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Core Theorems in The Generalized Statistical Sense

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Abstract. The main purpose of the paper is to give some results concerning with the generalized statistical core of a bounded sequence via $\mathfrak{B}$-statistical convergence where $\mathfrak{B} = (B_i)$ is a sequence of infinite matrices. We characterize the matrix class $(st_{\mathfrak{B}} \cap X, Y)$ for certain sequence spaces X and Y. Here $st_{\mathfrak{B}}$ is the set of all $\mathfrak{B}$-statistically convergent sequences. Finally we answer the multipliers and factorization problem for $\mathfrak{B}$-statistically convergent sequences.

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1. Introduction

The relationship between statistical convergence and strong summability has been studied by many authors. These two concepts have been further extended by defining $\mathfrak{B}$-density where $\mathfrak{B} = (B_i)$ is a sequence of infinite matrices [10]. It is worth for noting that $\mathfrak{B}$-statistical convergence reduces to statistical convergence, A-statistical convergence and uniform statistical convergence in some special cases. Therefore, approximation type theorems have also been studied with the use of this notion [5, 6]. In [12], $\mathfrak{B}$-statistical limit superior and limit inferior have been introduced and a characterization of $\mathfrak{B}$-statistical convergence for $\mathfrak{B}$-bounded sequences has been given. Also the Knopp core theorem has been studied by many authors with various directions [11, 13]. Since $\mathfrak{B}$-statistical convergence is more general than many well known convergences, our results are generalization of statistical versions of some theorems about the Knopp core. Among the main results, it is shown that the matrix class $(st_{\mathfrak{B}} \cap X, Y)$ for certain sequence spaces X and Y, where $st_{\mathfrak{B}}$ is the set of all $\mathfrak{B}$-statistically convergent sequences can be characterized. Finally we present answers to the multipliers and factorization problem for $\mathfrak{B}$-statistically convergent sequences.

2. Notations

Let $\mathfrak{B} = (B_i)$ be a sequence of infinite matrices with $B_i = (b_{nk}^{(i)})$, for i. Then the sequence x is said to be $\mathfrak{B}-$summable to the value L if

$$\lim_n (B_i x)_n = \lim_n \sum_k b_{nk}^{(i)} x_k = L, \text{ uniformly in } i.$$

The method $\mathfrak{B}$ is regular [1] if and only if

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- $\| \mathfrak{B} \| < \infty$
- $\lim_n b_{nk}^{(i)} = 0$, for all $k \geq 1$, uniformly in i
- $\lim_n \sum_k b_{nk}^{(i)} = 1$, uniformly in i

where $\| \mathfrak{B} \| := \sup_{n,i} \sum_k | b_{nk}^{(i)} | < \infty$ to mean that there exists a constant M such that $\sum_k | b_{nk}^{(i)} | \leq M$ for all n, i and the series $\sum_k | b_{nk}^{(i)} |$ converges uniformly in i for each n.

Kolk [10] introduced the following :

An index set K is said to have $\mathfrak{B}$-density by $\delta_{\mathfrak{B}}(K)$ equal to d, if the characteristic sequence of K is $\mathfrak{B}-$summable to d, i.e.

$$\lim_n \sum_{k \in K} b_{nk}^{(i)} = d, \text{ uniformly in } i,$$

where by an index set we mean a set $K = \{k_m\} \subseteq \mathbb{N}$, $k_m < k_{m+1}$ for all m.

By $\mathfrak{R}^+$ we denote the set of all regular methods $\mathfrak{B}$ with $b_{nk}^{(i)} \geq 0$ for all n, k and i. Let $\mathfrak{B} \in \mathfrak{R}^+$. A sequence $x = (x_k)$ is called generalized statistically convergent (or $\mathfrak{B}$-statistically convergent) to the number l, if for every $\varepsilon > 0$

$$\delta_{\mathfrak{B}}(\{k : |x_k - l| \geq \varepsilon\}) = 0$$

and we write $st_{\mathfrak{B}} - \lim x = l$. By $st_{\mathfrak{B}}$, the space of all $\mathfrak{B}-$statistically convergent sequences is denoted. In particular,

- if $\mathfrak{B} = (C_1)$, the Cesàro matrix, then $\mathfrak{B}$-statistical convergence is reduced to the usual statistical convergence,
- if $\mathfrak{B} = (I)$, the identity matrix, then $\mathfrak{B}$-statistical convergence is also reduced to the usual convergence,
- if $\mathfrak{B} = (A)$, then it is reduced to A-statistical convergence,
- if $\mathfrak{B} = \mathfrak{B}_1$, then it is reduced to uniform statistical convergence where $\mathfrak{B}_1 = (b_{nk}^{(i)})$

$$b_{nk}^{(i)} = \begin{cases} \frac{1}{n}, & \text{if } 1 + i \leq k \leq n + i \\ 0, & \text{otherwise.} \end{cases}$$

Neither of the two methods, statistical convergence and $\mathfrak{B}$-statistical convergence need to imply the other (see, Example of [6]). The following definitions have been given in [12].

