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Hybrid genetic algorithms for global optimization problems

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Abstract

In the last two decades the field evolutionary computation has become a mainstream and several types of evolutionary algorithms are developed for solving optimization and search problems. Evolutionary algorithms (EAs) are mainly inspired from the biological process of evolution. They do not demand for any concrete information such as continuity or differentiability and other information related to the problems to be solved. Due to population based nature, EAs provide a set of solutions and share properties of adaptation through an iterative process. The steepest descent methods and Broyden-Fletcher-Goldfarb-Shanno (BFGS),Hill climbing local search are quite often used for exploitation purposes in order to improve the performance of the existing EAs. In this paper, We have employed the BFGS as an additional operator in the framework of Genetic Algorithm. The idea of add-in BFGS is to sharpen the search around local optima and to speeds up the search process of the suggested algorithm. We have used 24 benchmark functions which was designed for the special session of the 2005 IEEE-Congress on Evolutionary Computation (IEEE-CEC 06) to examine the performance of the suggested hybrid GA. The experimental results provided by HGBA are much competitive and promising as compared to the stand alone GA for dealing with most of the used test problems

Keywords: Global Optimization, Evolutionary Computation (EC), Evolutionary Algorithm (EA), Genetic Algorithm (GA), Hybrid GA.

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1. Introduction

Global optimization has gained much attentions in both academia and industrial application over the past many years. In this regards, different test suites of optimization problems are designed in the existing literature of evolutionary computation. These problems are quite useful for thorough experimental computational testing and evaluation in order to design a powerful and robust optimization algorithm [1, 2]. The practical examples of optimization included the pooling/blending operations, heat exchanger network synthesis, phase and chemical reaction equilibrium, robust stability analysis, batch plant design under uncertainty, chemical reactor network synthesis, parameter estimation and data reconciliation, conformational problems in clusters of atoms and molecules, pump network synthesis, trim loss minimization, homogeneous azeotropic separation system, dynamic optimization problems in parameter estimation and in reactor network synthesis, and optimal control problems. In general, constrained minimization problem can be written as follows [3, 5]:

$$\text{Minimize } f(x), x = (x_1, x_2, \ldots, x_n)^T \in R^n \tag{1.1}$$

$$\text{Subject to} \begin{cases} g_i(x) \leq 0, i = 1, 2, \ldots, p \\ h_j(x) = 0, j = p+1, p+2, \ldots, q \end{cases} \tag{1.2}$$

where $x \in \Omega \subseteq S$, Ω is the feasible search space defined by p number of inequality constraints, q number of equality constraints and S is the search space defined by parametric constraints: $L_i \leq x \leq U_i$. The inequality constraints that satisfy $g_i(\mathbf{x}) = 0$ are said to be an active constraints. It is important to be mentioned here that an equality constraints are always active. There are many types of optimization problems including multi-quadratic programming, bilinear and biconvex, generalized geometric programming, general constrained nonlinear optimization, bilevel optimization, complementarity, semi-definite programming, mixed-integer nonlinear optimization, combinatorial optimization and optimal control problems [3]. Generally all the above mentioned optimization problems can be categorized into two class including the constrained and unconstrained one. In this paper, we are interested in solving the unconstrained optimization problems with continuous variables. They are called boxed constrained optimization problems.

The last two decades are witnessed for the significant improvement and developments of optimization methods.Technically, optimization methods can be categorized into deterministic and stochastic ones. They have tackled diverse set of problems with continuous, discrete, integer, mixed integer variables [4]. The deterministic approaches are required the analytical properties of the problems while finding their optimal solutions [7]. The interval optimization [6], branch-and-bound [8, 9] and algebraic techniques [10], Simplex method [11], Hill climbing [12], Newton-Raphson method [13] are leading examples of the some deterministic approaches.

