Mathematics and Geometry: Two Key Concepts in Plato's Theory of Ideas

"Let no one who is ungeometrical enter"

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Abstract

Mathematical reasoning, mathematical objects and mathematical knowledge are important aspects of Greek philosopher Plato's philosophy. In his Theory of Ideas, mathematical reasoning provides the truth and the method for achieving the true knowledge of reality. In his theory, Plato makes a distinction between two worlds. The first is the world of Ideas, which refers to the timeless, abstract and unchanged world of Being, and the second is the world of appearances, which refers to the visible, sensible and changeable world of Becoming. According to this dualist theory, mathematical objects belong to the world of Being and mathematical reasoning is the only method that enables us to conceive the idea of good and the truths in this real world. Plato frequently presents and expresses the priority of mathematics in his dialogues and he refers to mathematical objects as the main subjects of philosophy. Accordingly, he puts mathematics as a propaedeutic to philosophy. In this respect, evaluating and analyzing the place of mathematics in Plato's view and showing the importance of mathematical reasoning in philosophy are the main purposes of this study. This evaluation is valuable for both understanding Plato's Theory of Ideas and showing the inseparable relationship between mathematics and philosophy.

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1. Introduction

"Let no one who is ungeometrical enter". The legend says that Plato imposes this inscription over the entrance to his Academy. By this way, the famous Greek philosopher expresses his admiration for mathematical thinking and mathematical ontology. Plato, who is the most influential figure in the history of Western philosophy, presents his Theory of Ideas that includes all major philosophical subjects and questions of human thought. It can be said that the starting point of his metaphysics is mathematical objects, which are timeless, intellectual, real and behind the world of visible or sensible and physical objects. Accordingly, Plato defines two worlds; world of Ideas and world of appearances. The two worlds represent two distinct realms and for him only the realm of Ideas can represent reality and produce true knowledge. The realm of ideas involves intellectual and abstract entities and it does not involve any sensible or physical/material things. On the other hand, the realm of visible worlds is a copy of the realm of Ideas or reality and the visible world involves only unreal sensible-material things. Based on this distinction, mathematical entities, as intellectual, abstract and timeless objects, belong to the world of Ideas. Accordingly, mathematical reasoning and mathematical knowledge represents the true knowledge of reality. For this reason, mathematics is a very significant element of Plato's philosophy.

In his dialogues, Plato frequently uses the mathematical elements and assumptions in Greek mathematics in the fourth century BCE [8] and he contributes branches of mathematics as; arithmetic and number, plane geometry, solid geometry, astronomy and harmonics [10]. The dialogues of Meno, The Rebuplic, Theaetetus and Timaeus can be seen as the most popular dialogues on mathematical and geometrical reasoning in his theory. In the seventh book of The Republic, we can find the reasons for why he attaches importance to mathematics in his philosophical view. For him, to study arithmetic and geometry provides to conceive the idea of good and to organize the mind in order to reflect and to achieve the truths. On the other hand, to study arithmetic and geometry is important for the human soul, because being capable of calculation and knowledge of abstract numbers enables us to pass from the world of Becoming to the world of Being. This transition represents his distinction of two worlds and the world of Being or world of Ideas refers to reality as well [10].

On the other hand, according to Plato, mathematical reality exists as a distinct realm and mathematicians discover this mathematical reality. According to this view, mathematical objects have an ontological status between two realms of his dualist theory. In this respect,

the importance of mathematics, both as a method of grasping truth and as a distinct realm, reflects his acceptance in both ontology and epistemology [13]. On the other hand, he argues that to study mathematics and especially calculation provides the tools for mind or soul in order to reach the idea of good and well-organized daily life.

Against this background, the importance and the place of mathematics in Plato's philosophy deserves to be investigated. Accordingly, the aim of this study is to show and evaluate the place of mathematics in Plato's philosophy by analyzing his main dialogues and to show the important relationship between mathematics and philosophy. For this purpose, firstly, the general arguments on mathematics in the Theory of Ideas will be discussed. Secondly, one of the popular dialogues on epistemology, Meno will be analyzed in terms of the place of geometrical knowledge. Finally, one of the main analogy, "Divided Line" will be analyzed in terms of the ontological status of mathematical objects. The analysis in this context is valuable in order to understand the relationship between mathematical and philosophical reasoning in Plato's theory.

