Hacettepe Journal of Mathematics and Statistics
Volume 47 (3) (2018), 589 – 599

On properties of fuzzy soft locally connected spaces

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Abstract

The main aim of this paper is to continue investigate the further properties of fuzzy soft connected spaces defined in [26]. We also initiate and explore the concept of fuzzy soft locally connected spaces. We observe that every fuzzy soft locally connected space need not be fuzzy soft connected space. Moreover, we discuss its properties in general as well as with respect to fuzzy soft components. Examples are also provided to clarify and validate the defined notions. We believe that the findings in this paper will be the inspiration for many researchers and will yield more natural results towards applications in information science, decision making and medical diagnosis problems.

Keywords: Fuzzy soft sets, Fuzzy soft topology, Fuzzy soft open(closed), Fuzzy soft classes, Fuzzy soft connected, Fuzzy soft locally connected, Fuzzy soft components.

Received : 07.01.2017 *Accepted* : 02.05.2017 *Doi* : 10.15672/HJMS.2017.475

1. Introduction

It is usual to deal with uncertainties and imprecise data in various situations of different aspects of systems and its subsystems. So the mathematical tool was required to solve different types of complicated problems in real life problems such as sociology, economics, engineering, computer and medical sciences etc. with the issues of uncertainties and ambiguities. Fuzzy set theory initiated and studied by Zadeh[46] proved to be an important mathematical tool to solve different types of complicated problems having uncertainties in real life problems with ambiguous environment. Now a days, both mathematician and computer scientists are applying this theory, not limited to such as fuzzy control systems, fuzzy automats, fuzzy logic, fuzzy topology etc.
The contribution of different theories such as probability theory, rough set theory, vague set theory, interval mathematics theory and fuzzy set theory handled such problems associated with uncertainties but have their own inherent difficulties and limitations to deal with them. Moreover, these theories require the pre specification of some initial

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parameter to start. In [34], Molodtsov initiated soft sets as a new and enough approach for modelling the complicated problems with uncertainties, which resolved the problem of inadequacy of parameters. Now a days, many researchers including: mathematicians, computer scientists, economists, engineers and medical scientists are working widely in this topic and it has become the complete area of research to attract the attention of scientists. The research work and application of soft set theory in decision making, demand analysis, forecasting, information science, computer science, engineering, medical sciences and other fields of science are observed and studied in [3], [8], [9-10], [13], [15-16], [28], [30-31],[35-36],[38],[43],[48].
In [40], Shabbir and Naz initiated and discussed soft topological spaces. Further properties, results, structures, mappings and improvements in concepts of soft topological spaces have been studied in [2],[5-6],[17-21],[23].
In [32], Maji and Biswas generalized the soft sets and introduced fuzzy soft sets. The fuzzy soft set theoretic approach to decision making problems is observed by Z. Kong et. al[29]. Chang[4] defined and explored the basic properties of fuzzy topological spaces. In [41], Tanay and Kandemir defined and discussed the topological structures of fuzzy soft sets. Varol and Aygun[42] initiated fuzzy soft topology. Further structures of fuzzy soft sets and fuzzy soft topology and its applications are explored in [1],[7],[11-12],[14],[22], [24-27],[33],[37], [39],[43-44].

2. Preliminaries

2.1. Definition. **[46]** A fuzzy set f on X is a mapping $f : X \to I = [0, 1]$. The value $f(x)$ represents the degree of membership of $x \in X$ in the fuzzy set f, for $x \in X$.

2.2. Definition. **[34]** Let X be an initial universe and E be a set of parameters. Let $P(X)$ denotes the power set of X and A be a non-empty subset of E. A pair (F, A) is called a soft set over X, where F is a mapping given by $F : A \to P(X)$. In other words, a soft set over X is a parameterized family of subsets of the universe X. For $e \in A$, $F(e)$ may be considered as the set of e-approximate elements of the soft set (F, A).

2.3. Definition. **[32]** Let I^X denotes the set of all fuzzy sets on X and $A \subseteq X$. A pair (f, A) is called a fuzzy soft set over X, where $f : A \to I^X$ is a function. That is, for each $a \in A$, $f(a) = f_a : X \to I$, is a fuzzy set on X.

2.4. Definition. **[32]** For two fuzzy soft sets (f, A) and (g, B) over a common universe X, we say that (f, A) is a fuzzy soft subset of (g, B) if
(1) $A \subseteq B$ and
(2) for all $a \in A$, $f_a \leq g_a$; implies f_a is a fuzzy subset of g_a.
We denote it by $(f, A)\tilde{\leq}(g, B)$. (f, A) is said to be a fuzzy soft super set of (g, B), if (g, B) is a fuzzy soft subset of (f, A). We denote it by $(f, A)\tilde{\geq}(g, B)$.

2.5. Definition. **[32]** Two fuzzy soft sets (f, A) and (g, B) over a common universe X are said to be fuzzy soft equal, if (f, A) is a fuzzy soft subset of (g, B) and (g, B) is a fuzzy soft subset of (f, A).

2.6. Definition. **[32]** The union of two fuzzy soft sets of (f, A) and (g, B) over the common universe X is the fuzzy soft set (h, C), where $C = A \cup B$ and for all $c \in C$,

$$h_c = \begin{cases} f_c, & \text{if } c \in A - B \\ g_c, & \text{if } c \in B - A \\ f_c \vee g_c, & \text{if } c \in A \cap B. \end{cases}$$

We write $(f, A)\tilde{\vee}(g, B) = (h, C)$.

