Bending of Super-Elliptical Mindlin Plates by Finite Element Method

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ABSTRACT

Bending of shear deformable super-elliptical plates under transverse load was investigated using the Mindlin plate theory by means of the finite element method. Four-noded isoparametric quadrilateral plate bending element with three degrees of freedom per node was used. Parametric results for the maximum deflections were presented via sensitivity analysis for several geometric characteristics such as thickness, aspect ratio, and super-elliptical power. Good agreement with the solutions of elliptical and rectangular plates was obtained using fine mesh. The results revealed that the deflections of clamped and point supported super-elliptical plates lie in the range bounded by elliptical and rectangular plates. However, the bending response of simply supported plates was observed to be entirely different. It was shown that high rate of convergence is required to obtain such a relation and using insufficient number of degrees of freedom results in finding a totally different trend for the clamped case.

Keywords: Plate, bending, deflection, finite element method.

1. INTRODUCTION

Plates are basic structural members which are used extensively in many disciplines like mechanical, civil, aerospace, and marine engineering, and in offshore structures. Because of their practical importance, the analysis of plates has always received significant interest, and thousands of studies have been published [1-12]. These studies may be categorized into various ways with regard to (i) the shape of the plate, (ii) the plate theory, (iii) the solution method, (iv) the material properties, (v) the boundary conditions, (vi) the scope of research (bending, buckling, or vibration), (vii) the type of analysis (theoretical, experimental or computational), and (viii) the classifications based on the thickness (membrane, thin, moderately thick or thick [13].

Since dealing with 2-D equations is relatively simple than 3-D equations, and 3-D equations inevitably involve numerical errors of experimental nature as well as 2-D equations do [14], the extensive literature mostly involves solutions using 2-D equations. The publications on

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plates are basically focused on the analysis of thin and moderately thick plates based on Kirchhoff and Mindlin plate theories, respectively [15-76]. These plate theories have been two of the most applied models on the analysis of plates [37]. This is partly because these theories have been widely accepted among researchers [38], and partly because these models produce results with acceptable accuracy with less computational effort in comparison with three-dimensional elasticity solutions or higher order plate theories. Since closed form solutions are only available for a limited number of cases depending on (i) the geometry of the plate, (ii) the loading, (iii) the boundary conditions, and (iv) the plate model, numerical methods have almost always been employed in the solution of plate problems.

Bending is one of the most common and important mechanical behaviors of plates, which is often crucial for safety and performance of the structures [39]. Due to their geometrical simplicity rectangular plates have been widely studied in the literature. However, from the engineering point of view, sharp corners may be critical due to stress concentrations. Besides, plates with curved boundaries have been used in many industrial applications (e.g, platforms, wings of aircrafts, components of machines).

Despite their common practical importance, there is still lack of data on plates with curved perimeters. Furthermore, notwithstanding their prevalent usage in engineering applications (e.g., slabs supported by columns, solar panels, printed circuit boards, and telescope mirrors) the investigations on point supported plates are less common than those involving simply supported or clamped boundaries owing to mathematical difficulties.

Super-elliptical plates include a large variety of plate shapes ranging from an ellipse to a rectangle with rounded corners. Unlike plates with sharp corners, rectangular plates with rounded corners enable to diffuse and dilute stress concentrations [40]. Despite the recently published papers, the studies on the analysis of super-elliptical plates have mostly been made for the dynamic behavior, and generally thin plates have been investigated [41-59]. To the best of the author's knowledge, there are only nine published papers on the bending of super-elliptical plates all of which focused on thin plates only [60-68]. Thus, the current work was motivated by the lack of contributions on the bending analysis of shear deformable super-elliptical plates. As far as the author knows, in the literature devoted to the bending analysis of super-elliptical plates under transverse load, this is the first study solved by the finite element method, and also the first paper in which the Mindlin plate model was used to examine simply supported, clamped, and point supported super-elliptical plates. In the current study sensitivity analysis was made to determine the influence of the thickness, the super-elliptical power, the aspect ratio, and the boundary conditions on the maximum deflection of moderately thick super-elliptical plates. Convergence studies were performed for h-refinement (i.e., more of the same kind of elements [69]), and the results were checked with the solutions of the limiting cases which are elliptical (e) and rectangular (r) plates.

2. FORMULATION

The boundary of the homogeneous and isotropic plate with uniform thickness is defined by

$$\left(\frac{x}{a}\right)^{2k} + \left(\frac{y}{b}\right)^{2k} = 1, \qquad k = 1, 2, ..., \infty \tag{1}$$

As k is raised, the shape becomes a rectangle with rounded corners, and therefore, the area of the middle surface of the plate increases with increasing k (Fig. 1). Four-noded isoparametric quadrilateral plate bending element with straight boundaries developed by Hughes et al. (1977) [70] was used in discretiziting the plate domain. The geometry of the element is identified by [71].

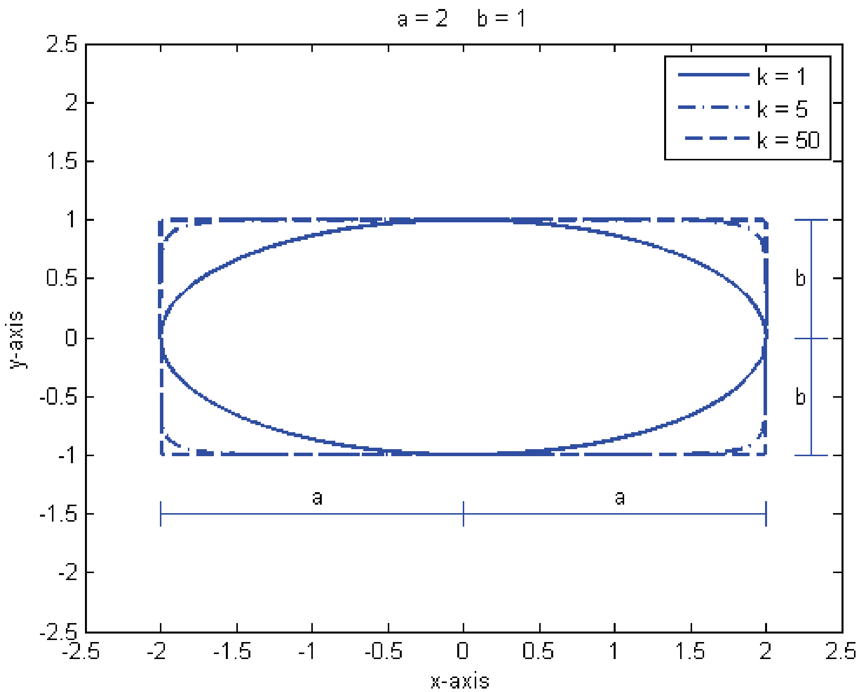

Figure 1. Geometry of a super-elliptical plate (c=2)

$$x = \sum_{i=1}^{4} N_i x_i, \qquad y = \sum_{i=1}^{4} N_i y_i, \qquad N_i = \frac{1}{4}(1 + rr_i)(1 + ss_i) \tag{2}$$

Each element has three field variables (i.e., degrees of freedom) per node. These field variables involve the deflection, and the rotations denoted by w, θ_x and θ_y, respectively.

$$w = \sum_{i=1}^{4} N_i w_i, \qquad \theta_x = \sum_{i=1}^{4} N_i \theta_{xi}, \qquad \theta_y = \sum_{i=1}^{4} N_i \theta_{yi} \tag{3}$$

The element shape functions are bilinear for transverse displacement and rotations. C^0 continuity for the displacement model was ensured based on the Mindlin's plate theory. The shear locking was prevented by separating the shear and bending energy terms and using selective integration procedure. The curvature and shear deformation vector $\{\varepsilon\}$ and the nodal displacement vector $\{d_i\}$ are related by [71]

$$\{d_i\}^T = \{w_i \quad \theta_{xi} \quad \theta_{yi}\}, \qquad \{\varepsilon\} = \sum_{i=1}^{4} [B_i]\{d_i\} \tag{4}$$

$$[B_i] = \begin{bmatrix} 0 & 0 & \frac{\partial N_i}{\partial x} \\ 0 & -\frac{\partial N_i}{\partial y} & 0 \\ 0 & -\frac{\partial N_i}{\partial x} & \frac{\partial N_i}{\partial y} \\ \frac{\partial N_i}{\partial x} & 0 & N_i \\ \frac{\partial N_i}{\partial y} & -N_i & 0 \end{bmatrix}, \qquad \{\varepsilon\} = \left\{ \begin{array}{c} k_x = \sum_{i=1}^{4} \theta_{yi} \frac{\partial N_i}{\partial x} \\ k_y = -\sum_{i=1}^{4} \theta_{xi} \frac{\partial N_i}{\partial y} \\ k_{xy} = \sum_{i=1}^{4} \theta_{yi} \frac{\partial N_i}{\partial y} - \sum_{i=1}^{4} \theta_{xi} \frac{\partial N_i}{\partial x} \\ \phi_x = \sum_{i=1}^{4} w_i \frac{\partial N_i}{\partial x} + \sum_{i=1}^{4} \theta_{yi} N_i \\ \phi_y = \sum_{i=1}^{4} w_i \frac{\partial N_i}{\partial y} - \sum_{i=1}^{4} \theta_{xi} N_i \end{array} \right\} \tag{5}$$