2. The Importance of Mathematics in the Theory of Ideas

Mathematical knowledge and the independent existence of mathematical objects are the contentious subjects in Plato's philosophy. Although, there are many different interpretations on this philosophical subject, we understand the importance of mathematics in Plato's philosophy and its effects on the contemporary philosophical-mathematical perspectives. Bogdan Dembinski explains this as follow;

> In contemporary philosophy, it is widely accepted that according to Plato mathematics is the domain of ideal beings (ideas) that are eternal and unalterable and exist independently from the subject's beliefs and decisions. Two issues seem to be important here. The first issue concerns the question: was Plato really a proponent of present-day mathematical Platonism? The second one is of greater importance: how mathematics influences our understanding of the nature of the world on its many ontological levels? [5]

We encounter the important character of mathematics in Plato's view especially in three ways. The first is on the value of mathematical education, which provides the process of true reasoning and grasping the idea of good in human life. The second is on the explanation of knowledge as a recollection, which is presented in the dialogue of Meno, where it is argued that mathematical knowledge has a priori or innate character in itself. The third is on the ontological status of the mathematical objects, which is presented in The Republic, where it is argued that mathematical objects have a special realm between two worlds of Plato's theory. Accordingly, we can say that both ontologically and epistemologically, mathematics is a very powerful and functional element of his theory.

In the Theory of Ideas, the importance of mathematics is essentially presented as a training of the soul. Plato defines the educational or pedagogical role of the study of mathematics as "drawing the soul away from the world of becoming towards the world of what is" [10]. As mentioned before, Plato argues that "the world as a whole consisted of two realms: the physical world, which people could comprehend with their five senses, and the world of Ideas or Forms, which people could comprehend only with their intellect" [1]. The teaching or education on the world of Ideas is the center of his philosophy and mathematical reasoning provides the true method for the soul or mind in order to conceive the realm of Ideas. In addition to this, Plato explains geometrical reasoning in the same way and he says that geometry, as a knowledge, "draws the soul towards truth and produces philosophical thought by directing upwards what we now wrongly direct downwards" [10]. It can be said that, learning and doing mathematics brings true reasoning process, which is necessary for doing philosophy.

> It is an essential part of a soldier's education, for the deployment of troops, and of a philosopher's education, as he attempts to rise above becoming. He needs to make contact with being if he is ever to become capable of calculation or reasoning... We should persuade those in the city who are going to have a hand in the most important decisions to take up arithmetical reasoning and practise it - not as a hobby, but until they reach the contemplation of the nature of numbers by means of thought alone." [10]

Thus, the prior role of mathematics is being a method of reasoning and this brings its educational value for all individuals in society. Mathematical reasoning provides "mental discipline and the development of abstract thought... [and] it involves turning away from sense perception but because it is constructive reasoning pursued without reference to immediate instrumental usefulness" [2]. On the other hand, one of the important aims of mathematical education and training the soul is to achieve the knowledge of Good. Plato explains this aim as follow;

> But for that sort of purpose you need only a very small part of geometry and arithmetic. What we must ask ourselves is whether the main body of the subject, the part which goes beyond that, is going to contribute to helping us see the form or character of the good. And what does contribute, in our view, is anything which forces the soul to turn towards that place where lies the most blessed part of what is, which the soul must do everything it can to see. [10]

The relationship between mathematical reasoning and the idea of Good brings out the question of whether there is any ethical dimension or ethical aim that is involved in mathematical education. The distinction between two worlds can be interpreted as between "Good and sensible world" [7]. Some thinkers argue that there is an ethical dimension in this context and it is the reflection of Pythagorean effects on Plato's mathematics. The ethical dimension in this interpretation is based on drawing a parallel between the aim of purification by means of mathematics in Pythagorean philosophy and the aim of achieving the idea of Good by means of mathematics in Plato's philosophy. However, some other thinkers argue that this case is not an ethical but an epistemological dimension of mathematical reasoning. The idea of Good does not refer to an ethical ideal but refers to the highest degree of the knowledge of the realm of Being or the world of Ideas. "The Form of the Good exists, but enjoys a higher status of existence than those dependent objects that rely upon it for their mode of existence" [6]. Accordingly, it can be said that Plato aims to preserve the position of mathematical reasoning in the epistemological and ontological contexts in his philosophy.

The other significant discussion on Plato's mathematical perspective is on the question of whether Plato was a mathematician. One the one hand, some scholars argues that Plato follows the mathematical discoveries in his time and uses technical mathematics in his works, on the other hand some other interpretations argue that his focusing on mathematics is only a part of his pedagogical plan. According to this, mathematics has a place only for doing philosophy and for his ontological ideals.

Most modern scholars have come to the conclusion that Plato, while he exercised great influence as an enthusiastic advocate of mathematical study and an intelligent critic of mathematical methods and aims, did little or no original work in technical mathematics himself; but extreme positions have been taken up by others... Others have insisted, however, upon ascribing to Plato an important part in the mathematical discoveries and developments of the fourth century. [4]

To sum up, from the philosophical point of view, Plato's emphasis on mathematics shows us the place of mathematical argumentation, reasoning, method and mathematical objects that represent the realm of Being and refer to the knowledge of ontological acceptances in his theory. Accordingly, mathematics has an educational role for the development of the soul and achieving good and also has a metaphysical role as an example for the description of ideal entities in the world of Being.