2.7. Definition. **[32]** The intersection (h, C) of two fuzzy soft sets (f, A) and (g, B) over a common universe X , denoted $(f, A)\tilde{\wedge}(g, B)$, is defined as $C = A \cap B$, and $h_c = f_c \wedge g_c$, for all $c \in C$.

2.8. Definition. **[32]** The difference (h, C) of two fuzzy soft sets (f, A) and (g, B) over X, denoted by $(f, A)\tilde{\setminus}(g, B)$, is defined as $(f, A)\tilde{\setminus}(g, B) = (f, A)\tilde{\wedge}(f, B)^c$.

For our convenience, we will use the notation f_A for fuzzy soft set instead of (f, A) in the sequel.

2.9. Definition. **[41]** Let τ be the collection of fuzzy soft sets over X, then τ is said to be a fuzzy soft topology on X, if
(1) $\tilde{0}_A$, $\tilde{1}_A$ belong to τ.
(2) If $(f_A)_i \in \tau$, for all $i \in I$, then $\tilde{\bigvee}_{i\in I}(f_A)_i \in \tau$.
(3) $f_a, g_b \in \tau$ implies that $f_a \tilde{\bigwedge} g_b \tilde{\in} \tau$.
The triplet (X, τ, A) is called a fuzzy soft topological space over X. Every member of τ is called fuzzy soft open set. A fuzzy soft set is fuzzy soft closed if and only if its complement is fuzzy soft open.

2.10. Definition. **[42]** Let (X, τ, A) be a fuzzy soft topological space over X and f_A be a fuzzy soft set over X. Then
(1) fuzzy soft interior of fuzzy soft set f_A over X is denoted by $(f_A)^\circ$ and is defined as the union of all fuzzy soft open sets contained in f_A. Thus $(f_A)^\circ$ is the largest fuzzy soft open set contained in f_A.
(2) fuzzy soft closure of f_A, denoted by $\overline{f_A}$ is the intersection of all fuzzy soft closed super sets of f_A. Clearly $\overline{f_A}$ is the smallest fuzzy soft closed set over X which contains f_A.

2.11. Definition. **[1]** Let $F(X, A)$ and $F(Y, B)$ be families of fuzzy soft sets. $u : X \to Y$ and $p : A \to B$ are mappings. Then a function $f_{pu} : F(X, A) \to F(Y, B)$ is defined as :
(1) Let f_A be a fuzzy soft set in $F(X, A)$. The image of f_A under f_{pu}, written as $f_{pu}(f_A)$, is a fuzzy soft set in $F(Y, B)$ such that for $\beta \in p(A) \subseteq B$ and $y \in Y$,

$$f_{pu}(f_A)(\beta)(y) = \begin{cases} \bigvee_{x\in u^{-1}(y)}(\bigvee_{\alpha\in p^{-1}(\beta)\cap A}(f_A(\alpha)), & u^{-1}(y) \neq \phi, p^{-1}(\beta) \cap A \neq \phi \\ 0, & \text{otherwise} \end{cases},$$

for all $y \in B$. $f_{pu}(f_A)$ is known as a fuzzy soft image of a fuzzy soft set f_A.
(2) Let g_B be a fuzzy soft set in $F(Y, B)$. Then the inverse image of g_B under f_{pu}, written as $f^{-1}_{pu}(g_B)$, is a fuzzy soft set in $F(X, A)$ such that

$$f^{-1}_{pu}(g_B)(\alpha)(x) = \begin{cases} g(p(\alpha))(u(x)), & p(\alpha) \in B \\ 0, & \text{otherwise} \end{cases},$$

for all $x \in A$. $f^{-1}_{pu}(g_B)$ is known as a fuzzy soft inverse image of a fuzzy soft set g_B.
The fuzzy soft function f_{pu} is called fuzzy soft surjective, if p and u are surjective. The fuzzy soft function f_{pu} is called fuzzy soft injective, if p and u are injective.

2.12. Definition. **[42]** Let (X, τ_1, A) and (Y, τ_2, B) are fuzzy soft topological spaces and $f_{pu} : F(X, A) \to F(Y, B)$ be a fuzzy soft mapping. Then fuzzy soft function $f_{pu} : F(X, A) \to F(Y, B)$ is fuzzy soft pu-continuous, if for any fuzzy soft open set g_B in (Y, τ_2, B), $f^{-1}_{pu}(g_B)$ is fuzzy soft open in (X, τ_1, A).

2.13. Definition. **[42]** Let (X, τ_1, A) and (Y, τ_2, B) are fuzzy soft topological spaces and $f_{pu} : F(X, A) \to F(Y, B)$ be a fuzzy soft function. Then f_{pu} is said to be fuzzy soft pu-open(resp. fuzzy soft pu-closed), if for any fuzzy soft open (resp. fuzzy soft closed) set h_A in (X, τ_1, A), $f_{pu}(h_A)$ is fuzzy soft open(resp. fuzzy soft closed) in (Y, τ_2, B).

2.14. Definition. [22] A fuzzy soft set f_A is said to be a fuzzy soft point in (X,τ,A) denoted by $e(f_A)$, if for the element $e\tilde{\in}A$, $f(e)\tilde{\neq}\tilde{0}$ and $f(e^c)\tilde{=}\tilde{0}$, for all $e^c\in A\backslash\{e\}$.