The stochastic approaches involve randomness to perform their search process. The simulated annealing [14], Monte Carlo sampling [15], stochastic tunneling [16], and parallel tempering [17], Genetic Algorithm (GA) [18], Evolutionary Strategies (ES) [19], Evolutionary Programming (EP) [20], Particle Swarm Optimization (PSO) [23], Ant Colony Optimization (ACO) [25] and differential evolution (DE) [26], Krill herd algorithms [35, 36, 37], Monarch butterfly optimization [38], Earthworm optimization algorithm [39], Plant propagation algorithm (PPA) [40, 41, 42, 43] are stochastic nature based optimization methods. Evolutionary computation is the collective name of these algorithms inspired by biological process of evolution, such as natural selection and genetic inheritance [44].

Hybrid evolutionary algorithms [45, 46, 51] have got much attention of the researchers and practitioners due to their high potentialities and capabilities in handling various real world problems and benchmark functions comprising high complexity, noisy environment, imprecision, uncertainty and vagueness [27, 28, 29, 47, 48, 49, 50].

In this paper, the suggested algorithm utilizes the Broyden-Fletcher-Goldfarb-Shanno (BFGS) algorithm [30, 31] in combination with GA for population evolution at each multiple of 10^{th} generations. The suggested algorithm is called HGBA have solved most of the test problems that were designed for the special session of the 2005 IEEE-congress on evolutionary computation (IEEE-CEC'05) [32]. HGBA have tackled most of the used test problems in an auspicious manner.

The rest of the paper is organized as follows. Section 2 presents the proposed algorithms. Section 3 demonstrates experimental results. Section 4 concludes this paper with some future plan.

2. Hybridization of Genetic Algorithm with BFGS

Genetic algorithm was first proposed by Holland inspired by the process of natural selection [33, 34]. GA is one of the most popular and well-known classical paradigms of evolutionary algorithms (EAs) [44]. This paradigm mainly relies on evolutionary operators such as mutation, crossover and selection to evolve their uniformly and randomly generated set of initial solutions called population. Due to population based nature, GA provides a set of optimal solutions in a single simulation run unlike traditional optimization methods. It simulates the survival of the fittest among the population over a consecutive generation. Since its inception [52, 53], several variants of GAs have been proposed and tackled different types of optimization and search problems, particularly in machine learning, scientific modeling, and artificial life and reviews a broad span of research, including the work of Mitchell and her colleagues [54].

The Local search algorithms like the GSAT and WalkSat, 2-opt algorithm, Iterated Local Search (ILS) perform search by applying local changes in the search space of solution to solution until stopping criteria is not satisfied. The combined use of efficient local search optimizers can speed up search process of the GA framework to locate the exact global optimum of the given optimization problems. The BFGS algorithm [30, 31] is one of best well known hill-climbing local search method. Due to their fast convergence speed behavior, BFGS is applied to solve different nonlinear global optimization functions.

In the recent few years, several modifications have been made in the of the original framework of the genetic algorithm (GA) aiming at to alleviate their drawbacks. GA has successfully tackled different real-world problems such as space allocation problems on different sample test like warehouse, shelf, building floors and container and many others [55]. Different benchmark functions with continuous and discrete variables are also solved by GAs with great success. The combination of global and local searching (LS) can appeared in the form of hybrid evolutionary algorithms. They are quite useful for reducing the likelihood of the premature convergence which is normally occurred in the basic EAs for dealing with various search and optimization problems [56, 57, 62].

The suggested algorithm calls the BFGS [30, 31] algorithm 2 at their algorithmic step 5 as explained in the algorithm 1, where the Hessian matrix of the BFGS algorithm is initialized with identity matrix and here after updated with gradient information of the current and previous iterations.

Algorithm 1 Framework of the HGBA.