## 3. The Dialogue of Meno: Theory of Recollection and Geometry

The dialogue of Meno is one of the essential works of Plato, which reflects his theory of knowledge as "the theory of recollection" and his main assumptions on innateness and immortality of the soul. The dialogue is an important work, because it shows the place of geometry and the nature of geometrical knowledge in Plato's philosophy. In this dialogue, Plato uses one of the problems of geometry as an example of innate knowledge. The main thesis of this work is based on the arguments that; the human soul has the knowledge of Ideas before birth; but this knowledge of the Ideas is lost or forgotten when the soul enters the world of appearances from the world of Ideas, and then it is recalled by the process of recollection. According to this, all true knowledge is knowledge of Ideas and we can only achieve this knowledge by means of recollection [9].

In this dialogue, Socrates meet a slave boy, who never gets education, and by asking him some main geometrical questions Socrates wants to show that geometrical knowledge is recollection. Through these questions, the slave boy can solve the geometry problem in a correct way. The questioning can be summarized as follow; "If the side of a square is two feet long, then its area is four feet. Doubling the area, we draw another square with an area of eight feet. How long is the side of the new square?" [11]. In Meno [9], Socrates shows the problem by drawing on a geometric figure and he presents the process of questioning on this geometric figure as follow;

Socrates draws a square ABCD. The "lines through the middle" are the lines joining the middle of these sides, which also go through the center of the square, namely EF and GH... the eight-foot square is double the four-foot square and half the sixteen-foot square—double the square based on a line two feet long, and half the square based on a four-foot side.
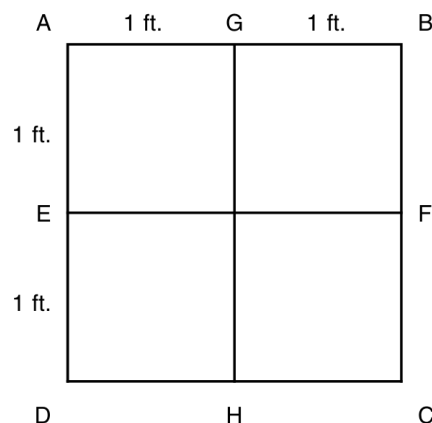
Socrates now builds up his sixteen-foot square by joining two four-foot squares, then a third... Filling "the space in the corner" will give another four-foot square, which completes the sixteen-foot square containing four four-foot squares.
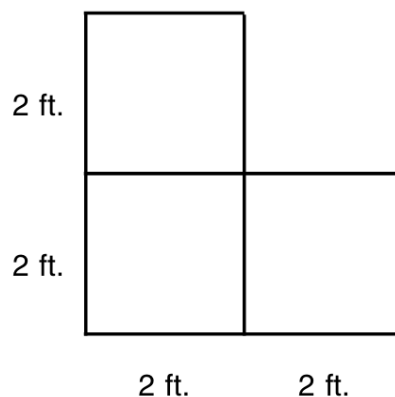
Socrates now draws the diagonals of the four inside squares, namely, FH, HE, EG, and GF, which together form the square GFHE. Socrates is asking how many of the triangles "cut off from inside" there are inside GFHE.
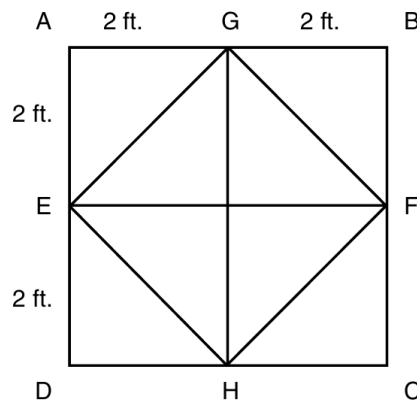
According to this example, Plato shows that if we ask true questions, the knowledge of ideas, such as geometrical principles, arises by means of recollection as it is in the slave boy cases. At this point, to solve a geometrical problem without any reference to the visible world represents the true intellectual knowledge of the world of Ideas. So, this geometrical reasoning process and geometrical ideas belong to the realm of Being in Plato's theory.