2.15. Definition. [22] Let (X,τ,A) be a fuzzy soft topological space over X and f_A be a fuzzy soft set over X. Then f_A is called a fuzzy soft neighborhood (briefly: nbd) of the fuzzy soft point $e(f_A)$, if there exists a fuzzy soft open set h_A such that $e(f_A)\tilde{\in}h_A\tilde{\leq}g_A$.

2.16. Definition. [22] Let (X,τ,A) be a fuzzy soft topological space over X, f_A be a fuzzy soft set over X and $e(g_A)$ be a fuzzy soft point in X. If every fuzzy soft nbd of $e(g_A)$ fuzzy soft intersects f_A in some fuzzy soft point other than $e(g_A)$ itself, then $e(g_A)$ is called a fuzzy soft limit point of f_A.

2.17. Definition. [27] Let (X,τ,A) be a fuzzy soft topological space over X and f_A be a fuzzy soft set over X. Then fuzzy soft boundary of fuzzy soft set f_A over X is denoted by $Bd(f_A)$ and is defined as $Bd(f_A)\tilde{=}\overline{f_A}\tilde{\bigwedge}\overline{f_A^c}$.

2.18. Definition. [33] Let (X,τ,A) be a fuzzy soft topological space over X. Then (X,τ,A) is said to be fuzzy soft T_2-space if and only for any two fuzzy soft point $e(g_A)$, $e(k_A)$ of fuzzy soft set f_A in (X,τ,A) with $e(g_A)\tilde{\neq}e(k_A)$, there exists fuzzy soft open sets h_A and s_A such that $e(g_A)\tilde{\in}h_A$, $e(k_A)\tilde{\in}s_A$ and $h_A\tilde{\bigwedge}s_A\tilde{=}\tilde{0}_A$.

3. Properties of fuzzy soft connected spaces

3.1. Definition. [26] Let (X,τ,A) be a fuzzy soft topological space over X and f_A be a fuzzy soft set over X. If there are two proper fuzzy soft open subsets g_A and k_A such that $f_A\tilde{\leq}g_A\tilde{\bigvee}k_A$ and $g_A\tilde{\bigwedge}k_A\tilde{=}\tilde{0_A}$, then the fuzzy soft set f_A is called fuzzy soft disconnected set. If there do not exist such two proper fuzzy soft open subsets, then the fuzzy soft set f_A is called fuzzy soft connected set.

In the above definition, if we take $\tilde{1}_A$ instead of f_A, then (X,τ,A) is called fuzzy soft disconnected space and the pair of fuzzy soft sets g_A and k_A is said to be the fuzzy soft disconnection of (X,τ,A).

3.2. Example. Let $X=\{h_1,h_2,h_3\}$, $A=\{e_1,e_2\}$ and $\tau=\{\tilde{0},\tilde{1},(f_A)_1,(f_A)_2,(f_A)_3,$ $(f_A)_4\}$ where $(f_A)_1$, $(f_A)_2$, $(f_A)_3$, $(f_A)_4$ are fuzzy soft sets over X, defined as follows
$f_1(e_1)(h_1)=0.5,\ f_1(e_1)(h_2)=0.3,\ f_1(e_1)(h_3)=0.2,$
$f_1(e_2)(h_1)=0.3,\ f_1(e_2)(h_2)=0.5,\ f_1(e_2)(h_3)=0.2,$
$f_2(e_1)(h_1)=1,\ f_2(e_1)(h_2)=0,\ f_2(e_1)(h_3)=0.5,$
$f_2(e_2)(h_1)=0.5,\ f_2(e_2)(h_2)=0.3,\ f_2(e_2)(h_3)=1,$
$f_3(e_1)(h_1)=0.5,\ f_3(e_1)(h_2)=0,\ f_3(e_1)(h_3)=0.2,$
$f_3(e_2)(h_1)=0.3,\ f_3(e_2)(h_2)=0.3,\ f_3(e_2)(h_3)=0.2,$
$f_4(e_1)(h_1)=1,\ f_4(e_1)(h_2)=0.3,\ f_4(e_1)(h_3)=0.5,$
$f_4(e_2)(h_1)=0.5,\ f_4(e_2)(h_2)=0.5,\ f_4(e_2)(h_3)=1.$
Then τ is a fuzzy soft topology on X and hence (X,τ,A) is a fuzzy soft topological space over X. Consider the fuzzy soft sets g_A and k_A over X defined by
$g(e_1)(h_1)=1,\ g(e_1)(h_2)=0.5,\ g(e_1)(h_3)=0.6,$
$g(e_2)(h_1)=0.6,\ g(e_2)(h_2)=0.7,\ g(e_2)(h_3)=1.$
$k(e_1)(h_1)=0.3,\ k(e_1)(h_2)=0,\ k(e_1)(h_3)=1,$
$k(e_2)(h_1)=0.2,\ k(e_2)(h_2)=0.3,\ k(e_2)(h_3)=0.1.$
That is, $g_A=\{\{h_1,h_{0.5},h_{0.6}\},\{h_{0.6},h_{0.7},h_1\}\}$, and
$k_A=\{\{h_{0.3},h_0,h_{0.1}\},\{h_{0.2},h_{0.3},h_{0.1}\}\}$.
Then k_A is fuzzy soft disconnected set because there exist two proper fuzzy soft open subsets $(f_A)_1$, $(f_A)_2$ such that $k_A\tilde{\leq}(f_A)_1\tilde{\bigvee}(f_A)_2$ and $(f_A)_1\tilde{\bigwedge}(f_A)_2\tilde{=}\tilde{0_A}$. But g_A is fuzzy soft connected set because there do not exist such two proper fuzzy soft open sets in (X,τ,A).