L: Lower bound;
U: Upper bound of the search space.
n: Dimension of search space;
N: Population Size.
T_G: Total Generations/Function Evaluations (FES);
$x \leftarrow (L + (U - L) \times rand(N, n)$; % Initial Population of size N.
$f(x) \leftarrow Eval_f(x)$; % Evaluate population set x of size N.
$G = 1$;% Initialize the generation counter.
while $G < T_G$ **do**
  if $rem(G, 10) == 0$ **then**
    $(y, f) = BFGS(x^b, f, tol)$
  else
    $x[G] \leftarrow Select - parents(x[G])$;
    $\acute{y}[t] \leftarrow Xovers(\acute{y}[G])$;
    $y[G] \leftarrow Mutation(\acute{y}[G])$;
    $f(y[G]) \leftarrow Evaluate(y[G])$;
  end if
  if $f(y[G]) < f(x[G])$ **then**
    $x[G]$=$y[G]$
  else
    $x[G] = x[G]$
  end if
  $G = G + 1$;
end while

Algorithm 2 The Framework of the BFGS Algorithm.

Initialize q best solutions;
Initialize Hessian Matrix with Identity Matrix;
Set $\epsilon = 0.00001$;% The tolerance value.
Define T_{FE}; % Total Function Evaluations.
for $i = 1 : T_{FE}$ **do**
  $d_x = x^{i+1} - x^i$;
  $d_g =: \nabla f(x^{i+1}) - \nabla f(x^i)$; %Compute the difference of gradients.
  if $d_x \neq 0 \& d_g \neq 0$ **then**
    $\lambda_1 = (d_g)' H^i d_g$
    $\lambda_2 = (d_x)' d_g$
    $H^{i+1} = H^i + (d_x d_x'(1 + \lambda_1/\lambda_2))/\lambda_2 - (H^i d_g d_x' + d_x d_g' H^i)/\lambda_2$; % Update the Hessian Matrix.
  end if
  $s^i = -H^i \nabla f(x^i)$; % The search direction s^i
  α^i; % Calculate the step size by using golden section search technique.
  $y^i = x^i + \alpha^i s^i$; % Update new solutions.
end for
y^q; % The q local solutions.
$f(x^q)$; % The objective function values of the q local solutions.

Table 1. Classification and properties of Tested functions

Unimodel	Multimodel	Separable	Non Separable
f01	f06	f01	f02
f02	f07	f09	f03
f03	f08	f15	f04
f04	f09	No	f05
f05	f10	No	f06
No	f11	No	f07
No	f12	No	f08
No	f13	No	f10
No	f14	No	f11
No	f15	No	f12
No	f16	No	f13
No	f17	No	f14
No	f18	No	f16
No	f19	No	f17
No	f20	No	f18
No	f21	No	f19
No	f22	No	f20
No	f23	No	f21
No	f24	No	f22
No	f25	No	f23
No	No	No	f24
No	No	No	f25

3. Discussion on Experimental Results

Continuous optimization problems have wide practical applications ranging from simple parameter identification in data-model fitting to intrinsic design-parameter optimization in complex technical systems in sciences and engineering fields. Different test suites of optimization with diverse characteristics as explained in the Table 1 are quite important for examining the overall performance of optimization algorithms in terms of convergence rate,precision and robustness.

In this paper, we have chosen test suite 25 benchmark functions [32] that comprising different characteristics. A brief summary regarding the used test functions denoted by $f_1 - f_{25}$ is hereby summarized in the Table 1.

The Table 1 provides the name of each test function, $f_1 - f_{24}$ and its variables' value range is recorded as appears in the original Technical Report [32]. The dimension N of each solution vector used in the experiments is also recorded together with the fitness value of the optimal solution f(x*). The CEC'05 test functions are characterized as follows: the first five functions $f_1 - f_5$ are unimodal and shifted; the second seven test functions $f_6 - f_{12}$ are basic multimodal and shifted; the third two functions $f_1 3 - f_1 4$ are expanded multimodal; and the fourth eleven functions $f_{15} - f_{25}$ are hybrid composition (i.e., all of them are non-separable, rotated, and multimodal functions containing a large number of local minima). Further details and evaluation criteria of the IEEE-CEC05 are given in [32].

We have carried out experiments at the platform:

- Operating system: Windows XP Professional.
- Programming language of the algorithms: Matlab.

- CPU: Core 2 Quad 2.4 GHz.
- RAM: 4 GB DDR2 1066 MHz.
- Execution: 30 times independent simulation of each algorithm with different random seeds.

