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Design of economically and statistically optimal sampling plans

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Abstract

This paper attempts to develop four types of acceptance sampling plans under the objectives of minimizing the total loss to the producer and consumer and maximizing the rate of approaching to the ideal operating characteristic (OC) curve when the quality characteristic follows a normal distribution and has lower specification limit. To provide the desired protection for both the producer and the consumer, two constraints are considered that balance both producer and consumer risks by using two specified points on OC curve. The first objective function of this model is constructed based on the total expected loss by incorporating the one-sided minimum specification Taguchi loss function, and the second one is constructed based on conformity to the ideal OC curve by using minimum angle method. Also, the optimal solution of the proposed plans is determined for 10 different scenarios of process parameters. Furthermore, the optimal solution of these plans is determined when only one of the objective functions is considered.

Keywords: Quality control, Acceptance sampling plan, Loss function, Minimum angle method, Acceptable quality level, Limiting quality level.

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1. Introduction

Acceptance sampling is a statistical procedure used for sentencing lots of items at contracts and procurement when destructive, time-consuming, or expensive testing should be used. Many types of acceptance sampling plans have been proposed. The types of sampling plans discussed here are variables repetitive group sampling plan, variables multiple dependent (or deferred) state sampling plan, cumulative count of conforming sampling plan, and sampling plan for resubmitted lot. Sherman (1965) proposed the repetitive group sampling (RGS) plan and the concept of repetitive group sampling for variables inspection is introduced by Balamurali and Jun (2006). Recently, the RGS plans have been studied by many authors. Examples include Aslam et al. (2011), Wu (2012), and Liu and Wu (2014). Balamurali and Jun (2007) developed a multiple dependent (or deferred) state (MDS) sampling plan for variables inspection of normally distributed quality characteristics. Zhang et al. (2008) proposed the cumulative count of conforming (CCC) charts under inspection by samples. Some other applications of CCC sampling plan are reported by Fallahnezhad et al. (2011) and Fallahnezhad and Niaki (2013). Govindaraju and Ganesalingam (1997) developed a resubmitted sampling plan and examined the situation where resampling is permitted on lots not accepted on original inspection. Liu et al. (2014) develop a sampling scheme by variables inspection for resubmitted lots based on the process yield index.

Different approaches are proposed for designing the acceptance sampling plans. In one of these approaches, a sampling plan is designed in such a way that minimize the producer and consumer loss simultaneously. Ferrell and Chhoker (2002) proposed economically optimal acceptance sampling plans based on the Taguchi loss function to determine the producer's tolerance that minimizes the total loss to the producer and consumer. Moskowitz and Tang (1992) developed a Bayesian analysis for variables acceptance sampling using quadratic loss function. Fallahnezhad and Aslam (2013) proposed a new acceptance sampling model to decide about the received lot based on cost objective function. They used the Bayesian inference to evaluate the expected cost of different decisions.

Another approach to design acceptance sampling plans is minimum angle method. The main purpose of this approach is to reach the ideal operating characteristic (OC) curve. In acceptance sampling plan with the ideal OC curve, the probability of accepting an acceptable lot is one and the probability of accepting an unacceptable lot is zero. Therefore, it is obvious that approaching to the ideal OC curve is favorable. Soundararajan and Christina (1997) were first authors who proposed a method for the selection of optimal single sampling plans based on the minimum angle between the lines that joins $[AQL,\ P_a(AQL)]$ to $[LQL,\ P_a(LQL)]$. This angle (θ) is denoted in Figure 1. By minimizing (θ), the OC curve will approach to the ideal OC curve. $tan(\theta)$ is obtained as follows.

$$tan\left(\theta\right) = \frac{LQL - AQL}{P_a\left(AQL\right) - P_a\left(LQL\right)}$$

Since (θ) should be minimized, thus by minimizing the value of $tan(\theta)$, (θ) will be minimized. Ahmadi Yazdi and Fallahnezhad (2014) presented a markove model for acceptance sampling plans based on the cumulative count of conforming run length using the minimum angle method.

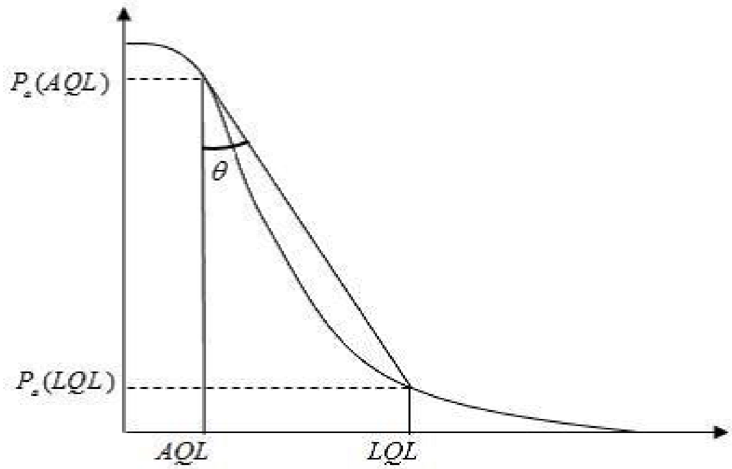

Figure 1. Tangant angle minimizing using AQL, LQL(Soundararajan and Christina (1997))

In our proposed approach, new sampling plans is presented by considering the total expected loss and conformity to the ideal OC curve as objective functions. Thus, according to this approach, not only the summation of the producer loss and the consumer loss has been minimized but also the OC curve of the proposed sampling plan approaches to the ideal OC curve. The cost function of the proposed model is formulated based on the application of rectifying inspection plan. In the rectifying inspection plan, the 100% inspection will be executed for the items of the rejected lot and the producer must repair or replace the nonconforming items. Also, the producer must repair or replace the nonconforming items detected in the sample of accepted lot. The second function is formulated based on the minimum angle method. So, by minimizing the value of $tan(\theta)$, the OC curve will approach to the ideal OC curve. As mentioned above, both the objective functions should be optimized simultaneously.

There are two kinds of risks. Producer risk is the probability of rejecting an acceptable lot and consumer risk is the probability of accepting an unacceptable lot. To design an appropriate sampling plan, these risks should be balanced. For this purpose, the designer specifies two points on the OC curve which define the acceptable and unacceptable quality levels (Pearn and Wu (2006)). Wu et al. (2012) developed a variables inspection scheme for resubmitted lots so that it could satisfy producers and consumers opposing requirements.

Therefore, the main purpose of this paper is to develop four types of acceptance sampling plans based on minimizing the total loss to the producer and consumer and maximizing the rate of approaching to the ideal OC curve simultaneously. In addition, two constraints are considered to balance both producer risk α and consumer risk β. In this research, the classical sampling plans are investigated based on the concept of variable sampling methods for the quality characteristic with one sided specification limit that usually occurs in practical application like canning problems. The main contributions of the model can be summarized as follows:

1. Developing cost objective function based on the application of rectifying inspection plan for the process that its quality characteristic has only lower specification limit.

2. Developing the minimum angle method for different types of acceptance sampling plan.
3. Developing a new objective function to consider economical and statistical properties of sampling plan.
4. Comparison study of different types of acceptance sampling plans in the presence of each objective function.

The rest of this paper is organized as follows. The required notations to model the problem are introduced in Section 2. The design and construction of the proposed models is presented in detail in Section 3. In Section 4, simulation studies are performed to study the performance of the proposed plans. Finally, the conclusions of our discussion are given in Section 5.

2. Notations

The following notations will be used in the rest of the paper:
N: The number of items in the submitted lot.
p: The nonconforming proportion of the submitted lot.
x: The quality characteristic.
$\acute{c}$: The cost of inspection.
$c_p(x)$: The loss function of producer.
B: The producer's cost to repair or replace an item.
$c_c(x)$: The loss function of consumer.
H: The coefficient of consumer loss function.
K: Average quality loss.
Δ: The consumer's tolerance.
L_0: The lower specification limit of the quality characteristic.
μ: Mean value of the quality characteristic (Process mean).
σ: Standard deviation of the quality characteristic (Process standard deviation).
n_σ: The sample size.
$f(x)$: The probability density function of the normal random variable with mean μ and standard deviation σ.
$\phi(y)$: The cumulative distribution function of a random variable y.
p_{ij}: The transition probability from state i to state j in a single step.
P: The transition probability matrix.
A: An identity matrix that its size is equal to the number of absorbing states.
O: A zero matrix.
R: A matrix that includes the transition probabilities from a transient states to an absorbing states.
Q: A square matrix that its size is equal to the number of transient states and includes the transition probabilities from a transient state to a transient state.
M: The fundamental matrix that includes the expected number of times in the long-run that the process resides in the transient states before absorption occurs.
$m_{ij}(p)$: The expected number of transitions from a transient state (i) to a transient state (j) for given lot quality, p, before absorption occurs.
F: The long-run absorption probability matrix.
$f_{ij}(p)$: Long-run transition probability from a transient state (i) to an absorbing state (j) for given lot quality, p.
$P_a(p)$: The proportion of lots that are expected to be accepted for given lot quality, p.
$P_r(p)$: The proportion of lots that are expected to be rejected for given lot quality, p.
ASN: Average sample number.
$E(loss)$: The expected value of total loss.