3.3. Example. Let $X = \{h_1, h_2, h_3\}$, $A = \{e_1, e_2\}$ and $\tau = \{\tilde{0}, \tilde{1}, (f_A)_1, (f_A)_2\}$, where $(f_A)_1$, $(f_A)_2$ are fuzzy soft sets over X, defined as follows
$f_1(e_1)(h_1) = 0.4$, $f_1(e_1)(h_2) = 0.3$, $f_1(e_1)(h_3) = 0.2$,
$f_1(e_2)(h_1) = 0.2$, $f_1(e_2)(h_2) = 0.1$, $f_1(e_2)(h_3) = 0.3$,
$f_2(e_1)(h_1) = 0.6$, $f_2(e_1)(h_2) = 0.7$, $f_2(e_1)(h_3) = 0.8$,
$f_2(e_2)(h_1) = 0.8$, $f_2(e_2)(h_2) = 0.9$, $f_2(e_2)(h_3) = 0.7$,
Then τ is a fuzzy soft topology on X and hence (X, τ, A) is a fuzzy soft topological space over X. Clearly (X, τ, A) is fuzzy soft connected, since there do not exist two proper fuzzy soft open sets g_A and k_A such that $\tilde{1}_A \tilde{\leq} g_A \tilde{\bigvee} k_A$ and $g_A \tilde{\bigwedge} k_A \tilde{=} \tilde{0}_A$.

3.4. Definition. Let (X, τ, A) be a fuzzy soft topological space over X and f_A, g_A are two fuzzy soft sets over X. Then f_A and g_A are fuzzy soft disjoint, if $f_A \tilde{\bigwedge} g_A \tilde{=} \tilde{0}_A$. That is, $0 = f(e) \bigwedge g(e)$, for all $e \in A$.

3.5. Definition. Let (X, τ, A) be a fuzzy soft topological space over X and f_A be a nonempty fuzzy soft subset over X. Then
$\tilde{\tau}_{f_A} \tilde{=} \{v_A \tilde{\bigwedge} f_A,$ *where* v_A *is fuzzy soft open set in* $\tilde{X}\}$
is said to be the fuzzy soft relative topology on f_A and $(f_A, \tilde{\tau}_{f_A}, A)$ is called a fuzzy soft subspace of (X, τ, A).

3.6. Theorem. *If (X, τ, A) be a fuzzy soft T_2-space and v_A be a nonempty fuzzy soft subset of (X, τ, A) containing finite number of fuzzy soft points, then v_A is fuzzy soft closed.*

Proof. Let us take $v_A \tilde{=} \{e(g_A)\}$. Now we show that v_A is fuzzy soft closed. If $e(f_A)$ is a fuzzy soft point of (X, τ, A) different from $e(g_A)$, then $e(g_A)$ and $e_(f_A)$ have disjoint fuzzy soft nbds h_A and k_A, respectively. Since h_A does not fuzzy soft intersect $\{e(f_A)\}$, fuzzy soft point $e_(g_A)$ cannot belong to the fuzzy soft closure of the set $\{e(f_A)\}$. As a result, the fuzzy soft closure of the fuzzy soft set $\{e(g_A)\}$ is $\{e(g_A)\}$ itself and so it is fuzzy soft closed. Since $e(g_A)$ is arbitrary fuzzy soft point, this is true for all fuzzy soft subsets of (X, τ, A) containing finite number of fuzzy soft points. Hence the proof. □

3.7. Theorem. *A fuzzy soft topological space (X, τ, A) is fuzzy soft connected if and only if there does not exist nonempty proper fuzzy soft subset f_A of (X, τ, A) which is both fuzzy soft open and fuzzy soft closed in (X, τ, A).*

Proof. $(\Rightarrow)$ Suppose that fuzzy soft topological space (X, τ, A) is fuzzy soft connected and let f_A be any proper fuzzy soft subset of (X, τ, A) which is both fuzzy soft open and fuzzy soft closed. Now $f_A \tilde{\bigvee} f_A^c \tilde{=} \tilde{1}_A$ and $f_A \tilde{\bigwedge} f_A^c \tilde{=} \tilde{0}_A$. Since (X, τ, A) is fuzzy soft connected, then $f_A \tilde{=} \tilde{0}_A$ or $f_A^c \tilde{=} \tilde{0}_A$. That is, $f_A \tilde{=} \tilde{1}_A$ or $f_A^c \tilde{=} \tilde{1}_A$. Hence there does not exist nonempty proper fuzzy soft subset f_A of (X, τ, A) which is both fuzzy soft open and fuzzy soft closed in (X, τ, A).
$(\Leftarrow)$ Suppose that there does not exist nonempty proper fuzzy soft subset of (X, τ, A) which is both fuzzy soft open and fuzzy soft closed in (X, τ, A). Suppose on the contrary that (X, τ, A) is fuzzy soft disconnected, then there exist pair f_A and g_A of nonempty fuzzy soft open sets such that $f_A \tilde{\bigvee} g_A \tilde{=} \tilde{1}_A$ and $f_A \tilde{\bigwedge} g_A \tilde{=} \tilde{0}_A$. Since $f_A \tilde{=} g_A^c$ and $g_A \tilde{=} f_A^c$, then f_A and g_A are fuzzy soft closed as well. Thus by hypothesis, either $f_A \tilde{=} \tilde{1}_A$ and $g_A \tilde{=} \tilde{0}_A$ or $g_A \tilde{=} \tilde{1}_A$ and $f_A \tilde{=} \tilde{0}_A$. This contradicts the fact that both f_A and g_A are nonempty. Thus (X, τ, A) is fuzzy soft connected. This completes the proof. □

Similarly we have the following theorem.