Evolutionary Algorithms (EAs) are searching for the global optimum in their search space R^n comprising n dimensions. Initially, EAs require a set of N solutions to evolve them user defined function evaluations (FES). In this paper, the experiments performed according to parameter settings described as follows:

- $N = 100$, the population size.
- $n = 2$, 5,10,20, 30 are dimensions of the search space.
- $FES = n \times N$, total function evaluations.
- $q = 5$, allowed best solutions for BFGS to works in the HGBA framework.

The algorithmic behavior of the proposed HGBA is verified by CEC05 problems with parameter settings as mentioned above. We have recorded all experimental results in minimum, mean, median, standard deviation and maximum values with different settings of$n = 2$, 5, 10, 20, $n = 30$ while solving each test problem of the IEEE-CEC05 test suite [32]. It is important to mentioned here that we did not include all experimental results obtained with different settings of n.

Table 2 provides the numerical results of problems solved with $n = 10$ dimension. Table 3 presents the objective function values each CEC'05 benchmark function with $n = 30$ dimension. Both these tables clearly indicate that the suggested hybrid version of GA has solved most functions with better performance as compared to the basic GA on most problems.

The convergence graph of the HGBA versus GA are illustrated in the figures 1-1 for benchmark functions with search space dimension $n = 10$ and $n = 30$ in their 25 independent runs of simulation. The figure 1 shows the evolution of average function values within allowed function evaluations (FES). While the 1 demonstrates the average evolution in the objective function values of some CEC '05 test problems solved with search space dimension $n = 30$. It can see from these figures, the convergence speed of the proposed algorithm is much better than the basic genetic algorithm (GA) while elapsing less function evaluations to reach near to the global optimum of the most CEC05 test problems.

4. Conclusion

Global optimization problems offer many challenges to evolutionary computing (EC) communities due to the existence of nonlinearity and multi-modality in their formulation structures. The stand-alone local search optimization methods are mostly unable to deal with such problems. Currently, hybridization of local search optimizers with existing meta-heuristic algorithms have got much attention of researchers in EC field . In this paper, a hybrid population-based global optimization algorithm is proposed that combines genetic algorithm (GA) with BFGS. The proposed algorithm denoted by HGBA are combined GA with BFGS in an ensemble manner to promote information sharing among the population and thus enhance the searching efficiency of basic GA. The performance of the proposed HGBA is evaluated on a diverse set of benchmark functions designed for the special session of the 2005 IEEE-CEC [32]. The experimental results show the proposed algorithm have performed better than GA in terms of better convergence speed near to the known optimal and hence not get stuck in local optimum of the most problems.

In future, we intend to analyze the impact of the proposed algorithm by employing some other effective local search optimizers and search operators such as improved variants differential evolution [26], particle swarm optimization (PSO) [23] and ant colony

Table 2. The Numerical Results of the HGBA versus GA for each CEC'20O5 test problems with $n = 10$ dimension [32].