δ:The minimum possible value for quality characteristic.
p_1: The acceptable quality level (AQL).
p_2: The limiting quality level (LQL).
α: The producer's risk.
β: The consumer's risk.

3. Description of the proposed sampling plans

Consider a lot of N items with a quality level p. Random samples are drawn from the lot for inspection and based on the difference between sample average and lower specification limit of the quality characteristic, the lot is accepted, rejected, or the inspection continues. If the lot is rejected during the decision-making process, then rectifying inspection will be applied. It is assumed that the cost of inspection, the producer's cost to repair or replace a nonconforming item, and the consumer's cost associated with a product which its quality characteristic deviates from the target value are known. The cost of inspection and cost of repair or replacement of an item, regardless of the value of the quality characteristic, are $\acute{c}$ and B respectively. Further, the one-sided minimum specification Taguchi loss function called the larger-is-better loss function is assumed to represent the consumer's loss. In this function, the deviation from the target is allowed in one direction only. Let $c_c(x)$ be the loss function of consumer that is defined as (Liao and Kao, 2010):

$$c_c(x) = \frac{H}{x^2}, \tag{3.1}$$

here, $c_c(x)$ represents the loss associated with a specified value of the quality characteristic x, and H is given by

$$H = K\Delta^2, \tag{3.2}$$

where K is average quality loss and Δ is the consumer's tolerance.

It is also assumed that the quality characteristic has the lower specification limit L_0 and follows a normal distribution with unknown mean μ and known standard deviation σ. In the following, four sampling plans are proposed under the same conditions.

An example for application of such models is the canning problem where the amount of an expensive ingredient put into a can is the quality characteristic (for example, the amount of vitamin C in a juice). Depending on whether the amount of an expensive ingredient in the can exceed the lower specification limit or not, the can is classified as conforming or nonconforming. In this problem, the lot of can with the size of N is submitted for inspection. A samples of size n_σ are randomly selected from the lot and based on the difference between sample average and lower specification limit of the quality characteristic, the lot is accepted, rejected, or the inspection continues. In the cumulative count of conforming (CCC) sampling plan, in addition to the criteria considered for other plans, making decisions is also based on the number of conforming samples before detecting r_{th} nonconforming sample. This means that, at first the inspected sample is classified as conforming or nonconforming based on the difference between sample average and lower specification limit of the quality characteristic and then based on the number of conforming samples before detecting r_{th} nonconforming sample the lot is accepted, rejected, or the inspection continues.

3.1. Proposed variables RGS plan. First, a variables RGS plan is proposed based on minimizing the total loss to the producer and consumer and maximizing the rate of approaching to the ideal operating characteristic (OC) curve simultaneously. The decision-making process is operated as following steps.

Step 1. From each submitted lot, take a random sample of size n_σ, say $(x_1, x_2, \ldots, x_{n_\sigma})$ and compute

$$v = \frac{\overline{x} - L_0}{\sigma}, \text{ where } \overline{x} = \frac{1}{n_\sigma}\sum_{i=1}^{n_\sigma} x_i.$$

Step 2. Accept the lot if $v \geq k_{a\sigma}$ and reject the lot if $v < k_{r\sigma}$. Otherwise, if $k_{r\sigma} \leq v < k_{a\sigma}$, repeat the steps 1 and 2 (Note: $k_{a\sigma} > k_{r\sigma}$) (Balamurali and Jun, 2006).

It is noted that this plan is characterized by three parameters, namely n_σ, $k_{a\sigma}$ and $k_{r\sigma}$, that should be determined.

The decision-making process involves three states, as follows:

state 1: $k_{r\sigma} \leq v < k_{a\sigma}$, taking a new sample for further judgment.

state 2: $v \geq k_{a\sigma}$, accepting the lot.

state 3: $v < k_{r\sigma}$, rejecting the lot.

So the transition probabilities from the first state to all states mentioned above, can be obtained as follows.

The probability of taking a new sample for further judgment $= p_{11} = P(k_{r\sigma} \leq v < k_{a\sigma})$.

The probability of accepting the lot $= p_{12} = P(v \geq k_{a\sigma})$.

The probability of rejecting the lot $= p_{13} = P(v < k_{r\sigma})$.

The item is nonconforming if its quality characteristic falls below the lower specification limit L_0, thus, the nonconforming proportion of the lot can be determined as:

$$p = P\left(x < L_0\right) = \int_{-\infty}^{L_0} f(x)dx = 1 - \phi\left(\frac{\mu - L_0}{\sigma}\right), \tag{3.3}$$

where $f(x)$ is the probability density function of the normal random variable with mean μ and standard deviation σ and $\phi(y)$ is the cumulative distribution function of a random variable y that is given by

$$\phi\left(y\right) = \int_{-\infty}^{y} \frac{1}{\sqrt{2\pi}} exp(\frac{-z^2}{2})dz. \tag{3.4}$$

Thus the mean value of quality characteristic μ is obtained using Eq. (3.3) as follows:

$$\mu = \sigma \times \phi^{-1}\left(1 - p\right) + L_0. \tag{3.5}$$

Since the quality characteristic is normally distributed with mean μ and standard deviation σ, thus the following is obtained,

$$P\left(v \geq k_{a\sigma}\right) = P\left(\frac{\overline{x} - L_0}{\sigma} \geq k_{a\sigma}\right) = P\left(\frac{\overline{x} - \mu}{\frac{\sigma}{\sqrt{n_\sigma}}} \geq (k_{a\sigma} + \frac{L_0 - \mu}{\sigma})\sqrt{n_\sigma}\right) \tag{3.6}$$

The probability of accepting the lot at each iteration in Eq. (3.6) can be written as

$$p_{12} = P\left(v \geq k_{a\sigma}\right) = 1 - \phi\left(w_1\right), \tag{3.7}$$

where

$$w_1 = \left(k_{a\sigma} + \frac{L_0 - \mu}{\sigma}\right)\sqrt{n_\sigma} \tag{3.8}$$

and

$$(3.9) \qquad P(v < k_{r\sigma}) = P\left(\frac{\overline{x} - L_0}{\sigma} < k_{r\sigma}\right) = P\left(\frac{\overline{x} - \mu}{\frac{\sigma}{\sqrt{n_\sigma}}} < (k_{r\sigma} + \frac{L_0 - \mu}{\sigma})\sqrt{n_\sigma}\right)$$

The probability of rejecting the lot at each iteration in Eq. (3.9) can be written as

$$(3.10) \qquad p_{13} = P(v < k_{r\sigma}) = \phi(w_2),$$

where

$$(3.11) \qquad w_2 = \left(k_{r\sigma} + \frac{L_0 - \mu}{\sigma}\right)\sqrt{n_\sigma}$$

Finally, the probability of taking a new sample for further judgment can be obtained using Eq. (3.7) and (3.10) as follows.

$$(3.12) \qquad p_{11} = 1 - p_{12} - p_{13} = \phi(w_1) - \phi(w_2)$$

Then, the transition probability matrix used to describe the transitions of a Markov chain is expressed as follows:

$$(3.13) \qquad P = \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

It can be realized that the first state is transient state and states 2 and 3 are absorbing states. Therefore, the matrix P is an absorbing Markov chain because each transient state reaches to an absorbing state.

To analyze the above absorbing Markov chain, the arrangement of transition probability matrix should be changed in to the following form:

$$(3.14) \qquad \begin{bmatrix} A & O \\ R & Q \end{bmatrix}$$

where A is an identity matrix, O is a zero matrix, R is a matrix that includes transition probabilities from non-absorbing states to absorbing states and Q is a transition probability matrix among non-absorbing states.

