

## TEST OF THE WEAK FORM EFFICIENT MARKET HYPOTHESIS FOR THE ISTANBUL STOCK EXCHANGE BY MARKOV CHAINS METHODOLOGY

**Öğr.Gör.Dr. Süleyman Bilgin KILIÇ**

Çukurova Üniversitesi  
İktisadi ve İdari Bilimler Fakültesi  
Ekonometri Bölümü

### ÖZET

Bu çalışma İstanbul Menkul Kıymetler Borsası 100 endeksine ait günlük getirilerinin rassal yürüyüş gösterip göstermediği Markov zincirleri yöntemi ile test edilmektedir. Eğer bir piyasada zayıf formda etkinlik hipotezi geçerli ise hisse senedi getirileri rassal yürüyüş özelliği gösterecektir. Rassal yürüyüş teorisi hisse senedi getirilerinin tarihsel fiyat verileriyle tahmin edilemeyeceğini öngörür. Böylece rassal yürüyüş özelliği gösteren bir piyasada tarihsel fiyat verilerine dayanılarak gerçekleştirilen teknik analiz yöntemleri geçersiz olacaktır.

### ABSTRACT

In this study, Markov chain methodology is used to test whether or not the daily returns of the Istanbul Stock Exchange (ISE) 100 index follows a martingale (random walk) process. If the Weak Form Efficient Market Hypothesis (EMH) holds in any stock market, stocks prices or returns follow a random walk process. The random walk theory asserts that price movements will not follow any patterns or trends and that past price movements cannot be used to predict future price movements hence, technical analysis is no use.

### 1. Introduction

Efficient Market Hypothesis (EMH) is an issue of intense debate among academics and financial professionals. Much of the theory on these subjects can be traced to French mathematician Louis Bachelier whose Ph.D. dissertation titled "The Theory of Speculation" (1900)<sup>1</sup>. EMH evolved by Fama (1965) who proposed three forms of the efficient market hypothesis: (1) The "Weak" form asserts that all past market prices and data are fully reflected in securities prices. In other words, technical analysis is of no use. (2) The "Semistrong" form asserts that all publicly available information is fully reflected in securities prices. In other words, fundamental analysis is of no use. (3) The

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<sup>1</sup> Bachelier came to the conclusion that "*The mathematical expectation of the speculator is zero*" and he described this condition as a "fair game." Unfortunately, his insights were so far ahead of the times that they went largely unnoticed for over 50 years until his paper was rediscovered and eventually translated into English and published in 1964 (<http://www.investorhome.com>).

"Strong" form asserts that all information is fully reflected in securities prices. In other words, even insider information is of no use.

The debate about EMH has resulted in hundreds of empirical studies attempting to determine whether specific markets are in fact "efficient" and if so to what degree. We summarize below only those studies which utilizes Markov Chain methodology in analyzing the stock prices and testing random walk hypothesis.

In an early study, Ryan (1973) explained the relevance of the theory of Markov processes to the analysis of stock price movements and stated that Markov theory is seen to be relevant to the analysis of stock prices in two ways: "1. As a useful tool for making probabilistic statements about future stock price levels. In this role it constitutes an alternative to the more traditional regression forecasting techniques to which it is, in many ways, superior. 2. As an extension of the random walk hypothesis.

McQueen and Thorley (1991) used Markov chain model to test the random walk hypothesis stock prices and showed that annual real returns exhibit significant non random walk behaviors in the post war period in the New York Stock Exchange (NSE).

Los (1998) investigated the nonparametric efficiency testing of Asian stock markets and illustrate that all six Asian stock markets have strong price trend behavior and can be profitably exploited by technical analysis with first-order Markov filters. Mills and Jordanov (2003) examined the predictability of size portfolio returns supplementing conventional autocorrelation analysis by Markov chain processes and found that predictabilities appear for the largest size portfolios rather than the smallest.