a)Hybrid Genetic Algorithm, b) Genetic Algorithm				
CEC'05 Functions	Best	Mean	Std	Algorithm
	0.000001	0.000001	0.521016	a
f01	**0.000000**	**0.000000**	**0.306146**	**b**
	0.000000	**0.000000**	**0.198374**	**a**
f02	0.000000	0.000000	0.445100	b
	0.000000	**0.000000**	637.666968	**a**
f03	0.000000	0.000105	14.057797	b
	0.000000	**0.000000**	**0.327159**	**a**
f04	0.000000	0.000000	0.039318	b
	0.000000	**0.000000**	**32.036672**	**a**
f05	0.000000	0.000002	40.692395	b
	0.000000	**0.000000**	**13.781630**	**a**
f06	0.000000	0.000283	10.536194	b
	0.000009	**3.557594**	**2.587509**	**a**
f07	0.004455	2.986633	1.413326	b
	0.000000	**0.000000**	**0.076302**	**a**
f08	0.000000	0.000000	0.197469	b
	0.000000	**0.000000**	**0.311911**	**a**
f09	0.000000	0.000000	0.426611	b
	0.000061	**0.007804**	**0.103723**	**a**
f10	0.004425	0.021671	0.120534	b
	0.000000	**0.000000**	**0.206919**	**a**
f11	0.000000	0.000000	2.123631	b
	0.000000	0.000000	0.012976	a
f12	**0.000000**	**0.000000**	**0.005400**	**b**
	0.000006	**0.000768**	**0.040584**	**a**
f13	0.000024	0.000056	0.041487	b
	1457.115466	**1457.115806**	**4.142761**	**a**
f14	1457.215625	1457.259106	7.013462	b
	1024.120243	**1024.157612**	**22.599572**	**a**
f15	1024.994920	1025.288941	22.631336	b
	1018.156033	**1018.192758**	**14.263865**	**a**
f16	1018.836790	1019.096324	12.314532	b
	827.203398	**827.205103**	**19.246432**	**a**
f17	827.217635	832.500164	15.346813	b
	1250.190437	**1250.106352**	**6.944744**	**a**
f18	1250.705684	1251.048538	8.034050	b
	885.666535	**916.629538**	**67.349620**	**a**
f19	894.251157	921.019167	54.960124	b
	1341.214724	1341.214724	0.000000	a
f20	1341.214724	1341.214724	0.000000	b
	1180.095532	**1185.909739**	**16.297264**	**a**
f21	1188.596456	1236.814978	21.359977	b

Table 3. The Numerical Results of the HGBA versus GA for each CEC'20O5 test problems with $n = 30$ dimension [32].

a) Hybrid Genetic Algorithm, b) Genetic Algorithm.				
CEC'05 Functions	Best	Mean	Std	Algorithm
	0.000000	**0.000000**	**605.382503**	**a**
f01	0.000000	0.000000	661.259167	b
	0.000003	**0.039364**	**942.982813**	**a**
f02	0.000119	0.330952	1124.284441	GA
	315225.984858	539100.620758	5887328.994022	a
f03	**112502.970129**	**425041.073907**	**1309109.898453**	**b**
	0.000040	**0.149584**	**1154.242033**	**a**
f04	1218.002421	9074.364177	12412.710359	b
	0.000000	**0.000003**	**1366.160967**	**a**
f05	0.000000	0.000333	1403.301756	b
	1.231123	**4.107561**	**47669318.286299**	**a**
f06	2.318733	4.922776	30969857.223058	b
	20.267766	20.279503	0.015343	a
f07	**20.204322**	**20.277736**	**0.036564**	**b**
	0.000000	**3.422413**	**10.595200**	**a**
f08	0.000026	5.327192	11.222839	b
	17.676048	17.676048	8.873088	a
f09	**16.273877**	**18.097991**	**9.534268**	**b**
	6.364467	**7.146139**	**0.286591**	**a**
f10	7.327944	7.471782	0.455529	b
	12.600003	**40.466356**	**3909.201355**	**a**
f11	57.433299	553.051436	7246.718863	b
	0.895147	**1.349706**	**1.421402**	**a**
f12	1.018349	1.283563	1.315277	b
	3.103262	**3.103262**	**0.068194**	**a**
f13	3.366543	3.456506	0.074904	b
	1360.491281	**1360.554486**	**26.255421**	**a**
f14	1360.522848	1360.720281	27.040415	b
	1287.077900	**1297.099559**	**22.730716**	**a**
f15	1288.507390	1299.373887	23.507829	a
	1290.781525	**1296.891223**	**20.723626**	**a**
f16	1293.072508	1298.831638	23.050568	b
	1255.746668	**1260.787053**	**14.183325**	**a**
f17	1258.658973	1259.159586	13.039324	b
	1342.330076	1379.304890	29.825873	a
f18	**1317.106976**	**1382.449478**	**30.816738**	**b**
	1153.289503	**1211.524349**	**25.341379**	**a**
f19	1189.855128	1238.001648	23.684564	b
	1391.379431	**1400.300400**	**27.610007**	**a**
f20	1395.714142	1410.221171	20.295410	b
	1356.522833	**1365.801357**	**12.111518**	**a**
f21	1372.117983	1441.648192	25.123694	b

optimization algorithms [25] with combined self-adaptive procedures. We also our proposed HGBA to solve constrained test suites of the IEE-CEC series [63].