So by rearranging rows and columns of the P matrix correspondingly, the following matrix is obtained:

$$(3.15) \qquad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ p_{12} & p_{13} & p_{11} \end{bmatrix}$$

The fundamental matrix M can be determined as follows (Bowling et al, 2004):

$$(3.16) \qquad M = m_{11}(p) = (I - Q)^{-1} = \frac{1}{1 - p_{11}} = \frac{1}{1 - P(k_{r\sigma} \leq v < k_{a\sigma})},$$

where I is the identity matrix. The value $m_{11}(p)$ denotes the expected number of times in the long-run that the process visits the transient state 1 (i.e., continue inspection) for given value of p, until one of the absorbing states occures (i.e., accepted or rejected), when the process starts from the transient state 1. The long-run absorption probability matrix, F, is calculated as follows (Bowling et al, 2004):

$$F = M \times R = \left[\begin{array}{cc} f_{12}(p) & f_{13}(p) \end{array}\right] = \left[\begin{array}{cc} \frac{p_{12}}{1-p_{11}} & \frac{p_{13}}{1-p_{11}} \end{array}\right], \tag{3.17}$$

where $f_{12}(p)$ and $f_{13}(p)$ denote the proportion of lots that are expected to be accepted and rejected for given lot quality, p, respectively, and can be written as

$$P_a(p) = f_{12}(p) = \frac{p_{12}}{1 - p_{11}} = \frac{P(v \geq k_{a\sigma})}{1 - P(k_{r\sigma} \leq v < k_{a\sigma})} = \frac{1 - \phi(w_1)}{1 - \phi(w_1) + \phi(w_2)} \tag{3.18}$$

and

$$P_r(p) = f_{13}(p) = \frac{p_{13}}{1 - p_{11}} = \frac{P(v < k_{r\sigma})}{1 - P(k_{r\sigma} \leq v < k_{a\sigma})} = \frac{\phi(w_2)}{1 - \phi(w_1) + \phi(w_2)} \tag{3.19}$$

In the repetitive group sampling plan, the number of items sampled is a random variable. Moreover, as already noted, $m_{11}(p)$ is the expected number of times in the long-run that the sampling continues for value of p, until one of the absorbing states occures, and in each sampling stage, the sample size is n_σ. Thus the average sample number (ASN) can be determined as follows:

$$ASN = n_\sigma \times m_{11}(p) \tag{3.20}$$

The objective functions of this model are written based on minimizing the total loss to the producer and consumer and approaching to the ideal OC curve simultaneously.

The total expected loss associated with the repetitive group sampling plan can be obtained as:

$$\begin{aligned} Z_{1RGS} = E(loss) = ASN \times \left\{ \acute{c} + \int_\delta^{L_0} c_p(x) f(x)\,dx + \int_{L_0}^\infty c_c(x) f(x)\,dx \right\} + \\ P_a(p) \times (N - ASN) \times \int_\delta^\infty c_c(x) f(x)\,dx + \\ P_r(p) \times (N - ASN) \times \left\{ \acute{c} + \int_\delta^{L_0} c_p(x) f(x) dx + \int_{L_0}^\infty c_c(x) f(x) dx \right\}, \end{aligned} \tag{3.21}$$

where

$$c_p(x) = B \tag{3.22}$$

Proof.

$$total\ expected\ loss = A_1 + A_2 + A_3$$

$$A_1 = E\,(cost\ of\ inspected\ items)$$

A_1 consists of three parts. The first part is related to the cost of inspecting the items. The second part presents the cost of the inspected items which their quality characteristic falls below the lower specification limit L_0, so these nonconforming items are returned to the producer and the costs of repair or replacement of them are the responsibility of the producer. The third part states that if the quality characteristic of the inspected items is more than L_0, then these items will be sent to the consumer and the consumer is exposed to the possible losses. Note that the consumer loss will be decreased with further increase in quality characteristics.

$$A_2 = E\{\text{cost of remained items in the lot} \mid \text{accepting the lot and sending to consumer}\} \times P(\text{accepting the lot and sending to consumer})$$

A_2 describes the expected cost of remained items in the lot conditioned on the decision of accepting the lot. When the lot is accepted, then remained items in the lot will not be inspected and they will be sent to the consumer. So there will be no costs of inspection for the remaining items. Also, the producer will not have any costs in this condition, because the consumer is responsible for cost of all remained items in the lot. That is why the integration limits are the parameter δ and ∞. The limits δ and ∞ denote that the item is not inspected and we do not have any information about variations interval of quality characteristics.

$$A_3 = E\{\text{cost of remained items in the lot} \mid \text{rejecting the lot and rectifying inspection}\} \times P(\text{rejecting the lot and rectifying inspection})$$

A_3 examines the cost of remained items in the lot if it is rejected. In this case, rectifying inspection will be executed. When rectifying inspection is carried out then all of the remained items in the lot will be inspected, nonconforming items will be returned to the producer and the producer must repair or replace them, conforming items will be sent to the consumer and the consumer is responsible for cost of them. So, we are faced with three types of costs, the cost of inspection, the producer's cost, and the consumer's cost.

It is obvious that each quality characteristic has a random value, but we can be sure that its value is more than zero, also in the industrial applications, each quality characteristic can not be less than a specified value like δ. According to equation (3.21), the first term represents the costs associated with inspecting the random samples, the second term denotes the consumer's loss when the lot is accepted and the third term represents the costs associated with inspecting remained items in the lot when the lot is rejected (Ferrell and Chhoker, 2002).

In order to approach to the ideal OC curve, the tangent of angle between the line that joins $[p_1,\ P_a(p_1)]$ to $[p_2,\ P_a(p_2)]$ is considered. The tangent of this angle, $\tan(\theta)$, is obtained as (Soundararajan and Christina, 1997):

$$Z_{2RGS} = tan(\theta) = \frac{p_2 - p_1}{P_a(p_1) - P_a(p_2)}, \tag{3.23}$$

where

$$P_a(p_1 = AQL) = \frac{1-\phi(w_{11})}{1-\phi(w_{11})+\phi(w_{21})},$$
$$P_a(p_2 = LQL\ ) = \frac{1-\phi(w_{12})}{1-\phi(w_{12})+\phi(w_{22})}, \tag{3.24}$$

here, w_{11} is the value of w_1 at $p = AQL$ (or p_1), w_{21} is the value of w_2 at $p = AQL$, w_{12} is the value of w_1 at $p = LQL$ (or p_2) and w_{22} is the value of w_2 at $p = LQL$. That are determined as following,

$$w_{11} = \left(k_{a\sigma} + \frac{L_0 - \mu_1}{\sigma}\right)\sqrt{n_\sigma}, \quad w_{21} = \left(k_{r\sigma} + \frac{L_0 - \mu_1}{\sigma}\right)\sqrt{n_\sigma},$$

$$w_{12} = \left(k_{a\sigma} + \frac{L_0 - \mu_2}{\sigma}\right)\sqrt{n_\sigma}, \quad w_{22} = \left(k_{r\sigma} + \frac{L_0 - \mu_2}{\sigma}\right)\sqrt{n_\sigma}, \tag{3.25}$$

where μ_1 is the value of μ at AQL and μ_2 is the value of μ at LQL, as following:

$$\mu_1 = \sigma \times \phi^{-1}(1 - p_1) + L_0,$$

$$\mu_2 = \sigma \times \phi^{-1}(1 - p_2) + L_0. \tag{3.26}$$

As noted by Soundararajan and Christina (1997), it is obvious that if $\tan(\theta)$ decreases then OC curve approaches to the ideal OC curve.

To design an appropriate sampling plan, the opposing requirements of both consumer and producer must be satisfied. Thus, the producer desires a high probability of acceptance $(1 - \alpha)$ when the nonconforming proportion of lot is equal to acceptable quality level (AQL) and the consumer desires that the probability of acceptance becomes less than β when the nonconforming proportion of lot is equal to limiting quality level (LQL). So the following constraints that cover these requirements are added to the model.

$$P_a(p_1 = AQL) \geq 1 - \alpha,$$

$$P_a(p_2 = LQL\ ) \leq \beta \tag{3.27}$$

Now the optimization model of the problem can be written as follows:

$$\begin{aligned}
&Minimize Z_{1RGS}\\
&Minimize Z_{2RGS}\\
&Subject\ to\\
&P_a(p_1) \geq 1 - \alpha,\\
&P_a(p_2) \leq \beta,\\
&k_{a\sigma} > k_{r\sigma} > 0,\\
&n_\sigma \geq 1.
\end{aligned} \tag{3.28}$$

3.2. Proposed variables MDS sampling plan. Balamurali and Jun (2007) originally proposed the MDS sampling plan for the inspection process. The proposed plan is the extension of their plan in which the total expected loss and the rate of approaching to the ideal OC curve are the objective functions. The plan procedure is stated as follows

Step 1: From each submitted lot, take a random sample of size n_σ, say $(x_1,\ x_2,\ \ldots,\ x_{n_\sigma})$ and compute

$v = \dfrac{\overline{x} - L_0}{\sigma}$, where $\overline{x} = \frac{1}{n_\sigma}\sum_{i=1}^{n_\sigma} x_i$.