Most of the other studies aimed to explore the underlying patterns of economic mechanisms that generate the time series of stock returns by using Markov chains regime-switching methodology; Hamilton (1989) first investigate to capture discrete changes in the underlying (unobservable) economic mechanism that generates the financial time series data by Markov regime switching model. Driffill and Sola (1998) show that a Markov-switching model is a more appropriate representation of stock dividends and that regime-switching provides a better explanation for stock prices than the bubble. Kanas (2003) examined the forecast performance of stock return of the Markov regime switching model for US stock market using annual observations and concluded that the Markov regime switching model is the most preferable non-linear empirical extension of the present-value model for the stock return forecasting. Takaki (2004) proposed a two-step procedure for predicting intraday returns consisting of the method of principal components and the EM algorithm to estimate the model parameters as well as the unobservable states. First, a rate of return of a 'stock' in a single day is assumed to be generated by several common factors plus some additive error ('intraday equation'). Secondly, the joint distribution of those common factors is assumed to depend on the hidden state of the day, which fluctuates according to a Markov chain.

In the next sections of this study, Markov chain methodology is used to test whether or not the daily returns of the ISE 100 index follows a random walk process. Daily returns of ISE 100 index is assumed to be a stochastic process with four discrete state space with Markov chain structure that, the conditional probability of any future return given any past return and the present return, is independent of the past return and depends only on the present return of the process. After determining the steady state

probabilities, given in any state, probabilities of going in either directions that are below and above expected return are tested.

## 2. The sample and normality test of the returns

Daily values of Istanbul Stock Exchange (ISE) 100 index were obtained from the electronic data delivery system of Central Bank of Turkey (<http://tcmbf40.tcmb.gov.tr/cbt-uk.html>). The ISE 100 index can be considered as a large diversified portfolio that covers the stocks of 100 leading firms, which are being traded in the ISE. Hence, the index sufficiently represents the ISE. The index values cover 4234 workdays of 17 years for the period 23.10.1987-2.11.2004.

Daily returns ( $R_n$ ) are computed as a percentage change of the ISE 100 index:

$$R_n = \frac{P_n - P_{n-1}}{P_{n-1}} \quad (1)$$

where,  $P_n$  is the ISE index value in day  $n$ , ( $n=1, \dots, 4234$ )

We initially estimated the daily expected return ( $\mu_R$ ) and standard deviation ( $\sigma_R$ ) of the ISE 100 index. We also investigated the distributional property and volatility of the daily returns and tested that the stock returns are normally distributed.

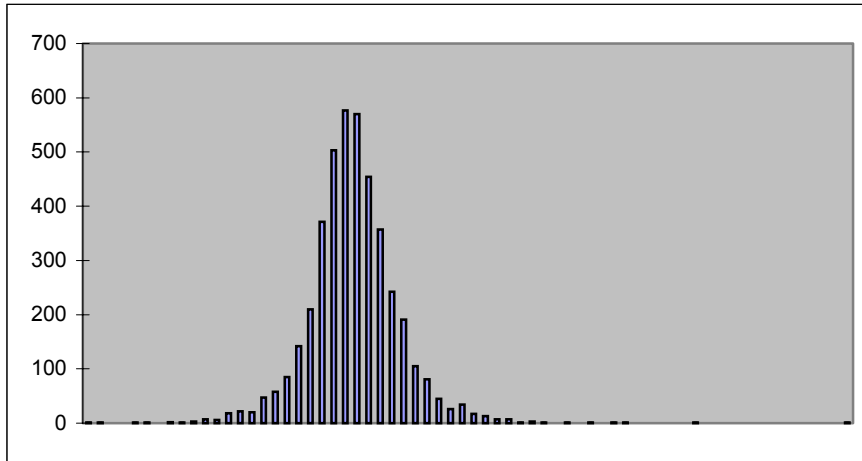
Table 1 gives the descriptive statistics for the daily stock returns. The expected value of the daily return is 0.18765% and standard deviation is 3.146%, that is relatively high when compared to the mean. This means that the daily returns exhibit high volatility. Minimum and maximum daily returns are -20% and 30% respectively for the period considered.

