I)^\beta$ are all classical (respectively, left, right) ideals of R .

Proof. Let I be a neutrosophic left ideal of R , $\beta \in [0, 1]$ and $r, s \in (t_I)_\beta$ (similarly $r, s \in (i_I)_\beta, (f_I)^\beta$). By the assumption,

$t_I(r-s) \geq t_I(r) \wedge t_I(s) \geq \beta \wedge \beta = \beta$ (and similarly, $i_I(r-s) \geq \beta$ and $f_I(r-s) \leq \beta$). Hence $r-s \in (t_I)_\beta$, (and similarly $r-s \in (i_I)_\beta, (f_I)^\beta$) for each $\beta \in [0, 1]$. In a similar way, we obtain $r \cdot s \in (t_I)_\beta$ (respectively, $r \cdot s \in (i_I)_\beta$ and $r \cdot s \in (f_I)^\beta$), for each $r \in R$ and $s \in (t_I)_\beta$ (respectively, $s \in (i_I)_\beta$ and $s \in (f_I)^\beta$). These mean that $(t_I)_\beta$ (and similarly $(i_I)_\beta, (f_I)^\beta$) is a classical ideal of R for each $\beta \in [0, 1]$.

Conversely, suppose $(t_I)_\beta, (i_I)_\beta$ and $(f_I)^\beta$ are classical ideals of R . Let $r, s \in R$ and $\beta = t_I(r) \wedge t_I(s)$, then $r, s \in (t_I)_\beta$. Since $(t_I)_\beta$ is a left ideal of R , then $r-s \in (t_I)_\beta$. This means that $t_I(r-s) \geq \beta = t_I(r) \wedge t_I(s)$.

Now let $r \in (t_I)_\beta$ and $s \in R$ such that $\beta = t_I(s)$. This shows that $t_I(r \cdot s) \geq \beta = t_I(s)$.

In similar computations, we obtain the desired inequalities as follows.

$$i_I(r-s) \geq i_I(r) \wedge i_I(s), i_I(r \cdot s) \geq i_I(s) \text{ and } f_I(r-s) \leq f_I(r) \vee f_I(s), f_I(r \cdot s) \leq f_I(s).$$

This completes the proof. \square

Theorem 3.7. Let I be a neutrosophic (left, right) ideal of R and $X_I = \{r \in R \mid I(r) = I(0)\}$, where 0 is the unit of the sum operation of R . Then the classical subset X_I of R is an (left, right) ideal of R .

Proof. Let I be a neutrosophic ideal of R and take $r, s \in X_I$. First we need to show that the set X_I is a subgroup of R under sum operation. By the assumption, $I(r) = I(0) = I(s)$ and by the condition (I1), the following inequality is true

$$I(r-s) \geq I(r) \wedge I(s) = I(0) \wedge I(0) = I(0).$$

Since, the inequality $I(0) \geq I(r-s)$ is always satisfied, we obtain that $I(r-s) = I(0)$. So, $r-s \in X_I$.

Now take $r \in X_I$ and $s \in R$. Second we need to show $r \cdot s \in X_I$, i.e., $I(r \cdot s) = I(0)$.

Since $I(r) = I(0)$ and by the condition (I2),

$$I(r \cdot s) \geq \max\{I(r), I(s)\} = \max\{I(0), I(s)\} = I(0).$$

Since always $I(0) \geq I(r \cdot s)$, then $I(r \cdot s) = I(0)$. Hence, $r \cdot s \in X_I$. Similarly, $s \cdot r \in X_I$.

In conclude, X_I is an ideal of R . \square

Let N and M be two neutrosophic sets on R , then $N \diamond M$ is a neutrosophic set on R and it is defined by

$$(N \diamond M)(z) = \left(\sup_{z=x \cdot y} \min\{t_N(x), t_M(y)\}, \sup_{z=x \cdot y} \min\{i_N(x), i_M(y)\}, \inf_{z=x \cdot y} \max\{f_N(x), f_M(y)\} \right),$$

otherwise, $(N \diamond M)(z) = (0, 0, 1)$, where $x, y, z \in R$.

Theorem 3.8. Let R be a ring and I be a neutrosophic left (right) ideal over R iff the followings are satisfied:

$$(1) I(r-s) \geq I(r) \wedge I(s), \text{ for each } r, s \in R.$$

$$(2) \chi_R \diamond I \leq I \text{ (respectively, } I \diamond \chi_R \leq I), \text{ where if } r \in R, \text{ then } \chi_R(r) = (1, 1, 0).$$

Proof. Suppose I is a neutrosophic left ideal over R and take $z \in R$, then

$$\begin{aligned} (\chi_R \diamond I)(z) &= (\sup_{z=r \cdot s} \min\{t_{\chi_R}(r), t_I(s)\}, \sup_{z=r \cdot s} \min\{i_{\chi_R}(r), i_I(s)\}, \inf_{z=r \cdot s} \max\{f_{\chi_R}(r), f_I(s)\}) \\ &= (\sup_{z=r \cdot s} t_I(s), \sup_{z=r \cdot s} i_I(s), \inf_{z=r \cdot s} f_I(s)) \\ &\leq I(r \cdot s) = I(z). \end{aligned}$$

Hence, $\chi_R \diamond I \leq I$.