Step 2: Accept the lot if $v \geq k_{a\sigma}$ and reject the lot if $v < k_{r\sigma}$. If $k_{r\sigma} \leq v < k_{a\sigma}$ and if the preceding m_σ lots were accepted on the condition that $v_j \geq k_{a\sigma}$ where $j = 1,\ 2,\ \ldots,\ m_\sigma$, then accept the current lot otherwise reject the lot.

Note that v_j is the statistic obtained in the previous m_σ lots and can be computed as follows

$v_j = \frac{\overline{x}_j - L_0}{\sigma}$, where $\overline{x}_j = \frac{1}{n_\sigma}\sum_{i=1}^{n_\sigma} x_{ij}$ and $(x_{1j},\ x_{2j},\ \ldots,\ x_{n_\sigma j})$ is the quality characteristic of sample withdrawn in the j-th lot, where $j = 1,\ 2,\ \ldots,\ m_\sigma$.

Thus, the parameters of this plan are the sample size n_σ, acceptance threshold $k_{a\sigma}$, rejection threshold $k_{r\sigma}$ and number of preceding lots m_σ.

The nonconforming proportion of the lot is determined as:

$$(3.29) \quad p = P\left\{x < L_0 \mid \mu\right\} = \int_{-\infty}^{L_0} f(x)dx = 1 - \phi\left(\frac{\mu - L_0}{\sigma}\right)$$

Consequently, mean value of quality characteristic μ is calculated as follows:

$$(3.30) \quad \mu = \sigma \times \phi^{-1}\left(1 - p\right) + L_0$$

According to the second step of the decision-making process, to calculate the proportion of lots that are expected to be accepted for given value of p, both the current sample from the lot and the sample results from preceding lots must be considered. Thus following is obtained

$$(3.31) \quad P_a\left(p\right) = P\left(v \geq k_{a\sigma} \mid p\right) + P\left(k_{r\sigma} \leq v < k_{a\sigma} \mid p\right) \times \prod_{j=1}^{m_\sigma} P\left(v_j \geq k_{a\sigma} \mid p\right),$$

Since the quality characteristic is normally distributed with mean μ and standard deviation σ, the probabilities can be obtained by

$$(3.32) \quad P\left(v \geq k_{a\sigma}\right) = P\left(\frac{\overline{x} - L_0}{\sigma} \geq k_{a\sigma}\right) = P\left(\frac{\overline{x} - \mu}{\frac{\sigma}{\sqrt{n_\sigma}}} \geq \left(k_{a\sigma} + \frac{L_0 - \mu}{\sigma}\right)\sqrt{n_\sigma}\right)$$

The probability of accepting the lot based on a current sample in Eq. (3.32) can be written as

$$(3.33) \quad P\left(v \geq k_{a\sigma}\right) = 1 - \phi\left(w_1\right),$$

and

$$(3.34) \quad P\left(v < k_{r\sigma}\right) = P\left(\frac{\overline{x} - L_0}{\sigma} < k_{r\sigma}\right) = P\left(\frac{\overline{x} - \mu}{\frac{\sigma}{\sqrt{n_\sigma}}} < \left(k_{r\sigma} + \frac{L_0 - \mu}{\sigma}\right)\sqrt{n_\sigma}\right)$$

The probability of rejecting the lot based on a current sample in Eq. (3.34) can be written as

$$(3.35) \quad P\left(v < k_{r\sigma}\right) = \phi\left(w_2\right),$$

where w_1 and w_2 are defined in Eq. (3.8) and (3.11) respectively.

Thus, the probability of considering the preceding lots for decision making can be obtained using Eq. (3.33) and (3.35) as follows.

$$(3.36) \quad P\left(k_{r\sigma} \leq v < k_{a\sigma}\right) = \phi\left(w_1\right) - \phi\left(w_2\right)$$

Also, the probability of accepting one of the preceding m_σ lots on the condition that $v_j \geq k_{a\sigma}$ can be obtained by

$$(3.37) \quad P\left(v_j \geq k_{a\sigma}\right) = P\left(\frac{\overline{x}_j - L_0}{\sigma} \geq k_{a\sigma}\right) = P\left(\frac{\overline{x}_j - \mu}{\frac{\sigma}{\sqrt{n_\sigma}}} \geq \left(k_{a\sigma} + \frac{L_0 - \mu}{\sigma}\right)\sqrt{n_\sigma}\right)$$

Equation (3.37) can be written as

$$P\left(v_j \geq k_{a\sigma}\right) = 1 - \phi\left(w_1\right) \tag{3.38}$$

Now, the acceptance probability of the lot in Eq. (3.31) can be written as

$$P_a\left(p\right) = \left[1 - \phi\left(w_1\right)\right] + \left[\phi\left(w_1\right) - \phi\left(w_2\right)\right]\left[1 - \phi\left(w_1\right)\right]^{m_\sigma} \tag{3.39}$$

In this problem, desired objectives are to minimize the total expected loss and to maximize the conformity to the ideal OC curve that can be formulated as following functions.

$$\begin{aligned} &Z_{1MDS} = E(loss) = n_\sigma \times \left\{\acute{c} + \int_\delta^{L_0} c_p\left(x\right) f\left(x\right) dx + \int_{L_0}^{\infty} c_c\left(x\right) f\left(x\right) dx\right\} + \\ &P_a\left(p\right) \times \left(N - n_\sigma\right) \times \int_\delta^{\infty} c_c\left(x\right) f\left(x\right) dx + \\ &P_r\left(p\right) \times \left(N - n_\sigma\right) \times \left\{\acute{c} + \int_\delta^{L_0} c_p\left(x\right) f\left(x\right) dx + \int_{L_0}^{\infty} c_c\left(x\right) f\left(x\right) dx\right\}, \end{aligned} \tag{3.40}$$

where

$$\begin{aligned} &c_p\left(x\right) = B, \\ &P_r\left(p\right) = 1 - P_a\left(p\right) \end{aligned} \tag{3.41}$$

and

$$Z_{2MDS} = tan\left(\theta\right) = \frac{p_2 - p_1}{P_a\left(p_1\right) - P_a\left(p_2\right)}\ , \tag{3.42}$$

where

$$\begin{aligned} &P_a\left(p_1 = AQL\right) = \left[1 - \phi\left(w_{11}\right)\right] + \left[\phi\left(w_{11}\right) - \phi\left(w_{21}\right)\right]\left[1 - \phi\left(w_{11}\right)\right]^{m_\sigma}, \\ &P_a\left(p_2 = LQL\right) = \left[1 - \phi\left(w_{12}\right)\right] + \left[\phi\left(w_{12}\right) - \phi\left(w_{22}\right)\right]\left[1 - \phi\left(w_{12}\right)\right]^{m_\sigma}, \end{aligned} \tag{3.43}$$

here, w_{11}, w_{12}, w_{21} and w_{22} are obtained in equation (3.25).

Also following constraints are considered in the model to balance producers and consumers risks.

$$\begin{aligned} &P_a\left(p_1 = AQL\right) \geq 1 - \alpha, \\ &P_a\left(p_2 = LQL\right) \leq \beta, \end{aligned} \tag{3.44}$$

Now the optimization model of the problem can be written as follows:

$$\begin{aligned} &Minimize Z_{1MDS} \\ &Minimize Z_{2MDS} \\ &Subject\ to \\ &P_a\left(p_1\right) \geq 1 - \alpha, \\ &P_a\left(p_2\right) \leq \beta, \\ &k_{a\sigma} > k_{r\sigma} > 0, \\ &n_\sigma \geq 1. \end{aligned} \tag{3.45}$$

3.3. Proposed CCC sampling plan. Applying the cumulative count of conforming (CCC) control chart under inspection by samples for designing a sampling plan is an effective method. In this plan, in addition to the criteria considered for other plans, making decisions is also based on the number of conforming samples before detecting r_{th} nonconforming sample. Let y denote the number of conforming samples till the detection of r_{th} nonconforming sample. It is obvious that y follows negative binomial distribution. The decision making method is as follows. If the y value is more than the critical threshold for acceptance U, then the lot is accepted. If the y value is less than the critical threshold for rejection L, then the lot is rejected. Otherwise, there is no sufficient information for decision making, so a new sample should be taken for further judgment. Thus, states of the decision making method are as follows,
State 1: $L < y < U$, continue inspecting.
State 2: $y \geq U$, the lot is accepted.
State 3: $y \leq L$, the lot is rejected.

