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Theory and Applications of the Triple Laplace Transform for Local Derivative with the Mittag Leffler Kernel

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ABSTRACT

This article aims to provide a practical and reliable approach to solving fractional M-derivative partial differential equations with nine parameters that involve the Mittag-Leffler function. Several theorems have been developed to describe and express the M-derivative triple Laplace transform. Furthermore, these defined concepts and theorems are demonstrated by applying them to fractional partial differential equations. This proposed transformation appears to efficiently enable finding solutions to partial differential equations with M-derivatives that match mathematical, engineering, and physical models.

1. Introduction

One of the most significant studies conducted by humanity to better comprehend nature is described in a letter that L'Hospital wrote to Leibniz in 1695 which establishes the terms derivative and integral, the building blocks of fractional calculations. Following this letter, it caught the interest of other scientists, leading to the substantial contributions of numerous mathematicians including Euler, Lagrange, Laplace, Lacroix, Fourier, Liouville, Riemann, Greer, Holmgren, Grünwald, Letnikov, Sonin, Laurent, Nekrassov, Krug, and Weyl (Kurt, 2018). Their contributions have helped physics, engineering, and other scientific fields advance (Spıegel, 1965). In recent years, there have been numerous advancements in the field of fractional calculus, as well as numerous definitions of fractional derivatives.

Riemann-Liouville, Caputo, Grüwald-Letnikov, Hadamard, Marchaud, Riesz, Wely, and Erdely-Kober are a few of the authors who defined fractional derivative (Kilbas, 2006). The Riemann-Liouville fractional nonlocal derivative had certain drawbacks, thus Caputo defined a new nonlocal fractional derivative in 1967 that was superior to many other fractional nonlocal derivatives. However, since the derivative of the resultant function, product, and quotient of two functions is not defined by these non-local fractional derivative definitions, Khalil and his colleagues defined the local conformable derivative (Khalil, et al., 2014) and the local conformable integral (Vanterler da Costa Sousa; & Capelas de Oliveira, 2017), which are close to the classical derivative. Additionally, Abdeljawad (2015) developed the chain rule, exponential functions, Gronwall's inequality, partial integration, Taylor series expansion, which are some important features of the harmonic derivative, and defined the Laplace transform in terms of the harmonic derivative (Abdeljawad, T., 2015). Another type of local derivative and integral was defined by Katugampola (2011). Atangana A. (2013) by the triple Laplace transform formulated, articulated, and demonstrated a number of theorems on this transformation. The Mboctara equations have been subjected to this novel alteration (Atangana ,2013). Thakur and Panda (2015) established and validated theorems pertaining to some characteristics of triple Laplace transformations. Jarad et al. (2017) investigated definitions and theorems in conformable fractional partial derivatives. The M −derivative, a brand-new local derivative using the Mittag-Leffler function, was published by Vanter da Costa Sousa and Capelas de Oliveira (2017). The characteristics of integer-order computations are satisfied by this newly found M-derivative. (Katugampola, 2011). In 2018, Ozkan and Kurt expressed and proved certain fundamental features of the conformable Laplace transform, and then they used these properties to get the solutions of conformable fractional integral and integro-differential equations. In a different work, Kurt and Ozkan (2018) defined the double conformable Laplace transform and discussed some of its characteristics. Using these characteristics, they were able to fully solve the conformable fractional-order heat and telegraph problem. Voltera Integro-Differential Equations were used to the triple Laplace transform by Mousa and Elzaki, Jarad and Abdeljawad (2020) examined the convolution theorem and Laplace transform in conformable derivatives. The spectrum solutions of differential equations with fractional M-derivatives under starting circumstances were studied by Bas et al (2020).

2. Description of Truncated M-Derivative and Some Fundamental Tools

2.1. Study site

Definition 2.1. For $0 < \beta \leq 1$, the fractional M −derivative is;

$$ {}_iD_M{}^{\beta\varphi} g(t) = \lim_{\epsilon\to 0} \frac{g\left(t_i E_\varphi(\varepsilon t^{-\beta})\right) - g(t)}{\varepsilon} \quad (1)$$

(Jarad, F., Uğurlu, E., Abdeljawad, T. and Baleanu, D., 2017).

Definition 2.2. An integrable function in the December $f, (a,t]$ $a \geq 0, t \geq a$ and $0 < \rho \leq 1$, the left-hand M −integral is defined as:

$$ {}_M\mathcal{J}_a^{\rho\gamma} f(t) = \int_a^t f(x) d_\rho(a,x)$$

$$= \Gamma(\gamma+1) \int_a^t f(x)(x-a)^{\rho-1} dx \quad (2)$$

for $d_\rho(x,a) = \Gamma(\gamma+1)(x-a)^{\rho-1} d_\rho x$ and assuming that the definition of the M-integral from the right at $a = 0$ is as follows

$$ {}^{\rho\gamma}_M\mathcal{J}_a f(t) = \int_a^t f(x) d_\rho(b,x)$$

$$= \Gamma(\gamma+1) \int_a^t f(x)(b-x)^{\rho-1} dx \quad (3)$$

(Bas et al., 2020).

Definition 2.3. Let f, g be differentiable functions with $0 < \alpha \leq 1$ and $f, g: [a,b] \to R$. Next, the M-derivative's partial integration from the left and right, respectively, is as follows:

$$\Gamma(\gamma+1) \int_a^b (t-a)^{\alpha-1} f(t) {}_iD_M{}^{\alpha\Upsilon} g(t) dt$$

$$= f(t)g(t) l_a^b - \Gamma(\gamma+1)$$

$$. \int_a^b (t-a)^{\alpha-1} \; x \, g(t) {}_iD_M{}^{\alpha\Upsilon} f(t) dt \quad (4)$$

and

$$\Gamma(\gamma+1) \int_a^b (t-a)^{\alpha-1} f(t) {}_iD_M{}^{\alpha\Upsilon} g(t) dt$$

$$= f(t)g(t) l_a^b + \Gamma(\gamma+1) \int_a^b (b-t)^{\alpha-1}$$

$. g(t)_i D_M{}^{\alpha\Upsilon} f(t)dt$ (5)

(Bas et al.,2020).

Definition 2.4. Consider a real-valued function $f:[a,\infty) \to R$ with $a \in R, \gamma > 0$, and $0 < \rho \le 1$. The Laplace transform of the function f's M-derivative in this instance is;

$$\mathcal{L}^{a}_{\rho,\gamma}\{f(t)\}(s) = F^{a}_{\rho,\gamma}(s)$$

$$= \Gamma(\rho+1)\int_b^\infty e^{-s\frac{\Gamma(\gamma+1)(t-a)^\rho}{\rho}} f(t)(t-a)^{\rho-1}dt \quad (6)$$

(Bas et al., 2020).

Theorem 2.1. Let f:$[a,\infty) \to \mathbb{R}$ be a defined function. Then $\mathcal{L}^{a}_{\rho,\gamma}\{f(t)\}(s) = F^{a}_{\rho,\gamma}(s)$, which is derived from the traditional Laplace transform as

$$\mathcal{L}^{a}_{\rho,\gamma}\{f(t)\}(s) = F^{a}_{\rho,\gamma}(s)$$

$$= \mathcal{L}\left\{f\left(a + \left(\frac{\rho t}{\Gamma(\gamma+1)}\right)^{\frac{1}{\rho}}\right)\right\} \quad (7)$$

(Bas et al., 2020).

Definition 2.4. Let the functions $f(t)$ and $g(t)$ have piecewise continuous and exponential order. Convolution integral of f and g functions over M-derivative

$$(f * g)(t) = \Gamma(\gamma+1)$$

$$.\int_a^t f(\tau)g(a + ((t-a)^\rho - (\tau-a)^\rho)^{\frac{1}{\rho}}(t-a)^{\rho-1}dt \quad (8)$$

(Bas, E., Acay, B., and T. Abdeljawad, 2020).

Theorem 2.2. The requirements $\mathcal{L}^{a}_{\rho,\gamma}[f(t)](s)=\mathcal{F}^{a}_{\rho,\gamma}(s)$, $\mathcal{L}^{a}_{\rho,\gamma}[g(t)](s)=\mathcal{G}^{a}_{\rho,\gamma}(s)$ are provided by $g:[a,\infty) \to \mathbb{R}$ $a \in \mathbb{R}$, $0<\rho \le 1$ $\Upsilon>0$, $s > 0$ and the functions $f(t)$ and $g(t)$.

$$\mathcal{L}^{a}_{\rho,\gamma}\{f * g\}(t)=\mathcal{F}^{a}_{\rho,\gamma}(s).\mathcal{G}^{a}_{\rho,\gamma}(s)$$

The convolution of the functions f and g is described by the equation $\{f * g\}$.

Laplace transformations of some functions can be expressed using the M −derivative, as shown in the table below (Bas, E., Acay, B., and T. Abdeljawad, 2020):

- $\mathcal{L}^{a}_{\rho,\gamma}\{1\}(s) = \frac{1}{s}, s > 0.$
- $\mathcal{L}^{a}_{\rho,\gamma}\left\{\left(\Gamma(\gamma+1)\frac{(t-a)^\rho}{\rho}\right)^\beta\right\}(s)$

  $= \frac{\Gamma(\beta+1)}{s^{1+\beta}}, Re(\beta) > 0, s > 0.$
- $\mathcal{L}^{a}_{\rho,\gamma}\{t\}(s) = \frac{\Gamma\left(\frac{1}{\rho}+1\right)\left(\frac{\rho}{\Gamma(\gamma+1)}\right)^{\frac{1}{\rho}}}{s^{\frac{\rho+1}{\rho}}}, s > 0.$
- $\mathcal{L}^{a}_{\rho,\gamma}\{t^k\}(s) = \frac{\Gamma\left(\frac{k}{\rho}+1\right)\left(\frac{\rho}{\Gamma(\gamma+1)}\right)^{\frac{k}{\rho}}}{s^{\frac{\rho+k}{\rho}}}$, $s > 0$ and k is any constant.
- $\mathcal{L}^{a}_{\rho,\gamma}\{e^{c\Gamma(\gamma+1)\frac{t^\rho}{\rho}}\}(s) = \frac{1}{s-c}$, $s > c$ and c is any constant.
- $\mathcal{L}^{a}_{\rho,\gamma}\left\{\Gamma(\gamma+1)\frac{t^\rho}{\rho}e^{c\Gamma(\gamma+1)\frac{t^\rho}{\rho}}\right\}(s)=\frac{1}{(s-c)^2}, c$ is any constant.
- $\mathcal{L}^{a}_{\rho,\gamma}\left\{\sin(b\,\Gamma(\gamma+1)\frac{t^\rho}{\rho})\right\}(s)=\frac{b}{b^2+s^2}, b$ is any constant.
- $\mathcal{L}^{a}_{\rho,\gamma}\left\{\cos(b\,\Gamma(\gamma+1)\frac{t^\rho}{\rho}\right\}(s)=\frac{s}{b^2+s^2}, b$ is any constant.
- $\mathcal{L}^{a}_{\rho,\gamma}\left\{e^{-c\Gamma(\gamma+1)\frac{t^\rho}{\rho}}\sin(b\Gamma(\gamma+1)\frac{t^\rho}{\rho}\right\}(s)$

$=\frac{b}{b^2+(s+c)^2}$, b and c are any constant.

- $\mathcal{L}^{a}_{\rho,\gamma}\left\{e^{-c\Gamma(\gamma+1)\frac{t^\rho}{\rho}}\cos(b\,\Gamma(\gamma+1)\frac{t^\rho}{\rho}\right\}(s)$

$=\frac{s+c}{b^2+(s+c)^2}$, b and $c$are any constant.

Utilizing the double Laplace and triple Laplace conformable derivatives as articulated in Kurt's (2018) thesis, a correlation with the M derivative was established through the incorporation of the Mittag-Leffler function. Consequently, the triple and double Laplace representations of the M derivative were derived. It is anticipated that the Mittag-Leffler function will yield superior and more accurate results concerning the M derivative, which delineates the distinction between the conformable derivative and the M-derivative.

3. Main Theorical Result and Applications

Definition 3.1. Let $v(x,y,z)$ be a continuous and exponential order function with a defined piecewise on it, and consider the function as$[0,\infty)\times[0,\infty)\times[0,\infty)$. If

$$sup_{x>0,y>0,z>0}\frac{|v(x,y,z)|}{e^{\Gamma(\gamma+1)\frac{t^{\rho}}{\rho}+\Gamma(\sigma+1)\frac{x^{\zeta}}{\zeta}+\Gamma(\mu+1)\frac{y^{\theta}}{\theta}}}<\infty$$

is satisfied for any $\mu,\gamma,\sigma>0$, then the function $V(p,r,s)$ is a triple M-derivative Laplace transform of the function $v(x,y,z)$. The function $V(p,r,s)$ is the function $v(x,y,z)$'s triple M-derivative Laplace transform.

$${}^{M}\mathcal{L}_{z}^{\rho}{}^{M}\mathcal{L}_{x}^{\zeta}{}^{M}\mathcal{L}_{y}^{\theta}\{v(x,y,z)\}=V(p,r,s)$$

$$=\int_0^\infty\int_0^\infty\int_0^\infty\Gamma(\sigma+1)\,e^{-p\Gamma(\sigma+1)\frac{x^{\zeta}}{\zeta}}$$

$$.\Gamma(\gamma+1)e^{-s\Gamma(\gamma+1)\frac{z^{\rho}}{\rho}}\,\Gamma(\mu+1)e^{-r\Gamma(\mu+1)\frac{y^{\theta}}{\theta}}$$

$$.(v(x,y,z))d_{\rho}zd_{\zeta}xd_{\theta}y \qquad (9)$$

İn the form. The integrals are for $p,s,r\in$ ₵,

$0<\rho,\zeta,\theta\leq 1$ and are fractional integrals in the sense of the M-integral depending on the variables z,y,x respectively.