The element stiffness matrix is given by

$$[k_e] = \iint_A [B]^T [C][B] dxdy, \qquad [k_e] = [k_B] + [k_S] \tag{6}$$

where [71]

$$[B] = \big[[B_1] \quad [B_2] \quad [B_3] \quad [B_4]\big]_{5\times12}, \qquad [C] = \begin{bmatrix} [C_B]_{3\times3} & [0]_{3\times2} \\ [0]_{2\times3} & [C_S]_{2\times2} \end{bmatrix}_{5\times5} \tag{7}$$

$$D = \frac{Eh^3}{12(1-\nu^2)}, \qquad [C_B] = D \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{(1-\nu)}{2} \end{bmatrix}, \qquad G = \frac{E}{2(1+\nu)} \tag{8}$$

$$[C_S] = D_S \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \qquad D_s = Gh\kappa \tag{9}$$

The element stiffness matrix $[k_e]$ shown in Eq. (6) is composed of the bending stiffness $[k_B]$ and the shear stiffness $[k_S]$ parts for which numerical integrations are performed using Gauss quadrature with 2x2 [8] and 1x1 schemes, respectively [71].

The strain-displacement matrix [B] shown in Eq. (7) is formed by the matrix $[B_i]$ where i=1,2,3,4 such that the first three rows of $[B_i]$ relate the curvatures to displacements, and the last two rows of $[B_i]$ relate the shear deformations to displacements [73]. $[C_B]$ and $[C_S]$ are the bending and shear parts of the constitutive matrix [C] which represents rigidity.

3. ANALYSIS

Due to the two fold symmetry with respect to x and y axes the quarter of the plate was considered in the solution. The mesh pattern used in the paper is composed of nonuniform quadrilateral elements. The process of automatic mesh generation was executed by an algorithm which was coded in Python by the author (Fig. 2), and the figures were plotted using Matlab. Six cases including clamped (C), simply supported (S), and point-supported (PS) plates were investigated in the paper (Tables 1-2). For cases 1-2, the plate was considered to be resting on symmetrically distributed four point supports located on the plate contour and on the diagonals defined by $y = \pm(b/a)x$ such that for a large value of the super-elliptical power (such as k=200) the plate approximates to a corner supported rectangular plate. The transverse displacement was prevented at the point supports each of which was modelled by a line support of length $\Delta = 0.5/(3p)$ on the above mentioned diagonal lines.

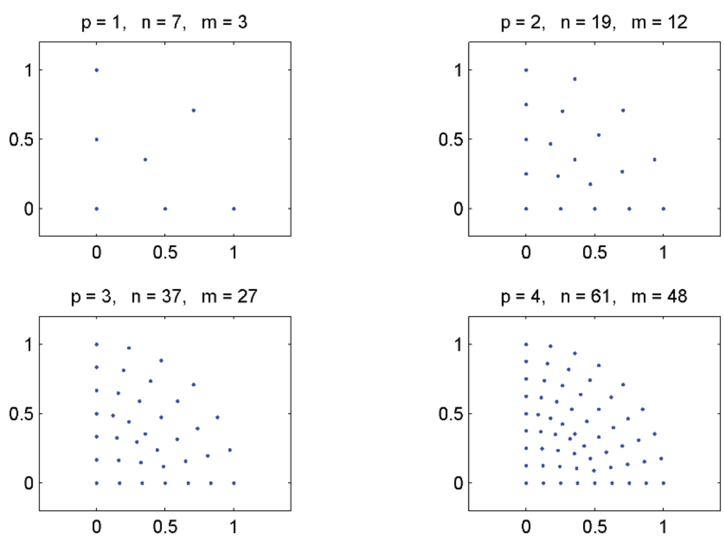

Figure 2. Location of the nodes in a quarter of the plate (k=1, c=1)

Two types of loading were considered: (i) uniformly distributed transverse pressure q, and (ii) a transverse central point load Q. The geometrical boundary conditions were satisfied exactly. The global nodal displacements were obtained by

$$[K]\{U\}=\{F\} \tag{10}$$

The simulation was carried out using the parameters defined by

$$c=\frac{a}{b},\qquad \eta=\frac{h}{b},\qquad \kappa=\frac{5}{6},\qquad \nu=0.3,\qquad W=w\frac{D}{qa^4},\qquad \lambda=w\frac{D}{Qa^2} \tag{11}$$

$$\mu=\frac{W_{(PS)}}{W_{(C)}}\frac{1}{c},\qquad \beta=\frac{\lambda_{(PS)}}{\lambda_{(C)}}\frac{1}{c},\qquad \Omega=Wa^4 \tag{12}$$

Table 1. Details of the numerical investigations

Support configuration	(PS)	(PS)	(C)	(C)	(S)	(S)
Type of loading	q	Q	q	Q	q	Q
Case number	1	2	3	4	5	6

Table 2. Number of meshes and nodes in a quarter of the plate

Support configuration	(PS)	(C)	(S)
Case number	1-2	3-4	5-6
m	4332	2028	2028
n	4447	2107	2107
p	38	26	26

4. NUMERICAL RESULTS

Convergence studies by h-refinement showed that the use of fine mesh is required for admissable accuracy. The accuracy of the results was validated by comparing the nondimensional maximum deflection with those of elliptical, rectangular, and super-elliptical plates (Tables 3-6). Some of the results cited in Tables 3-6 were scaled by the author, and some of them were computed by the author using the formulations given in the cited references. Good agreement was obtained in the comparison tests which were made for thin and moderately thick plates.

Basic information such as the method of solution and the range of the super-elliptical power used in the previous studies on the bending of super-elliptical plates was shown in Table 7.

Table 3. Comparison of nondimensional maximum deflection of thin elliptical and rectangular plates (η=0.002)

		Case 3	Case 3	Case 5	Case 5	Case 4	Case 6	
		(C)	(C)	(S)	(S)	(C)	(S)	
p	2k	W (c=1)	W (c=2)	W (c=1)	W (c=2)	λ (c=1)	λ (c=1)	Reference
	(e)	0.015625		0.063702		0.019894	0.050501	[27]
	(e)	0.015625		0.064103	0.009043			[72]
26	2	0.0156202	0.00211820	0.0636745	0.0088940	0.0198863	0.0504901	
25	2	0.0156198	0.00211820	0.0636723	0.0088937	0.0198855	0.0504891	
24	2	0.0156193	0.00211820	0.0636697	0.0088934	0.0198846	0.0504879	
	(r)	0.02016					0.0464	[72]
	(r)	0.02032						[2]
	(r)			0.064992	0.01013		0.0464	[27]
26	400	0.0202462	0.0025333	0.0650483	0.0101323	0.0224422	0.0464279	
25	400	0.0202463	0.00253330	0.0650475	0.0101322	0.0224415	0.0464270	
24	400	0.0202463	0.00253330	0.0650469	0.0101322	0.0224408	0.0464265	

Table 4. Comparison of nondimensional maximum deflection W of shear deformable elliptical and rectangular plates

p	c	2k	η=0.010	η=0.020	η=0.050	η=0.100	Reference	Case
	1	(e)			0.01578		[21]	3
	1	(e)	0.01561	0.01564	0.01578	0.01633	[29]	3
26	1	2	0.0156271	0.0156485	0.0157985	0.0163341		3
25	1	2	0.0156266	0.0156481	0.0157981	0.0163337		3
24	1	2	0.0156262	0.0156476	0.0157976	0.0163332		3
	1	(r)	0.020256	0.0202864		0.0212368	[26]	3
26	1	400	0.020256	0.0202865	0.0204978	0.0212373		3
25	1	400	0.0202561	0.0202865	0.0204978	0.0212374		3
24	1	400	0.0202561	0.0202866	0.0204979	0.0212375		3
	2	(r)	0.0025339	0.0025366		0.0026236	[26]	3
26	2	400	0.0025342	0.0025369	0.0025560	0.0026239		3
25	2	400	0.0025342	0.0025369	0.0025560	0.0026239		3
24	2	400	0.0025342	0.0025370	0.0025560	0.0026239		3
	1	(e)				0.06442	[22]	5
26	1	2	0.0636814	0.0637028	0.0638528	0.0643885		5
25	1	2	0.0636791	0.0637005	0.0638505	0.0643862		5
24	1	2	0.0636766	0.0636980	0.0638480	0.0643836		5
	1	(r)		0.06496		0.06576	[24]	5
	1	(r)		0.065031		0.06584	[20]	5