If p_{kl} denotes the probability of going from state k to state l then transition probabilities are obtained as follows,

$$p_{11} = P\left(L < y < U\right), p_{12} = P\left(y \geq U\right), p_{13} = P\left(y \leq L\right)$$

where $P\left(y = i \mid r, p_0\right) = \binom{i+r-1}{r-1}\left(1-p_0\right)^i {p_0}^r$; for $i = 0,\ 1,\ 2,\ \ldots$ is the negative binomial distribution and p_0 denotes the proportion of nonconforming samples.

To determine the proportion of nonconforming samples, first, from each submitted lot, take a random sample of size n_σ, say $(x_1, x_2, \ldots, x_{n_\sigma})$ and compute

$$v = \frac{\overline{x} - L_0}{\sigma}\ , \text{ where } \overline{x} = \tfrac{1}{n_\sigma}\textstyle\sum_{i=1}^{n_\sigma} x_i,$$

if $v \geq k_{a\sigma}$, then the sample is classified as conforming. Otherwise, the sample is classified as nonconforming.

Note that each observation is a sample instead of item and v is a criterion to determine that an inspected sample is conforming or nonconforming, so for each inspected sample if $v \geq k_{a\sigma}$, then the sample is classified as conforming, otherwise, the sample is classified as nonconforming. By identifying each conforming sample, one unit is added to the value of y and the inspection process will continue till the detection of r_{th} nonconforming sample.

Since the quality characteristic is normally distributed with mean μ and standard deviation σ, thus following is obtained

$$p_0 = P\left(v < k_{a\sigma}\right) = P\left(\tfrac{\overline{x}-L_0}{\sigma} < k_{a\sigma}\right) = P\left(\frac{\frac{\overline{x}-\mu}{\frac{\sigma}{\sqrt{n_\sigma}}}}{} < \left(k_{a\sigma} + \tfrac{L_0-\mu}{\sigma}\right)\sqrt{n_\sigma}\right) \tag{3.46}$$

The proportion of nonconforming samples, p_0, in Eq. (3.46) can be written as

$$p_0 = P\left(v < k_{a\sigma}\right) = \phi\left(w_1\right) \tag{3.47}$$

where w_1 is defined in Eq. (3.8).

Mean value of quality characteristic μ can be obtained based on the equation of determining the proportion of nonconforming items as following

$$p = P\left\{x < L_0 \mid \mu\right\} = \int_{-\infty}^{L_0} f(x)dx = 1 - \phi\left(\frac{\mu - L_0}{\sigma}\right), \tag{3.48}$$

So, mean value of quality characteristic μ is obtained as follows:

$$\mu = \sigma \times \phi^{-1}\left(1 - p\right) + L_0 \tag{3.49}$$

The transition probability matrix is as follows:

$$(3.50) \quad P = \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

States 2 and 3 are absorbing state and state 1 is transient. Also, it is possible to go to an absorbing state from each transient state. As a result, the matrix P is an absorbing Markov chain. For analyzing this absorbing Markov chain, the matrix is rewritten in following form.

$$(3.51) \quad \begin{bmatrix} A & O \\ R & Q \end{bmatrix}$$

By doing so, the following matrix is obtained:

$$(3.52) \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ p_{12} & p_{13} & p_{11} \end{bmatrix}$$

The fundamental matrix M can be determined as follows (Bowling et al, 2004):

$$(3.53) \quad M = m_{11}(p) = (I - Q)^{-1} = \frac{1}{1 - p_{11}} = \frac{1}{1 - P(L < y < U)},$$

where I is the identity matrix and $m_{11}(p)$ denotes the expected number of visits to transient state 1 (i.e., continue inspection) starting from transient state 1 until being absorbed (i.e., accepted or rejected). The long-run absorption probability matrix, F, is calculated as follows (Bowling et al, 2004):

$$(3.54) \quad F = M \times R = \begin{bmatrix} f_{12}(p) & f_{13}(p) \end{bmatrix} = \begin{bmatrix} \frac{p_{12}}{1-p_{11}} & \frac{p_{13}}{1-p_{11}} \end{bmatrix},$$

where $f_{12}(p)$ and $f_{13}(p)$ denote the proportion of lots that are expected to be accepted and rejected for given value of p respectively, and can be written as

$$(3.55) \quad P_a(p) = f_{12}(p) = \frac{p_{12}}{1 - p_{11}} = \frac{P(y \geq U)}{1 - P(L < y < U)}$$

and

$$(3.56) \quad P_r(p) = f_{13}(p) = \frac{p_{13}}{1 - p_{11}} = \frac{P(y \leq L)}{1 - P(L < y < U)}$$

In this sampling plan, due to the random feature of the number of samples inspected, an important performance measure of sampling plans, the average sample number (ASN), is used. As mentioned before, $m_{11}(p)$ is the expected number of visits to transient state 1 until absorption occurs. Moreover, in each visit to transient state, the average number of samples inspected is $\frac{r}{p_0}$ (the mean value of negative binomial distribution), and the sample size is n_σ. Thus the average sample number (ASN) can be determined as follows:

$$(3.57) \quad ASN = n_\sigma \left(\frac{r}{p_0} \right) m_{11}(p)$$

The objectives of this model are dependent on two factors: minimizing the total loss and approaching to the ideal OC curve.

The total expected loss can be obtained as:

$$\begin{aligned} & Z_{1CCC} = E(loss) = ASN \times \left\{ \acute{c} + \int_{\delta}^{L_0} c_p(x) f(x)\, dx + \int_{L_0}^{\infty} c_c(x) f(x)\, dx \right\} + \\ & P_a(p) \times (N - ASN) \times \int_{\delta}^{\infty} c_c(x) f(x)\, dx + \\ & P_r(p) \times (N - ASN) \times \left\{ \acute{c} + \int_{\delta}^{L_0} c_p(x) f(x) dx + \int_{L_0}^{\infty} c_c(x) f(x) dx \right\}, \end{aligned} \tag{3.58}$$

where

$$c_p(x) = B \tag{3.59}$$

The tangent of angle between the line that joins $[p_1,\ P_a(p_1)]$ to $[p_2,\ P_a(p_2)]$ is considered as the second objective function.

$$Z_{2CCC} = \tan(\theta) = \frac{p_2 - p_1}{P_a(p_1) - P_a(p_2)} \tag{3.60}$$

where $P_a(p)$ has been defined in Eq. (3.55) and, therefore, it can be deduced that

$$P_a(p_1 = AQL) = \frac{P\left(y \geq U \mid r, \phi\left[\left(k_{a\sigma} - \phi^{-1}(1-p_1)\right) \times \sqrt{n_\sigma}\right]\right)}{1 - P\left(L < y < U \mid r, \phi\left[\left(k_{a\sigma} - \phi^{-1}(1-p_1)\right) \times \sqrt{n_\sigma}\right]\right)} \tag{3.61}$$

and

$$P_a(p_2 = LQL) = \frac{P\left(y \geq U \mid r, \phi\left[\left(k_{a\sigma} - \phi^{-1}(1-p_2)\right) \times \sqrt{n_\sigma}\right]\right)}{1 - P\left(L < y < U \mid r, \phi\left[\left(k_{a\sigma} - \phi^{-1}(1-p_2)\right) \times \sqrt{n_\sigma}\right]\right)} \tag{3.62}$$

To ensure that the opposing requirements of both consumer and producer are satisfied, following constraints are added to the model.

$$\begin{aligned} & P_a(p_1 = AQL) \geq 1 - \alpha, \\ & P_a(p_2 = LQL\ ) \leq \beta \end{aligned} \tag{3.63}$$

In short, the optimization model of the problem can be written as follows:

$$\begin{aligned} & Minimize Z_{1CCC} \\ & Minimize Z_{2CCC} \\ & Subject\ to \\ & P_a(p_1) \geq 1 - \alpha, \\ & P_a(p_2) \leq \beta, \\ & U > L > 0. \end{aligned} \tag{3.64}$$

3.4. Proposed sampling plan for resubmitted lot. In this case, the situation of resampling from lots is examined under the objectives of minimizing the total expected loss and maximizing the rate of approaching to the ideal OC curve simultaneously. The operating procedure of the developed resubmitted sampling plan is given as follows

Step 1: From each submitted lot, take a random sample of size n_σ, say $(x_1,\ x_2,\ \ldots,\ x_{n_\sigma})$ and compute

$$v = \frac{\overline{x} - L_0}{\sigma},\ \text{where}\ \overline{x} = \tfrac{1}{n_\sigma}\textstyle\sum_{i=1}^{n_\sigma} x_i.$$

Step 2: Accept the lot if $v \geq k_{a\sigma}$ otherwise, go to step 3.