Theorem 3.1. $u(x,y,z)$ and $h(x,y,z)$ are two functions that provide a triple M −Laplace transform, $a_1,a_2\in\mathbb{R}$ such that

$${}^{M}\mathcal{L}_{z}^{\rho}{}^{M}\mathcal{L}_{x}^{\zeta}{}^{M}\mathcal{L}_{y}^{\theta}\{a_1v(x,y,z)+a_2h(x,y,z)\}$$

$$=a_1{}^{M}\mathcal{L}_{z}^{\rho}{}^{M}\mathcal{L}_{x}^{\zeta}{}^{M}\mathcal{L}_{y}^{\theta}\{v(x,y,z)\}$$

$$+a_2{}^{M}\mathcal{L}_{z}^{\rho}{}^{M}\mathcal{L}_{x}^{\zeta}{}^{M}\mathcal{L}_{y}^{\theta}\{h(x,y,z)\} \qquad (10)$$

there is equality. So, the ${}^{M}\mathcal{L}_{z}^{\rho}{}^{M}\mathcal{L}_{x}^{\zeta}{}^{M}\mathcal{L}_{y}^{\theta}$operator is linear.

Proof.

If we consider the linearity property of the triple Laplace transform with the definition of triple M-Laplace, theorem 3.1. İs easily proved.

Theorem 3.2. The translation property of the${}^{M}\mathcal{L}_{z}^{\rho}{}^{M}\mathcal{L}_{x}^{\zeta}{}^{M}\mathcal{L}_{y}^{\theta}$transformation, which is $v(x,y,z)$ function that provide a triple M-Laplace transform, c_1,c_2ve$c_3\in\mathbb{R}$, is as follows.

Proof.

In the formulation of the triple M-derivative Laplace transform, the function $e^{-c_1\Gamma(\sigma+1)\frac{x^{\zeta}}{\zeta}-c_2\Gamma(\gamma+1)\frac{z^{\rho}}{\rho}-c_3\Gamma(\mu+1)\frac{y^{\theta}}{\theta}}v(x,y,z)$ is written if it is substituted for the function $v(x,y,z)$ represented in equation (9).

$$\int_0^\infty\int_0^\infty\int_0^\infty\Gamma(\sigma+1)\,e^{-p\Gamma(\sigma+1)\frac{x^{\zeta}}{\zeta}}\,\Gamma$$

$$.\Gamma(\gamma+1)e^{-s\Gamma(\gamma+1)\frac{z^{\rho}}{\rho}}(\mu+1)e^{-r\Gamma(\mu+1)\frac{y^{\theta}}{\theta}}$$

$$.(e^{-c_1\Gamma(\sigma+1)\frac{x^{\zeta}}{\zeta}-c_2\Gamma(\gamma+1)\frac{z^{\rho}}{\rho}-c_3\Gamma(\mu+1)\frac{y^{\theta}}{\theta}}v(x,y,z))d_{\rho}zd_{\zeta}xd_{\theta}y$$

$$=\int_0^\infty\Gamma(\gamma+1)e^{-s\Gamma(\gamma+1)\frac{z^{\rho}}{\rho}-c_2\Gamma(\gamma+1)\frac{z^{\rho}}{\rho}}$$

$$.\left[\int_0^\infty\Gamma(\sigma+1)e^{-p\Gamma(\sigma+1)\frac{x^{\zeta}}{\zeta}--c_1\Gamma(\sigma+1)\frac{x^{\zeta}}{\zeta}}\right.$$

$$.\left[\int_0^\infty\Gamma(\mu+1)e^{-r\Gamma(\mu+1)\frac{y^{\theta}}{\theta}-c_3\Gamma(\mu+1)\frac{y^{\theta}}{\theta}}\right.$$

$$.v(x,y,z)d_{\theta}y]\,d_{\zeta}x\big]d_{\rho}z$$

is acquired. Given that the M-Laplace transform has the translational property

$$=\int_0^\infty\Gamma(\gamma+1)e^{-s\Gamma(\gamma+1)\frac{z^{\rho}}{\rho}--c_2\Gamma(\gamma+1)\frac{z^{\rho}}{\rho}}$$

$$.[[V(p+c_1,r+c_3,z)]]d_{\rho}z$$

$$=V(p+c_1,r+c_3,s+c_2)$$

the proof is complete.

Theorem 3.3. Allow ${}^{M}\mathcal{L}_{z}^{\rho}{}^{M}\mathcal{L}_{x}^{\zeta}{}^{M}\mathcal{L}_{y}^{\theta}[v(x,y,z)]$

$=V(p,r,s)$, $v(x,y,z)$ to represent the function ζ in relation to x order, ρ in relation to z order, and θ in relation to y. The triple M-derivative Laplace transforms of fractional order partial derivatives are respectively

$$(i){}^{M}\mathcal{L}_{z}^{\rho}{}^{M}\mathcal{L}_{x}^{\zeta}{}^{M}\mathcal{L}_{y}^{\theta}\{{}_{M}D_{x}{}^{\zeta}v(x,y,z)\}$$

$$=p\,V(p,r,s)-V(0,r,s) \qquad (11)$$

$(ii)^{M}\mathcal{L}_z^{\rho}{}^{M}\mathcal{L}_x^{\zeta}{}^{M}\mathcal{L}_y^{\theta}\{{}_{M}D_y{}^{\theta}v(x,y,z)\}$

$$=rV(p,r,s)-V(p,0,s) \quad (12)$$

$(iii)^{M}\mathcal{L}_z^{\rho}{}^{M}\mathcal{L}_x^{\zeta}{}^{M}\mathcal{L}_y^{\theta}\{{}_{M}D_z{}^{\rho}v(x,y,z)\}$

$$=sV(p,r,s)-V(p,r,0) \quad (13)$$

$(iv)^{M}\mathcal{L}_z^{\rho}{}^{M}\mathcal{L}_x^{\zeta}{}^{M}\mathcal{L}_y^{\theta}\{{}_{M}D_z{}^{\rho}{}_{M}D_x{}^{\zeta}v(x,y,z)\}$

$$=psV(p,r,s)-pV(p,r,0)-s\,V(0,r,s)+V(0,r,0) \quad (14)$$

$(v)^{M}\mathcal{L}_z^{\rho}{}^{M}\mathcal{L}_x^{\zeta}{}^{M}\mathcal{L}_y^{\theta}\{{}_{M}D_y{}^{\theta}{}_{M}D_z{}^{\rho}v(x,y,z)\}$

$$=rsV(p,r,s)-sV(p,0,s)-rV(p,r,0)+V(p,0,0) \quad (15)$$

$(vi)^{M}\mathcal{L}_z^{\rho}{}^{M}\mathcal{L}_x^{\zeta}{}^{M}\mathcal{L}_y^{\theta}\left\{{}_{M}D_x{}^{\zeta}{}_{M}D_y{}^{\theta}v(x,y,z)\right\}$

$$=prV(p,r,s)-rV(0,r,s)-pV(p,0,s)+V(0,0,s) \quad (16)$$

$(vii)^{M}\mathcal{L}_z^{\rho}{}^{M}\mathcal{L}_x^{\zeta}{}^{M}\mathcal{L}_y^{\theta}\{{}_{M}D_x{}^{\zeta}{}_{M}D_y{}^{\theta}{}_{M}D_z{}^{\rho}v(x,y,z)\}$

$$=prs\,V(p,r,s)-pr\,V(p,r,0)-ps\,V(p,0,s)+p\,V(p,0,0)-rs\,V(0,r,s)+r\,V(0,r,0)+s\,V(0,0,s)-V(0,0,0) \quad (17)$$

it is expressed in the form.

Proof.

(i) (9) if the function $v(x,y,z)$ is substituted with ${}_{M}D_x{}^{\zeta}v(x,y,z)$, the triple M-Laplace transform provided by the definition

$${}^{M}\mathcal{L}_z^{\rho}{}^{M}\mathcal{L}_x^{\zeta}{}^{M}\mathcal{L}_y^{\theta}\{{}_{M}D_x{}^{\zeta}v(x,y,z)\}$$

$$=\int_0^\infty\int_0^\infty\int_0^\infty \Gamma(\sigma+1)\,e^{-p\Gamma(\sigma+1)\frac{x^\zeta}{\zeta}}$$

$$.\,\Gamma(\gamma+1)e^{-s\Gamma(\gamma+1)\frac{z^\rho}{\rho}}\,\Gamma(\mu+1)e^{-r\Gamma(\mu+1)\frac{y^\theta}{\theta}}$$

$$.\,({}_{M}D_x{}^{\zeta}v(x,y,z))d_\rho z d_\zeta x d_\theta y$$

$$=\int_0^\infty\int_0^\infty \Gamma(\mu+1)e^{-r\Gamma(\mu+1)\frac{y^\theta}{\theta}}\Gamma(\gamma+1)e^{-s\Gamma(\gamma+1)\frac{z^\rho}{\rho}}$$

$$.\left(v(x,y,z)e^{-p\Gamma(\sigma+1)\frac{x^\zeta}{\zeta}}\Big|_0^\infty+p\int_0^\infty\Gamma(\sigma+1)\right.$$

$$\left..\,e^{-p\Gamma(\sigma+1)\frac{x^\zeta}{\zeta}}(v(x,y,z))d_\zeta x\right)d_\rho z d_\theta y$$

$$=\int_0^\infty\int_0^\infty \Gamma(\mu+1)e^{-r\Gamma(\mu+1)\frac{y^\theta}{\theta}}\Gamma(\gamma+1)e^{-s\Gamma(\gamma+1)\frac{z^\rho}{\rho}}$$

$$.\,[pV(p,y,z)-V(0,y,z)]d_\rho z d_\theta y$$

$$=pV(p,r,s)-V(0,r,s)$$

is obtained.

(ii) The proof will be produced if the function $v(x,y,z)$ is written ${}_{M}D_y{}^{\theta}u(x,y,z)$instead of the function $v(x,y,z)$ in the triple M-Laplace transform given by the definition, and the operations of Theorem 3.3 (i) are repeated.

(iii) The proof will be produced if the function $v(x,y,z)$ is written ${}_{M}D_z{}^{\rho}v(x,y,z)$instead of the function $v(x,y,z)$ in the triple M-Laplace transform given by the definition, and the operations of Theorem 3.3 (i) are repeated.

(iv) (13) If the function $v(x,y,z)$ is substituted with ${}_{M}D_x{}^{\zeta}{}_{M}D_z{}^{\rho}v(x,y,z)$, the triple M-Laplace transform provided by the definition

$${}^{M}\mathcal{L}_z^{\rho}{}^{M}\mathcal{L}_x^{\zeta}{}^{M}\mathcal{L}_y^{\theta}\{{}_{M}D_x{}^{\zeta}{}_{M}D_z{}^{\rho}v(x,y,z)\}$$

$$=\int_0^\infty\int_0^\infty\int_0^\infty \Gamma(\sigma+1)\,e^{-p\Gamma(\sigma+1)\frac{x^\zeta}{\zeta}}\Gamma(\gamma+1)$$

$$.\,e^{-s\Gamma(\gamma+1)\frac{z^\rho}{\rho}}\Gamma(\mu+1)e^{-r\Gamma(\mu+1)\frac{y^\theta}{\theta}}$$

$$.\,({}_{M}D_x{}^{\zeta}{}_{M}D_z{}^{\rho}v(x,y,z))d_\rho z d_\zeta x d_\theta y$$

$$=s\int_0^\infty\int_0^\infty \Gamma(\mu+1)e^{-r\Gamma(\mu+1)\frac{y^\theta}{\theta}}$$

$$.\,\Gamma(\sigma+1)\,e^{-p\Gamma(\sigma+1)\frac{x^\zeta}{\zeta}}{}_{M}D_x{}^{\zeta}V(x,y,s)d_\zeta x d_\theta y$$

$$-\int_0^\infty\int_0^\infty \Gamma(\mu+1)e^{-r\Gamma(\mu+1)\frac{y^\theta}{\theta}}\Gamma(\sigma+1)\,e^{-p\Gamma(\sigma+1)\frac{x^\zeta}{\zeta}}$$

$$.\,{}_{M}D_x{}^{\zeta}V(x,y,0)d_\zeta x d_\theta y$$

be located. By performing procedures similar to the proof of parts (i) and (iii) of Theorem 3.3., it follows that

$$= s\int_0^{\infty} \Gamma(\mu+1)e^{-r\Gamma(\mu+1)\frac{y^\theta}{\theta}}$$

$$.\left[\int_0^{\infty} \Gamma(\sigma+1)\, e^{-p\Gamma(\sigma+1)\frac{x^\zeta}{\zeta}} {}_{M}D_x{}^{\zeta} V(x,y,s) d_\zeta x\right] d_\theta y$$

$$-\int_0^{\infty} \Gamma(\mu+1)e^{-r\Gamma(\mu+1)\frac{y^\theta}{\theta}}$$

$$.\left[\int_0^{\infty} \Gamma(\sigma+1)\, e^{-p\Gamma(\sigma+1)\frac{x^\zeta}{\zeta}} {}_{M}D_x{}^{\zeta} V(x,y,0) d_\zeta x\right] d_\theta y$$

$$= s\int_0^{\infty} \Gamma(\mu+1)e^{-r\Gamma(\mu+1)\frac{y^\theta}{\theta}}$$

$$.\,[pV(p,y,s) - V(0,y,s)]d_\theta y - \int_0^{\infty} \Gamma(\mu+1)e^{-r\Gamma(\mu+1)\frac{y^\theta}{\theta}}$$

$$.\,[pV(p,y,0) - V(0,y,0)]d_\theta y$$

$$= psV(p,r,s) - pV(p,r,0) - sV(0,r,s)$$

$$+ V(0,r,0)$$

Thus, the proof is completed.

(v) The proof will be produced if the function $v(x,y,z)$ is written ${}_{M}D_y{}^{\theta}{}_{M}D_z{}^{\rho} v(x,y,z)$ instead of the function $v(x,y,z)$ in the triple M-Laplace transform given by the definition, and the operations of Theorem 3.3 (iv) are repeated.

(vi) The proof will be produced if the function $v(x,y,z)$ is written ${}_{M}D_x{}^{\zeta}{}_{M}D_y{}^{\theta} v(x,y,z)$ instead of the function $v(x,y,z)$ in the triple M-Laplace transform given by the definition, and the operations of Theorem 3.3 (iv) are repeated.