**Table 1: Descriptive statistics for the normality test**

	Sample size	Min.Val.	Max.Val.	Exp. Return ( $\mu_R$ )	Std. Dev. ( $\sigma_R$ )
Return ( $R_n$ )	4234	-.20	.30	0.001876	0.03146

We applied the One-Sample Kolmogorov-Smirnov normality test to determine whether or not the daily returns are normally distributed. This test compares the observed cumulative distribution function for the stock return with the cumulative normal distribution; the Kolmogorov-Smirnov Z statistic is computed from the largest difference (in absolute value) between the observed and theoretical cumulative normal distribution functions. This goodness-of-fit test tests whether the observations could reasonably have come from the normal distribution.

**Figure 1: Histogram of the daily returns**



**Table 2: One-Sample Kolmogorov-Smirnov Test results**

Kolmogorov-Smirnov Z statistic	3.706
Asymp. Sig. (2-tailed)	.000

Figure 1 shows the histogram of the daily returns, and the Table 2 gives the normality test results. In this test the null hypothesis is that the distribution is not normally distributed. Calculated Z statistics in Table 2 is 3.706 and corresponding two-tailed significant level is almost zero that we can strongly reject the null hypothesis. The daily returns of the ISE 100 index are normally distributed. The results of normality test above support the well-known empirical evidence for stock markets that the distributions of longer-horizon returns are closer to the normal, (Takaki, 2004).

### 3. Markov chain analysis for testing of random walk hypothesis

After the calculation of mean ( $\mu_R$ ) and standard deviation ( $\sigma_R$ ) of the daily returns of ISE 100 index ( $R_n$ ), we transformed the returns into four discrete state space and analyzed these states as Markov chains. Table 3, presents the return states ( $S_j$ ), return intervals and descriptions of the states. We assumed that one standard deviation above the mean return as high the return.

**Table 3: Return interval and descriptions of the states**

Return states ( $S_j$ )	Return interval	Description
$S_1$	$R_n < 0$	Negative return
$S_2$	$0 \leq R_n < \mu_R$	Positive low return
$S_3$	$\mu_R \leq R_n \leq \mu_R + \sigma_R$	Between mean and high return
$S_4$	$R_n > \mu_R + \sigma_R$	Above high return

We transformed the daily returns to the four states according to the function 2 below and computed the frequencies of the occurrence of the transitions from states  $i$  to  $j$  in Table 4.

$$S_j = \begin{cases} S_1, & \text{if } R_n < 0 \\ S_2, & \text{if } 0 \leq R_n < \mu_R \\ S_3, & \text{if } \mu_R \leq R_n \leq \mu_R + \sigma_R \\ S_4, & \text{if } R_n > \mu_R + \sigma_R \end{cases} \quad (2)$$

**Table 4: Frequencies of the occurrence of the transitions from states  $i$  to  $j$**

	$S_1$	$S_2$	$S_3$	$S_4$	Row totals
$S_1$	1029	66	682	226	2003
$S_2$	66	7	68	13	154
$S_3$	696	69	619	162	1546
$S_4$	212	12	177	130	531

From the frequencies table (Table 4) we can compute the four state (4x4), one step (one day) transition probability matrix ( $P_{ij}^1$ ) from state  $i$  to  $j$  by dividing the row elements to row totals:

$$P_{ij}^1 = \begin{matrix} & \begin{matrix} S_1 & S_2 & S_3 & S_4 \end{matrix} \\ \begin{matrix} S_1 \\ S_2 \\ S_3 \\ S_4 \end{matrix} & \begin{bmatrix} 0.514 & 0.033 & 0.340 & 0.113 \\ 0.429 & 0.045 & 0.442 & 0.084 \\ 0.450 & 0.045 & 0.400 & 0.105 \\ 0.399 & 0.023 & 0.333 & 0.245 \end{bmatrix} \end{matrix} \quad (3)$$