3.8. Theorem. *A fuzzy soft topological space (X, τ, A) is fuzzy soft disconnected if and only if there exists nonempty proper fuzzy soft subset f_A of (X, τ, A) which is both fuzzy soft open and fuzzy soft closed in (X, τ, A).*

3.9. Theorem. *Let h_A be a fuzzy soft connected subspace with respect to fuzzy soft relative topology of fuzzy soft topological space (X, τ, A) and the pair of f_A and g_A be the fuzzy soft disconnection of (X, τ, A). Then $h_A \tilde{\leq} f_A$ or $h_A \tilde{\leq} g_A$.*

Proof. Suppose on the contrary that neither $h_A \tilde{\nleq} f_A$ nor $h_A \tilde{\nleq} g_A$. Then $h_A \tilde{\bigwedge} f_A$ and $h_A \tilde{\bigwedge} g_A$ are both nonempty fuzzy soft disjoint with $(h_A \tilde{\bigwedge} f_A) \tilde{\leq} h_A$ and $(h_A \tilde{\bigwedge} g_A) \tilde{\leq} h_A$ such that $(h_A \tilde{\bigwedge} f_A) \tilde{\bigvee} (h_A \tilde{\bigwedge} g_A) \tilde{=} h_A$. This implies that the pair of $h_A \tilde{\bigwedge} f_A$ and $h_A \tilde{\bigwedge} g_A$ is a fuzzy soft disconnection for h_A. A contradiction. Hence the proof. □

3.10. Theorem. *Let g_A be a fuzzy soft connected subset of a fuzzy soft topological space (X, τ, A) and f_A be fuzzy soft subset of (X, τ, A) such that $g_A \tilde{\leq} f_A \tilde{\leq} \overline{g_A}$. Then f_A is fuzzy soft connected.*

Proof. To prove our result, it is enough to prove that $\overline{g_A}$ is fuzzy soft connected. We suppose contrarily that $\overline{g_A}$ is fuzzy soft disconnected. Then there exists a pair of h_A and k_A of fuzzy soft sets which forms the fuzzy soft disconnection of $\overline{g_A}$. That is, there are $(h_A \tilde{\bigwedge} g_A)$, $(k_A \tilde{\bigwedge} g_A)$ fuzzy soft open sets in g_A such that $(h_A \tilde{\bigwedge} g_A) \tilde{\bigwedge} (k_A \tilde{\bigwedge} g_A) \tilde{=} (h_A \tilde{\bigwedge} k_A) \tilde{\bigwedge} g_A \tilde{=} 0_A$, and $(h_A \tilde{\bigwedge} g_A) \tilde{\bigvee} (k_A \tilde{\bigwedge} g_A) \tilde{=} (h_A \tilde{\bigvee} k_A) \tilde{\bigwedge} g_A \tilde{=} g_A$. This implies that pair $(h_A \tilde{\bigwedge} g_A)$ and $(k_A \tilde{\bigwedge} g_A)$ of fuzzy soft sets is a fuzzy soft disconnection of g_A. This contradiction proves the theorem. □

3.11. Corollary. *If f_A is fuzzy soft connected subspace with respect to fuzzy soft relative topology of a fuzzy soft topological space (X, τ, E), then $\overline{f_A}$ is fuzzy soft connected.*

Next we characterize fuzzy soft connectedness in terms of fuzzy soft boundary as:

3.12. Theorem. *A fuzzy soft topological space (X, τ, A) is fuzzy soft connected if and only if every nonempty proper fuzzy soft subspace with respect to fuzzy soft topology has a nonempty fuzzy soft boundary.*

Proof. Contrarily suppose that a nonempty proper fuzzy soft subspace f_A with respect to fuzzy soft topology of a fuzzy soft connected space (X, τ, A) has empty fuzzy soft boundary. Then f_A is fuzzy soft open and $\overline{f_A} \tilde{\bigwedge} \overline{f_A^c} \tilde{=} \tilde{0}_A$. Let $e(g_A)$ be a fuzzy soft limit point of f_A. Then $e(g_A) \tilde{\in} \overline{f_A}$ but $e(g_A) \notin \overline{f_A^c}$. In particular, $e(g_A) \tilde{\notin} f_A^c$ and so $e(g_A) \tilde{\in} f_A$. Thus f_A is fuzzy soft closed and fuzzy soft open. By Theorem 3.8, (X, τ, A) is fuzzy soft disconnected. This contradiction proves that f_A has a nonempty fuzzy soft boundary.
Conversely, suppose that (X, τ, A) is fuzzy soft disconnected. Then by Theorem 3.8, (X, τ, A) has a proper fuzzy soft subset f_A which is both fuzzy soft closed and fuzzy soft open. Then $\overline{(f_A)} \tilde{=} (f_A)$, $\overline{(f_A)^c} \tilde{=} (f_A)^c$ and $\overline{(f_A)} \tilde{\bigwedge} \overline{(f_A)^c} \tilde{=} \tilde{0}_A$. So f_A has empty fuzzy soft boundary, a contradiction. Hence (X, τ, A) is fuzzy soft connected. This completes the proof. □