(vii) (9) if the function $v(x,y,z)$ is substituted with ${}_{M}D_x{}^{\zeta}{}_{M}D_y{}^{\theta}{}_{M}D_z{}^{\rho} v(x,y,z)$, the triple M-Laplace transform provided by the definition

$${}^{M}\mathcal{L}_z^{\rho}\,{}^{M}\mathcal{L}_x^{\zeta}\,{}^{M}\mathcal{L}_y^{\theta}\{{}_{M}D_x{}^{\zeta}{}_{M}D_y{}^{\theta}{}_{M}D_z{}^{\rho} v(x,y,z)\}$$

$$=\int_0^{\infty}\int_0^{\infty}\int_0^{\infty} \Gamma(\sigma+1)\, e^{-p\Gamma(\sigma+1)\frac{x^\zeta}{\zeta}} \Gamma(\gamma+1)$$

$$.\, e^{-s\Gamma(\gamma+1)\frac{z^\rho}{\rho}} \Gamma(\mu+1) e^{-r\Gamma(\mu+1)\frac{y^\theta}{\theta}}$$

$$.\,({}_{M}D_x{}^{\zeta}{}_{M}D_y{}^{\theta}{}_{M}D_z{}^{\rho} v(x,y,z)) d_\rho z d_\zeta x d_\theta y$$

$$=\int_0^{\infty}\int_0^{\infty} \Gamma(\sigma+1)e^{-p\Gamma(\sigma+1)\frac{x^\zeta}{\zeta}} \Gamma(\mu+1)e^{-r\Gamma(\mu+1)\frac{y^\theta}{\theta}}$$

$$.\left(\int_0^{\infty} \Gamma(\gamma+1)e^{-s\Gamma(\gamma+1)\frac{z^\rho}{\rho}} {}_{M}D_x{}^{\zeta}{}_{M}D_y{}^{\theta}{}_{M}D_z{}^{\rho}\right.$$

$$\left. .\, v(x,y,z)\right) d_\rho z$$

$$=\int_0^{\infty}\int_0^{\infty} \Gamma(\sigma+1)e^{-p\Gamma(\sigma+1)\frac{x^\zeta}{\zeta}} \Gamma(\mu+1)e^{-r\Gamma(\mu+1)\frac{y^\theta}{\theta}}$$

$$.\,[s\,{}_{M}D_x{}^{\zeta}{}_{M}D_y{}^{\theta} v(x,y,s) - {}_{M}D_x{}^{\zeta}{}_{M}D_y{}^{\theta} v(x,y,0)] d_\zeta x d_\theta y$$

be located. By performing procedures similar to the proof of part (vi) of Theorem 3.3.,

$$s\int_0^{\infty}\int_0^{\infty} \Gamma(\sigma+1)e^{-p\Gamma(\sigma+1)\frac{x^\zeta}{\zeta}} \Gamma(\mu+1)e^{-r\Gamma(\mu+1)\frac{y^\theta}{\theta}}$$

$$.\left({}_{M}D_x{}^{\zeta}{}_{M}D_y{}^{\theta} v(x,y,s)\right) d_\zeta x d_\theta y$$

$$-\int_0^{\infty}\int_0^{\infty} \Gamma(\sigma+1)e^{-p\Gamma(\sigma+1)\frac{x^\zeta}{\zeta}}$$

$$.\,\Gamma(\mu+1)e^{-r\Gamma(\mu+1)\frac{y^\theta}{\theta}} \left({}_{M}D_x{}^{\zeta}{}_{M}D_y{}^{\theta} v(x,y,0)\right) d_\zeta x d_\theta y$$

$$= sr\left(pV(p,r,s) - V(0,r,s)\right) - s\left(pV(p,0,s) - \right.$$
$$\left. - V(0,0,s)\right) - r\left(pV(p,r,0) - V(0,r,0)\right)$$

$$+ (pV(p,0,0) - V(0,0,0))$$

Theorem 3.4. Let $v(x,y,z) \in \mathbb{C}^1(\mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}^+)$ and $\lfloor = max(f,l,u)\ with\ 0 < \rho,\zeta,\theta \leq 1$ and $f,l,o \in \mathbb{N}$. Also, let $v(x,y,z)$, ${}_{M}D_x{}^{(i)\zeta} v(x,y,z)$, ${}_{M}D_z{}^{(j)\rho} v(x,y,z)$, ${}_{M}D_y{}^{(k)\theta} v(x,y,z) i = 1,2,\ldots,f, j = 1,2,\ldots,l$, and $k = 1,2,\ldots,o$ functions that have triple M-Laplace transformations. In this case,

$$(i)\,{}^{M}\mathcal{L}_z^{\rho}\,{}^{M}\mathcal{L}_x^{\zeta}\,{}^{M}\mathcal{L}_y^{\theta}\{{}_{M}D_x{}^{(f)\zeta} v(x,y,z)\}$$

$$= p^f V(p,r,s) - p^{f-1} V(0,r,s)$$

$$-\sum_{i=1}^{f-1} p^{f-1-i}\, {}^{M}\mathcal{L}_z^{\rho}\,{}^{M}\mathcal{L}_y^{\theta}[{}_{M}D_x{}^{(i)\zeta} u(0,y,z)]$$

$$(ii)\,{}^{M}\mathcal{L}_z^{\rho}\,{}^{M}\mathcal{L}_x^{\zeta}\,{}^{M}\mathcal{L}_y^{\theta}\{{}_{M}D_z{}^{(l)\rho} u(x,y,z)\}$$

$=s^{l}U(p,r,s)-\ s^{l-1}U(p,r,0)$

$$-\sum_{j=1}^{l-1}s^{l-1-j}\,{}^{M}\mathcal{L}_{x}^{\beta}\,{}^{M}\mathcal{L}_{y}^{\theta}[{}_{M}D_{z}{}^{(j)\rho}v(x,y,0)]$$

$(iii)\ {}^{M}\mathcal{L}_{z}^{\rho}\ {}^{M}\mathcal{L}_{x}^{\zeta}\,{}^{M}\mathcal{L}_{y}^{\theta}\{{}_{M}D_{y}{}^{(\mathrm{o})\theta}u(x,y,z)\}$

$=r^{o}U(p,r,s)-\ r^{o-1}U(p,0,s)$

$$-\sum_{k=1}^{o-1}r^{o-1-k}\,{}^{M}\mathcal{L}_{z}^{\rho}\,{}^{M}\mathcal{L}_{x}^{\zeta}[{}_{M}D_{y}{}^{(\mathrm{k})\theta}u(x,0,z)]$$

$(iv)\,{}^{M}\mathcal{L}_{z}^{\rho}\,{}^{M}\mathcal{L}_{x}^{\zeta}\,{}^{M}\mathcal{L}_{y}^{\theta}\{{}_{M}D_{x}{}^{(\mathrm{f})\zeta}v(x,y,z)\,{}_{M}D_{z}{}^{(\mathrm{l})\rho}u(x,y,z)$

$\cdot\,{}_{M}D_{y}{}^{(\mathrm{o})\theta}v(x,y,z)\}$

$=p^{f}\big[r^{o}\big(s^{l}V(p,r,s)-\ s^{l-1}V(p,r,0)$

$$-\sum_{j=1}^{l-1}s^{l-1-j}\left[\,{}_{M}D_{z}{}^{(\mathrm{j})\rho}V(p,r,0)\right]\Bigg)$$

$-r^{o-1}\big(s^{l}V(p,0,s)-\ s^{l-1}V(p,0,0)$

$$-\sum_{j=1}^{l-1}s^{l-1-j}\left[{}_{M}D_{z}{}^{(\mathrm{j})\rho}V(p,0,0)\right]\Bigg)$$

$$-s^{l}\sum_{k=1}^{o-1}r^{o-1-k}\left[{}_{M}D_{y}{}^{(\mathrm{k})\theta}V(p,0,s)\right]$$

$$-s^{l-1}\sum_{k=1}^{o-1}r^{o-1-k}\left[{}_{M}D_{y}{}^{(\mathrm{k})\theta}V(p,0,0)\right]$$

$$-\sum_{j=1}^{l-1}\sum_{k=1}^{o-1}r^{o-1-k}\,s^{l-1-j}\left[{}_{M}D_{z}{}^{(\mathrm{j})\rho}\,{}_{M}D_{y}{}^{(\mathrm{k})\theta}V(p,0,0)\right]\Bigg]$$

$-p^{f-1}\big[r^{o}\big(s^{l}V(0,r,s)-\ s^{l-1}V(0,r,0)$

$-\sum_{j=1}^{l-1}s^{l-1-j}\left[{}_{M}D_{z}{}^{(\mathrm{j})\rho}V(0,r,0)\right]\big)$

$-r^{o-1}\big(s^{l}V(0,0,s)-\ s^{l-1}V(0,0,0)$

$$-\sum_{j=1}^{l-1}s^{l-1-j}\left[{}_{M}D_{z}{}^{(\mathrm{j})\rho}V(0,0,0)\right]\Bigg)$$

$$-s^{l}\sum_{k=1}^{o-1}r^{o-1-k}\left[{}_{M}D_{y}{}^{(\mathrm{k})\theta}V(p,0,s)\right]$$

$$-s^{l-1}\sum_{k=1}^{o-1}r^{o-1-k}\left[{}_{M}D_{y}{}^{(\mathrm{k})\theta}V(p,0,0)\right]$$

$$-\sum_{j=1}^{l-1}\sum_{k=1}^{o-1}r^{o-1-k}\,s^{l-1-j}\left[{}_{M}D_{z}{}^{(\mathrm{j})\rho}\,{}_{M}D_{y}{}^{(\mathrm{k})\theta}V(0,0,0)\right]\Bigg]$$

$$-r^{o}\left(s^{l}\sum_{i=1}^{f-1}p^{f-1-i}\left[{}_{M}D_{x}{}^{(\mathrm{i})\zeta}V(0,r,s)\right]\right.$$

$$-s^{l-1}\sum_{i=1}^{f-1}p^{f-1-i}\left[{}_{M}D_{x}{}^{(\mathrm{i})\zeta}V(0,r,0)\right]$$

$$\left.-\sum_{j=1}^{l-1}\sum_{i=1}^{f-1}p^{f-1-i}\,s^{l-1-j}\left[{}_{M}D_{z}{}^{(\mathrm{i})\rho}\,{}_{M}D_{x}{}^{(\mathrm{i})\zeta}V(0,r,0)\right]\right)$$

$$-r^{o-1}\left(s^{l}\sum_{i=1}^{f-1}p^{f-1-i}\left[{}_{M}D_{x}{}^{(\mathrm{i})\zeta}V(0,0,s)\right]\right.$$

$$-s^{l-1}\sum_{i=1}^{f-1}p^{f-1-i}\left[{}_{M}D_{x}{}^{(\mathrm{i})\zeta}V(0,0,0)\right]$$

$$\left.-\sum_{j=1}^{l-1}\sum_{i=1}^{f-1}p^{f-1-i}\,s^{l-1-j}\left[{}_{M}D_{z}{}^{(\mathrm{j})\rho}\,{}_{M}D_{x}{}^{(\mathrm{i})\zeta}V(0,0,0)\right]\right)$$

$$+s^{l}\sum_{i=1}^{f-1}\sum_{k=1}^{o-1}r^{o-1-k}\,p^{f-1-i}\,{}_{M}D_{y}{}^{(\mathrm{k})\theta}\,{}_{M}D_{x}{}^{(\mathrm{i})\zeta}V(0,0,s)$$

$$-s^{l-1}\sum_{i=1}^{f-1}\sum_{k=1}^{o-1}r^{o-1-k}\,p^{f-1-i}\,{}_{M}D_{y}{}^{(\mathrm{k})\theta}\,{}_{M}D_{x}{}^{(\mathrm{i})\zeta}V(0,0,s)$$

$$-\sum_{j=1}^{l-1}\sum_{i=1}^{f-1}\sum_{k=1}^{o-1}r^{o-1-i}\,p^{f-1-i}\ {}_{M}D_{z}{}^{(\mathrm{j})\rho}\,{}_{M}D_{y}{}^{(k)\theta}\,{}_{M}D_{x}{}^{(\mathrm{i})\zeta}$$

$$.V(0,0,0) \qquad (18)$$

It can be written.

Proof.

(i) The entire technique requires us to first demonstrate its efficacy for $f=1$. We have demonstrated its accuracy for $f=1$ in Theorem 3.3. (i) Now accept the accuracy for the $f-1$. We will show that it is true for f. The triple M-Laplace transform is defined and, in accordance with Theorem 3.3

$$^{M}\mathcal{L}_t^{\alpha}\,^{M}\mathcal{L}_x^{\beta}\{_{M}D_x{}^{(m)\beta}u(x,t)\}$$

$$= p\,^{M}\mathcal{L}_t^{\alpha}\,^{M}\mathcal{L}_x^{\beta}\{_{M}D_x{}^{(m-1)\beta}u(x,t)\}- \\ -\,^{M}\mathcal{L}_t^{\alpha}\{_{M}D_x{}^{(m-1)\beta}u(0,t)\}$$

can be written. Since equality is considered true for $f-1$,

$$^{M}\mathcal{L}_z^{\rho}\,^{M}\mathcal{L}_x^{\zeta}\,^{M}\mathcal{L}_y^{\theta}\{_{M}D_x{}^{(f)\zeta}v(x,y,z)\}$$

$$= p\Big(p^{f-1}U(p,r,s) - p^{f-2}U(0,r,s)$$

$$-\sum_{i=1}^{f-2} p^{f-2-i}\,^{M}\mathcal{L}_z^{\rho}\,^{M}\mathcal{L}_y^{\theta}[_{M}D_x{}^{(\mathrm{i})\zeta}u(0,y,z)]\Big)$$

$$-\,^{M}\mathcal{L}_z^{\rho}\,^{M}\mathcal{L}_y^{\theta}\{_{M}D_x{}^{(f-1)\zeta}u(0,y,z)\}$$

İf the final equality achieved undergoes the required processes, then

$$^{M}\mathcal{L}_z^{\rho}\,^{M}\mathcal{L}_x^{\zeta}\,^{M}\mathcal{L}_y^{\theta}\{_{M}D_x{}^{(f)\zeta}u(x,y,z)\}$$

$$= p^{f}U(p,r,s) - p^{f-1}U(0,r,s)$$

$$-\sum_{i=1}^{f-1} p^{m-1-i}\,^{M}\mathcal{L}_z^{\rho}\,^{M}\mathcal{L}_y^{\theta}\{_{M}D_x{}^{(\mathrm{i})\zeta}u(0,y,z)\}$$

İt is obtained.