In the above one step transition probability matrix, the daily return states of ISE 100 index ( $S_j$ ) is assumed to be a stochastic process with four discrete state space  $\{S_1, S_2, S_3$  and  $S_4\}$  with Markov chain structure that, the conditional probability of any future return state ( $S_{n+1} = j$ ), given any past return state ( $S_0 = i_0, \dots, S_{n-1} = i_{n-1}$ ) and the present return state ( $S_n = i$ ), is independent of the past return and depends only on the present return of the state:

$$\begin{aligned} & P\{S_{n+1} = j | S_0 = i_0, S_1 = i_1, \dots, S_{n-1} = i_{n-1}, S_n = i\} \\ & = P\{S_{n+1} = j | S_n = i\} \quad (4) \\ & \text{for all } i, j, \text{ and for } n=0, 1, \dots \end{aligned}$$

Markov chain methodology do not require that the daily stock returns to be normally distributed but require the Markov chain to be stationary which is defined as constant transition probabilities in the long run.

After the modeling the system as Markov chain we can analyze the long run behavior of the return states to determine the steady state probabilities. Since the above one step probability matrix (3) have all positive probability values it has the property of regular ergodic chain. An ergodic Markov chain can have only one invariant distribution, which is also referred to as its equilibrium distribution (see Neal (1993) for the properties of ergodic Markov chains). This means that after enough number of steps ( $n$  days) a given return state will tend to occur a fixed percent of time.

The steady state probabilities can be stated as follows:

For a regular ergodic Markov chain:

$$\lim_{n \rightarrow \infty} P_{ij}^{(n)} = \pi_j$$

where  $\pi_j$ 's are the steady state probabilities and this limit is independent of  $i$ . The  $\pi_j$ 's satisfy the following steady state equations:

$$\pi_j > 0,$$

$$\sum_{j=1}^M \pi_j = 1,$$

$$\pi_j = \sum_{i=1}^M \pi_i P_{ij}, \quad \text{For } j = 1, \dots, M.$$

We can find the value of  $\pi_j$ 's that is independent of the initial probability distribution after an enough number of transitions. As  $n$  becomes larger, the values of the  $P_{ij}^n$  moves to fixed limit and each probability vector of tend to become equal for all values of  $i$ . Thus, each of the four rows of  $P_{ij}^n$  has identical probabilities:

$$P^1 = \begin{matrix} & S_1 & S_2 & S_3 & S_4 \\ S_1 & \begin{bmatrix} 0.514 & 0.033 & 0.340 & 0.113 \end{bmatrix} \\ S_2 & \begin{bmatrix} 0.429 & 0.045 & 0.442 & 0.084 \end{bmatrix} \\ S_3 & \begin{bmatrix} 0.450 & 0.045 & 0.400 & 0.105 \end{bmatrix} \\ S_4 & \begin{bmatrix} 0.399 & 0.023 & 0.333 & 0.245 \end{bmatrix} \end{matrix} \quad (5)$$

$$P^2 = \begin{matrix} & S_1 & S_2 & S_3 & S_4 \\ S_1 & \begin{bmatrix} 0.476 & 0.036 & 0.363 & 0.124 \end{bmatrix} \\ S_2 & \begin{bmatrix} 0.472 & 0.038 & 0.371 & 0.119 \end{bmatrix} \\ S_3 & \begin{bmatrix} 0.472 & 0.037 & 0.368 & 0.122 \end{bmatrix} \\ S_4 & \begin{bmatrix} 0.463 & 0.035 & 0.361 & 0.142 \end{bmatrix} \end{matrix} \quad (6)$$