Conversely, let I be a neutrosophic set on R which satisfies the corresponding two conditions.

$$(1) I(r \cdot s) \geq I(r) \wedge I(s) \quad (2) \chi_R \diamond I \leq I.$$

Take arbitrary $r, s \in R$, then

$$\begin{aligned} I(r \cdot s) &\geq (\chi_R \diamond I)(r \cdot s) \\ &= (\sup_{r \cdot s=p \cdot q} \min\{t_{\chi_R}(p), t_I(q)\}, \sup_{r \cdot s=p \cdot q} \min\{i_{\chi_R}(p), i_I(q)\}, \inf_{r \cdot s=p \cdot q} \max\{f_{\chi_R}(p), f_I(q)\}) \\ &\geq (\min\{t_{\chi_R}(r), t_I(s)\}, \min\{i_{\chi_R}(r), i_I(s)\}, \max\{f_{\chi_R}(r), f_I(s)\}) \\ &= (t_I(s), i_I(s), f_I(s)) = I(s). \end{aligned}$$

This implies the neutrosophic set I is a neutrosophic left ideal over R .

The other situations are proved similarly. □

Theorem 3.9. Let R_1, R_2 be the classical rings and $g : R_1 \rightarrow R_2$ be a homomorphism of rings. If J is a left (respectively, right) ideal of R_2 , then the preimage $g^{-1}(J)$ is a left (respectively, right) ideal of R_1 .

Proof. Suppose that J is a neutrosophic left ideal of R_2 and $r_1, r_2 \in R_1$. Since g is a homomorphism of rings, the following inequality is obtained.

$$\begin{aligned} g^{-1}(J)(r_1 - r_2) &= (t_J(g(r_1 - r_2)), i_J(g(r_1 - r_2)), f_J(g(r_1 - r_2))) \\ &= (t_J(g(r_1) - g(r_2)), i_J(g(r_1) - g(r_2)), f_J(g(r_1) - g(r_2))) \\ &\geq (t_J(g(r_1)) \wedge t_J(g(r_2)), i_J(g(r_1)) \wedge i_J(g(r_2)), f_J(g(r_1)) \vee f_J(g(r_2))) \\ &= (t_J(g(r_1)), i_J(g(r_1)), f_J(g(r_1))) \wedge (t_J(g(r_2)), i_J(g(r_2)), f_J(g(r_2))) \\ &= g^{-1}(J)(r_1) \wedge g^{-1}(J)(r_2). \end{aligned}$$

In similar computations, it is clear that $g^{-1}(J)(r \cdot s) \geq g^{-1}(J)(s)$, for each $r, s \in R$.

Therefore, $g^{-1}(J)$ is a neutrosophic left ideal of R_1 . □

Theorem 3.10. Let R_1, R_2 be the classical rings and $g : R_1 \rightarrow R_2$ be a homomorphism of rings. If I is a neutrosophic left (respectively, right) ideal of R_1 , then $g(I)$, the image of I , is a neutrosophic left (respectively, right) ideal of R_2 .

Proof. The proof is obtained by using the definitions of a left (respectively, right) ideal of a classical ring, and the image of a neutrosophic set. □

In the following, we introduce the neutrosophic ideal of a neutrosophic subring.

Definition 3.11. Let N be a neutrosophic subring of a classical ring R . A non-null neutrosophic set M is called a neutrosophic ideal of N , if the following conditions are valid for each $r, s \in R$,

- (1) $M(r - s) \geq M(r) \wedge M(s)$.
- (2) $M(r \cdot s) \geq M(r) \wedge M(s)$.
- (3) $M(r) \leq N(r)$.

Theorem 3.12. Let M_1 and M_2 be the neutrosophic ideals of the neutrosophic subrings of N_1 and N_2 , respectively. Then the intersection $M_1 \cap M_2$ is a neutrosophic ideal of $N_1 \cap N_2$.

Proof. Similar to the proof of Theorem 3.5. □

4. Conclusion

Just as normal subgroups played a crucial role in the theory of groups, so ideals play an analogous role in the study of rings. A single valued neutrosophic set is a kind of neutrosophic set which is suitable to use in real world applications. Therefore, the study of single valued neutrosophic sets and their properties have a considerable significance in the sense of applications as well as in understanding the fundamentals of uncertainty. So, we decided to propose the definitions of a neutrosophic ideals of a classical ring and of a neutrosophic subring, in the sense of [4, 5], and observe their fundamental properties. For further research one can handle cyclic (respectively, symmetric, abelian) neutrosophic group structure, and some of other algebraic structures.

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