(ii) The proof will be made in the same manner as Theorem 3.4 (i).

(iii) The proof will be made in the same manner as Theorem 3.4 (i).

(iv) We shall make use of equations (i), (ii), and (iii) to demonstrate this theorem. From equality, in (i)

$$^{M}\mathcal{L}_z^{\rho}\,^{M}\mathcal{L}_y^{\theta}\,^{M}\mathcal{L}_x^{\zeta}\{\,_{M}D_x{}^{(f)\zeta}v(x,y,z)\,_{M}D_z{}^{(\mathrm{l})\rho}u(x,y,z)$$

$$_{M}D_y{}^{(\mathrm{o})\theta}v(x,y,z)\}$$

$$= \,^{M}\mathcal{L}_z^{\rho}\,^{M}\mathcal{L}_y^{\theta}\{_{M}D_z{}^{(\mathrm{l})\rho}u(x,y,z)\,_{M}D_y{}^{(\mathrm{o})\theta}v(x,y,z)V(p,y,z)\}$$

$$.p^{f} - p^{f-1}\,^{M}\mathcal{L}_z^{\rho}\,^{M}\mathcal{L}_y^{\theta}$$

$$\{_{M}D_z{}^{(\mathrm{l})\rho}u(x,y,z)\,_{M}D_y{}^{(\mathrm{o})\theta}v(x,y,z)V(0,y,z)\}-$$

$$-\,^{M}\mathcal{L}_z^{\rho}\,^{M}\mathcal{L}_y^{\theta}\left\{\sum_{i=1}^{f-1}p^{f-1-i}\left[_{M}D_z{}^{(\mathrm{l})\rho}\,_{M}D_y{}^{(\mathrm{o})\theta}\,_{M}D_x{}^{(\mathrm{i})\zeta}V(0,y,z)\right]\right\}$$
(19)

It can be set up like this. (19) The expressions on the right side of the equation are according to the property of the triple M-Laplace transform

$$^{M}\mathcal{L}_z^{\rho}\,^{M}\mathcal{L}_y^{\theta}\{_{M}D_z{}^{(\mathrm{l})\rho}\,_{M}D_y{}^{(\mathrm{o})\theta}V(p,y,z)\}$$

$$= r^{o}\,^{M}\mathcal{L}_z^{\rho}\{_{M}D_z{}^{(\mathrm{l})\rho}V(p,r,z)\}$$

$$-\,r^{o-1}\,^{M}\mathcal{L}_z^{\rho}\{_{M}D_z{}^{(\mathrm{l})\rho}V(p,0,z)\}$$

$$-\,^{M}\mathcal{L}_z^{\rho}\{\textstyle\sum_{k=1}^{o-1}r^{o-1-i}\left[_{M}D_z{}^{(\mathrm{l})\rho}\,_{M}D_y{}^{(\mathrm{k})\theta}V(0,0,z)\right]\} \qquad (20)$$

$$^{M}\mathcal{L}_z^{\rho}\,^{M}\mathcal{L}_y^{\theta}\{_{M}D_z{}^{(\mathrm{l})\rho}\,_{M}D_y{}^{(\mathrm{o})\theta}V(0,y,z)\}$$

$$= r^{o}\,^{M}\mathcal{L}_z^{\rho}\{_{M}D_z{}^{(\mathrm{l})\rho}V(0,r,z)\}$$

$$-\,r^{o-1}\,^{M}\mathcal{L}_z^{\rho}\{_{M}D_z{}^{(\mathrm{l})\rho}V(0,0,z)\}$$

$$-\,^{M}\mathcal{L}_z^{\rho}\{\textstyle\sum_{k=1}^{o-1}r^{o-1-i}\left[_{M}D_z{}^{(\mathrm{l})\rho}\,_{M}D_y{}^{(\mathrm{k})\theta}V(0,0,z)\right]\} \qquad (21)$$

$$^{M}\mathcal{L}_z^{\rho}\,^{M}\mathcal{L}_y^{\theta}\left\{\textstyle\sum_{i=1}^{f-1}p^{f-1-i}\left[_{M}D_z{}^{(\mathrm{l})\rho}\,_{M}D_y{}^{(\mathrm{o})\theta}\,_{M}D_x{}^{(\mathrm{i})\zeta}V(0,y,z)\right]\right\}$$

$$= r^{o}\,^{M}\mathcal{L}_z^{\rho}\left\{\sum_{i=1}^{f-1}p^{f-1-i}\left[_{M}D_z{}^{(\mathrm{l})\rho}\,_{M}D_x{}^{(\mathrm{i})\zeta}V(0,r,z)\right]\right\}$$

$$-\,r^{o-1}\,^{M}\mathcal{L}_z^{\rho}\{\sum_{i=1}^{f-1}p^{f-1-i}\left[\,_{M}D_z{}^{(\mathrm{l})\rho}\,_{M}D_x{}^{(\mathrm{i})\zeta}V(0,0,z)\right]\}$$

$$-\,^{M}\mathcal{L}_z^{\rho}\{\sum_{i=1}^{f-1}\sum_{k=1}^{o-1}r^{o-1-k}\,p^{f-1-i}\,_{M}D_z{}^{(\mathrm{l})\rho}\,_{M}D_y{}^{(\mathrm{k})\theta}$$

$$\cdot\,_{M}D_x{}^{(\mathrm{i})\zeta}V(0,0,z)\} \qquad (22)$$

it is written. If the equation's right-hand expressions (20), (21) and (22) are expanded;

$$^{M}\mathcal{L}_z^{\rho}\{_{M}D_z{}^{(\mathrm{l})\rho}V(0,r,z)\}$$

$$= s^l V(0,r,s) - s^{l-1} V(0,r,0)$$

$$-\sum_{j=1}^{l-1} s^{l-1-j} \left[{}_M D_z{}^{(j)\rho} V(0,r,0)\right] \quad (23)$$

$${}^{M}\mathcal{L}_z^{\rho}\left\{{}_M D_z{}^{(l)\rho} V(0,0,z)\right\}$$

$$= s^l V(0,0,s) - s^{l-1} V(0,0,0)$$

$$-\sum_{j=1}^{l-1} s^{l-1-j} \left[{}_M D_z{}^{(j)\rho} V(0,0,0)\right] \quad (24)$$

$${}^{M}\mathcal{L}_z^{\rho}\left\{{}_M D_z{}^{(l)\rho} V(p,r,z)\right\}$$

$$= s^l V(p,r,s) - s^{l-1} V(p,r,0)$$

$$-\sum_{j=1}^{l-1} s^{l-1-j} \left[{}_M D_z{}^{(j)\rho} V(p,r,0)\right] \quad (25)$$

$${}^{M}\mathcal{L}_z^{\rho}\left\{{}_M D_z{}^{(l)\rho} V(p,0,z)\right\}$$

$$= s^l V(p,0,s) - s^{l-1} V(p,0,0)$$

$$-\sum_{j=1}^{l-1} s^{l-1-j} \left[{}_M D_z{}^{(j)\rho} V(p,0,0)\right] \quad (26)$$

$${}^{M}\mathcal{L}_z^{\rho}\left\{\sum_{k=1}^{o-1} r^{o-1-k} \left[{}_M D_z{}^{(l)\rho}\, {}_M D_y{}^{(k)\theta} V(p,0,z)\right]\right\}$$

$$= s^l \sum_{k=1}^{o-1} r^{o-1-k} \left[{}_M D_y{}^{(k)\theta} V(p,0,s)\right]$$

$$- s^{l-1} \sum_{k=1}^{o-1} r^{o-1-k} \left[{}_M D_y{}^{(k)\theta} V(p,0,0)\right]$$

$$-\sum_{j=1}^{l-1}\sum_{k=1}^{o-1} r^{o-1-k} s^{l-1-j} \left[{}_M D_z{}^{(j)\rho}\, {}_M D_y{}^{(k)\theta} V(p,0,0)\right]$$
(27)

$${}^{M}\mathcal{L}_z^{\rho}\left\{\sum_{k=1}^{o-1} r^{o-1-k} \left[{}_M D_z{}^{(l)\rho}\, {}_M D_y{}^{(k)\theta} V(0,0,z)\right]\right\}$$

$$= s^l \sum_{k=1}^{o-1} r^{o-1-k} \left[{}_M D_y{}^{(k)\theta} V(0,0,s)\right]$$

$$- s^{l-1} \sum_{k=1}^{o-1} r^{o-1-k} \left[{}_M D_y{}^{(k)\theta} V(0,0,0)\right]$$

$$-\sum_{j=1}^{l-1}\sum_{k=1}^{o-1} r^{o-1-k} s^{l-1-j} \left[{}_M D_z{}^{(j)\rho}\, {}_M D_y{}^{(k)\theta} V(0,0,0)\right]$$

(28)

$${}^{M}\mathcal{L}_z^{\rho}\left\{\sum_{i=1}^{f-1} p^{f-1-i} \left[{}_M D_z{}^{(l)\rho}\, {}_M D_x{}^{(i)\zeta} V(0,r,z)\right]\right\}$$

$$= s^l \sum_{i=1}^{f-1} p^{f-1-i} \left[{}_M D_x{}^{(i)\zeta} V(0,r,s)\right]$$

$$- s^{l-1} \sum_{i=1}^{f-1} p^{f-1-i} \left[{}_M D_x{}^{(i)\zeta} V(0,r,0)\right]$$

$$-\sum_{j=1}^{l-1}\sum_{i=1}^{f-1} p^{f-1-i} s^{l-1-j} \left[{}_M D_z{}^{(j)\rho}\, {}_M D_x{}^{(i)\zeta} V(0,r,0))\right]$$

(29)

$${}^{M}\mathcal{L}_z^{\rho}\left\{\sum_{i=1}^{f-1} p^{f-1-i} \left[{}_M D_z{}^{(l)\rho}\, {}_M D_x{}^{(i)\zeta} V(0,0,z)\right]\right\}$$

$$= s^l \sum_{i=1}^{f-1} p^{f-1-i} \left[{}_M D_x{}^{(i)\zeta} V(0,0,s)\right]$$

$$- s^{l-1} \sum_{i=1}^{f-1} p^{f-1-i} \left[{}_M D_x{}^{(i)\zeta} V(0,0,0)\right]$$

$$-\sum_{j=1}^{l-1}\sum_{i=1}^{f-1} p^{f-1-i} s^{l-1-j} \left[{}_M D_z{}^{(j)\rho}\, {}_M D_x{}^{(i)\zeta} V(0,0,0)\right] \quad (30)$$

$${}^{M}\mathcal{L}_z^{\rho}\left\{\sum_{i=1}^{f-1}\sum_{k=1}^{o-1} r^{o-1-k} p^{f-1-i}\, {}_M D_z{}^{(l)\rho}\, {}_M D_y{}^{(k)\theta}\right.$$

$$\left. \cdot {}_M D_x{}^{(i)\zeta} V(0,0,z)\right\}$$

$$= s^l \sum_{i=1}^{f-1}\sum_{k=1}^{o-1} r^{o-1-k} p^{f-1-i}\, {}_M D_y{}^{(k)\theta}\, {}_M D_x{}^{(i)\zeta} V(0,0,s)$$

$$- s^{l-1} \sum_{i=1}^{f-1}\sum_{k=1}^{o-1} r^{o-1-k} p^{f-1-i}\, {}_M D_y{}^{(k)\theta}\, {}_M D_x{}^{(i)\zeta} V(0,0,s)$$

$$-\sum_{j=1}^{l-1}\sum_{i=1}^{f-1}\sum_{k=1}^{o-1} r^{o-1-k} p^{f-1-i} s^{l-1-j}\, {}_M D_z{}^{(j)\rho}\, {}_M D_y{}^{(k)\theta}\, {}_M D_x{}^{(i)\zeta}$$

$$. V(0,0,0) \quad (31)$$

It is obtained. The (18) equality is produced if the appropriate procedures are carried out and these discovered equations are substituted for the (19) equality.

Theorem 3.5. Using the M-derivative, we can express the triple Laplace transformations of various functions as follow:

$i)\ {}^{M}\mathcal{L}_{z}^{\rho}\,{}^{M}\mathcal{L}_{x}^{\zeta}\,{}^{M}\mathcal{L}_{y}^{\theta}\{1\}=\frac{1}{prs}p>0,s>0,r>0$

Proof.

If $v(x,y,z)=1$ is taken instead of the function $v(x,y,z)$ in the equation (13) in the definition of the m-derived Laplace transform, the triple M-Laplace transform is;

$${}^{M}\mathcal{L}_{z}^{\rho}\,{}^{M}\mathcal{L}_{x}^{\zeta}\,{}^{M}\mathcal{L}_{y}^{\theta}\{1\}$$

$$=\int_0^\infty\int_0^\infty\int_0^\infty \Gamma(\sigma+1)\mathrm{e}^{-\mathrm{p}\Gamma(\sigma+1)\frac{x^\zeta}{\zeta}}\ \Gamma(\gamma+1)$$

$$.\,[\mathrm{e}^{-\mathrm{s}\Gamma(\gamma+1)\frac{z^\rho}{\rho}}\Gamma(\mu+1)\mathrm{e}^{-\mathrm{r}\Gamma(\mu+1)\frac{y^\theta}{\theta}}d_\rho z d_\zeta x d_\theta y]$$

$$=\frac{1}{s}\int_0^\infty \Gamma(\sigma+1)e^{-\mathrm{p}\Gamma(\sigma+1)\frac{x^\zeta}{\zeta}}$$

$$.\left[\int_0^\infty \Gamma(\mu+1)\mathrm{e}^{-\mathrm{r}\Gamma(\mu+1)\frac{y^\theta}{\theta}}d_\theta y\right]d_\zeta x$$

$$=\frac{1}{s}\int_0^\infty \Gamma(\sigma+1)e^{-\mathrm{p}\Gamma(\sigma+1)\frac{x^\zeta}{\zeta}}\left[\frac{-\mathrm{e}^{-\mathrm{r}\Gamma(\mu+1)\frac{y^\theta}{\theta}}}{r}\Bigg|_0^\infty\right]d_\zeta x$$

$$=\frac{1}{sr}\int_0^\infty \Gamma(\sigma+1)e^{-\mathrm{p}\Gamma(\sigma+1)\frac{x^\zeta}{\zeta}}d_\zeta x=\frac{1}{psr}$$

as obtained.