Definition 2.1. $\mathfrak{B}$-statistical limit superior and $\mathfrak{B}$-statistical limit inferior are respectively defined as below:

$$st_{\mathfrak{B}} - \limsup x = \begin{cases} \sup G_x & , \quad G_x \neq \emptyset \\ -\infty & , \quad G_x = \emptyset, \end{cases}$$

and

$$st_{\mathfrak{B}} - \liminf x = \begin{cases} \inf F_x & , \quad F_x \neq \emptyset \\ \infty & , \quad F_x = \emptyset, \end{cases}$$

where $G_x = \{g \in \mathbb{R} : \ \delta_{\mathfrak{B}}(\{k : \ x_k > g\}) \neq 0\}$ and $F_x = \{f \in \mathbb{R} : \ \delta_{\mathfrak{B}}(\{k : \ x_k < f\}) \neq 0\}$.

Definition 2.2. x is said to be $\mathfrak{B}$-statistically bounded if there is a number M such that $\delta_{\mathfrak{B}}(\{k : \ |x_k| > M\}) = 0$.

It is known that every $\mathfrak{B}$-statistically convergent sequence is also $\mathfrak{B}$-statistically bounded.

The Knopp core of a sequence x is defined as follows:

$$K - core\{x\} := \bigcap_{n \in \mathbb{N}} C_n(x)$$

where $C_n(x)$ is the closed convex hull of $\{x_k\}_{k \geq n}$. In [7], Fridy and Orhan have introduced the concept of statistical core of a sequence and have proved the statistical core theorem. Then Demirci [3, 4] has extended this concept to A-statistical core and I-core, and has proved the A-statistical and ideal analogues of the theorems in [7]. In [16] it is shown that for every bounded x,

$$K - core\{x\} = \bigcap_{z \in \mathbb{C}} B_x^*(z)$$

where $B_x^*(z) := \{w \in \mathbb{C} : \ |w - z| \leq \limsup_k |x_k - z|\}$.

Now we can give the following definition in a similar way. First note that if x and y are sequences such that

$$\delta_{\mathfrak{B}}(\{k \in \mathbb{N} : \ x_k = y_k\}) = 1$$

then we write "$x_k = y_k$" $\mathfrak{B} - a.a.k$.

Definition 2.3. For any complex sequence x, let $H_{\mathfrak{B}}(x)$ be the collection of all closed half-planes that contain x_k $\mathfrak{B} - a.a.k$; i.e.

$$H_{\mathfrak{B}}(x) := \{H : \ H \ is \ closed \ half-plane \ and \ \delta_{\mathfrak{B}}(\{k \in \mathbb{N} : \ x_k \notin H\}) = 0\},$$

then the $\mathfrak{B}$-statistical core of x is given by

$$st_{\mathfrak{B}} - core\{x\} := \bigcap_{H \in H_{\mathfrak{B}}(x)} H.$$

It is clear that for any $\mathfrak{B}$-statistically bounded real sequence x

$$st_{\mathfrak{B}} - core\{x\} = [st_{\mathfrak{B}} - \liminf x, st_{\mathfrak{B}} - \limsup x].$$

3. Main Results

In this section we present our results concerning with the generalized statistical core of a bounded sequence. A characterization theorem for the matrix class $(st_{\mathfrak{B}} \cap X, Y)$ where X, Y are certain sequence spaces has been obtained and answers to the multipliers and factorization problem for $\mathfrak{B}$-statistical convergence have been provided. By l_∞, we denote the set of all bounded sequences.
In [4] the concepts of I-limit superior and I-limit inferior have been defined and some results concerning with I-core have also been given. One can immediately obtain the following $\mathfrak{B}$-statistical anologues of those results.

Theorem 3.1. *Let T satisfy $\sup_n \sum_k |t_{nk}| < \infty$. Then $K - core\{Tx\} \subset st_{\mathfrak{B}} - core\{x\}$ for every $x \in l_\infty$ if and only if*
(i) $T \in \tau^*_{\mathfrak{B}}$ i.e. T is regular and $\lim_n \sum_{k \in E} |t_{nk}| = 0$ whenever $\delta_{\mathfrak{B}}(E) = 0$,
(ii) $\lim_n \sum_k |t_{nk}| = 1$.