3.13. Theorem. *Let $\{f_{A_i} : i \in I\}$ be a collection of fuzzy soft connected fuzzy soft subspaces with respect to fuzzy soft relative topology of fuzzy soft topological space and $\tilde{\bigwedge}_{i \in I} f_{A_i} \tilde{=} \tilde{0}_A$, then $\tilde{1}_A \tilde{=} \bigvee_{i \in I} f_{A_i}$ is fuzzy soft connected.*

Proof. Contrarily suppose that $\tilde{1}_A \tilde{=} \bigvee_{i \in I} f_{A_i}$ is fuzzy soft disconnected and let the pair of h_A and k_A be a fuzzy soft disconnection. Since f_{A_i} is fuzzy soft connected then by Theorem 3.9, $f_{A_i} \tilde{\leq} h_A$ or $f_{A_i} \tilde{\leq} k_A$. Also $\bigwedge_{i \in I} f_{A_i} \tilde{\neq} \tilde{0}_A$ implies that all f_{A_i} contained in h_A or k_A . That is $\tilde{1}_A \tilde{=} \bigvee_{i \in I} f_{A_i}$ is contained either in h_A or k_A. If $\tilde{1}_A \tilde{\leq} h_A$, then $k_A \tilde{=} \tilde{0}_A$ and if

$\tilde{1}_A \tilde{\leq} k_A$, then $h_A \tilde{=} \tilde{0_A}$. A contradiction. Thus $\tilde{1}_A \tilde{=} \bigvee_{i \in I} f_{A_i}$ is fuzzy soft connected. Hence the proof. □

4. Fuzzy soft locally connected spaces

Here we initiate and explore the concept of fuzzy soft locally connected spaces. Moreover we discuss its properties in general as well as with respect to newly defined concept of fuzzy soft component.

4.1. Definition. Let (X, τ, A) be a fuzzy soft topological space. Then (X, τ, A) is said to be fuzzy soft locally connected at fuzzy soft point $e(f_A)$, if for fuzzy soft point $e(f_A)$ in (X, τ, A) and any fuzzy soft open nbd g_A of $e(f_A)$, there exist a fuzzy soft open fuzzy soft connected set h_A such that $e(f_A) \tilde{\in} h_A \tilde{\leq} g_A$.

4.2. Example. Let X be any universal set, A be the set of any parameters and τ_D is a fuzzy soft discrete topology on X, then (X, τ_D, A) is a fuzzy soft discrete topological space over X. Clearly (X, τ_D, A) is fuzzy soft locally connected space. Because, if we take any fuzzy soft point $e(f_A)$ in (X, τ_D, A) and g_A any fuzzy soft open nbd of $e(f_A)$ such that $h_A \tilde{=} \{e(f_A)\}$. Then h_A is fuzzy soft open fuzzy soft connected set with $e(f_A) \tilde{\in} h_A \tilde{\leq} g_A$.

In the following example, we observe that fuzzy soft locally connected space does not imply fuzzy soft connected space.

4.3. Example. Let $X = \{h_1, h_2, h_3\}$, $A = \{e_1, e_2\}$ and $\tau = \{\tilde{0}, \tilde{1}, (f_A)_1, (f_A)_2\}$, where $(f_A)_1$, $(f_A)_2$ are fuzzy soft sets over X, defined as follows
$f_1(e_1)(h_1) = 1$, $f_1(e_1)(h_2) = 0$, $f_1(e_1)(h_3) = 1$,
$f_1(e_2)(h_1) = 0$, $f_1(e_2)(h_2) = 1$, $f_1(e_2)(h_3) = 0$,
$f_2(e_1)(h_1) = 0$, $f_2(e_1)(h_2) = 1$, $f_2(e_1)(h_3) = 0$,
$f_2(e_2)(h_1) = 1$, $f_2(e_2)(h_2) = 0$, $f_2(e_2)(h_3) = 1$,
Clearly τ is a fuzzy soft topology on X and hence (X, τ, A) is a fuzzy soft topological space over X. Then (X, τ, A) is fuzzy soft locally connected. If we take the fuzzy soft point $e(f_A) \tilde{=} \{\{h_1, h_0, h_1\}\}$ and $g_A \tilde{=} \{\{h_1, h_0, h_1\}, \{h_0, h_1, h_0\}\}$ or $\tilde{1}_A$, which is fuzzy soft open nbd of $e(f_A)$ also take $k_A \tilde{=} \{\{h_1, h_0, h_1\}, \{h_0, h_1, h_0\}\}$. Then k_A is fuzzy soft open fuzzy soft connected and $e(f_A) \tilde{\in} k_A \tilde{\leq} g_A$. Also if we take the fuzzy soft point $e(f_A) \tilde{=} \{\{h_0, h_1, h_0\}\}$ and $g_A \tilde{=} \{\{h_0, h_1, h_0\}, \{h_1, h_0, h_1\}\}$ or $\tilde{1}_A$, which is fuzzy soft open nbd of $e(f_A)$ also take $k_A \tilde{=} \{\{h_0, h_1, h_0\}, \{h_1, h_0, h_1\}\}$. Then k_A is fuzzy soft open fuzzy soft connected and $e(f_A) \tilde{\in} k_A \tilde{\leq} g_A$. Moreover, (X, τ, A) is not fuzzy soft connected.