$ii)\ {}^{M}\mathcal{L}_{z}^{\rho}\,{}^{M}\mathcal{L}_{x}^{\zeta}\,{}^{M}\mathcal{L}_{y}^{\theta}\{x^m z^n y^k\}$

$$=\frac{1}{prs}\left(\frac{\rho}{s\Gamma(\gamma+1)}\right)^{\frac{n}{\rho}}$$

$$.\left(\frac{\zeta}{p\Gamma(\sigma+1)}\right)^{\frac{m}{\zeta}}\left(\frac{\theta}{r\Gamma(\mu+1)}\right)^{\frac{k}{\theta}}\Gamma\left(1+\frac{k}{\theta}\right)\Gamma\left(1+\frac{n}{\rho}\right)$$

$.\Gamma\left(1+\frac{m}{\zeta}\right)p>0,s>0r>0$ m, n and k are any constants.

Proof.

When the function $v(x,y,z)$ in equation (9) is substituted by $x^m z^n y^k$, the derived M-Laplace transform definition and the M-Laplace transformation formulas are considered:

$${}^{M}\mathcal{L}_{z}^{\rho}\,{}^{M}\mathcal{L}_{x}^{\zeta}\,{}^{M}\mathcal{L}_{y}^{\theta}\{x^m z^n y^k\}$$

$$=\int_0^\infty\int_0^\infty\int_0^\infty \Gamma(\sigma+1)\,\mathrm{e}^{-\mathrm{p}\Gamma(\sigma+1)\frac{x^\zeta}{\zeta}}\Gamma(\gamma+1)\mathrm{e}^{-\mathrm{s}\Gamma(\gamma+1)\frac{z^\rho}{\rho}}$$

$$.\,\Gamma(\mu+1)\mathrm{e}^{-\mathrm{r}\Gamma(\mu+1)\frac{y^\theta}{\theta}}\left(x^m z^n y^k\right)d_\rho z d_\zeta x d_\theta y$$

$$=\int_0^\infty \Gamma(\sigma+1)\,\mathrm{e}^{-\mathrm{p}\Gamma(\sigma+1)\frac{x^\zeta}{\zeta}}x^m$$

$$.\left[\int_0^\infty \Gamma(\gamma+1)\,\mathrm{e}^{-\mathrm{p}\Gamma(\sigma+1)\frac{x^\zeta}{\zeta}}\mathrm{e}^{-\mathrm{s}\Gamma(\gamma+1)\frac{z^\rho}{\rho}}z^n\right.$$

$$.\left(\int_0^\infty \Gamma(\mu+1)\mathrm{e}^{-\mathrm{r}\Gamma(\mu+1)\frac{y^\theta}{\theta}}y^k d_\theta y\right)d_\rho z)d_\zeta x$$

$$=\int_0^\infty \Gamma(\sigma+1)\,\mathrm{e}^{-\mathrm{p}\Gamma(\sigma+1)\frac{x^\zeta}{\zeta}}x^m$$

$$.\left(\int_0^\infty \Gamma(\gamma+1)\mathrm{e}^{-\mathrm{s}\Gamma(\gamma+1)\frac{z^\rho}{\rho}}z^n\left({}^{M}\mathcal{L}_{y}^{\theta}\{y^k\}\right)d_\rho z\right)d_\zeta x$$

$$=\frac{1}{p}\left(\frac{\theta}{r\Gamma(\mu+1)}\right)^{\frac{k}{\theta}}\Gamma\left(1+\frac{k}{\theta}\right)\int_0^\infty \Gamma(\sigma+1)\,\mathrm{e}^{-\mathrm{p}\Gamma(\sigma+1)\frac{x^\zeta}{\zeta}}$$

$$.\,x^m\left(\int_0^\infty \Gamma(\gamma+1)\mathrm{e}^{-\mathrm{s}\Gamma(\gamma+1)\frac{z^\rho}{\rho}}z^n\,d_\rho z\right)d_\zeta x$$

$$=\left(\frac{\theta}{r\Gamma(\mu+1)}\right)^{\frac{k}{\theta}}\Gamma\left(1+\frac{k}{\theta}\right)\int_0^\infty \Gamma(\sigma+1)\,\mathrm{e}^{-\mathrm{p}\Gamma(\sigma+1)\frac{x^\zeta}{\zeta}}$$

$$.x^m\left({}^{M}\mathcal{L}_{z}^{\rho}\{z^n\}\right)d_\zeta x$$

$$=\frac{1}{p}\left(\frac{\theta}{r\Gamma(\mu+1)}\right)^{\frac{k}{\theta}}\Gamma\left(1+\frac{k}{\theta}\right)\frac{1}{s}\left(\frac{\rho}{s\Gamma(\gamma+1)}\right)^{\frac{n}{\rho}}\Gamma\left(1+\frac{n}{\rho}\right)$$

$${}^{M}\mathcal{L}_{x}^{\zeta}\{x^m\}=\frac{1}{prs}\left(\frac{\rho}{s\Gamma(\gamma+1)}\right)^{\frac{n}{\rho}}\left(\frac{\zeta}{p\Gamma(\sigma+1)}\right)^{\frac{m}{\zeta}}\left(\frac{\theta}{r\Gamma(\mu+1)}\right)^{\frac{k}{\theta}}$$

$$.\Gamma\left(1+\frac{k}{\theta}\right)\Gamma\left(1+\frac{n}{\rho}\right)\Gamma\left(1+\frac{m}{\zeta}\right)$$

$iii)\ {}^{M}\mathcal{L}_{z}^{\rho}\,{}^{M}\mathcal{L}_{x}^{\zeta}\,{}^{M}\mathcal{L}_{y}^{\theta}\left\{\mathrm{e}^{\mathrm{c}\Gamma(\gamma+1)\frac{z^\rho}{\rho}+d\Gamma(\sigma+1)\frac{x^\zeta}{\zeta}+f\Gamma(\mu+1)\frac{y^\theta}{\theta}}\right\}$

$= \frac{1}{(p-d)(s-c)(r-f)}$ $\quad p > c, s > d, r > f,$ c and f are any constants.

Proof.

(9) If the triple M-Laplace transform given by the definition is written $\mathrm{e}^{c\Gamma(\gamma+1)\frac{z^{\rho}}{\rho}+d\Gamma(\sigma+1)\frac{x^{\zeta}}{\zeta}+f\Gamma(\mu+1)\frac{y^{\theta}}{\theta}}$ instead of the function $v(x,y,z)$ and the M-Laplace transformation formulas and the definition of the M-Laplace transform are taken into account, then, after performing the necessary steps:

$$ {}^{M}\mathcal{L}_{z}^{\rho}\,{}^{M}\mathcal{L}_{x}^{\zeta}\,{}^{M}\mathcal{L}_{y}^{\theta}\left\{\mathrm{e}^{c\Gamma(\gamma+1)\frac{z^{\rho}}{\rho}+d\Gamma(\sigma+1)\frac{x^{\zeta}}{\zeta}+f\Gamma(\mu+1)\frac{y^{\theta}}{\theta}}\right\}$$

$$= \int_0^\infty\int_0^\infty\int_0^\infty \Gamma(\sigma+1)\,\mathrm{e}^{-p\Gamma(\sigma+1)\frac{x^{\zeta}}{\zeta}}\Gamma(\gamma+1)$$

$$.\ \mathrm{e}^{-s\Gamma(\gamma+1)\frac{z^{\rho}}{\rho}}\Gamma(\mu+1)\mathrm{e}^{-r\Gamma(\mu+1)\frac{y^{\theta}}{\theta}}$$

$$.\ (\mathrm{e}^{c\Gamma(\gamma+1)\frac{z^{\rho}}{\rho}+d\Gamma(\sigma+1)\frac{x^{\zeta}}{\zeta}+f\Gamma(\mu+1)\frac{y^{\theta}}{\theta}})d_{\rho}z d_{\zeta}x d_{\theta}y$$

$$= \int_0^\infty \Gamma(\sigma+1)\,\mathrm{e}^{-p\Gamma(\sigma+1)\frac{x^{\zeta}}{\zeta}}\mathrm{e}^{d\Gamma(\sigma+1)\frac{x^{\zeta}}{\zeta}}$$

$$.\left[\int_0^\infty \Gamma(\gamma+1)\mathrm{e}^{-s\Gamma(\gamma+1)\frac{z^{\rho}}{\rho}}\mathrm{e}^{c\Gamma(\gamma+1)\frac{z^{\rho}}{\rho}}\right.$$

$$.\left(\int_0^\infty \Gamma(\mu+1)\mathrm{e}^{-r\Gamma(\mu+1)\frac{y^{\theta}}{\theta}}\mathrm{e}^{f\Gamma(\mu+1)\frac{y^{\theta}}{\theta}}d_{\theta}y\right)$$

$$.\left.d_{\rho}z\right]d_{\zeta}x$$

$$= \int_0^\infty \Gamma(\sigma+1)\,\mathrm{e}^{-p\Gamma(\sigma+1)\frac{x^{\zeta}}{\zeta}}\mathrm{e}^{d\Gamma(\sigma+1)\frac{x^{\zeta}}{\zeta}}$$

$$.\left[\int_0^\infty \Gamma(\gamma+1)\mathrm{e}^{-s\Gamma(\gamma+1)\frac{z^{\rho}}{\rho}}\mathrm{e}^{c\Gamma(\gamma+1)\frac{z^{\rho}}{\rho}}\right.$$

$$.\left.\left({}^{M}\mathcal{L}_{y}^{\theta}\left\{\mathrm{e}^{f\Gamma(\mu+1)\frac{y^{\theta}}{\theta}}\right\}\right)d_{\rho}z\right]d_{\zeta}x$$

$$= \int_0^\infty \Gamma(\sigma+1)\,\mathrm{e}^{-p\Gamma(\sigma+1)\frac{x^{\zeta}}{\zeta}}\mathrm{e}^{d\Gamma(\sigma+1)\frac{x^{\zeta}}{\zeta}}$$

$$.\left(\int_0^\infty \Gamma(\gamma+1)\mathrm{e}^{-s\Gamma(\gamma+1)\frac{z^{\rho}}{\rho}}\mathrm{e}^{c\Gamma(\gamma+1)\frac{z^{\rho}}{\rho}}\left(\frac{1}{(r-f)}\right)d_{\rho}z\right)d_{\zeta}x$$

$$= \frac{1}{(r-f)}\int_0^\infty \Gamma(\sigma+1)\,\mathrm{e}^{-p\Gamma(\sigma+1)\frac{x^{\zeta}}{\zeta}}\mathrm{e}^{d\Gamma(\sigma+1)\frac{x^{\zeta}}{\zeta}}$$

$$.\left(\int_0^\infty \Gamma(\gamma+1)\mathrm{e}^{-s\Gamma(\gamma+1)\frac{z^{\rho}}{\rho}}\mathrm{e}^{c\Gamma(\gamma+1)\frac{z^{\rho}}{\rho}}d_{\rho}z\right)d_{\zeta}x$$

$$= \int_0^\infty \Gamma(\sigma+1)\,\mathrm{e}^{-p\Gamma(\sigma+1)\frac{x^{\zeta}}{\zeta}}\mathrm{e}^{d\Gamma(\sigma+1)\frac{x^{\zeta}}{\zeta}}d_{\zeta}x$$

$$.\ \frac{1}{(r-f)(s-c)}$$

$$= \frac{1}{(r-f)(s-c)}\left({}^{M}\mathcal{L}_{x}^{\zeta}\{\mathrm{e}^{d\Gamma(\sigma+1)\frac{x^{\zeta}}{\zeta}}\}\right)$$

$$= \frac{1}{(p-d)(s-c)(r-f)}\ .$$

$iv)$ $${}^{M}\mathcal{L}_{z}^{\rho}\,{}^{M}\mathcal{L}_{x}^{\zeta}\,{}^{M}\mathcal{L}_{y}^{\theta}\left\{\sin\left(a\Gamma(\gamma+1)\frac{z^{\rho}}{\rho}\right)\right.$$

$$.\left.\sin\,(b\Gamma(\sigma+1)\frac{x^{\zeta}}{\zeta})\sin\,(c\Gamma(\mu+1)\frac{y^{\theta}}{\theta})\right\}$$

$= \frac{abc}{(a^2+s^2)(b^2+p^2)(c^2+r^2)}$ a, b, c are any constants.

Proof.