Step 3: Repeat step 1 and step 2 for $m-1$ times. If the lot was accepted then stop sampling else if the lot was not accepted after $m-1$ iterations of sampling then reject the lot.

Note that v_j is the statistic obtained in the $m-1$ iterations of sampling and can be computed as follows

$$v_j = \frac{\overline{x}_j - L_0}{\sigma},\ \text{where}\ \overline{x}_j = \tfrac{1}{n_\sigma}\textstyle\sum_{i=1}^{n_\sigma} x_{ij}\ \text{and}\ j = 1,\ 2,\ \ldots,\ m-1.$$

For this sampling plan, there are three plan parameters, n_σ, $k_{a\sigma}$ and m, needed to be determined.

The nonconforming proportion of the lot will be determined as:

$$p = P\left\{x < L_0 \mid \mu\right\} = \int_{-\infty}^{L_0} f(x)dx = 1 - \phi\left(\frac{\mu - L_0}{\sigma}\right), \tag{3.65}$$

Consequently, mean value of quality characteristic μ is calculated as follows:

$$\mu = \sigma \times \phi^{-1}(1-p) + L_0 \tag{3.66}$$

Due to this feature of the plan that resubmission is allowed $m-1$ times and in the case of nonacceptance in each inspection, resampling is done, thus the probability of acceptance is easily calculated as follows.

$$P_a(p) = 1 - P(v < k_{a\sigma}) \times \prod_{j=1}^{m-1} P(v_j < k_{a\sigma}) \tag{3.67}$$

The probability of rejecting the lot based on a single sample is as follows

$$P(v < k_{a\sigma}) = P\left(\frac{\overline{x} - L_0}{\sigma} < k_{a\sigma}\right) = P\left(\frac{\overline{x} - \mu}{\frac{\sigma}{\sqrt{n_\sigma}}} < (k_{a\sigma} + \frac{L_0 - \mu}{\sigma})\sqrt{n_\sigma}\right) \tag{3.68}$$

It can be written as

$$P(v < k_{a\sigma}) = \phi(w_1) \tag{3.69}$$

where w_1 is defined in Eq. (3.8).

It can be easily deduced that

$$P(v_j < k_{a\sigma}) = \phi(w_1) \tag{3.70}$$

Now, the probability of accepting the lot in Eq. (3.67) can be written as

$$(3.71)\quad P_a(p) = 1 - [\phi(w_1)]^m$$

The average sample number (ASN) of the proposed plan for given value of p, can be determined as follows (Govindaraju and Ganesalingam, 1997)

$$(3.72)\quad ASN = \frac{n_\sigma P_a(p)}{P(v \geq k_{a\sigma})} = \frac{n_\sigma [1 - [\phi(w_1)]^m]}{1 - \phi(w_1)}$$

The objective functions for the problem are written based on the total expected loss and conformity to the ideal OC curve as given below:
The first objective function follows:

$$Z_{1resubmitted\ lot} = E(loss) = ASN \times \left\{ \acute{c} + \int_\delta^{L_0} c_p(x) f(x)\, dx + \int_{L_0}^\infty c_c(x) f(x)\, dx \right\} +$$
$$P_a(p) \times (N - ASN) \times \int_\delta^\infty c_c(x) f(x)\, dx +$$
$$(3.73)\quad P_r(p) \times (N - ASN) \times \left\{ \acute{c} + \int_\delta^{L_0} c_p(x) f(x) dx + \int_{L_0}^\infty c_c(x) f(x) dx \right\},$$

where

$$(3.74)\quad c_p(x) = B$$

And the second objective function follows:

$$(3.75)\quad Z_{2resubmitted\ lot} = \tan(\theta) = \frac{p_2 - p_1}{P_a(p_1) - P_a(p_2)}$$

where

$$P_a(p_1 = AQL) = 1 - [\phi(w_{11})]^m$$
$$(3.76)\quad P_a(p_2 = LQL) = 1 - [\phi(w_{12})]^m$$

where w_{11} and w_{12} are defined in equation (3.25).
The same constraints are considered for the optimization model of this plan, as:

$$P_a(p_1 = AQL) \geq 1 - \alpha,$$
$$(3.77)\quad P_a(p_2 = LQL\ ) \leq \beta$$

Now the optimization model of the problem can be written as follows:

$$
\begin{aligned}
& Minimize Z_{1resubmitted\ lot} \\
& Minimize Z_{2resubmitted\ lot} \\
& Subject\ to \\
& P_a\left(p_1\right) \geq 1-\alpha, \\
& P_a\left(p_2\right) \leq \beta, \\
& n_\sigma \geq 1. \qquad (3.78)
\end{aligned}
$$

4. Simulation studies

In this study, minimizing the total expected loss, $E(loss)$, and the tangent of angle between the line that joins $[p_1,\ P_a\left(p_1\right)]$ to $[p_2,\ P_a\left(p_2\right)]$, $\tan\left(\theta\right)$, are the objectives that must be met simultaneously. As mentioned, $\tan\left(\theta\right)$ is obtained as (Soundararajan and Christina, 1997):

$$
tan\left(\theta\right)=\frac{p_2-p_1}{P_a\left(p_1\right)-P_a\left(p_2\right)}
$$

In fact, due to the constancy of the value of (p_2-p_1) in each Scenario, our goal is to maximize the value of $\left(P_a\left(p_1\right)-P_a\left(p_2\right)\right)$ where $P_a\left(p_1\right)$ and $P_a\left(p_2\right)$ are the probabilities of accepting the lot when the nonconforming proportion of lot is equal to AQL and LQL respectively. Thus, these two objectives are combined in one as follows:

$$
Z=\frac{E(loss)}{P_a\left(p_1\right)-P_a\left(p_2\right)},
$$

and it is obvious that this function should be minimized.

10 different scenarios of process parameters are randomly generated by uniform distribution, and the values are listed in Table 1. For them, the optimal solution of the proposed plans is determined by solving the related nonlinear optimization model using MATLAB R2015a software and applying a grid search procedure. Also, the optimal solution of these plans is determined when only one of the objective functions is considered for all the proposed plans. The results are reported in Tables 2, 3, 4, 5, 6 and 7.

It is important to be noted that the design parameters are determined with an accuracy of one decimal place.

Table 1. Random process parameters

Scenarios	p_1	p_2	p	H	B	$\acute{c}$	N	L_0	σ	α	β
1	0.013	0.083	0.141	176	37	6	107	5	4	0.061	0.145
2	0.001	0.176	0.006	88	19	3	147	4	2	0.104	0.200
3	0.017	0.170	0.055	172	23	4	145	3	4	0.082	0.243
4	0.018	0.057	0.009	86	29	9	96	3	2	0.079	0.121
5	0.013	0.047	0.019	189	19	4	91	3	3	0.112	0.120
6	0.015	0.134	0.164	102	31	8	129	1	1	0.055	0.231
7	0.015	0.240	0.139	79	22	6	175	2	1	0.098	0.251
8	0.008	0.098	0.063	87	33	9	114	2	2	0.097	0.217
9	0.013	0.154	0.190	142	34	3	138	2	3	0.064	0.161
10	0.004	0.071	0.006	120	35	6	102	3	2	0.081	0.097

Table 2. The results of proposed variables RGS plan

Scenarios	Proposed plan with two objective functions						Proposed plan with the first objective function					Proposed plan with the second objective function				
	n_σ	$k_{r\sigma}$	$k_{a\sigma}$	Z_1	Z_2	ASN	n_σ	$k_{r\sigma}$	$k_{a\sigma}$	Z_1	ASN	n_σ	$k_{r\sigma}$	$k_{a\sigma}$	Z_2	ASN
1	70	1.2	2.4	1275.34	0.07	82.30	5	1.4	2.0	1273.00	6.37	60	1.1	2.5	0.07	104.49
2	6	1.2	2.3	566.60	0.18	8.58	2	0.5	2.0	564.83	2.60	22	0.0	2.7	0.18	116.32
3	43	1.2	1.5	1023.89	0.15	57.74	8	0.8	1.4	1022.64	11.04	127	1.4	1.5	0.15	144.56
4	13	1.4	2.2	528.60	0.04	17.93	8	1.4	2.2	489.60	11.71	90	1.4	2.2	0.04	95.55
5	67	1.8	2.0	615.36	0.03	90.27	29	1.8	2.2	615.24	90.74	67	1.8	2.0	0.03	90.27
6	53	1.0	2.3	3965.18	0.12	94.11	2	0.7	2.1	3826.64	4.96	53	1.0	2.3	0.12	94.11
7	74	1.1	2.1	2746.88	0.23	134.07	15	0.9	3.3	2746.88	63.28	61	1.1	1.8	0.23	111.48
8	27	1.4	1.6	1403.45	0.10	44.43	27	1.3	1.5	1277.24	39.82	84	1.6	2.2	0.09	113.63
9	37	0.8	2.4	1607.16	0.14	116.42	22	0.9	2.6	1607.16	40.64	37	0.8	2.4	0.14	116.42
10	9	1.6	2.3	572.16	0.07	12.15	4	1.2	2.4	556.49	6.74	80	0.0	2.4	0.07	95.00