Table 4. Comparison of nondimensional maximum deflection W of shear deformable elliptical and rectangular plates (continue)

p	c	2k	η=0.010	η=0.020	η=0.050	η=0.100	Reference	Case
26	1	400	0.0652761	0.0655787	0.0665933	0.0686209		5
25	1	400	0.0652752	0.0655777	0.0665922	0.0686198		5
24	1	400	0.0652743	0.0655766	0.0665909	0.0686184		5
	2	(r)		0.01013		0.01020	[24]	5
26	2	400	0.0101487	0.0101708	0.0102472	0.0104068		5
25	2	400	0.0101486	0.0101707	0.0102471	0.0104067		5
24	2	400	0.0101485	0.0101706	0.0102470	0.0104065		5
	1	(r)		0.40928			[25]	1
38	1	400	0.403840	0.405156	0.410184	0.422200		1
36	1	400	0.403740	0.405052	0.410063	0.422025		1
34	1	400	0.403629	0.404935	0.409928	0.421832		1
32	1	400	0.403503	0.404804	0.409776	0.421618		1

Table 5. Comparison of nondimensional maximum deflection of (PS) thin super-elliptical plates (η=0.002)

p	c	2k	W (Case 1)	λ (Case 2)	Reference
	1	2	0.0828		[60]
	1	2	0.0831	0.0564	[65]
	1	2	0.084		[19]
38	1	2	0.0828439	0.0567861	
36	1	2	0.0828088	0.0567855	
34	1	2	0.0827695	0.0567847	
	1	40	0.3739		[60]
38	1	40	0.372567	0.147073	
36	1	40	0.372474	0.147074	
34	1	40	0.372370	0.147076	
	1	100	0.3939	0.1522	[65]
38	1	100	0.392393	0.152677	
36	1	100	0.392299	0.152680	
34	1	100	0.392193	0.152683	
	1	(r)	0.3984		[72]
	1	(r)	0.4052		[18]
	1	(r)	0.40799	0.15654	[17]
	1	(r)	0.4081		[31, 28]

Table 5. Comparison of nondimensional maximum deflection of (PS) thin super-elliptical plates (η=0.002) (continue)

p	c	2k	W (Case 1)	λ (Case 2)	Reference
	1	(r)	0.4171		[15]
	1	(r)	0.4208		[16]
	1	(r)		0.15657	[36]
38	1	400	0.402925	0.155642	
36	1	400	0.402828	0.155645	
34	1	400	0.402720	0.155649	
	2	2	0.0497		[60]
38	2	2	0.0496860	0.0650117	
36	2	2	0.0496704	0.0650103	
34	2	2	0.0496531	0.0650088	
	2	40	0.2140		[60]
38	2	40	0.213463	0.170878	
36	2	40	0.213428	0.170876	
34	2	40	0.213388	0.170873	

Table 6. Comparison of the central deflection Ω of thin (C) super-elliptical plates under q (η=0.002)

c	d	k=1	k=2	k=4	k=6	k=8	k=10	Reference
1	2	0.01563	0.01375	0.00696	0.00335	0.00172	0.00096	[68]
1	2	0.01563	0.01375	0.00696	0.00335	0.00172	0.00096	[61]
1	4	0.01563	0.01817	0.01683	0.01404	0.01129	0.00900	[61]
1	6	0.01563	0.01945	0.02009	0.01991	0.01964	0.01934	[61]
1	8	0.01563	0.01971	0.02027	0.02024	0.02019	0.02017	[61]
1							0.02017	[63]
1			0.01971	0.02027			0.02017	[64]
1		0.0156202	0.0197669	0.0202216	0.0202425	0.0202450	0.0202456	p=26
1		0.0156198	0.0197666	0.0202215	0.0202425	0.0202450	0.0202456	p=25
1		0.0156193	0.0197662	0.0202214	0.0202425	0.0202450	0.0202456	p=24
2	2	0.03390	0.02783	0.01531	0.00864	0.00512	0.00319	[68]
2	2	0.03390	0.02783	0.01531	0.00864	0.00512	0.00319	[61]
2	4	0.03390	0.03682	0.03562	0.03198	0.02785	0.02391	[61]
2	6	0.03390	0.03927	0.04073	0.04101	0.04110	0.04102	[61]
2	8	0.03390	0.03973	0.04063	0.04063	0.04062	0.04064	[61]
2		0.03390	0.03973	0.04063			0.04064	[64]
2		0.0338912	0.039824	0.0404944	0.0405264	0.0405312	0.0405312	p=26
2		0.0338912	0.039824	0.0404944	0.0405264	0.0405312	0.0405312	p=25
2		0.0338912	0.039824	0.0404944	0.040528	0.0405312	0.0405328	p=24
3	2	0.03835	0.02983	0.01778	0.01167	0.00813	0.00587	[61]

Table 6. Comparison of the central deflection Ω of thin (C) super-elliptical plates under q (η=0.002) (continue)

c	d	k=1	k=2	k=4	k=6	k=8	k=10	Reference
3	4	0.03835	0.03891	0.03919	0.03747	0.03460	0.03139	[61]
3	6	0.03835	0.04116	0.04251	0.04338	0.04411	0.04462	[61]
3	8	0.03835	0.04157	0.04198	0.04191	0.04124	0.04189	[61]
3		0.0383535	0.0416583	0.0418777	0.0418851	0.0418851	0.0418851	p=26
3		0.0383535	0.0416583	0.0418851	0.0418851	0.0418851	0.0418851	p=25
3		0.0383535	0.0416583	0.0418851	0.0418851	0.0418851	0.0418851	p=24

Table 7. Publications on the bending of isotropic super-elliptical plates

Reference	k considered	Method	d	Support configuration
[60]	1, 2, 3, .., 19, 20, 300	Ritz	12	(PS)
[61]	1, 2, 4, 6, 8, 10	Galerkin	8	(C)
[65]	1, 2, 3, .., 19, 20, 50, 250	Ritz	18	(PS)
[62]	1, 2, 4, 6	New Double Side Approach		(C)
[63]	10	Galerkin	8	(C)
[64]	1, 2, 4, 10	Ritz	8	(S), (C)
[66]	1, 2, 3, .., 19, 20, 50, 200	Ritz	8	(PS)
[67]	2, 4, 6	Galerkin, Double Side Approach		(C)
[68]	1, 2, 4, 6, 8, 10	Ritz	2	(C)

Since the maximum deflection develops at the center of the plate, the numerical investigations for the cases presented in Table 1 regarding the nondimensional central deflection of shear deformable super-elliptical plates were made for several values of the parameter of thickness from η=0.002 to η=0.100 in Appendix A (Tables A1-A15). Each table was constructed to observe how the deflection trend -from an elliptical to a rectangular plate- is affected with the shape (the shape of the plate is defined by the super-elliptical power which controls the roundness of the corner) and with the aspect ratio. The deflection trend corresponding to two types of loading was depicted in the same table (Tables A1-A15). Clamped (C), simply supported (S), and point-supported (PS) super-elliptical plates were analyzed in Tables A1-A5, A6-A10, and A11-A15, respectively. A decreasing incrementation in the nondimensional central deflection of clamped (C) and point-supported (PS) plates with increasing super-elliptical power was identified in Tables A1-A5 and A11-A15. The bending behavior of simply supported (S) super-elliptical plates was examined in Tables A6-A10, and it was shown that for $k>3$ the nondimensional central deflection decreases with increasing super-elliptical power. It was detected that the deflection trend is not affected by the thickness of the plate.

5. CONCLUSIONS

Bending of moderately thick super-elliptical plates was examined based on the first order shear deformation theory (FSDT). The numerical simulation was made using the finite element method. Influence of the geometric properties of the plate on the maximum deflection was investigated via sensitivity analysis. Three support configurations were considered within the scope of the paper.

The bending results obtained for clamped super-elliptical plates under transverse uniform pressure in the current study were compared with those presented by Ceribasi et al. (2008) [61], Ceribasi (2012) [63], Zhang (2013) [64], and Altunsaray (2017) [68] (Table 7). In these publications, the trial function was constructed as the product of the basic function and a complete two dimensional polynomial function of degree d. Since the results corresponding to k>10 have not been presented in these publications, a relation between the maximum deflection of super-elliptical plates and that of rectangular plates is not available in the literature. Relatively fewer terms in the trial function were considered in these studies, and therefore the deflection parameter in the studies [61, 68] was found to be decreasing for c=1 and k>4 (Table 6).