4. The Analogy of the Divided Line: The Ontological Status of Mathematical Objects

It is generally thought that the pedagogical plan of the seventh book of The Republic presents mathematics as a propaedeutic for metaphysics. However, some other interpretations argue that in contrast to this general opinion, the metaphysical assumptions provide new and important mathematical assumptions in his theory. Cherniss argues that

> Mathematics being more often the end of metaphysical reflection than its point of departure, and that he recommended mathematical study to his pupils not merely as a propaedeutic for dialectic but primarily because from his own experience he believed that subsequent philosophical meditation upon the fund of knowledge thus acquired would often lead to mathematical discoveries. [4]

According to this perspective, the other important argument of Plato in the sixth book of The Republic refers to the distinction of two worlds and the place of mathematics in both epistemological and ontological perspectives. The analogy of "the divided line" describes four kinds of ontological entities and four kinds of cognition of them or states of mind in the same line. "The divided line passage seems to compare the four states of the soul and their objects to the four segments of a line" [12]. In the famous mathematical passage, this analogy is presented as follow;

> Imagine taking a line which has been divided into two unequal sections, and dividing each section - the one representing the category of the seen and the one representing the category of the understood — again in the same proportion. The clearness or obscurity of the sections of the line, relative to one another, you will find to be as follows. In the category of the seen the first section is images, by which I mean in the first place shadows, and in the second place reflections in water, or any dense, smooth, shiny surface. [10].

The line includes two worlds and two subfields of them. "The visible world is divided into visible images [A-shadows or mirrors] and their originals [B-sensible-material things], and the intelligible world is divided into the objects dealt with in mathematics [C-mathematical forms and symbols] and the objects dealt with in dialectic [D-pure intelligible ideas] [12]. The objects of the visible world are easily understood because they refer to objects of our perception and we can grasp their difference in between sensible objects and their shadows. However, the objects of the conceived world or the categories of the understood are not grasped easily because of their abstract nature. Plato defines this difference as follows;

> You're aware, I imagine, that when people are doing things like geometry and arithmetic, there are some things they take for granted in their respective disciplines. Odd and even, figures and the three types of angle. That sort of thing. Taking these as known, they make them into assumptions. They see no need to justify them either to themselves or to anyone else. They regard them as plain to anyone. Starting from these, they then go through the rest of their argument, and finally reach, by agreed steps, that which they set out to investigate. [10]

In the category of the understood, Plato separates visible forms of mathematical objects for example the square that mathematicians draw or "mathematical imagery" [3] and abstract mathematical objects itself (Ideas), for example, the square that mathematicians think. Plato says that reasoning or reflection of mathematicians "has in view the square itself and the diagonal itself, not the diagonal they have drawn" [10]. According to this distinction, the square that mathematicians draw is an intellectual copy of the square in itself (the idea of square). According to this definition, the square that mathematicians draw is inferred from the square that mathematicians think. For this reason, the ideas of square is the object of pure rational intuition, but the second object is an inference from the ideas in itself. The basic aim in this analogy is to explain mathematical reasoning by using mathematical format and examples.

To sum up, mathematical objects are located at the imaginary side of the category of the understood. In this way, while the ontological status of mathematical objects are distinguished from visible object, at the same time the status of them are different from highest abstract objects of the world of Ideas. Accordingly, these two categories of the world of Ideas form the very specific distinction between pure conceivable Ideas and their inferential assumptions. This deliberate description of the ontological status of mathematical objects represents the privileged place of mathematics in Plato's Theory of Ideas.

5. Conclusion

Throughout the history of philosophy, from Thales to Descartes and from Leibniz to Analytic tradition in twentieth century, mathematics has always played a dominant role. Mathematical reasoning, as a starting point, stands in an indispensable place in philosophical reasoning. The two disciplines represent the abstract thought and the knowledge derived from this kind of abstract reasoning. Accordingly, these two intellectual productions of human thought refer to the exclusive ability of human beings. The great Greek philosopher Plato represents the first systematical philosophical theory, which involves all major questions of philosophical thought. In his Theory of Ideas, Plato pays attention to mathematics and focuses on mathematical reasoning as a method for philosophy. His theory reflects his investigation on timeless, intellectual, unchangeable, insensible and immaterial substance and mathematics is the first example of this category of existence.

From this point of view, the great thinker constitutes his philosophical system by means of mathematical reasoning, mathematical truth and the ontological status of mathematical objects. Therefore, mathematics is a basic structure of his system both epistemologically and ontologically. Plato refers to mathematical reasoning as one of the necessary elements of education and the best way in order to achieve true knowledge of reality. In addition, for Plato mathematical knowledge represents the a priori knowledge, which exists in our minds before birth and which refers to necessarily true and non-contradictory knowledge. The last and most important expression of mathematics in his theory is about the existence of mathematical objects. Mathematical objects directly refer to the conception of substance as Ideas in the world of Being. Therefore, mathematics represents realm of real existences in the world. All these reasons show that Plato focuses on the conception of mathematical philosophy and he conceptualizes mathematical objects as real members of the world of Ideas in the late period of his philosophical life.

Article Information

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