Table 3. The results of proposed variables MDS sampling plan

Scenarios	Proposed plan with two objective functions						Proposed plan with the first objective function					Proposed plan with the second objective function				
	n_σ	$k_{r\sigma}$	$k_{a\sigma}$	m_σ	Z_1	Z_2	n_σ	$k_{r\sigma}$	$k_{a\sigma}$	m_σ	Z_1	n_σ	$k_{r\sigma}$	$k_{a\sigma}$	m_σ	Z_2
1	107	1.6	1.9	1	1275.34	0.07	7	1.3	1.8	2	1272.71	107	1.6	1.9	1	0.07
2	9	1.4	2.0	1	566.87	0.18	3	0.0	1.5	2	564.96	43	0.0	2.2	1	0.18
3	45	1.2	1.5	1	1023.71	0.15	11	0.0	1.2	2	1022.64	145	0.0	1.6	2	0.15
4	19	1.4	1.9	2	536.00	0.05	17	0.0	1.9	2	523.02	96	1.7	1.9	1	0.04
5	91	1.8	2.0	1	615.19	0.03	91	0.0	1.8	3	615.19	91	1.8	2.0	1	0.03
6	129	0.0	1.7	2	3965.18	0.12	3	0.0	1.6	2	3807.08	129	0.0	1.7	2	0.12
7	162	0.0	1.7	1	2746.88	0.23	80	0.0	2.1	1	2746.88	100	0.0	1.6	1	0.23
8	35	1.3	1.6	1	1395.99	0.09	28	1.2	1.5	1	1239.40	114	1.6	2.0	1	0.09
9	138	0.0	1.7	2	1607.16	0.14	42	1.1	2.1	1	1607.16	138	0.0	1.7	2	0.14
10	12	1.6	2.2	1	576.15	0.07	7	0.0	2.0	2	559.44	102	1.8	2.2	1	0.07

Table 4. The results of proposed CCC sampling plan for r=1

Scenarios	Proposed plan with two objective functions							Proposed plan with the first objective function						Proposed plan with the second objective function					
	n_σ	L	U	$k_{a\sigma}$	Z_1	Z_2	ASN	n_σ	L	U	$k_{a\sigma}$	Z_1	ASN	n_σ	L	U	$k_{a\sigma}$	Z_2	ASN
1	35	0	2	1.7	1275.34	0.07	35.01	7	1	2	1.5	1273.74	8.18	35	0	2	1.7	0.07	35.01
2	2	0	5	1.4	579.65	0.18	43.19	2	0	1	1.7	573.96	17.92	67	0	1	2.5	0.18	145.52
3	47	0	1	1.6	1025.57	0.15	93.08	2	0	3	0.9	1023.50	18.87	72	0	1	1.6	0.15	142.26
4	No solution							No solution						No solution					
5	17	0	2	1.9	615.69	0.04	88.05	3	1	13	1.2	616.28	89.54	17	0	2	1.9	0.04	88.05
6	43	0	2	1.5	3965.18	0.12	43.03	7	0	1	1.4	3837.29	9.11	43	0	2	1.5	0.12	43.03
7	87	0	1	1.6	2746.88	0.23	87.00	7	0	23	1.4	2746.88	11.00	58	0	2	1.3	0.23	64.18
8	8	0	4	1.4	1512.95	0.09	42.51	5	0	2	1.4	1360.41	21.00	38	0	2	1.7	0.09	50.98
9	46	0	2	1.5	1607.16	0.14	46.00	4	0	32	1.1	1607.16	8.87	46	0	2	1.5	0.14	46.00
10	2	0	6	1.5	671.21	0.07	37.63	2	0	4	1.5	654.40	35.06	2	0	16	1.2	0.07	100.22

Table 5. The results of proposed CCC sampling plan for r=2

Scenarios	Proposed plan with two objective functions							Proposed plan with the first objective function						Proposed plan with the second objective function					
	n_σ	L	U	$k_{a\sigma}$	Z_1	Z_2	ASN	n_σ	L	U	$k_{a\sigma}$	Z_1	ASN	n_σ	L	U	$k_{a\sigma}$	Z_2	ASN
1	15	0	5	1.7	1275.34	0.07	30.71	8	0	1	1.9	1273.66	16.48	15	0	5	1.7	0.07	30.71
2	2	0	4	1.9	573.83	0.18	26.25	2	0	1	2.0	571.42	18.63	34	0	1	2.5	0.18	144.14
3	17	0	2	1.6	1025.28	0.15	90.31	2	0	4	1.1	1023.20	27.07	36	1	2	1.6	0.15	142.77
4	2	0	12	1.4	854.42	0.05	66.62	2	0	11	1.4	834.51	66.62	2	0	18	1.3	0.04	94.07
5	8	0	4	1.9	616.31	0.04	85.44	2	10	23	1.0	615.91	89.67	8	0	4	1.9	0.04	85.44
6	25	0	3	1.6	3965.18	0.12	50.14	4	0	1	1.7	3827.21	9.91	25	0	3	1.6	0.12	50.14
7	43	0	2	1.6	2746.88	0.23	86.09	2	0	47	1.2	2746.88	22.21	29	0	4	1.3	0.23	85.95
8	7	0	3	1.6	1490.29	0.09	45.22	4	0	3	1.5	1339.13	37.72	22	0	3	1.8	0.09	60.61
9	23	0	4	1.5	1607.16	0.14	46.20	2	1	35	1.2	1607.16	7.87	23	0	4	1.5	0.14	46.20
10	2	0	5	1.9	635.33	0.07	29.18	2	0	2	2.2	611.73	17.15	2	0	19	1.4	0.07	101.67

Table 6. The results of proposed CCC sampling plan for r=3

Scenarios	Proposed plan with two objective functions							Proposed plan with the first objective function						Proposed plan with the second objective function					
	n_σ	L	U	$k_{a\sigma}$	Z_1	Z_2	ASN	n_σ	L	U	$k_{a\sigma}$	Z_1	ASN	n_σ	L	U	$k_{a\sigma}$	Z_2	ASN
1	7	0	12	1.6	1275.34	0.07	29.67	3	0	2	2.0	1273.59	11.25	7	0	12	1.6	0.07	29.67
2	2	0	3	2.2	572.37	0.18	21.90	2	0	1	2.2	570.98	19.62	34	0	1	2.6	0.18	146.60
3	9	0	3	1.6	1024.99	0.15	86.36	2	0	3	1.4	1023.11	24.64	24	2	3	1.6	0.15	142.99
4	2	0	9	1.7	735.60	0.04	48.72	2	0	8	1.7	713.86	48.72	2	0	18	1.5	0.04	87.82
5	5	0	6	1.9	615.24	0.04	90.78	2	17	26	1.1	615.70	90.54	7	0	4	2.0	0.04	86.69
6	16	0	5	1.6	3965.18	0.12	49.26	4	0	1	1.8	3832.10	14.50	16	0	5	1.6	0.12	49.26
7	19	0	6	1.5	2746.88	0.23	65.78	2	1	43	1.3	2746.88	19.02	25	0	4	1.4	0.23	95.04
8	5	1	4	1.6	1492.77	0.09	41.27	2	2	6	1.3	1337.32	28.86	14	0	5	1.8	0.09	82.67
9	15	0	6	1.5	1607.16	0.14	46.47	2	5	43	1.1	1607.16	10.00	15	0	6	1.5	0.14	46.47
10	2	0	4	2.2	622.82	0.07	25.16	2	0	2	2.3	603.74	20.43	3	0	12	1.8	0.07	101.95