However, the results in the current study reveal that the central deflection of a clamped super-elliptical plate lies in the range bounded by elliptical and rectangular plates as expected because the maximum deflection of a clamped square plate is larger than the central deflection of a clamped circular plate. This statement was also verified for super-elliptical plates under a transverse central point load (Tables A1-A5). Therefore, for both types of loading, W and λ increase with increasing k. It is worth noting that slight discrepancies which may possibly arise from truncation or rounding-off errors may be neglected for c=3 (Table A4, Case 3).

However, the case is different for a simply supported super-elliptical plate (Tables A6-A10). The answer may be found in the fact that “the central deflection of the circular plate is larger than that of the corresponding square plate. This result may be attributed to the action of the reactive forces concentrated at the corners of the square plate which have the tendency to produce deflection of the plate convex upward” [72]. Therefore, based on the results of this study, it can be stated that interpolation may be used for the prediction of the deflection of clamped super-elliptical plates, but it should not be used for simply supported super-elliptical plates. The super-elliptical power corresponding to the maximum deflection increases with increasing aspect ratio. Beyond these specific value of k as k is raised, W and λ decrease.

As far as the author knows there has been no published paper on point-supported shear deformable super-elliptical plates. Therefore, the accuracy of the results in the current study was validated through comparison with the results of thin plates, and good agreement was obtained (Tables 4-5). The maximum deflection of a corner-supported super-elliptical plate lies in the range bounded by elliptical and rectangular plates (Table A11-A15). Consequently, interpolation may be used. Compared to simply supported plates, the transverse displacement of corner supported plates is larger, but the difference gets smaller with decreasing super-elliptical power.

The numerical simulations reveal that for the loadings and for the support configurations considered in the study, the linear bending response of (C) and (PS) super-elliptical plates is similar to each other (Figs. 3-4).

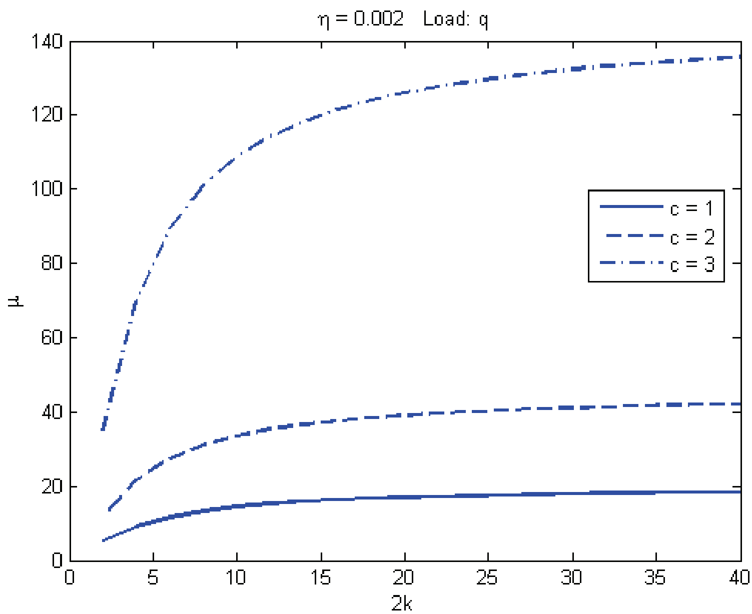

Figure 3. Bending response of (PS) and (C) super-elliptical plates under q

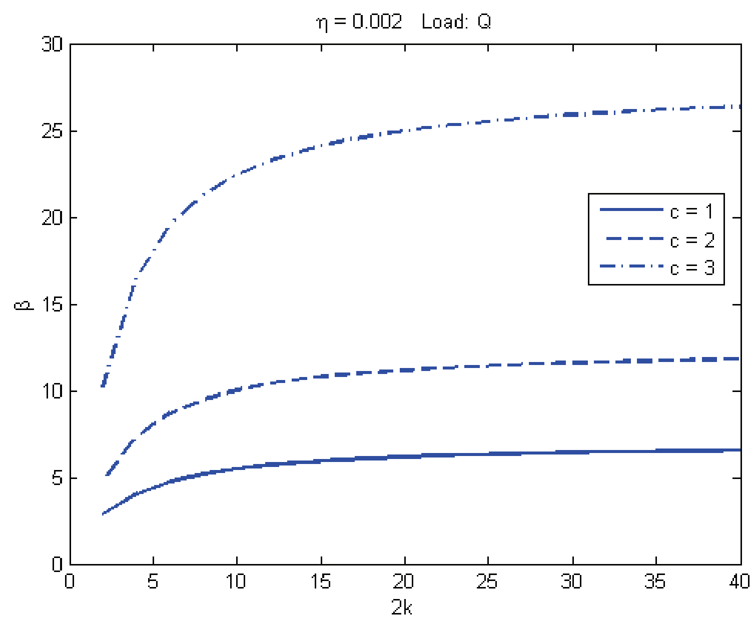

Figure 4. Bending response of (PS) and (C) super-elliptical plates under Q

It was shown that computation with high rate of convergence is required to determine the trend of the relation between the maximum deflection and the super-elliptical power (Tables 2-4). Especially for (S) and (C) plates, considering relatively fewer terms in the trial function, may lead to loss of precision due to low rate of convergence, and therefore the aforementioned trend may not be obtained with admissible accuracy.

The quadrilateral element and the mesh pattern used in the study are capable of simulating the bending response of super-elliptical plates efficiently. The boundary conditions have considerable importance on the results. The results reveal that super-elliptical plates require extensive computational effort in comparison with elliptical and rectangular plates.

Symbols

a, b, c	: semi-major, and semi-minor axes of the plate, aspect ratio
d	: degree of the complete two dimensional polynomial function
h	: thickness of the plate
k, m	: super-elliptical power, number of meshes in the quarter of the plate
n, p	: number of nodes and number of partitions in the quarter of the plate
q, w	: uniform transverse pressure, deflection
D, E, G, Q	: flexural rigidity, Young's modulus, shear modulus, transverse point load
W	: nondimensional deflection under uniform transverse pressure q
k_x , k_y, k_{xy}	: curvatures, twist
r_i, s_i	: local coordinates of the i-th node (i=1, 2, 3, 4)
D_s, N_i	: shear rigidity, shape function (i=1, 2, 3, 4)
κ, η, ν	: shear correction factor , parameter of thickness, Poisson's ratio
θ_x, θ_y	: rotations
λ	: nondimensional deflection under central point load Q
ϕ_x, ϕ_y	: average shear deformations
$[k_e]$, [K]	: element, and global stiffness matrices
[B], [C]	: strain-displacement matrix, constitutive matrix
{F}, {U}	: global nodal load vector, global displacement vector
$[k_B]$, $[k_S]$	: bending, and shear stiffness part of $[k_e]$
$[C_B]$, $[C_S]$	: bending, and shear deformation part of [C]
$W_{(PS)}$, $W_{(C)}$	: nondimensional deflection of (PS) and (C) plates under q
$\lambda_{(PS)}$, $\lambda_{(C)}$	: nondimensional deflection of (PS) and (C) plates under Q
$\{\varepsilon\}$, $\{d_i\}$	: curvature and shear deformation vector, nodal displacement vector

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Appendix A

Table A1. Nondimensional central deflection of (C) super-elliptical plates (p=26, η=0.002)

	W	W	W	λ	λ	λ
2k	c=1	c=2	c=3	c=1	c=2	c=3
2	0.0156202	0.0021182	0.0004735	0.0198863	0.0068442	0.0031454
4	0.0197669	0.002489	0.0005143	0.0222393	0.0072021	0.0032137
6	0.0201555	0.0025246	0.0005168	0.0224094	0.0072232	0.0032157
8	0.0202216	0.0025309	0.0005170	0.0224342	0.007226	0.0032159
10	0.0202377	0.0025324	0.0005171	0.0224396	0.0072265	0.0032159
12	0.0202425	0.0025329	0.0005171	0.0224411	0.0072267	0.0032159
14	0.0202443	0.0025331	0.0005171	0.0224416	0.0072267	0.0032159
16	0.020245	0.0025332	0.0005171	0.0224418	0.0072267	0.0032159
18	0.0202454	0.0025332	0.0005171	0.0224419	0.0072267	0.0032159
20	0.0202456	0.0025332	0.0005171	0.0224419	0.0072267	0.0032159
22	0.0202457	0.0025332	0.0005171	0.0224419	0.0072267	0.0032159
24	0.0202457	0.0025332	0.0005171	0.022442	0.0072267	0.0032159
26	0.0202458	0.0025332	0.0005171	0.022442	0.0072267	0.0032159
28	0.0202458	0.0025332	0.0005171	0.022442	0.0072267	0.0032159
30	0.0202458	0.0025332	0.0005171	0.022442	0.0072267	0.0032159
32	0.0202459	0.0025333	0.0005171	0.022442	0.0072267	0.0032159
34	0.0202459	0.0025333	0.0005171	0.022442	0.0072267	0.0032159
36	0.0202459	0.0025333	0.0005171	0.022442	0.0072267	0.0032159
38	0.0202459	0.0025333	0.0005171	0.022442	0.0072267	0.0032159
40	0.020246	0.0025333	0.0005171	0.022442	0.0072267	0.0032159
100	0.0202461	0.0025333	0.0005171	0.0224421	0.0072268	0.0032159
400	0.0202462	0.0025333	0.0005171	0.0224422	0.0072268	0.0032159