$$P^3 = \begin{matrix} & S_1 & S_2 & S_3 & S_4 \\ S_1 & \begin{bmatrix} 0.473 & 0.036 & 0.365 & 0.125 \end{bmatrix} \\ S_2 & \begin{bmatrix} 0.473 & 0.037 & 0.366 & 0.124 \end{bmatrix} \\ S_3 & \begin{bmatrix} 0.473 & 0.036 & 0.365 & 0.125 \end{bmatrix} \\ S_4 & \begin{bmatrix} 0.472 & 0.036 & 0.365 & 0.128 \end{bmatrix} \end{matrix} \quad (7)$$

$$P^4 = \begin{matrix} & S_1 & S_2 & S_3 & S_4 \\ S_1 & \begin{bmatrix} 0.473 & 0.036 & 0.365 & 0.125 \end{bmatrix} \\ S_2 & \begin{bmatrix} 0.473 & 0.036 & 0.365 & 0.125 \end{bmatrix} \\ S_3 & \begin{bmatrix} 0.473 & 0.036 & 0.365 & 0.125 \end{bmatrix} \\ S_4 & \begin{bmatrix} 0.473 & 0.036 & 0.365 & 0.126 \end{bmatrix} \end{matrix} \quad (8)$$

$$P^5 = \begin{matrix} & S_1 & S_2 & S_3 & S_4 \\ S_1 & \begin{bmatrix} 0.473 & 0.036 & 0.365 & 0.125 \end{bmatrix} \\ S_2 & \begin{bmatrix} 0.473 & 0.036 & 0.365 & 0.125 \end{bmatrix} \\ S_3 & \begin{bmatrix} 0.473 & 0.036 & 0.365 & 0.125 \end{bmatrix} \\ S_4 & \begin{bmatrix} 0.473 & 0.036 & 0.365 & 0.125 \end{bmatrix} \end{matrix} \quad (9)$$

We can see from the above probability matrixes that after the five transitions ( $n=5$ ), the values of the  $P_{ij}^n$  moves to fixed limit and each of the four rows of  $P_{ij}^5$  has identical probabilities. These results indicate that there is a limiting probability that the return states will be in steady state condition after the 5 days and this probability is independent of the initial state of  $i$ .

From  $P_{ij}^5$ , the steady state probabilities are:

$$\pi_j = \begin{matrix} S_1 & S_2 & S_3 & S_4 \\ \begin{bmatrix} 0.473 & 0.036 & 0.365 & 0.125 \end{bmatrix} \end{matrix} \quad (10)$$

From the steady state probabilities we can compute the expected recurrence time for each of the return states ( $T_j$ ). Expected recurrence times are equal to the reciprocal of the expected steady state probabilities,

$$T_j = \frac{1}{\pi_j}, \text{ for } j=1,2,3,4.$$

or,

$$T_j = \frac{1}{\pi_j} = \left( \frac{1}{0.473} \quad \frac{1}{0.036} \quad \frac{1}{0.365} \quad \frac{1}{0.125} \right) = (2.1 \quad 27.5 \quad 2.7 \quad 8)$$

Hence, expected recurrence times for the states  $S_1$ ,  $S_2$ ,  $S_3$  and  $S_4$  are 2.1, 27.5, 2.7 and 8 days respectively. This result indicates that the state of negative return ( $S_1$ ) occurs most frequently that stock returns experiences negative return for each of the 2 days; when the ISE is in negative return state, it can be expected that it will be in negative state after two days later. Similarly, when the ISE is in  $S_3$ , it can be expected that it will be in state  $S_3$  after 2.74 days later. When ISE is in high return state ( $S_4$ ) is expected that it will be in that state after approximately 8 days later. The state  $S_2$  have the least frequency and expected to be occurring for each of the 27.49 days.