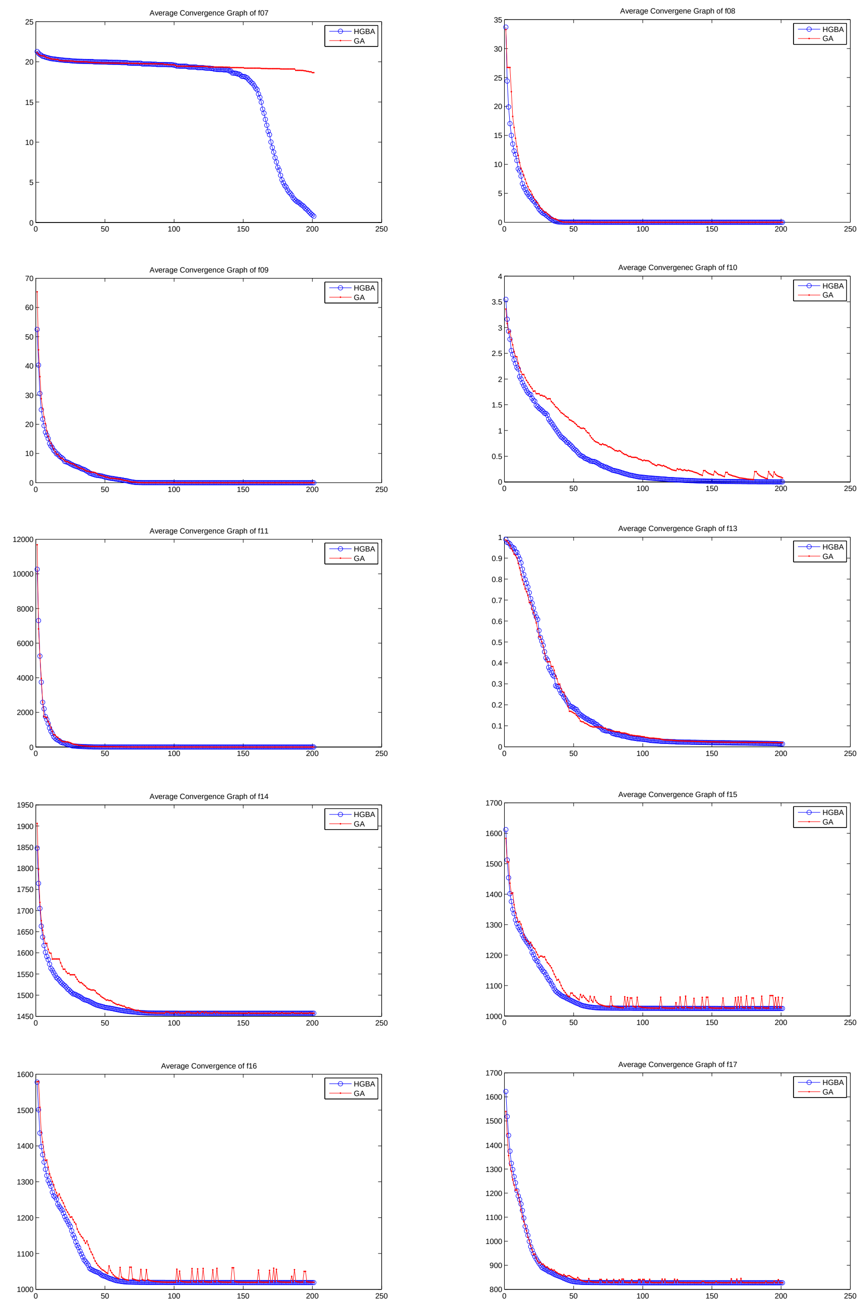

Figure 1. The convergence graph displayed by HGBA versus GA for CEC'05 [32] with ten variables.