Table A2. Nondimensional central deflection of (C) super-elliptical plates (p=26, η=0.010)

	W	W	W	λ	λ	λ
2k	c=1	c=2	c=3	c=1	c=2	c=3
2	0.0156271	0.002119	0.0004736	0.0199322	0.0068554	0.0031502
4	0.0197758	0.0024899	0.0005145	0.0222873	0.0072138	0.0032187
6	0.0201651	0.0025255	0.0005169	0.0224581	0.007235	0.0032207
8	0.0202314	0.0025318	0.0005172	0.0224833	0.0072378	0.0032209
10	0.0202475	0.0025333	0.0005173	0.0224888	0.0072384	0.0032209
12	0.0202524	0.0025338	0.0005173	0.0224904	0.0072385	0.0032209
14	0.0202541	0.002534	0.0005173	0.022491	0.0072386	0.0032209
16	0.0202549	0.002534	0.0005173	0.0224912	0.0072386	0.0032209
18	0.0202552	0.0025341	0.0005173	0.0224914	0.0072386	0.0032209
20	0.0202554	0.0025341	0.0005173	0.0224914	0.0072386	0.0032209

Table A2. Nondimensional central deflection of (C) super-elliptical plates (p=26, η=0.010) (continue)

	W	W	W	λ	λ	λ
2k	c=1	c=2	c=3	c=1	c=2	c=3
22	0.0202555	0.0025341	0.0005173	0.0224915	0.0072386	0.0032209
24	0.0202556	0.0025341	0.0005173	0.0224915	0.0072386	0.0032209
26	0.0202556	0.0025341	0.0005173	0.0224916	0.0072386	0.0032209
28	0.0202557	0.0025341	0.0005173	0.0224916	0.0072386	0.0032209
30	0.0202557	0.0025341	0.0005173	0.0224916	0.0072386	0.0032209
32	0.0202557	0.0025341	0.0005173	0.0224916	0.0072387	0.0032209
34	0.0202557	0.0025341	0.0005173	0.0224916	0.0072387	0.0032209
36	0.0202557	0.0025341	0.0005173	0.0224917	0.0072387	0.0032209
38	0.0202558	0.0025341	0.0005173	0.0224917	0.0072387	0.0032209
40	0.0202558	0.0025341	0.0005173	0.0224917	0.0072387	0.0032209
100	0.0202559	0.0025342	0.0005173	0.0224918	0.0072387	0.0032209
400	0.020256	0.0025342	0.0005173	0.0224919	0.0072387	0.0032209

Table A3. Nondimensional central deflection of (C) super-elliptical plates (p=26, η=0.020)

	W	W	W	λ	λ	λ
2k	c=1	c=2	c=3	c=1	c=2	c=3
2	0.0156485	0.0021213	0.0004741	0.0200758	0.0068904	0.0031651
4	0.0198036	0.0024925	0.0005150	0.0224373	0.007250	0.0032341
6	0.0201946	0.0025282	0.0005175	0.0226103	0.0072716	0.0032363
8	0.0202615	0.0025345	0.0005178	0.0226363	0.0072745	0.0032365
10	0.0202778	0.002536	0.0005178	0.0226423	0.0072752	0.0032366
12	0.0202828	0.0025365	0.0005178	0.0226442	0.0072754	0.0032366
14	0.0202846	0.0025367	0.0005178	0.022645	0.0072755	0.0032366
16	0.0202853	0.0025368	0.0005178	0.0226454	0.0072756	0.0032366
18	0.0202857	0.0025368	0.0005178	0.0226457	0.0072756	0.0032366
20	0.0202859	0.0025368	0.0005178	0.0226458	0.0072756	0.0032366
22	0.020286	0.0025369	0.0005178	0.022646	0.0072756	0.0032367
24	0.020286	0.0025369	0.0005178	0.0226461	0.0072757	0.0032367
26	0.0202861	0.0025369	0.0005178	0.0226461	0.0072757	0.0032367
28	0.0202861	0.0025369	0.0005178	0.0226462	0.0072757	0.0032367
30	0.0202862	0.0025369	0.0005178	0.0226463	0.0072757	0.0032367
32	0.0202862	0.0025369	0.0005178	0.0226463	0.0072757	0.0032367
34	0.0202862	0.0025369	0.0005178	0.0226464	0.0072757	0.0032367
36	0.0202862	0.0025369	0.0005178	0.0226464	0.0072757	0.0032367
38	0.0202862	0.0025369	0.0005178	0.0226465	0.0072757	0.0032367
40	0.0202863	0.0025369	0.0005178	0.0226465	0.0072757	0.0032367
100	0.0202864	0.0025369	0.0005178	0.0226469	0.0072758	0.0032367
400	0.0202865	0.0025369	0.0005178	0.0226471	0.0072758	0.0032367

Table A4. Nondimensional central deflection of (C) super-elliptical plates (p=26, η=0.050)

	W	W	W	λ	λ	λ
2k	c=1	c=2	c=3	c=1	c=2	c=3
2	0.0157985	0.0021374	0.0004776	0.0210805	0.007135	0.0032692
4	0.0199967	0.002511	0.0005188	0.0234863	0.0075038	0.0033422
6	0.0203998	0.0025472	0.0005212	0.023674	0.0075277	0.0033453
8	0.0204702	0.0025535	0.0005215	0.0237062	0.0075316	0.0033459
10	0.0204879	0.0025551	0.0005215	0.0237155	0.0075329	0.0033461
12	0.0204934	0.0025556	0.0005216	0.0237195	0.0075335	0.0033463
14	0.0204955	0.0025558	0.0005216	0.0237218	0.0075338	0.0033464
16	0.0204964	0.0025559	0.0005215	0.0237232	0.0075341	0.0033465
18	0.0204968	0.0025559	0.0005215	0.0237243	0.0075343	0.0033465
20	0.0204971	0.0025559	0.0005215	0.0237251	0.0075344	0.0033466
22	0.0204972	0.002556	0.0005215	0.0237258	0.0075346	0.0033466
24	0.0204973	0.002556	0.0005215	0.0237264	0.0075347	0.0033467
26	0.0204974	0.002556	0.0005215	0.0237268	0.0075347	0.0033467
28	0.0204974	0.002556	0.0005215	0.0237272	0.0075348	0.0033467
30	0.0204974	0.002556	0.0005215	0.0237276	0.0075349	0.0033467
32	0.0204975	0.002556	0.0005215	0.0237279	0.0075349	0.0033467
34	0.0204975	0.002556	0.0005215	0.0237282	0.007535	0.0033467
36	0.0204975	0.002556	0.0005215	0.0237284	0.007535	0.0033468
38	0.0204975	0.002556	0.0005215	0.0237286	0.0075351	0.0033468
40	0.0204975	0.002556	0.0005215	0.0237288	0.0075351	0.0033468
100	0.0204977	0.002556	0.0005215	0.0237311	0.0075355	0.0033469
400	0.0204978	0.002556	0.0005215	0.0237324	0.0075357	0.003347

Table A5. Nondimensional central deflection of (C) super-elliptical plates (p=26, η=0.100)

	W	W	W	λ	λ	λ
2k	c=1	c=2	c=3	c=1	c=2	c=3
2	0.0163341	0.002195	0.0004902	0.0246689	0.0080081	0.0036407
4	0.0206759	0.0025769	0.0005322	0.0272251	0.0084088	0.0037277
6	0.0211185	0.0026145	0.0005347	0.027463	0.008441	0.003734
8	0.021201	0.0026212	0.0005349	0.0275167	0.0084486	0.0037359
10	0.0212231	0.0026229	0.0005350	0.0275379	0.008452	0.0037369
12	0.0212306	0.0026235	0.0005350	0.0275494	0.0084539	0.0037375
14	0.0212337	0.0026237	0.0005350	0.0275568	0.0084553	0.0037379
16	0.0212351	0.0026238	0.0005350	0.0275622	0.0084563	0.0037382
18	0.0212358	0.0026238	0.0005350	0.0275662	0.008457	0.0037385
20	0.0212362	0.0026238	0.0005350	0.0275694	0.0084576	0.0037387
22	0.0212365	0.0026239	0.0005350	0.027572	0.0084581	0.0037388
24	0.0212366	0.0026239	0.0005350	0.0275742	0.0084585	0.003739