(9) If the triple M-Laplace transform given by the definition is written $\sin\left(a\Gamma(\gamma+1)\frac{z^{\rho}}{\rho}\right))$

$.\ \sin\left(b\Gamma(\sigma+1)\frac{x^{\zeta}}{\zeta}\right)\ \sin\,(c\Gamma(\mu+1)\frac{y^{\theta}}{\theta})$

instead of the function $v(x,y,z)$ and the M-Laplace transformation formulas and the definition of the M-Laplace transform are taken into account then,

$${}^{M}\mathcal{L}_{z}^{\rho}\,{}^{M}\mathcal{L}_{x}^{\zeta}\,{}^{M}\mathcal{L}_{y}^{\theta}\left\{\sin\left(a\Gamma(\gamma+1)\frac{z^{\rho}}{\rho}\right)\right.$$

$$.\left.\sin\left(b\Gamma(\sigma+1)\frac{x^{\zeta}}{\zeta}\right)\ \sin\,(c\Gamma(\mu+1)\frac{y^{\theta}}{\theta})\right\}$$

$$= \int_0^\infty\int_0^\infty\int_0^\infty \Gamma(\sigma+1)\,\mathrm{e}^{-p\Gamma(\sigma+1)\frac{x^{\zeta}}{\zeta}}\Gamma(\gamma+1)$$

$$.\ \mathrm{e}^{-s\Gamma(\gamma+1)\frac{z^{\rho}}{\rho}}\Gamma(\mu+1)\mathrm{e}^{-r\Gamma(\mu+1)\frac{y^{\theta}}{\theta}}\sin\left(a\Gamma(\gamma+1)\frac{z^{\rho}}{\rho}\right))$$

$$.\ \sin\left(b\Gamma(\sigma+1)\frac{x^{\zeta}}{\zeta}\right)\sin\left(c\Gamma(\mu+1)\frac{y^{\theta}}{\theta}\right)d_{\rho}z d_{\zeta}x d_{\theta}y$$

$$= \int_0^\infty \Gamma(\sigma+1)\,\mathrm{e}^{-p\Gamma(\sigma+1)\frac{x^{\zeta}}{\zeta}}\sin\,(b\Gamma(\sigma+1)\frac{x^{\zeta}}{\zeta})$$

$$.\left[\int_0^{\infty}\Gamma(\sigma+1)\,e^{-s\Gamma(\gamma+1)\frac{z^{\rho}}{\rho}}\sin\left(a\Gamma(\gamma+1)\frac{z^{\rho}}{\rho}\right)\right.$$

$$.\left({}^{M}\mathcal{L}_{y}^{\theta}\left\{\sin\left(c\Gamma(\mu+1)\frac{y^{\theta}}{\theta}\right\}\right)d_{\rho}z\right]d_{\zeta}x$$

$$=\int_0^{\infty}\Gamma(\sigma+1)\,e^{-p\Gamma(\sigma+1)\frac{x^{\zeta}}{\zeta}}\sin\left(b\Gamma(\sigma+1)\frac{x^{\zeta}}{\zeta}\right)d_{\zeta}x$$

$$.\frac{a}{(a^2+s^2)}\frac{c}{(c^2+r^2)}$$

$$=\frac{ac}{(a^2+s^2)(c^2+r^2)}\left({}^{M}\mathcal{L}_{x}^{\zeta}\{\sin\left(b\Gamma(\sigma+1)\frac{x^{\zeta}}{\zeta}\right)\}\right)$$

$$=\frac{abc}{(a^2+s^2)(b^2+p^2)(c^2+r^2)}$$

$v)$ $${}^{M}\mathcal{L}_{z}^{\rho}\,{}^{M}\mathcal{L}_{x}^{\zeta}\,{}^{M}\mathcal{L}_{y}^{\theta}\left\{\cos\left(a\Gamma(\gamma+1)\frac{z^{\rho}}{\rho}\right)\right.$$

$$\left..\cos\left(b\Gamma(\sigma+1)\frac{x^{\zeta}}{\zeta}\right)\cos\left(c\Gamma(\mu+1)\frac{y^{\theta}}{\theta}\right)\right\}$$

$$=\frac{psr}{(a^2+s^2)(b^2+p^2)(c^2+r^2)}$$ $a, b, c$are any constants

Proof.

In the theorem's (IV) form, the proof will be finished if the $\cos\left(a\Gamma(\gamma+1)\frac{z^{\rho}}{\rho}\right)\cos\left(c\Gamma(\mu+1)\frac{y^{\theta}}{\theta}\right)$

$\cos\left(b\Gamma(\sigma+1)\frac{x^{\zeta}}{\zeta}\right)$is written in place of

$\sin\left(a\Gamma(\gamma+1)\frac{z^{\rho}}{\rho}\right)\sin\left(b\Gamma(\sigma+1)\frac{x^{\zeta}}{\zeta}\right)$

$\sin\left(c\Gamma(\mu+1)\frac{y^{\theta}}{\theta}\right)$ and the required operations are carried out.

$vi)$ $${}^{M}\mathcal{L}_{z}^{\rho}\,{}^{M}\mathcal{L}_{x}^{\zeta}\,{}^{M}\mathcal{L}_{y}^{\theta}\left\{\Gamma(\gamma+1)\frac{z^{\rho}}{\rho}(\sigma+1)\frac{x^{\zeta}}{\zeta}\Gamma(\mu+1)\frac{y^{\theta}}{\theta}\right.$$

$$\left..\,e^{c\Gamma(\gamma+1)\frac{z^{\rho}}{\rho}+d\Gamma(\sigma+1)\frac{x^{\zeta}}{\zeta}+f\Gamma(\mu+1)\frac{y^{\theta}}{\theta}}\right\}=\frac{1}{(s-c)^2(p-d)^2(r-f)^2}$$

$p > c, s > d, r > f$, c, d and $f$are any constants.

Proof.

(9) If the triple M-Laplace transform given by the definition is written

$\Gamma(\gamma+1)\frac{z^{\rho}}{\rho}\Gamma(\sigma+1)\frac{x^{\zeta}}{\zeta}\Gamma(\mu+1)\frac{y^{\theta}}{\theta}$

$.\,e^{c\Gamma(\gamma+1)\frac{z^{\rho}}{\rho}+d\Gamma(\sigma+1)\frac{x^{\zeta}}{\zeta}+f\Gamma(\mu+1)\frac{y^{\theta}}{\theta}}$ instead of the function $v(x, y, z)$ and the M-Laplace transformation formulas and the definition of the M-Laplace transform are taken into account,

$${}^{M}\mathcal{L}_{z}^{\rho}\,{}^{M}\mathcal{L}_{x}^{\zeta}\,{}^{M}\mathcal{L}_{y}^{\theta}\left\{\Gamma(\gamma+1)\frac{z^{\rho}}{\rho}\Gamma(\sigma+1)\frac{x^{\zeta}}{\zeta}\Gamma(\mu+1)\frac{y^{\theta}}{\theta}\right.$$

$$\left..\,e^{c\Gamma(\gamma+1)\frac{z^{\rho}}{\rho}+d\Gamma(\sigma+1)\frac{x^{\zeta}}{\zeta}+f\Gamma(\mu+1)\frac{y^{\theta}}{\theta}}\right\}$$

$$=\int_0^{\infty}\int_0^{\infty}\int_0^{\infty}\Gamma(\sigma+1)\,e^{-p\Gamma(\sigma+1)\frac{x^{\zeta}}{\zeta}}\Gamma(\gamma+1)$$

$$.\,e^{-s\Gamma(\gamma+1)\frac{z^{\rho}}{\rho}}\Gamma(\mu+1)e^{-r\Gamma(\mu+1)\frac{y^{\theta}}{\theta}}\Gamma(\gamma+1)\frac{z^{\rho}}{\rho}$$

$$.\Gamma(\sigma+1)\frac{x^{\zeta}}{\zeta}\Gamma(\mu+1)\frac{y^{\theta}}{\theta}$$

$$.\,e^{c\Gamma(\gamma+1)\frac{z^{\rho}}{\rho}+d\Gamma(\sigma+1)\frac{x^{\zeta}}{\zeta}+f\Gamma(\mu+1)\frac{y^{\theta}}{\theta}}d_{\rho}z\,d_{\zeta}x\,d_{\theta}y$$

$$=\int_0^{\infty}\Gamma(\sigma+1)\,e^{-p\Gamma(\sigma+1)\frac{x^{\zeta}}{\zeta}}$$

$$.\left(\Gamma(\sigma+1)\frac{x^{\zeta}}{\zeta}e^{d\Gamma(\sigma+1)\frac{x^{\zeta}}{\zeta}}\right)$$

$$.\left[\int_0^{\infty}\Gamma(\gamma+1)e^{-s\Gamma(\gamma+1)\frac{z^{\rho}}{\rho}}\Gamma(\gamma+1)\frac{z^{\rho}}{\rho}e^{c\Gamma(\gamma+1)\frac{z^{\rho}}{\rho}}\right.$$

$$.\left({}^{M}\mathcal{L}_{y}^{\theta}\left\{\Gamma(\mu+1)\frac{y^{\theta}}{\theta}e^{f\Gamma(\mu+1)\frac{y^{\theta}}{\theta}}\right\}\right)d_{\rho}z\Big]d_{\zeta}x$$

$$=\int_0^{\infty}\Gamma(\sigma+1)\,e^{-p\Gamma(\sigma+1)\frac{x^{\zeta}}{\zeta}}\left(\Gamma(\sigma+1)\frac{x^{\zeta}}{\zeta}e^{d\Gamma(\sigma+1)\frac{x^{\zeta}}{\zeta}}\right)$$

$$=\frac{1}{(r-f)^2}\int_0^{\infty}\Gamma(\sigma+1)\,e^{-p\Gamma(\sigma+1)\frac{x^{\zeta}}{\zeta}}$$

$$.\left(\Gamma(\sigma+1)\frac{x^{\zeta}}{\zeta}e^{d\Gamma(\sigma+1)\frac{x^{\zeta}}{\zeta}}\right)\left(\frac{1}{(s-c)^2}\right)d_{\zeta}x$$

$$=\frac{1}{(r-f)^2}\frac{1}{(s-c)^2}\int_0^{\infty}\Gamma(\sigma+1)\,e^{-p\Gamma(\sigma+1)\frac{x^{\zeta}}{\zeta}}$$

$$.\left(\Gamma(\sigma+1)\frac{x^{\zeta}}{\zeta}e^{d\Gamma(\sigma+1)\frac{x^{\zeta}}{\zeta}}\right)d_{\zeta}x$$

$$=\frac{1}{(r-f)^2}\frac{1}{(s-c)^2}\left({}^{M}\mathcal{L}_x^{\zeta}\{\Gamma(\sigma+1)\frac{x^\zeta}{\zeta}\,\mathrm{e}^{d\Gamma(\sigma+1)\frac{x^\zeta}{\zeta}}\}\right)$$

$$=\frac{1}{(s-c)^2(p-d)^2(r-f)^2}$$

vii) $${}^{M}\mathcal{L}_z^{\rho}\,{}^{M}\mathcal{L}_x^{\zeta}\,{}^{M}\mathcal{L}_y^{\theta}\left\{\sinh\left(\Gamma(\gamma+1)\frac{z^\rho}{\rho}\right)\right.$$

$$\left..\sinh\left(\Gamma(\sigma+1)\frac{x^\zeta}{\zeta}\right)\sinh\left(\Gamma(\mu+1)\frac{y^\theta}{\theta}\right)\right\}$$

$$=\frac{1}{(s^2-1)(p^2-1)(r^2-1)}$$

Proof.

(9) If the triple M-Laplace transform given by the definition is written $\sinh\left(\Gamma(\gamma+1)\frac{z^\rho}{\rho}\right))$

$.\sin h\left(\Gamma(\sigma+1)\frac{x^\zeta}{\zeta}\right)\sinh\left(\Gamma(\mu+1)\frac{y^\theta}{\theta}\right)$

instead of the function $v(x,y,z)$ and the M-Laplace transformation formulas and the definition of the M-Laplace transform are taken into account;

$${}^{M}\mathcal{L}_z^{\rho}\,{}^{M}\mathcal{L}_x^{\zeta}\,{}^{M}\mathcal{L}_y^{\theta}\left\{\sinh\left(\Gamma(\gamma+1)\frac{z^\rho}{\rho}\right)\right)$$

$$\left..\sinh\left(\Gamma(\sigma+1)\frac{x^\zeta}{\zeta}\right)\ \sin\ h(\Gamma(\mu+1)\frac{y^\theta}{\theta})\right\}$$

$${}^{M}\mathcal{L}_z^{\rho}\,{}^{M}\mathcal{L}_x^{\zeta}\,{}^{M}\mathcal{L}_y^{\theta}\left\{\sinh\left(\Gamma(\gamma+1)\frac{z^\rho}{\rho}\right)\right)$$

$$\left..\sinh\left(\Gamma(\sigma+1)\frac{x^\zeta}{\zeta}\right)\ \sin\ h(\Gamma(\mu+1)\frac{y^\theta}{\theta})\right\}$$

$$=\int_0^\infty\int_0^\infty\int_0^\infty\Gamma(\sigma+1)\,\mathrm{e}^{-p\Gamma(\sigma+1)\frac{x^\zeta}{\zeta}}\Gamma(\gamma+1)$$

$$.\,\mathrm{e}^{-s\Gamma(\gamma+1)\frac{z^\rho}{\rho}}\Gamma(\mu+1)\mathrm{e}^{-r\Gamma(\mu+1)\frac{y^\theta}{\theta}}\sin h\left(\Gamma(\gamma+1)\frac{z^\rho}{\rho}\right))$$

$$.\sinh\left(\Gamma(\sigma+1)\frac{x^\zeta}{\zeta}\right)\sinh\left(\Gamma(\mu+1)\frac{y^\theta}{\theta}\right)d_\rho z d_\zeta x d_\theta y$$

$$=\int_0^\infty\Gamma(\sigma+1)\,\mathrm{e}^{-p\Gamma(\sigma+1)\frac{x^\zeta}{\zeta}}\,sinh\left(\Gamma(\sigma+1)\frac{x^\zeta}{\zeta}\right)$$

$$.\left(\int_0^\infty\Gamma(\gamma+1)\mathrm{e}^{-s\Gamma(\gamma+1)\frac{z^\rho}{\rho}}sinh\left(\Gamma(\gamma+1)\frac{z^\rho}{\rho}\right)\right)$$

$$.\left(\Gamma(\mu+1)\mathrm{e}^{-r\Gamma(\mu+1)\frac{y^\theta}{\theta}}sinh\left(\Gamma(\mu+1)\frac{y^\theta}{\theta}\right)d_\theta y\right)$$

$$.\,d_\rho z d_\zeta x$$

$$=\int_0^\infty\Gamma(\sigma+1)\,\mathrm{e}^{-p\Gamma(\sigma+1)\frac{x^\zeta}{\zeta}}\sinh\left(\Gamma(\sigma+1)\frac{x^\zeta}{\zeta}\right)$$

$$.\left(\int_0^\infty\Gamma(\gamma+1)\mathrm{e}^{-s\Gamma(\gamma+1)\frac{z^\rho}{\rho}}sinh\left(\Gamma(\gamma+1)\frac{z^\rho}{\rho}\right)\right)$$

$$.\left(\frac{1}{(r^2-1)}\right)d_\rho z d_\zeta x$$

$$=\frac{1}{(r^2-1)}\int_0^\infty\Gamma(\sigma+1)\,\mathrm{e}^{-p\Gamma(\sigma+1)\frac{x^\zeta}{\zeta}}\,sinh\left(\Gamma(\sigma+1)\frac{x^\zeta}{\zeta}\right)$$

$$.\left({}^{M}\mathcal{L}_z^{\rho}\left\{sinh\left(\Gamma(\gamma+1)\frac{z^\rho}{\rho}\right)\right\}\right)d_\zeta x$$

$$=\frac{1}{(s^2-1)(p^2-1)(r^2-1)}$$

viii) $${}^{M}\mathcal{L}_z^{\rho}\,{}^{M}\mathcal{L}_x^{\zeta}\,{}^{M}\mathcal{L}_y^{\theta}\left\{\cosh\left(\Gamma(\gamma+1)\frac{z^\rho}{\rho}\right)\right.$$

$$\left..\cosh\left(\Gamma(\sigma+1)\frac{x^\zeta}{\zeta}\right)\cosh\left(\Gamma(\mu+1)\frac{y^\theta}{\theta}\right)\right\}$$

$$=\frac{psr}{(s^2-1)(p^2-1)(r^2-1)}$$

Proof.