Table 7. The results of proposed sampling plan for resubmitted lot

Scenarios	Proposed plan with two objective functions						Proposed plan with the first objective function					Proposed plan with the second objective function				
	n_σ	$k_{a\sigma}$	m	Z_1	Z_2	ASN	n_σ	$k_{a\sigma}$	m	Z_1	ASN	n_σ	$k_{a\sigma}$	m	Z_2	ASN
1	107	1.8	1	1275.34	0.07	107.00	4	2.4	7	1273.25	27.66	107	1.8	1	0.07	107.00
2	8	2.2	4	567.00	0.18	9.85	2	2.2	6	564.88	2.98	22	2.7	12	0.18	106.92
3	48	1.5	2	1024.08	0.15	59.91	9	1.5	5	1022.71	14.49	145	1.5	1	0.15	145.00
4	16	2.2	10	551.80	0.04	21.44	11	2.2	6	512.68	15.51	95	2.1	22	0.04	95.46
5	72	2.0	2	615.21	0.03	90.91	91	1.8	1	615.19	91.00	72	2.0	2	0.03	90.91
6	129	1.6	1	3965.18	0.12	129.00	4	1.7	2	3826.13	7.70	129	1.6	1	0.12	129.00
7	175	1.4	1	2746.88	0.23	175.00	66	2.1	2	2746.88	132.00	143	1.4	1	0.23	143.00
8	48	1.5	1	1419.19	0.10	48.00	16	1.6	2	1314.85	25.76	114	1.9	1	0.09	114.00
9	131	1.6	1	1607.16	0.14	131.00	23	2.2	6	1607.16	138.00	131	1.6	1	0.14	131.00
10	10	2.3	4	575.64	0.07	13.30	5	2.4	5	562.01	8.26	80	2.4	9	0.07	95.00

According to Tables 2, 3, 4, 5, 6 and 7, the best proposed plan for each random scenario is provided in terms of average sample number and objective function value in Table 8. Then, the proposed plan selected as the best plan in most of the scenarios, is presented as the best.

Table 8. The best proposed plan

Scenarios	The best proposed plan with two objective functions in each scenario		The best proposed plan with the first objective function in each scenario		The best proposed plan with the second objective function in each scenario	
	Z	ASN	Z_1	ASN	Z_2	ASN
1	All of the proposed plans	CCC sampling plan for r=3	MDS sampling plan	RGS plan	All of the proposed plans	CCC sampling plan for r=3
2	RGS plan	RGS plan	RGS plan	RGS plan	All of the proposed plans	MDS sampling plan
3	MDS sampling plan	MDS sampling plan	RGS plan, MDS sampling plan	MDS sampling plan	All of the proposed plans	CCC sampling plan for r=1
4	RGS plan	RGS plan	RGS plan	RGS plan	All of the proposed plans except CCC sampling plan for r=1	CCC sampling plan for r=3
5	MDS sampling plan	CCC sampling plan for r=2	MDS sampling plan, sampling plan for resubmitted lot	CCC sampling plan for r=1	RGS plan, MDS sampling plan, sampling plan for resubmitted lot	CCC sampling plan for r=2
6	All of the proposed plans	CCC sampling plan for r=1	MDS sampling plan	MDS sampling plan	All of the proposed plans	CCC sampling plan for r=1
7	All of the proposed plans	CCC sampling plan for r=3	All of the proposed plans	CCC sampling plan for r=1	All of the proposed plans	CCC sampling plan for r=1
8	MDS sampling plan	MDS sampling plan	MDS sampling plan	CCC sampling plan for r=1	All of the proposed plans	CCC sampling plan for r=1
9	All of the proposed plans	CCC sampling plan for r=1	All of the proposed plans	CCC sampling plan for r=2	All of the proposed plans	CCC sampling plan for r=1
10	RGS plan	MDS sampling plan	RGS plan	RGS plan	All of the proposed plans	RGS plan, sampling plan for resubmitted lot
The best	MDS sampling plan, RGS plan	MDS sampling plan	MDS sampling plan	RGS plan	RGS plan, MDS sampling plan, sampling plan for resubmitted lot	CCC sampling plan for r=1

As can be seen in Table 8, the proposed variables MDS sampling plan has better performance than the other proposed plans in most of the scenarios and the proposed variables RGS plan is the second method that has the best performance.

The average sample number of the proposed plans is additionally shown in graphical form in Figure 2 and 3.

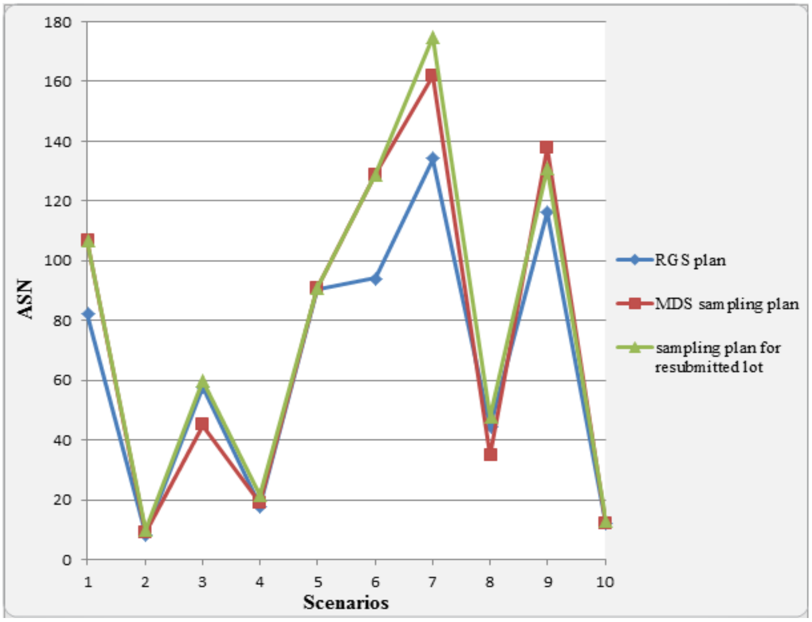

Figure 2. The average sample number of the proposed plans for different scenarios

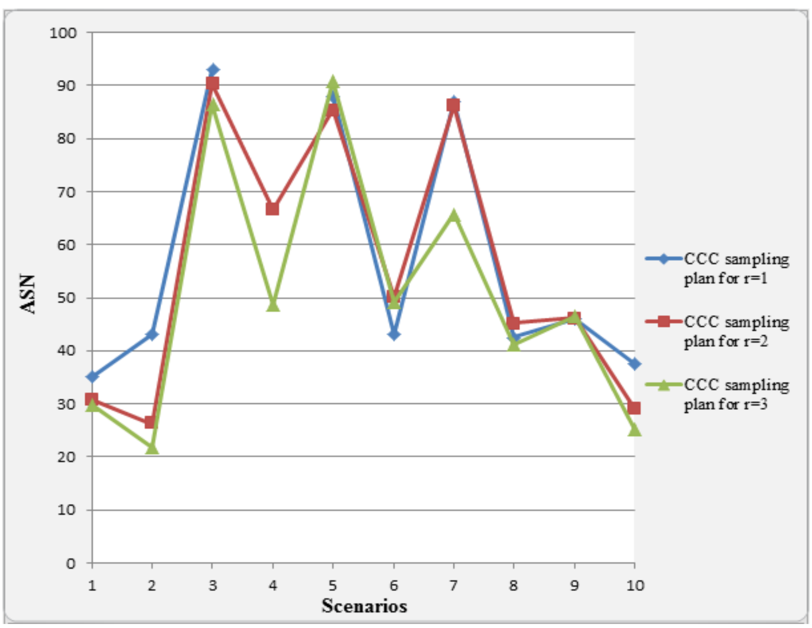

Figure 3. The average sample number of the proposed plans for different scenarios

5. Conclusion

In this paper, four nonlinear optimization models are proposed for four types of acceptance sampling plans. The design parameters are determined by minimizing the total loss to the producer and consumer and maximizing the rate of approaching to the ideal OC curve simultaneously with two constraints which satisfy the opposing requirements of both consumer and producer. 10 different scenarios of process parameters are prepared and the optimal solution of the proposed plans is determined for them. Also, the optimal solution of these plans is determined for the scenarios when only one of the objective functions is considered. The results indicate that the proposed variables MDS sampling plan and the proposed variables RGS plan ranked as the first and second method in terms of having a good performance respectively.

As a future research, proposed acceptance sampling models can be extended by considering the assumption that the standard deviation of the quality characteristic is unknown. In this condition, we can take a random sample of size n_s from the submitted lot, say $(x_1,\ x_2,\ \ldots,\ x_{n_s})$ and compute $v=\frac{\overline{x}\ -\ L_0}{s}$ where s is the sample standard deviation.

Here $\overline{x} = \frac{1}{n_s}\sum_{i=1}^{n_s} x_i$ and $s = \sqrt{\frac{\sum (x_i - \overline{x})^2}{n_s - 1}}$.

All other formulations are the same and only t distribution should be employed for evaluating the required probabilities. Thus all of the sampling plans can be developed based on t distribution and their performance can be compared.

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