Table A5. Nondimensional central deflection of (C) super-elliptical plates (p=26, η=0.100) (continue)

	W	W	W	λ	λ	λ
2k	c=1	c=2	c=3	c=1	c=2	c=3
26	0.0212367	0.0026239	0.0005350	0.0275761	0.0084588	0.0037391
28	0.0212368	0.0026239	0.0005350	0.0275776	0.0084591	0.0037392
30	0.0212369	0.0026239	0.0005350	0.027579	0.0084593	0.0037393
32	0.0212369	0.0026239	0.0005350	0.0275802	0.0084596	0.0037393
34	0.021237	0.0026239	0.0005350	0.0275812	0.0084598	0.0037394
36	0.021237	0.0026239	0.0005350	0.0275822	0.0084599	0.0037394
38	0.021237	0.0026239	0.0005350	0.027583	0.0084601	0.0037395
40	0.021237	0.0026239	0.0005350	0.0275838	0.0084602	0.0037395
100	0.0212372	0.0026239	0.0005350	0.0275928	0.0084617	0.003740
400	0.0212373	0.0026239	0.0005350	0.0275979	0.0084627	0.0037403

Table A6. Nondimensional central deflection of (S) super-elliptical plates (p=26, η=0.002)

	W	W	W	W	λ	λ	λ	λ
2k	c=1	c=2	c=3	c=5	c=1	c=2	c=3	c=5
2	0.0636745	0.008894	0.0021009	0.0003069	0.0504901	0.0165024	0.0074299	0.0026975
4	0.0729315	0.0105591	0.0024185	0.0003301	0.0514505	0.0170786	0.0075629	0.0027121
6	0.0702026	0.0104952	0.0024387	0.0003319	0.0493006	0.0168586	0.0075473	0.0027114
8	0.0684491	0.0103899	0.0024357	0.0003322	0.048222	0.016735	0.0075382	0.0027111
10	0.0674217	0.0103182	0.0024315	0.0003322	0.0476429	0.0166673	0.0075333	0.002711
12	0.0667878	0.0102711	0.0024282	0.0003322	0.0473018	0.0166272	0.0075306	0.002711
14	0.0663739	0.0102393	0.0024258	0.0003322	0.0470854	0.0166018	0.0075289	0.0027109
16	0.0660904	0.0102171	0.0024241	0.0003322	0.0469402	0.0165847	0.0075277	0.0027109
18	0.0658883	0.0102011	0.0024228	0.0003322	0.0468381	0.0165727	0.0075269	0.0027109
20	0.0657395	0.0101892	0.0024218	0.0003322	0.0467637	0.016564	0.0075264	0.0027109
22	0.0656269	0.0101801	0.0024211	0.0003322	0.0467079	0.0165574	0.0075259	0.0027108
24	0.0655398	0.010173	0.0024205	0.0003322	0.0466649	0.0165524	0.0075256	0.0027108
26	0.0654709	0.0101674	0.002420	0.0003322	0.0466312	0.0165485	0.0075254	0.0027108
28	0.0654157	0.0101629	0.0024196	0.0003322	0.0466043	0.0165453	0.0075252	0.0027108
30	0.0653707	0.0101592	0.0024193	0.0003322	0.0465824	0.0165427	0.007525	0.0027108
32	0.0653335	0.0101561	0.002419	0.0003322	0.0465644	0.0165406	0.0075248	0.0027108
34	0.0653025	0.0101536	0.0024188	0.0003322	0.0465494	0.0165389	0.0075247	0.0027108
36	0.0652763	0.0101514	0.0024186	0.0003322	0.0465368	0.0165374	0.0075246	0.0027108
38	0.0652541	0.0101496	0.0024185	0.0003322	0.0465261	0.0165362	0.0075246	0.0027108
40	0.065235	0.010148	0.0024183	0.0003322	0.0465169	0.0165351	0.0075245	0.0027108
100	0.0650823	0.0101352	0.0024172	0.0003321	0.046444	0.0165265	0.0075239	0.0027108
400	0.0650483	0.0101323	0.002417	0.0003321	0.0464279	0.0165246	0.0075238	0.0027108

Table A7. Nondimensional central deflection of (S) super-elliptical plates (p=26, η=0.010)

	W	W	W	W	λ	λ	λ	λ
2k	c=1	c=2	c=3	c=5	c=1	c=2	c=3	c=5
2	0.0636814	0.0088986	0.002102	0.0003070	0.0505361	0.0165219	0.0074394	0.002701
4	0.0730054	0.0105663	0.0024195	0.0003301	0.0515375	0.0171014	0.0075729	0.0027157
6	0.0703163	0.0105047	0.0024398	0.0003320	0.0494094	0.0168836	0.0075575	0.0027149
8	0.0685856	0.0104007	0.0024369	0.0003323	0.0483427	0.0167611	0.0075484	0.0027147
10	0.0675735	0.0103299	0.0024328	0.0003323	0.0477714	0.0166941	0.0075437	0.0027146
12	0.0669505	0.0102835	0.0024296	0.0003323	0.0474359	0.0166546	0.0075409	0.0027145
14	0.066545	0.0102523	0.0024273	0.0003323	0.0472238	0.0166295	0.0075392	0.0027145
16	0.0662681	0.0102305	0.0024255	0.0003323	0.0470818	0.0166128	0.0075381	0.0027145
18	0.0660714	0.0102148	0.0024243	0.0003323	0.0469823	0.016601	0.0075374	0.0027144
20	0.0659271	0.0102031	0.0024233	0.0003322	0.0469101	0.0165925	0.0075368	0.0027144
22	0.0658182	0.0101942	0.0024226	0.0003322	0.0468562	0.0165861	0.0075364	0.0027144
24	0.0657343	0.0101874	0.002422	0.0003322	0.0468148	0.0165812	0.0075361	0.0027144
26	0.0656682	0.0101819	0.0024216	0.0003322	0.0467824	0.0165774	0.0075358	0.0027144
28	0.0656154	0.0101775	0.0024212	0.0003322	0.0467566	0.0165743	0.0075356	0.0027144
30	0.0655725	0.010174	0.0024209	0.0003322	0.0467358	0.0165719	0.0075355	0.0027144
32	0.0655372	0.010171	0.0024206	0.0003322	0.0467187	0.0165699	0.0075353	0.0027144
34	0.0655078	0.0101686	0.0024204	0.0003322	0.0467045	0.0165682	0.0075352	0.0027144
36	0.0654832	0.0101665	0.0024202	0.0003322	0.0466926	0.0165668	0.0075352	0.0027144
38	0.0654623	0.0101647	0.0024201	0.0003322	0.0466825	0.0165656	0.0075351	0.0027144
40	0.0654444	0.0101632	0.0024199	0.0003322	0.046674	0.0165645	0.007535	0.0027144
100	0.0653053	0.0101513	0.0024189	0.0003322	0.0466076	0.0165566	0.0075345	0.0027143
400	0.0652761	0.0101487	0.0024187	0.0003322	0.0465938	0.0165548	0.0075344	0.0027143

Table A8. Nondimensional central deflection of (S) super-elliptical plates (p=26, η=0.020)

	W	W	W	W	λ	λ	λ	λ
2k	c=1	c=2	c=3	c=5	c=1	c=2	c=3	c=5
2	0.0637028	0.0089057	0.0021037	0.0003072	0.0506796	0.0165669	0.007460	0.0027082
4	0.0731182	0.010577	0.0024212	0.0003302	0.0517381	0.0171516	0.0075944	0.0027232
6	0.0704814	0.0105183	0.0024416	0.0003321	0.0496391	0.0169366	0.0075793	0.0027225
8	0.0687806	0.0104162	0.0024389	0.0003324	0.0485882	0.0168157	0.0075704	0.0027222
10	0.0677882	0.0103466	0.0024348	0.0003324	0.0480272	0.0167497	0.0075658	0.0027221
12	0.0671794	0.010301	0.0024317	0.0003324	0.0476989	0.0167109	0.0075631	0.0027221
14	0.0667846	0.0102705	0.0024294	0.0003324	0.0474922	0.0166863	0.0075615	0.0027221
16	0.066516	0.0102492	0.0024277	0.0003324	0.0473544	0.01667	0.0075604	0.002722
18	0.0663261	0.0102339	0.0024265	0.0003324	0.0472584	0.0166585	0.0075597	0.002722
20	0.0661872	0.0102225	0.0024256	0.0003323	0.0471889	0.0166503	0.0075591	0.002722
22	0.066083	0.010214	0.0024249	0.0003323	0.0471372	0.0166441	0.0075587	0.002722
24	0.0660029	0.0102073	0.0024243	0.0003323	0.0470978	0.0166394	0.0075584	0.002722
26	0.0659401	0.0102021	0.0024239	0.0003323	0.0470671	0.0166357	0.0075582	0.002722
28	0.0658901	0.0101979	0.0024235	0.0003323	0.0470427	0.0166328	0.007558	0.002722
30	0.0658497	0.0101945	0.0024232	0.0003323	0.0470231	0.0166304	0.0075578	0.002722
32	0.0658166	0.0101916	0.002423	0.0003323	0.047007	0.0166285	0.0075577	0.002722
34	0.0657891	0.0101893	0.0024228	0.0003323	0.0469938	0.0166269	0.0075576	0.002722
36	0.0657662	0.0101873	0.0024226	0.0003323	0.0469827	0.0166255	0.0075575	0.002722
38	0.0657467	0.0101857	0.0024224	0.0003323	0.0469734	0.0166244	0.0075575	0.002722
40	0.0657302	0.0101842	0.0024223	0.0003323	0.0469655	0.0166234	0.0075574	0.002722
100	0.0656044	0.0101732	0.0024213	0.0003323	0.0469057	0.0166161	0.0075569	0.0027219
400	0.0655787	0.0101708	0.0024211	0.0003323	0.0468937	0.0166146	0.0075568	0.0027219