The following theorem implies that fuzzy soft local connectedness is fuzzy soft open hereditary property:

4.4. Theorem. *Let (X, τ, A) be a fuzzy soft locally connected space. If f_A be a fuzzy soft open subset of (X, τ, A). Then f_A is fuzzy soft locally connected .*

Proof. Let f_A be a fuzzy soft open set in a fuzzy soft locally connected space (X, τ, A). We show that f_A is fuzzy soft locally connected. Given $e(g_A) \tilde{\in} f_A$ and an arbitrary fuzzy soft open nbd v_A of $e(g_A)$ in f_A. Then v_A is also fuzzy soft open nbd in (X, τ, A). Since (X, τ, A) is fuzzy soft locally connected, then there exists fuzzy soft open fuzzy soft connected nbd v_{0_A} of $e(g_A)$ such that $e(g_A) \tilde{\in} v_{0_A} \tilde{\leq} v_A$. In this way, v_{0_A} is also fuzzy soft open connected nbd of $e(g_A)$ in f_A such that $e(g_A) \tilde{\in} v_{0_A} \tilde{\leq} v_A \tilde{\leq} f_A$. Therefore, $e(g_A) \tilde{\in} v_{0_A} \tilde{\leq} f_A$. Thus f_A is fuzzy soft locally connected. Hence the proof. □

4.5. Definition. Let (X, τ, A) be a fuzzy soft topological space and f_A be fuzzy soft subset in (X, τ, A). Then f_A is said to be a fuzzy soft component in (X, τ, A), if it is maximal fuzzy soft connected subset of (X, τ, A).

4.6. Theorem. *Let (X, τ, A) be a fuzzy soft topological space. Then*
(1) Each fuzzy soft component is fuzzy soft closed.
(2) A fuzzy soft connected subset of (X, τ, A) which is both fuzzy soft open and fuzzy soft closed is a fuzzy soft component in (X, τ, A).

Proof. (1) Let f_A be a fuzzy soft component in (X, τ, A), then by Corollary 3.11, $\overline{f_A}$ is fuzzy soft connected. Since f_A is maximal fuzzy soft connected, then $\overline{f_A}\tilde{\leq} f_A$. Also $f_A\tilde{\leq}\overline{f_A}$. Thus $\overline{f_A}\tilde{=}f_A$. This follows that f_A is fuzzy soft closed.
(2) Let f_A be a nonempty fuzzy soft connected subset of (X, τ, A) which is both fuzzy soft open and fuzzy soft closed. Suppose on the contrary that f_A is not fuzzy soft component. Then $f_A\tilde{\leq}k_A$, where k_A is some fuzzy soft component in (X, τ, A). But $k_A\tilde{=}(k_A\tilde{\bigwedge}f_A)\tilde{\bigvee}[k_A\tilde{\bigwedge}f_A^c]$ and $(k_A\tilde{\bigwedge}f_A)\tilde{\bigwedge}[k_A\tilde{\bigwedge}f_A^c]\tilde{=}\tilde{0}_A$. This implies that k_A is fuzzy soft disconnected. A contradiction. Hence f_A is fuzzy soft component. This completes the proof. □

4.7. Theorem. *Let (X, τ, A) be a fuzzy soft locally connected space. Then any fuzzy soft component f_A in (X, τ, A) is fuzzy soft open.*

Proof. Since (X, τ, A) is fuzzy soft locally connected, for each $e(g_A)\tilde{\in}f_A$, taking $(h_{e(g_A)})_A\tilde{=}\tilde{1}_A$, there exists a fuzzy soft open fuzzy soft connected set $(k_{e(g_A)})_A$ such that $e(g_A)\tilde{\in}(k_{e(g_A)})_A\tilde{\leq}\tilde{1}_A$. Since f_A is fuzzy soft component, then by the fuzzy soft maximality, $(k_{e(g_A)})_A\tilde{\leq}f_A$. Now
$f_A\tilde{=}\tilde{\bigvee}_{e(g_A)\in f_A}\{e(g_A)\}\tilde{\leq}\bigvee_{e(g_A)\in f_A}(k_{(e(g_A))})_A\tilde{\leq}f_A$. Therefore, $f_A\tilde{=}\bigvee_{e(g_A)\in f_A}(k_{(e(g_A))})_A$, which is the fuzzy soft union of fuzzy soft open sets. Therefore, f_A is fuzzy soft open. Hence the proof. □

Now we give the characterization of fuzzy soft locally connected spaces.

4.8. Theorem. *Let (X, τ, A) be a fuzzy soft topological space. Then the following statements are equivalent:*
(1) (X, τ, A) is fuzzy soft locally connected.
(2) For each fuzzy soft open set f_A in (X, τ, A), every fuzzy soft component of f_A is a fuzzy soft open set in (X, τ, A).