Since $st_{\mathfrak{B}} - core\{x\} \subset K - core\{x\}$, we have the following corollary.

Corollary 3.2. *If the matrix T satisfies $\sup_n \sum_k |t_{nk}| < \infty$ and properties (i) and (ii) of the above theorem, then*

$$st_{\mathfrak{B}} - core\{Tx\} \subset st_{\mathfrak{B}} - core\{x\}.$$

We will show that the converse of the above corollary is not true but we first need the following lemma.
Initially recall that the number γ is said to be a $\mathfrak{B}$-statistical cluster point of a sequence x if for every $\varepsilon > 0$ the set $\{k : \ |x_k - \gamma| < \varepsilon\}$ does not have $\mathfrak{B}$-density zero. By $\Gamma^{\mathfrak{B}}_x$, we denote the set of all $\mathfrak{B}$-statistical cluster points of a sequence x and the set does not have $\mathfrak{B}$-density zero means either the $\mathfrak{B}$-density does not exist or it is not zero.

Lemma 3.3. *If x and y are sequences such that $x_k = y_k$ for $\mathfrak{B} - a.a.k$ then $\Gamma^{\mathfrak{B}}_x = \Gamma^{\mathfrak{B}}_y$.*

Proof. Let $\gamma \in \Gamma^{\mathfrak{B}}_x$. Define $K = \{k \in \mathbb{N} : \ x_k = y_k\}$. For every $\varepsilon > 0$, the set $\{k \in K : \ |y_k - \gamma| < \varepsilon\}$ does not have $\mathfrak{B}$-density zero. Since

$$\{k \in K : \ |y_k - \gamma| < \varepsilon\} \subset \{k \in \mathbb{N} : \ |y_k - \gamma| < \varepsilon\},$$

it follows that for every $\varepsilon > 0$, the set $\{k \in \mathbb{N} : \ |y_k - \gamma| < \varepsilon\}$ does not have $\mathfrak{B}$-density zero. Hence $\gamma \in \Gamma^{\mathfrak{B}}_y$. By the symmetry we see that $\Gamma^{\mathfrak{B}}_y \subset \Gamma^{\mathfrak{B}}_x$ whence $\Gamma^{\mathfrak{B}}_x = \Gamma^{\mathfrak{B}}_y$. □

In order to see the converse of the above corollary is not true, let $\mathfrak{B} = (b^{(i)}_{nk}) = (b_{nk})$ for all i, where b_{nk} is defined as follows

$$b_{nk} = \begin{cases} 1 & , \quad k = n^2 + 1 \\ 0 & , \quad otherwise. \end{cases}$$

Consider $T = (t_{nk})$ such that $(Tx)_n = x_n$ for $\mathfrak{B} - a.a.k$. From the above lemma, we obtain $st_{\mathfrak{B}} - core\{x\} = st_{\mathfrak{B}} - core\{Tx\}$. Let

$$t_{nk} = \begin{cases} 1 & , \quad k = n, \ n \text{ is not square} \\ 1 & , \quad k \leq n, \ n \text{ is square} \\ 0 & , \quad \text{otherwise}. \end{cases}$$

One can get

$$(Tx)_n = \begin{cases} x_n & , \quad n \text{ is not square} \\ \sum\limits_{k=1}^{n} x_k & , \quad n \text{ is square}. \end{cases}$$

For the matrix $T = (t_{nk})$, the following holds

$$\sum_{k=1}^{\infty} |t_{nk}| = \begin{cases} 1 & , \quad n \text{ is not square} \\ n & , \quad n \text{ is square}. \end{cases}$$

Since $\sup\limits_{n} \sum\limits_{k=1}^{\infty} |t_{nk}| = \infty$, the limit of the sequence $\left(\sum\limits_{k=1}^{\infty} |t_{nk}| \right)$ does not exist. If we consider $E = \{k = n^2 : \ n = 1, 2, ...\}$, we obtain $\delta_{\mathfrak{B}}(E) = 0$. On the other hand it is easy to see that

$$\sum_{k \in E} |t_{nk}| = \begin{cases} 0 & , \quad n \text{ is not square} \\ \sqrt{n} & , \quad n \text{ is square}, \end{cases}$$

and the limit of the sequence $\left(\sum\limits_{k \in E} |t_{nk}| \right)$ does not exist. Therefore (i) and (ii) do not hold.