Table A9. Nondimensional central deflection of (S) super-elliptical plates (p=26, η=0.050)

	W	W	W	W	λ	λ	λ	λ
2k	c=1	c=2	c=3	c=5	c=1	c=2	c=3	c=5
2	0.0638528	0.0089362	0.0021108	0.0003079	0.0516844	0.01684	0.0075802	0.0027496
4	0.0735879	0.0106203	0.0024283	0.0003308	0.0529488	0.0174451	0.0077195	0.0027663
6	0.0711233	0.0105711	0.0024492	0.0003327	0.0509485	0.0172402	0.0077058	0.0027658
8	0.0695174	0.0104746	0.0024469	0.0003329	0.0499498	0.0171247	0.0076976	0.0027657
10	0.0685858	0.0104087	0.0024431	0.0003330	0.0494214	0.0170621	0.0076934	0.0027657
12	0.0680194	0.0103658	0.0024402	0.0003330	0.0491157	0.0170255	0.007691	0.0027657
14	0.0676557	0.0103372	0.002438	0.0003329	0.0489254	0.0170026	0.0076895	0.0027657
16	0.0674107	0.0103174	0.0024365	0.0003329	0.04880	0.0169875	0.0076886	0.0027657
18	0.0672392	0.0103032	0.0024353	0.0003329	0.0487136	0.0169771	0.0076879	0.0027657
20	0.067115	0.0102929	0.0024345	0.0003329	0.0486518	0.0169696	0.0076875	0.0027657
22	0.0670225	0.010285	0.0024338	0.0003329	0.0486064	0.016964	0.0076871	0.0027657
24	0.0669521	0.010279	0.0024333	0.0003329	0.048572	0.0169598	0.0076869	0.0027657
26	0.0668974	0.0102743	0.0024329	0.0003329	0.0485455	0.0169566	0.0076867	0.0027657
28	0.0668542	0.0102706	0.0024326	0.0003329	0.0485246	0.016954	0.0076865	0.0027657
30	0.0668194	0.0102675	0.0024323	0.0003329	0.048508	0.016952	0.0076864	0.0027657
32	0.0667911	0.010265	0.0024321	0.0003329	0.0484945	0.0169503	0.0076863	0.0027657
34	0.0667678	0.010263	0.0024319	0.0003329	0.0484834	0.0169489	0.0076863	0.0027657
36	0.0667484	0.0102613	0.0024318	0.0003329	0.0484743	0.0169478	0.0076862	0.0027657
38	0.0667321	0.0102598	0.0024317	0.0003329	0.0484666	0.0169468	0.0076861	0.0027657
40	0.0667183	0.0102586	0.0024315	0.0003329	0.0484601	0.016946	0.0076861	0.0027657
100	0.0666152	0.0102492	0.0024307	0.0003329	0.048413	0.0169401	0.0076858	0.0027657
400	0.0665933	0.0102472	0.0024305	0.0003329	0.0484038	0.016939	0.0076857	0.0027657

Table A10. Nondimensional central deflection of (S) super-elliptical plates (p=26, η=0.100)

	W	W	W	W	λ	λ	λ	λ
2k	c=1	c=2	c=3	c=5	c=1	c=2	c=3	c=5
2	0.0643885	0.009017	0.002129	0.0003101	0.0552728	0.017755	0.0079749	0.0028843
4	0.0747886	0.010729	0.0024473	0.0003328	0.0569857	0.0184107	0.0081293	0.0029066
6	0.0726465	0.0106973	0.0024692	0.0003345	0.0551806	0.0182273	0.0081196	0.0029072
8	0.071204	0.0106105	0.0024675	0.0003348	0.054278	0.0181226	0.0081132	0.0029074
10	0.0703689	0.0105507	0.0024642	0.0003348	0.0538061	0.0180664	0.008110	0.0029076
12	0.0698649	0.0105118	0.0024615	0.0003348	0.0535368	0.018034	0.0081083	0.0029077
14	0.0695437	0.010486	0.0024596	0.0003348	0.0533716	0.0180141	0.0081073	0.0029078
16	0.069329	0.0104683	0.0024582	0.0003348	0.0532641	0.0180011	0.0081067	0.0029079
18	0.0691795	0.0104557	0.0024572	0.0003348	0.053191	0.0179923	0.0081063	0.0029079
20	0.0690719	0.0104466	0.0024565	0.0003348	0.0531394	0.0179861	0.008106	0.002908
22	0.0689921	0.0104397	0.0024559	0.0003348	0.0531018	0.0179815	0.0081058	0.002908
24	0.0689316	0.0104344	0.0024555	0.0003348	0.0530736	0.0179781	0.0081057	0.002908
26	0.0688847	0.0104303	0.0024551	0.0003348	0.0530521	0.0179755	0.0081056	0.002908
28	0.0688476	0.010427	0.0024548	0.0003348	0.0530353	0.0179735	0.0081056	0.0029081
30	0.0688179	0.0104244	0.0024546	0.0003348	0.0530221	0.0179719	0.0081055	0.0029081
32	0.0687938	0.0104223	0.0024544	0.0003348	0.0530114	0.0179707	0.0081055	0.0029081
34	0.0687739	0.0104205	0.0024542	0.0003348	0.0530027	0.0179696	0.0081055	0.0029081
36	0.0687573	0.010419	0.0024541	0.0003348	0.0529956	0.0179688	0.0081055	0.0029081
38	0.0687434	0.0104177	0.002454	0.0003348	0.0529896	0.0179681	0.0081055	0.0029081
40	0.0687316	0.0104167	0.0024539	0.0003348	0.0529846	0.0179675	0.0081055	0.0029081
100	0.0686419	0.0104086	0.0024531	0.0003348	0.0529503	0.0179636	0.0081055	0.0029082
400	0.0686209	0.0104068	0.002453	0.0003348	0.0529454	0.0179633	0.0081057	0.0029083

Table A11. Nondimensional central deflection of (PS) super-elliptical plates (p=38, η=0.002)

	W	W	W	λ	λ	λ
2k	c=1	c=2	c=3	c=1	c=2	c=3
2	0.0828439	0.0496860	0.0498087	0.0567861	0.0650117	0.0965172
4	0.180261	0.107651	0.107750	0.0900652	0.105834	0.158468
6	0.234681	0.138874	0.138519	0.106860	0.125763	0.188297
8	0.268146	0.157613	0.156803	0.116866	0.137362	0.205496
10	0.290677	0.170016	0.168817	0.123500	0.144927	0.216637
12	0.306864	0.178814	0.177292	0.128222	0.150247	0.224432
14	0.319056	0.185377	0.183586	0.131756	0.154192	0.230189
16	0.328572	0.190458	0.188441	0.134503	0.157234	0.234613
18	0.336207	0.194510	0.192301	0.136699	0.159652	0.238120
20	0.342471	0.197816	0.195442	0.138496	0.161620	0.240968
22	0.347704	0.200565	0.198049	0.139994	0.163254	0.243327
24	0.352142	0.202888	0.200246	0.141262	0.164631	0.245313
26	0.355953	0.204875	0.202124	0.142349	0.165808	0.247007
28	0.359263	0.206596	0.203747	0.143293	0.166826	0.248471
30	0.362163	0.208100	0.205164	0.144118	0.167716	0.249747
32	0.364727	0.209426	0.206412	0.144847	0.168499	0.250870
34	0.367010	0.210605	0.207519	0.145496	0.169194	0.251865
36	0.369055	0.211658	0.208508	0.146077	0.169815	0.252754
38	0.370898	0.212606	0.209397	0.146600	0.170373	0.253553
40	0.372567	0.213463	0.210200	0.147073	0.170878	0.254275
100	0.392393	0.223530	0.219583	0.152677	0.176788	0.262675
400	0.402925	0.228789	0.224440	0.155642	0.179862	0.267007