In the theorem's (vii) form, the proof will be finished if the

$\cosh\left(\Gamma(\gamma+1)\frac{z^\rho}{\rho}\right)\cosh\left(\Gamma(\mu+1)\frac{y^\theta}{\theta}\right)$

$.\cosh\left(\Gamma(\sigma+1)\frac{x^\zeta}{\zeta}\right)$is written in place of

$\sinh\left(\Gamma(\gamma+1)\frac{z^\rho}{\rho}\right)\sinh\left(\Gamma(\sigma+1)\frac{x^\zeta}{\zeta}\right)$

$.\sinh\left(\Gamma(\mu+1)\frac{y^\theta}{\theta}\right)$ and the required operations are carried out.

xi) $${}^{M}\mathcal{L}_z^{\rho}\,{}^{M}\mathcal{L}_x^{\zeta}\,{}^{M}\mathcal{L}_y^{\theta}\left\{\left(\Gamma(\gamma+1)\frac{z^\rho}{\rho}\right)^a\left(\Gamma(\sigma+1)\frac{x^\zeta}{\zeta}\right)^b\right.$$

$$\left.\left(\Gamma(\mu+1)\frac{y^\theta}{\theta}\right)^c\right\}=\frac{\Gamma(1+a)}{s^{1+a}}\frac{\Gamma(1+b)}{p^{1+b}}\frac{\Gamma(1+c)}{r^{1+c}}$$

$Re\,(a) > 0, Re\,(b) > 0, Re\,(c) > 0, s > 0, r > 0$ and $p > 0$.

Proof.

(9) If the triple M-Laplace transform given by the definition is written$\left(\Gamma(\gamma+1)\frac{z^\rho}{\rho}\right)^a\left(\Gamma(\mu+1)\frac{y^\theta}{\theta}\right)^c$ $.\left(\Gamma(\sigma+1)\frac{x^\zeta}{\zeta}\right)^b$ instead of the function $v(x,y,z)$ and the M-Laplace transformation formulas and the definition of the M-Laplace transform are taken into account,

$${}^M\mathcal{L}_z^\rho\,{}^M\mathcal{L}_x^\zeta\,{}^M\mathcal{L}_y^\theta\left\{\left(\Gamma(\gamma+1)\frac{z^\rho}{\rho}\right)^a\left(\Gamma(\sigma+1)\frac{x^\zeta}{\zeta}\right)^b .\left(\Gamma(\mu+1)\frac{y^\theta}{\theta}\right)^c\right\}$$

$$=\int_0^\infty\int_0^\infty\int_0^\infty \Gamma(\sigma+1)\,e^{-p\Gamma(\sigma+1)\frac{x^\zeta}{\zeta}}\Gamma(\gamma+1)e^{-s\Gamma(\gamma+1)\frac{z^\rho}{\rho}}$$

$$.\Gamma(\mu+1)e^{-r\Gamma(\mu+1)\frac{y^\theta}{\theta}}\left(\Gamma(\gamma+1)\frac{z^\rho}{\rho}\right)^a\left(\Gamma(\sigma+1)\frac{x^\zeta}{\zeta}\right)^b$$

$$.\left(\Gamma(\mu+1)\frac{y^\theta}{\theta}\right)^c d_\rho z d_\zeta x d_\theta y$$

$$=\int_0^\infty \Gamma(\sigma+1)\,e^{-p\Gamma(\sigma+1)\frac{x^\zeta}{\zeta}}\left(\Gamma(\sigma+1)\frac{x^\zeta}{\zeta}\right)^b$$

$$.\left[\int_0^\infty \Gamma(\gamma+1)\,e^{-s\Gamma(\gamma+1)\frac{z^\rho}{\rho}}\left(\Gamma(\gamma+1)\frac{z^\rho}{\rho}\right)^a\right.$$

$$.\left(\int_0^\infty \Gamma(\mu+1)\,e^{-r\Gamma(\mu+1)\frac{y^\theta}{\theta}}\left(\Gamma(\mu+1)\frac{y^\theta}{\theta}\right)^c d_\theta y\right)$$

$$\left.. \, d_\rho z\right]d_\zeta x$$

$$=\int_0^\infty \Gamma(\sigma+1)\,e^{-p\Gamma(\sigma+1)\frac{x^\zeta}{\zeta}}\left(\Gamma(\sigma+1)\frac{x^\zeta}{\zeta}\right)^b$$

$$.\left[\int_0^\infty \Gamma(\gamma+1)\,e^{-s\Gamma(\gamma+1)\frac{z^\rho}{\rho}}\left(\Gamma(\gamma+1)\frac{z^\rho}{\rho}\right)^a\right.$$

$$\left..\left({}^M\mathcal{L}_y^\theta\left\{\left(\Gamma(\mu+1)\frac{y^\theta}{\theta}\right)^c\right\}\right)d_\rho z\right]d_\zeta x$$

$$=\frac{\Gamma(1+c)}{r^{1+c}}\int_0^\infty \Gamma(\sigma+1)\,e^{-p\Gamma(\sigma+1)\frac{x^\zeta}{\zeta}}\left(\Gamma(\sigma+1)\frac{x^\zeta}{\zeta}\right)^b$$

$$.\left[\int_0^\infty \Gamma(\gamma+1)\,e^{-s\Gamma(\gamma+1)\frac{z^\rho}{\rho}}\left(\Gamma(\gamma+1)\frac{z^\rho}{\rho}\right)^a d_\rho z\right]d_\zeta x$$

$$=\frac{\Gamma(1+c)}{r^{1+c}}\frac{\Gamma(1+a)}{s^{1+a}}\,{}^M\mathcal{L}_x^\zeta\left\{\left(\Gamma(\sigma+1)\frac{x^\zeta}{\zeta}\right)^b\right\}$$

$$=\frac{\Gamma(1+a)}{s^{1+a}}\frac{\Gamma(1+b)}{p^{1+b}}\frac{\Gamma(1+c)}{r^{1+c}}.$$

Definition 3.2. The triple convolution of $f(x,y,z)$ and $g(x,y,z)$ as continuous functions in the sense of M-derivative

$$(f *** g)(x,y,z) = \Gamma(\gamma+1)\,\Gamma(\sigma+1)\Gamma(\mu+1)$$

$$.\int_0^x\int_0^z\int_0^y f(x-v, y-\tau, z-\varphi)\,g(v,\tau,\varphi)$$

$$.\,(d_\rho\varphi d_\zeta v d_\theta\tau)$$

is determined by this integral.

Theorem 3.6. (Convolution Theorem) The functions $u(x,y,z)$ and $v(x,y,z)$ with $U(p,r,s) = {}^M\mathcal{L}_z^\rho\,{}^M\mathcal{L}_x^\zeta\,{}^M\mathcal{L}_y^\theta\{u(x,y,z)\}$ and $V(p,r,s) = {}^M\mathcal{L}_z^\rho\,{}^M\mathcal{L}_x^\zeta\,{}^M\mathcal{L}_y^\theta\{v(x,y,z)\}$ allow the triple M-Laplace transform to exist for $p > 0$, $s > 0$ and $r > 0$. Here, the triple convolutions of the functions $u(x,y,z) *** v(x,y,z)$ must be expressed in order to

$${}^M\mathcal{L}_z^\rho\,{}^M\mathcal{L}_x^\zeta\,{}^M\mathcal{L}_y^\theta\{u(x,y,z) *** v(x,y,z)\}$$

$$= U(p,r,s)\,V(p,r,s)$$

Which is expressed in this form.

Proof.

$${}^M\mathcal{L}_z^\rho\,{}^M\mathcal{L}_x^\zeta\,{}^M\mathcal{L}_y^\theta\{u(x,y,z) *** v(x,y,z)\}$$

$$={}^M\mathcal{L}_z^\rho\,{}^M\mathcal{L}_x^\zeta\,{}^M\mathcal{L}_y^\theta[u\left(\left(\frac{\zeta x}{\Gamma(\sigma+1)}\right)^\zeta,\left(\frac{\theta y}{\Gamma(\mu+1)}\right)^\theta\left(\frac{\rho z}{\Gamma(\gamma+1)}\right)^\rho\right)$$

$$*** \, v\left(\left(\frac{\zeta x}{\Gamma(\sigma+1)}\right)^\zeta,\left(\frac{\theta y}{\Gamma(\mu+1)}\right)^\theta\left(\frac{\rho z}{\Gamma(\gamma+1)}\right)^\rho\right)]$$

It can be written. Since the convolution property is known to be provided by the triple Laplace transform on the right side of the preceding equation

$${}^{M}\mathcal{L}_z^{\rho}{}^{M}\mathcal{L}_x^{\zeta}{}^{M}\mathcal{L}_y^{\theta}\{u\left(\left(\frac{\zeta x}{\Gamma(\sigma+1)}\right)^{\zeta},\left(\frac{\theta y}{\Gamma(\mu+1)}\right)^{\theta}\left(\frac{\rho z}{\Gamma(\gamma+1)}\right)^{\rho}\right)$$

$$*** \, v\left(\left(\frac{\zeta x}{\Gamma(\sigma+1)}\right)^{\zeta},\left(\frac{\theta y}{\Gamma(\mu+1)}\right)^{\theta}\left(\frac{\rho z}{\Gamma(\gamma+1)}\right)^{\rho}\right)\}$$

$$= {}^{M}\mathcal{L}_z^{\rho}{}^{M}\mathcal{L}_x^{\zeta}{}^{M}\mathcal{L}_y^{\theta}\left\{u\left(\left(\frac{\zeta x}{\Gamma(\sigma+1)}\right)^{\zeta},\left(\frac{\theta y}{\Gamma(\mu+1)}\right)^{\theta}\left(\frac{\rho z}{\Gamma(\gamma+1)}\right)^{\rho}\right)\right\}$$

$$.{}^{M}\mathcal{L}_z^{\rho}{}^{M}\mathcal{L}_x^{\zeta}{}^{M}\mathcal{L}_y^{\theta}\{v\left(\left(\frac{\zeta x}{\Gamma(\sigma+1)}\right)^{\zeta},\left(\frac{\theta y}{\Gamma(\mu+1)}\right)^{\theta}\left(\frac{\rho z}{\Gamma(\gamma+1)}\right)^{\rho}\right)$$

$$= U(p,r,s)\,V(p,r,s)$$

it can be written. This is where the proof ends.

Applications 3.1.

$${}_{M}D_x{}^{\zeta}\big(u(x,y,z)\big) + {}_{M}D_y{}^{\theta}\big(u(x,y,z)\big)$$

$$+{}_{M}D_z{}^{\rho}\big(u(x,y,z)\big)$$

$$= -\left(\Gamma(\gamma+1)\frac{z^{\rho}}{\rho}\right)^2\left(\Gamma(\sigma+1)\frac{x^{\zeta}}{\zeta}\right)^2\left(\Gamma(\mu+1)\frac{y^{\theta}}{\theta}\right)^2$$

$$+4\,\Gamma(\sigma+1)\frac{x^{\zeta}}{\zeta}\,\Gamma(\mu+1)\frac{y^{\theta}}{\theta}+4\Gamma(\gamma+1)\frac{z^{\rho}}{\rho}\,\Gamma(\sigma+1)\frac{x^{\zeta}}{\zeta}$$

$$+4\Gamma(\gamma+1)\frac{z^{\rho}}{\rho}\,\Gamma(\mu+1)\frac{y^{\theta}}{\theta}+2\Gamma(\gamma+1)\,\Gamma(\sigma+1)$$

$$.\,\Gamma(\mu+1)\int_0^x\int_0^y\int_0^z U(\vartheta,\tau,\delta)d_{\xi}\vartheta d_{\theta}\tau d_{\rho}\delta \qquad (32)$$

$$u(x,y,0) = u(x,0,z) = u(0,y,z) = 0 \qquad (33)$$

Let us solve the differential equation given by the initial conditions:

Solution.