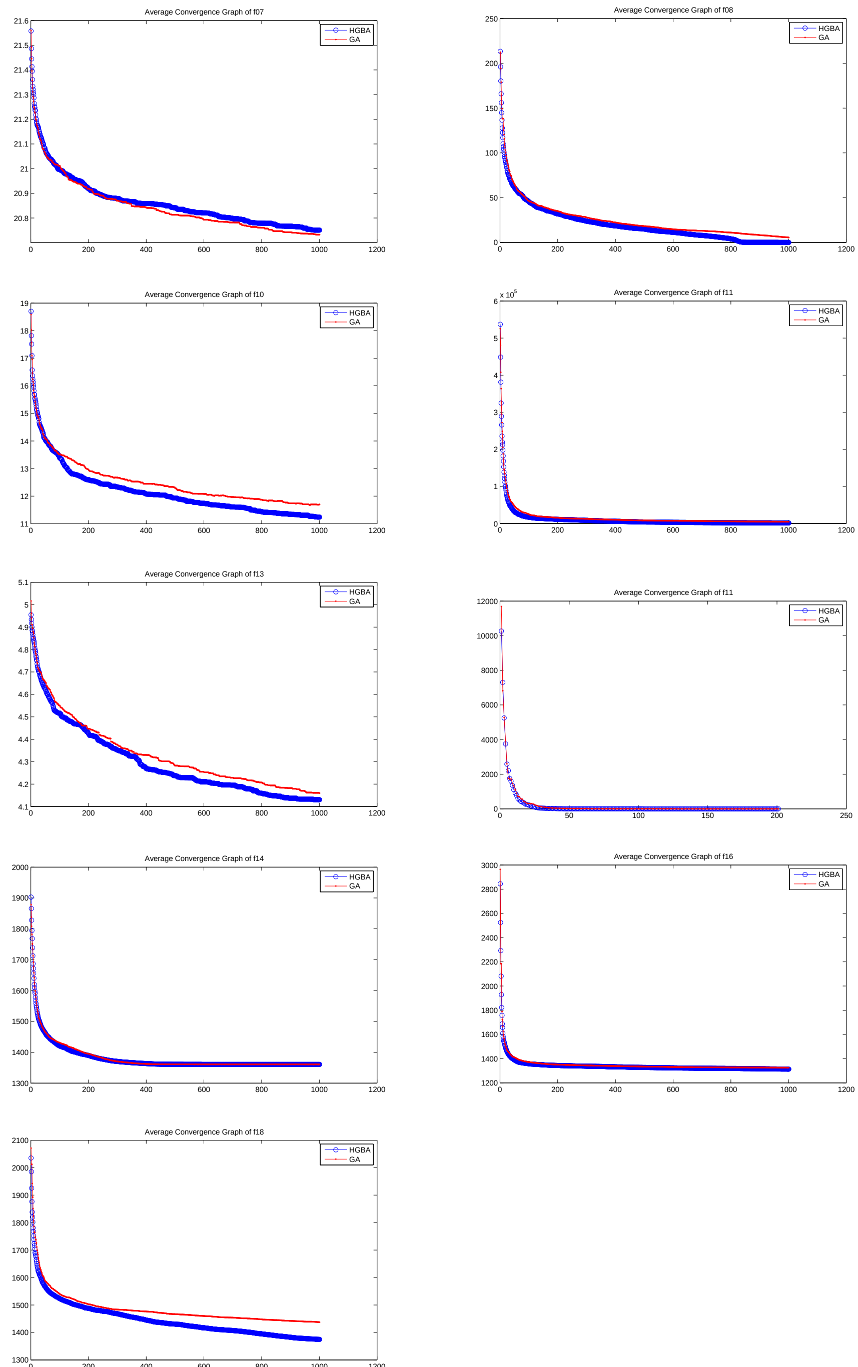

Figure 2. The convergence graph displayed by HGBA versus GA for CEC'05 [32] with thirty variables.

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