Table A12. Nondimensional central deflection of (PS) super-elliptical plates (p=38, η=0.010)

	W	W	W	λ	λ	λ
2k	c=1	c=2	c=3	c=1	c=2	c=3
2	0.0829997	0.0497447	0.0498487	0.0568923	0.0650605	0.0965594
4	0.180465	0.107733	0.107802	0.0901769	0.105890	0.158516
6	0.234941	0.138966	0.138575	0.106984	0.125824	0.188348
8	0.268461	0.157711	0.156859	0.117003	0.137427	0.205548
10	0.291041	0.170118	0.168873	0.123649	0.144994	0.216689
12	0.307270	0.178920	0.177348	0.128382	0.150316	0.224484
14	0.319500	0.185485	0.183640	0.131926	0.154262	0.230240
16	0.329048	0.190569	0.188495	0.134680	0.157305	0.234664
18	0.336712	0.194622	0.192354	0.136884	0.159725	0.238170
20	0.343002	0.197930	0.195495	0.138687	0.161694	0.241018
22	0.348258	0.200681	0.198101	0.140191	0.163328	0.243376

Table A12. Nondimensional central deflection of (PS) super-elliptical plates (p=38, η=0.010) (continue)

	W	W	W	λ	λ	λ
2k	c=1	c=2	c=3	c=1	c=2	c=3
24	0.352716	0.203004	0.200298	0.141464	0.164706	0.245362
26	0.356545	0.204994	0.202175	0.142556	0.165884	0.247056
28	0.359871	0.206716	0.203798	0.143504	0.166903	0.248519
30	0.362788	0.208221	0.205214	0.144333	0.167793	0.249795
32	0.365365	0.209548	0.206462	0.145066	0.168577	0.250918
34	0.367660	0.210728	0.207569	0.145718	0.169273	0.251914
36	0.369717	0.211782	0.208558	0.146302	0.169894	0.252803
38	0.371571	0.212731	0.209447	0.146827	0.170453	0.253602
40	0.373250	0.213589	0.210250	0.147303	0.170959	0.254323
100	0.393214	0.223675	0.219638	0.152944	0.176880	0.262729
400	0.403840	0.228952	0.224505	0.155935	0.179965	0.267071

Table A13. Nondimensional central deflection of (PS) super-elliptical plates (p=38, η=0.020)

	W	W	W	λ	λ	λ
2k	c=1	c=2	c=3	c=1	c=2	c=3
2	0.0832763	0.0498034	0.0498795	0.0571537	0.0651435	0.0966129
4	0.180826	0.107809	0.107842	0.0904526	0.105978	0.158572
6	0.235381	0.139054	0.138618	0.107279	0.125918	0.188407
8	0.268975	0.157810	0.156906	0.117317	0.137526	0.205609
10	0.291620	0.170227	0.168922	0.123980	0.145098	0.216753
12	0.307907	0.179036	0.177398	0.128728	0.150424	0.224548
14	0.320186	0.185608	0.183692	0.132285	0.154374	0.230306
16	0.329778	0.190699	0.188548	0.135051	0.157421	0.234731
18	0.337480	0.194758	0.192408	0.137264	0.159843	0.238238
20	0.343803	0.198070	0.195550	0.139077	0.161815	0.241087
22	0.349089	0.200826	0.198157	0.140589	0.163452	0.243446
24	0.353573	0.203153	0.200355	0.141869	0.164832	0.245432
26	0.357427	0.205146	0.202233	0.142968	0.166013	0.247127
28	0.360775	0.206871	0.203857	0.143921	0.167033	0.248591
30	0.363711	0.208380	0.205274	0.144756	0.167925	0.249868
32	0.366306	0.209710	0.206522	0.145493	0.168710	0.250992
34	0.368618	0.210892	0.207630	0.146150	0.169407	0.251988
36	0.370690	0.211949	0.208620	0.146737	0.170030	0.252878
38	0.372558	0.212900	0.209509	0.147267	0.170591	0.253677
40	0.374251	0.213760	0.210313	0.147747	0.171097	0.254399
100	0.394398	0.223877	0.219711	0.153439	0.177036	0.262814
400	0.405156	0.229177	0.224587	0.156469	0.180135	0.267164

Table A14. Nondimensional central deflection of (PS) super-elliptical plates (p=38, η=0.050)

	W	W	W	λ	λ	λ
2k	c=1	c=2	c=3	c=1	c=2	c=3
2	0.0846261	0.0500438	0.0499834	0.0587831	0.0656061	0.0968665
4	0.182576	0.108113	0.107970	0.0921728	0.106465	0.158835
6	0.237433	0.139403	0.138760	0.109082	0.126428	0.188681
8	0.271287	0.158197	0.157059	0.119194	0.138058	0.205892
10	0.294156	0.170647	0.169084	0.125921	0.145650	0.217044
12	0.310634	0.179485	0.177568	0.130723	0.150992	0.224847
14	0.323076	0.186082	0.183869	0.134327	0.154957	0.230611
16	0.332810	0.191194	0.188732	0.137134	0.158017	0.235042
18	0.340636	0.195273	0.192598	0.139383	0.160450	0.238554
20	0.347068	0.198603	0.195745	0.141227	0.162432	0.241407
22	0.352449	0.201373	0.198356	0.142767	0.164077	0.243770
24	0.357020	0.203715	0.200559	0.144073	0.165466	0.245760
26	0.360952	0.205720	0.202441	0.145194	0.166653	0.247459
28	0.364370	0.207457	0.204068	0.146168	0.167680	0.248926
30	0.367369	0.208976	0.205488	0.147022	0.168578	0.250206
32	0.370023	0.210315	0.206740	0.147777	0.169369	0.251332
34	0.372388	0.211506	0.207850	0.148449	0.170071	0.252331
36	0.374510	0.212571	0.208842	0.149052	0.170699	0.253223
38	0.376423	0.213529	0.209734	0.149595	0.171264	0.254024
40	0.378159	0.214397	0.210540	0.150087	0.171775	0.254748
100	0.398929	0.224617	0.219973	0.155971	0.177776	0.263196
400	0.410184	0.230003	0.224883	0.159160	0.180932	0.267580

Table A15. Nondimensional central deflection of (PS) super-elliptical plates (p=38, η=0.100)

	W	W	W	λ	λ	λ
2k	c=1	c=2	c=3	c=1	c=2	c=3
2	0.0885912	0.0506813	0.0502333	0.0643120	0.0670971	0.0976216
4	0.187676	0.108918	0.108279	0.0980058	0.108040	0.159629
6	0.243230	0.140307	0.139099	0.115132	0.128065	0.189502
8	0.277631	0.159180	0.157421	0.125419	0.139745	0.206736
10	0.300946	0.171696	0.169466	0.132289	0.147378	0.217907
12	0.317795	0.180590	0.177967	0.137211	0.152757	0.225727
14	0.330551	0.187235	0.184282	0.140918	0.156752	0.231505
16	0.340554	0.192389	0.189157	0.143813	0.159838	0.235948
18	0.348613	0.196503	0.193034	0.146139	0.162295	0.239471
20	0.355251	0.199865	0.196191	0.148051	0.164297	0.242334

Table A15. Nondimensional central deflection of (PS) super-elliptical plates (p=38, η=0.100) (continue)

	W	W	W	λ	λ	λ
2k	c=1	c=2	c=3	c=1	c=2	c=3
22	0.360815	0.202664	0.198812	0.149651	0.165961	0.244706
24	0.365549	0.205031	0.201022	0.151012	0.167366	0.246704
26	0.369628	0.207060	0.202911	0.152182	0.168569	0.248410
28	0.373180	0.208817	0.204545	0.153201	0.169610	0.249883
30	0.376302	0.210355	0.205972	0.154097	0.170520	0.251169
32	0.379069	0.211712	0.207228	0.154890	0.171323	0.252301
34	0.381539	0.212919	0.208344	0.155598	0.172036	0.253305
36	0.383758	0.213999	0.209341	0.156233	0.172674	0.254202
38	0.385763	0.214972	0.210238	0.156808	0.173248	0.255008
40	0.387583	0.215852	0.211048	0.157329	0.173768	0.255736
100	0.409704	0.226284	0.220552	0.163679	0.179914	0.264261
400	0.422200	0.231875	0.225540	0.167300	0.183214	0.268733