The triple M-Laplace transform on both sides of the integral differential equation given by (32) and Theorem 6., if we apply,

$${}^{M}\mathcal{L}_z^{\rho}{}^{M}\mathcal{L}_x^{\zeta}{}^{M}\mathcal{L}_y^{\theta}\{{}_{M}D_x{}^{\zeta}v(x,y,z)\}$$

$$=pU(p,r,s)-U(0,r,s)$$

$$= pU(p,r,s) \qquad (34)$$

$${}^{M}\mathcal{L}_z^{\rho}{}^{M}\mathcal{L}_x^{\zeta}{}^{M}\mathcal{L}_y^{\theta}\{{}_{M}D_y{}^{\theta}v(x,y,z)\}$$

$$= rU(p,r,s) - U(p,0,s) = rU(p,r,s) \qquad (35)$$

$${}^{M}\mathcal{L}_z^{\rho}{}^{M}\mathcal{L}_x^{\zeta}{}^{M}\mathcal{L}_y^{\theta}\{{}_{M}D_z{}^{\rho}v(x,y,z)\}$$

$$= sU(p,r,s) - U(p,r,0) = sU(p,r,s) \qquad (36)$$

$${}^{M}\mathcal{L}_z^{\rho}{}^{M}\mathcal{L}_x^{\zeta}{}^{M}\mathcal{L}_y^{\theta}\{\left(\Gamma(\gamma+1)\frac{z^{\rho}}{\rho}\right)^2\left(\Gamma(\sigma+1)\frac{x^{\zeta}}{\zeta}\right)^2$$

$$\left(\Gamma(\mu+1)\frac{y^{\theta}}{\theta}\right)^2\}=\frac{2}{p^3}\frac{2}{s^3}\frac{2}{r^3}=\frac{-8}{p^3s^3r^3} \qquad (37)$$

$${}^{M}\mathcal{L}_z^{\rho}{}^{M}\mathcal{L}_x^{\zeta}{}^{M}\mathcal{L}_y^{\theta}\left\{1\,\Gamma(\sigma+1)\frac{x^{\zeta}}{\zeta}\,\Gamma(\mu+1)\frac{y^{\theta}}{\theta}\right\}$$

$$=\frac{1}{sp^2r^2} \qquad (38)$$

$${}^{M}\mathcal{L}_z^{\rho}{}^{M}\mathcal{L}_x^{\zeta}{}^{M}\mathcal{L}_y^{\theta}\left\{1\,\Gamma(\gamma+1)\frac{z^{\rho}}{\rho}\,\Gamma(\sigma+1)\frac{x^{\zeta}}{\zeta}\right\}$$

$$=\frac{1}{rp^2s^2} \qquad (39)$$

$${}^{M}\mathcal{L}_z^{\rho}{}^{M}\mathcal{L}_x^{\zeta}{}^{M}\mathcal{L}_y^{\theta}\left\{1\,\Gamma(\gamma+1)\frac{z^{\rho}}{\rho}\,\Gamma(\mu+1)\frac{y^{\theta}}{\theta}\right\}$$

$$=\frac{1}{pr^2s^2} \qquad (40)$$

is obtained.

$${}^{M}\mathcal{L}_z^{\rho}{}^{M}\mathcal{L}_x^{\zeta}{}^{M}\mathcal{L}_y^{\theta}\{\int_0^x\int_0^y\int_0^z 1U(\vartheta,\tau,\delta)d_{\xi}\vartheta d_{\theta}\tau d_{\rho}\delta\}$$

Let us use the convolution integral to find the M-Laplace transform. Then

$${}^{M}\mathcal{L}_z^{\rho}{}^{M}\mathcal{L}_x^{\zeta}{}^{M}\mathcal{L}_y^{\theta}\{\Gamma(\gamma+1)\,\Gamma(\sigma+1)\Gamma(\mu+1)$$

$$.\int_0^x\int_0^y\int_0^z 1\,U(\vartheta,\tau,\delta)d_{\xi}\vartheta d_{\theta}\tau d_{\rho}\delta\}$$

$V(\vartheta-a,\tau-b,\delta-c)=1$ and

$U(\vartheta,\tau,\delta)$ given that,

$${}^{M}\mathcal{L}_z^{\rho}{}^{M}\mathcal{L}_x^{\zeta}{}^{M}\mathcal{L}_y^{\theta}\{\Gamma(\gamma+1)\,\Gamma(\sigma+1)\Gamma(\mu+1)$$

$$.\int_0^x\int_0^y\int_0^z 1\,U(\vartheta,\tau,\delta)d_{\xi}\vartheta d_{\theta}\tau d_{\rho}\delta\}$$

$$={}^{M}\mathcal{L}_z^{\rho}{}^{M}\mathcal{L}_x^{\zeta}{}^{M}\mathcal{L}_y^{\theta}\{1\}\,.\,{}^{M}\mathcal{L}_z^{\rho}{}^{M}\mathcal{L}_x^{\zeta}{}^{M}\mathcal{L}_y^{\theta}\{U(\vartheta,\tau,\delta)\}$$

$$=\frac{1}{prs}\,U(p,r,s)$$

$$ {}^{M}\mathcal{L}_z^{\rho}\,{}^{M}\mathcal{L}_x^{\zeta}\,{}^{M}\mathcal{L}_y^{\theta}\left\{\int_0^x\int_0^y\int_0^z 1\; U(\vartheta,\tau,\delta)\,d_\xi\vartheta\, d_\theta\tau\, d_\rho\delta\right\} $$

$$ =\frac{1}{prs}\,U(p,r,s) \tag{41} $$

If equations (34), (35), (36), (37), (38), (39), (40), and (41) are taken into consideration in the differential equation (32), then the following result is obtained;

$$ pU(p,r,s)+\ rU(p,r,s)+sU(p,r,s) $$

$$ =\frac{-8}{p^3s^3r^3}+\frac{4}{sp^2r^2}+\frac{4}{rp^2s^2}+\frac{4}{pr^2s^2}+\frac{2}{prs}\,U(p,r,s) $$

$$ U(p,r,s)\left[p+r+s-\frac{2}{prs}\right] $$

$$ =\frac{-8}{p^3s^3r^3}+\frac{4}{sp^2r^2}+\frac{4}{rp^2s^2}+\frac{4}{pr^2s^2} $$

$$ =\frac{4}{p^2s^2r^2} \tag{42} $$

If the inverse M-Laplace transform is applied to both sides of the equation

$$ u(x,y,z)=\ 4\Gamma(\gamma+1)\frac{z^{\rho}}{\rho}\,\Gamma(\sigma+1)\frac{x^{\zeta}}{\zeta} $$

$$.\,\Gamma(\mu+1)\frac{y^{\theta}}{\theta} \tag{43} $$

the solution of the problem is obtained.

4. Visual Results and Scientific Discussion

Figure 1 shows the change in the solution of equation (43) according to the values of ρ= ζ=θ=0.5,γ= σ=μ=0. From the figure, it appears that the shape resembles a quarter sphere. When the graphs in Figure 1, Figure 2, Figure 3, where the ρ= ζ=θ=0.5 values are the same but the γ,σ,μ values are different, are compared, it is observed that there is no significant change, although the blue color intensity of the blue part shown between the outer part of the quadrant-like shape and the cube-like outer shape has increased.

When we compare Figure 5 of equation (43) with Figure 1, Figure 2 and Figure 3, it is observed that the part shown in yellow on the surface of the quadrant-like shape turns blue. The source of this change is the change in ρ, ζ, θ values.

The difference between the M-derivative triple Laplace transform and the conformable triple Laplace transform lies in the gamma function depending on the parameters γ,σ,μ. Figure 1,

Figure 2 and Figure 3 show the comparison of this difference on the figure, where . ρ= ζ=θ=0.5 and γ,σ,μ vary.

The density varies based on the provided values, and the drawings exhibit geometric similarity to an ellipse. Despite their positional differences, they exhibit geometric similarity to a modified ellipsoid. Upon meticulous examination of Figures 1, 2, and 3, it is evident that variations in the parameters γ, σ, and μ influence the density of the outer region. Conversely, Figures 4, 5, 6, and 7 demonstrate that modifications in the parameters ρ, ζ, and θ impact the density of the inner part of the ellipsoidal form.

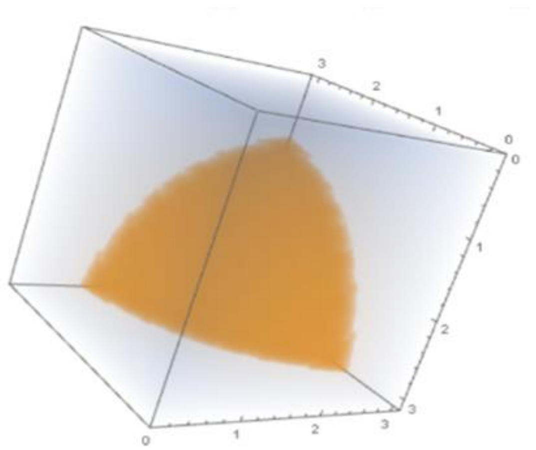

Figure 1: The image of the solution of equation (43) for different x,y and z values with values ρ = ζ= θ = 0.5,γ = σ = μ = 0

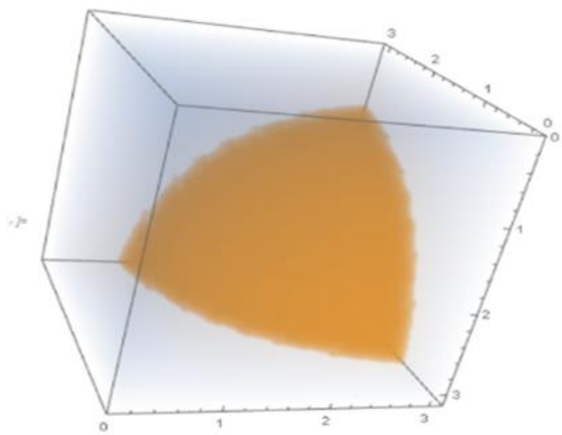

Figure 2: The image of the solution of equation (43) for different x,y and z values with values ρ= ζ= θ = 0.5,γ = σ = μ = 0.5

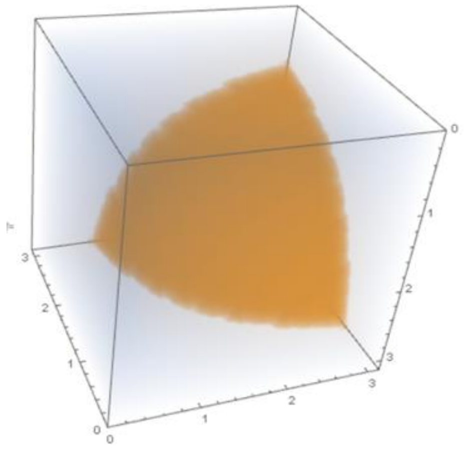

Figure 3: The image of the solution of equation (43) for different x,y and z values with values $\rho = \zeta = \theta = 0.5$, $\gamma = \sigma = \mu = 0.8$

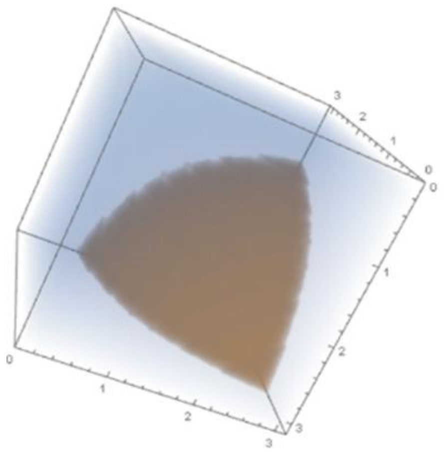

Figure 4: The image of the solution of equation (43) for different x, y and z values with values $\rho = \zeta = \theta = 0.8, \gamma = \sigma = \mu = 0$

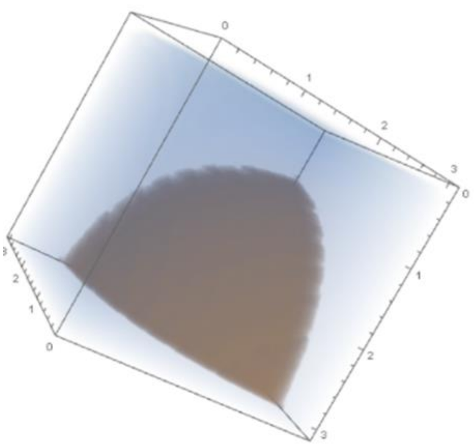

Figure 5: The image of the solution of equation (43) for different x, y and z values with values $\rho = \zeta = \theta = 0.8, \gamma = \sigma = \mu = 0.5$

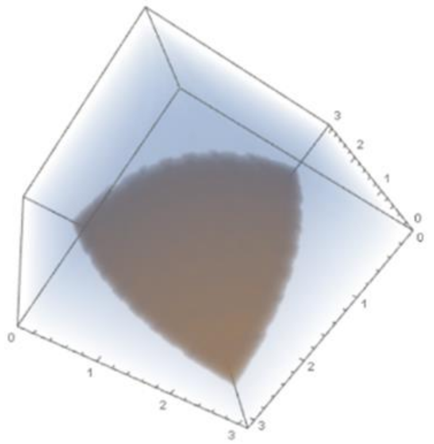

Figure 6: The image of the solution of equation (43) for different x,y and z values with values $\rho = \zeta = \theta = 0.8$, $\gamma = \sigma = \mu = 0.8$

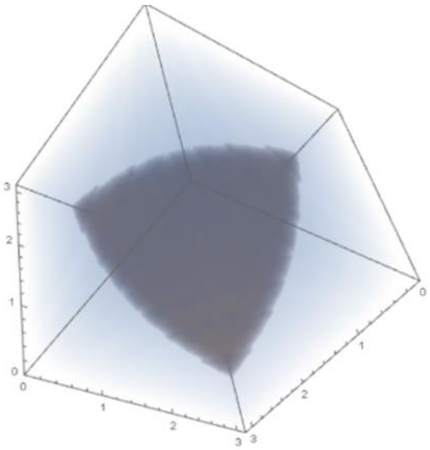

Figure 7: The image of the solution of equation (43) for different x, y and z values with values $\rho = \zeta = \theta = 1$, $\gamma = \sigma = \mu = 0.8$

5. Conclusions

In this paper, we present the M-derivative triple Laplace transform in conjunction with the partial derivatives M-derivative triple Laplace transform. Using the specified M-derivative triple Laplace transform, we apply this to the inhomogeneous differential equation and obtain the solution. The graph displays the variations in this equation's solution, as determined by the same approach, for various values. This transformation is therefore predicted to be a useful method for solving M-derivative fractional partial differential equations, which may be related to solving problems in a variety of fields, particularly with regard to physical final engineering models. In this article, we introduce the triple Laplace transform of M-derivative by applying the local derivatives containing the Mittag-Leffler function to the fractional M-derivative to the triple Laplace transform of partial derivatives of M-derivative and then applying the classical sense to the triple Laplace transform of local derivatives containing the Mittag-Leffler function. This defined M-derivative is applied to inhomogeneous using the triple Laplace transform and we give its solution by applying it to differential equations containing the initial conditions. The changes in the solution of this equation obtained by the same method at different values are shown on the graph, and interpretations of these figures are provide. It has been shown that with this method, a more practical and easier solution of partial fractional differential equations containing M-derivatives can be obtained. It inspire the use of a multilevel Laplace transform for solving generalized n-unknown fractional partial derivative-containing differential equations containing an M-derivative. Using this multilevel Laplace transform, fractional-order differential equations with partial derivatives that seem complex can be solved more conveniently. This transformation is therefore predicted to be a useful method for solving M-derivative fractional partial differential equations, which may be related to solving poblem in a variety of fields, particularly in engineering application involving physical models (for instance, in application involing ordered models pertaining to viscoelastic materials, anomalous diffusion, or electrical circuits, among others). Consequently, it is believed that systems of differential equations may be resolved more accurately and effortlessly using mathematical models with three variables, facilitated by the triple M-Laplace transform.

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