However, given in any state, probabilities of going in either directions that are below and above expected return ( $\mu_R$ ) are appears to be same. From the steady state probabilities ( $\pi_j$ ) we can see that the sum of the probability of the states that are below the expected return  $\{P(S_1)+P(S_2)\}$  and above the expected return  $\{P(S_3)+P(S_4)\}$  are 0.509 and 0.491 respectively. These probabilities are very close to each other and suggest that given in any state; probabilities of going in either directions below and above expected return are same. This situation can be treated as a [one-dimensional random walk](#) or a martingale with steps equally likely in either direction:

Let  $N$  represents the number of days. Let  $p$  be the probability of taking a step to the below expected return states,  $q$  the probability of taking a step to the above expected return states,  $n_1$  the number of steps taken to the below, and  $n_2$  the number of steps taken to the above expected return states. The expected quantities  $p$ ,  $q$ ,  $n_1$ ,  $n_2$ , and  $N$  are related by  $p+q=1$ ,  $p=q=1/2=0.50$ ,  $n_1+n_2=N$  and  $n_1=n_2=1/N$ .

By applying the maximum likelihood goodness of fitness test, we can rigorously determine whether or not the probabilities of return states going below and above the expected return are equal to 0.50.

From Table 4, we can calculate observed and expected number of frequencies of return states:

$$N=4234 \text{ (total number observations)}$$

$$E_{n_1} = E_{n_2} = \frac{N}{2} = \frac{4234}{2} = 2117 \text{ (Expected number of frequencies of } n_1$$

and  $n_2$ )



$O_{n_1} = 2157$  (Observed number of frequencies of  $n_1$ )

$O_{n_2} = 2077$  (Observed number of frequencies of  $n_2$ )

The null hypothesis is  $n_1=n_2=2117$  and that we can calculate chi-square ( $\chi^2$ ) statistics to test the null hypothesis:

$$\begin{aligned}\chi^2 &= \frac{(O_{n_1} - E_{n_1})}{E_{n_1}} + \frac{(O_{n_2} - E_{n_2})}{E_{n_2}} \\ &= \frac{(2157 - 2117)}{2117} + \frac{(2077 - 2117)}{2117} = 0.0000\end{aligned}$$

Calculated  $\chi^2$  statistic is almost zero and statistically insignificant that we can not reject the null hypothesis of equal probabilities. The daily return states of ISE 100 index follow a random walk (martingale) process with steps equally likely in either direction of below and above expected return.

This results are consistent with the our previous study (Kılıç, 1997) that we applied Augmented Dickey-Fuller test to the series of ISE index, and found that the series have a unit root and follow a random walk process. Existence of random walk in ISE supports weak form efficiency hypothesis.

The results of this paper do not indicate that the existence of semi strong form or strong form efficiency in ISE; Muradoğlu and Metin (1996) applied cointegration test in ISE and found that the stock prices and monetary variables cointegrate; ISE assimilates publicly available information on monetary variables with a lag. Hence, stock returns could be predicted by monetary variables. This suggest that ISE is inefficient in the semi strong form until 1993 because the data of the study covers the period of 1986-1993.

In another our previous study, Canbaş et al. (2002) we investigated the relationship between financial characteristics of the industrial firms and their annual stock returns in ISE, and found that three financial characteristics (liquidity, profitability to shareholders and growth) are useful for predicting stock returns; publicly available financial data did not reflected in the stock prices and it is possible to outperform in ISE by fundamental stock analysis.

#### **4. Conclusion**

Result of this study hold the weak-form EMH that at any given time, stock prices fully reflect all the available historical information. Under a random walk, historical data on prices and volume have no value in predicting future stock return. In other words, statistical analysis and "technical analysis" is useless. Buying and selling stocks by just depending only on historical stock prices in an attempt to outperform above the market return will effectively be a game of chance rather than skill.

Further research could be conducted toward the high frequency returns, such as five minute intraday returns. In this way we can see if there is an opportunity to outperform by intraday buying and selling strategy.

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