Proof. (1) $\Rightarrow$ (2) Suppose that (X, τ, A) is fuzzy soft locally connected space. Let f_A be a fuzzy soft open set in (X, τ, A) and let h_A be the fuzzy soft component of f_A in (X, τ, A). We prove that h_A is fuzzy soft open. Let $e(g_A)\tilde{\in}h_A$. Now $h_A\tilde{\leq}f_A$ implies that $e(g_A)\tilde{\in}f_A$. Since (X, τ, A) is fuzzy soft locally connected, then there exist a fuzzy soft open connected set v_A in (X, τ, A) such that $e(g_A)\tilde{\in}v_A\tilde{\leq}f_A$. As $e(g_A)\tilde{\in}h_A$ and h_A is the maximal fuzzy soft connected subset of f_A such that $e(g_A)\tilde{\in}f_A$. Thus we have $v_A\tilde{\leq}h_A$. Hence $e(g_A)\tilde{\in}v_A\tilde{\leq}h_A$, where v_A is fuzzy soft open set in (X, τ, A). This follows that h_A is fuzzy soft open in (X, τ, A).
(2) $\Rightarrow$ (1) Suppose that for each fuzzy soft open set f_A in (X, τ, A), every fuzzy soft component of f_A is a fuzzy soft open set in (X, τ, A). To show that (X, τ, A) is fuzzy soft locally connected, let $e(g_A)$ be fuzzy soft point in (X, τ, A) and f_A be a fuzzy soft open nbd of (X, τ, A). Let h_A be a fuzzy soft component of f_A such that $e(g_A)\tilde{\in}f_A$. Then h_A is fuzzy soft connected and by (2) is also fuzzy soft open. This implies that $e(g_A)\tilde{\in}h_A\tilde{\leq}f_A$, where h_A is fuzzy soft open and fuzzy soft connected set. Hence (X, τ, A) is fuzzy soft locally connected. This completes the proof. □

4.9. Theorem. *Let (X, τ_1, A) and (Y, τ_2, B) be two fuzzy soft topological spaces and $u : X \to Y$ and $p : A \to B$ are mappings such that (X, τ_1, A) be a fuzzy soft locally* *connected space. If $f_{pu} : F(X,A) \to F(Y,B)$ be fuzzy soft pu-continuous, fuzzy soft pu-closed and fuzzy soft onto, then (Y,τ_2,B) is fuzzy soft locally connected.*

Proof. Let v_B be fuzzy soft open set in (Y,τ_2,B) and h_B a fuzzy soft component of v_B . In view of Theorem 4.8, we just need to show that h_B is fuzzy soft open set in (Y,τ_2,B). First we show that $f_{pu}^{-1}(h_B)$ is fuzzy soft open in (X,τ,A). For each $e(f_A)\tilde{\in}f_{pu}^{-1}(h_B)$, let $(g_{(e(f_A))})_A$ be fuzzy soft component of $f_{pu}^{-1}(v_B)$ containing $e(f_A)$. Since (X,τ,A) is fuzzy soft locally connected , therefore by Theorem 4.7, $(g_{(e(f_A))})_A$ is fuzzy soft open in (X,τ,A). Since $(g_{(e(f_A))})_A$ is fuzzy soft connected in $f_{pu}^{-1}(v_B)$ and f_{pu} is fuzzy soft pu-continuous, so $f_{pu}((g_{(e(f_A))})_A)$ is fuzzy soft connected in $f_{pu}(f_{pu}^{-1}(v_B))\tilde{=}v_B$. Since $f_{pu}(e(f_A))\tilde{\in}f_{pu}((g_{(e(f_A))})_A)$ and also $f_{pu}(e(f_A))\tilde{\in}h_B$, by the maximality of h_B, $f_{pu}((g_{(e(f_A))})_A)\tilde{\leq}h_B$. Hence $(g_{(e(f_A))})_A\tilde{\leq}f_{pu}^{-1}(h_B)$, for each $e(f_A)\tilde{\in}f_{pu}^{-1}(h_B)$. We can write $f_{pu}^{-1}(h_B)\tilde{=}\tilde{\bigvee}_{e(f_A)\in f^{-1}(h_B)}\{e(f_A)\}\tilde{\leq}\bigvee_{e(f_A)\in f^{-1}(h_B)}(g_{(e(f_A))})_A\tilde{\leq}f_{pu}^{-1}(h_B)$. Therefore, $f_{pu}^{-1}(h_B)\tilde{=}\tilde{\bigvee}_{e(f_A)\in f^{-1}(h_B)}(g_{(e(f_A))})_A$. Which is union of fuzzy soft open set and thus is fuzzy soft open in (X,τ,A). Now $(f_{pu}^{-1}(h_B))^c$ is fuzzy soft closed in (X,τ,A) and f_{pu} is fuzzy soft closed function follows that $f_{pu}((f_{pu}^{-1}(h_B))^c)$ is fuzzy soft closed in (Y,τ_2,B). Also f_{pu} is fuzzy soft onto, so $f_{pu}((f_{pu}^{-1}(h_B))^c)\tilde{=}f_{pu}(f_{pu}^{-1}((h_B)^c))\tilde{=}(h_B)^c$. This implies that $(h_B)^c$ is fuzzy soft closed. Thus h_B is fuzzy soft open in (Y,τ_2,B). Therefore by Theorem 4.8, (Y,τ_2,B) is fuzzy soft locally connected. Hence the proof. □

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