Now we give the relationship between $\mathfrak{B}$-statistical convergence and strong $\mathfrak{B}$-summability. Recall that a sequence x is said to be strongly $\mathfrak{B}$-summable to L if the following limit exists

$$\lim_{n} \sup_{i} \sum_{k} b_{nk}^{(i)} |x_k - L| = 0.$$

Theorem 3.4. *Let $\mathfrak{B} = (b_{nk}^{(i)}) \in \mathfrak{R}^{+}$. If the sequence x is strongly $\mathfrak{B}$-summable, then x is $\mathfrak{B}$-statistically convergent to the same value.*

Proof. Without loss of generality we may take $L = 0$. Let $\lim\limits_{n} \sup\limits_{i} \sum\limits_{k} b_{nk}^{(i)} |x_k - 0| = 0$. Hence for every $\varepsilon > 0$, we have

$$\begin{aligned} \sum_{k} b_{nk}^{(i)} |x_k| &= \sum_{k:|x_k| \geq \varepsilon} b_{nk}^{(i)} |x_k| + \sum_{k:|x_k| < \varepsilon} b_{nk}^{(i)} |x_k| \\ &\geq \sum_{k:|x_k| \geq \varepsilon} b_{nk}^{(i)} |x_k| \\ &\geq \varepsilon \sum_{k:|x_k| \geq \varepsilon} b_{nk}^{(i)} \geq 0. \end{aligned}$$

Since $\lim\limits_{n} \sup\limits_{i} \sum\limits_{k:|x_k| \geq \varepsilon} b_{nk}^{(i)} = 0$ and from the last inequality, we get x is $\mathfrak{B}$-statistically convergent to 0. This completes the proof. □

Theorem 3.5. *Let $x = (x_k)$ be a bounded sequence. If the sequence x is $\mathfrak{B}$-statistically convergent, then x is strongly $\mathfrak{B}$-summable to the same value.*

Proof. Without loss of generality we may take $L = 0$. Then we get

$$\begin{aligned} \sum_{k} b_{nk}^{(i)} |x_k| &= \sum_{k:|x_k| \geq \varepsilon} b_{nk}^{(i)} |x_k| + \sum_{k:|x_k| < \varepsilon} b_{nk}^{(i)} |x_k| \\ &\leq \|x\|_{\infty} \sum_{k:|x_k| \geq \varepsilon} b_{nk}^{(i)} + \varepsilon \sum_{k} b_{nk}^{(i)}. \end{aligned}$$

Hence we obtain $\lim\limits_{n} \sup\limits_{i} \sum\limits_{k} b_{nk}^{(i)} |x_k| = 0$ which completes the proof. □

Notice that Theorem 3.4 and Theorem 3.5 indicate the equivalence of $\mathfrak{B}$-statistical convergence and strong $\mathfrak{B}$-summability on bounded sequences.
In order to characterize the matrix class $(st_{\mathfrak{B}} \cap X, Y)$ where X, Y are certain sequence spaces, we pause to collect some notation.
For arbitrary index set $K = \{k_m\}$ the sequence $x^{[K]} = (y_k)$, where

$$y_k = \begin{cases} x_k & , \quad k \in K \\ 0 & , \quad otherwise \end{cases}$$

will be called the K-section of x. This notion has been introduced by Kolk [9]. A sequence space X will be called section-closed if $x^{[K]} \in X$ for all $x \in X$ and for every index set K. Also we denote by $A^{[K]} = (d_{nk})$ the K-column-section of a matrix A is defined by

$$d_{nk} = \begin{cases} a_{nk} & , \quad k \in K \\ 0 & , \quad otherwise. \end{cases}$$

and the sequence $(1, 1, ...)$ by e.

Theorem 3.6. *Let X be a section-closed sequence space containing e and Y be an arbitrary sequence space. Then $A \in (st_{\mathfrak{B}} \cap X, Y)$ if and only if $A \in (c \cap X, Y)$ and $A^{[K]} \in (X, Y)$ whenever $\delta_{\mathfrak{B}}(K) = 0$.*

Proof. Let $A \in (st_{\mathfrak{B}} \cap X, Y)$. Since $c \subset st_{\mathfrak{B}}$, we immediately have $A \in (c \cap X, Y)$. Now consider a subset K of $\mathbb{N}$ with $\delta_{\mathfrak{B}}(K) = 0$ and let $x \in X$. Then the K-section y of x converges $\mathfrak{B}$-statistically to 0 in addition $y \in X$. Hence $y \in st_{\mathfrak{B}} \cap X$ and therefore $Ay \in Y$. By $A_n^{[K]}x = A_n y$, $n \in \mathbb{N}$, this implies $A^{[K]}x \in Y$. Thus $A^{[K]} \in (X, Y)$ for every index set K with $\delta_{\mathfrak{B}}(K) = 0$.
Conversely, let $x \in st_{\mathfrak{B}} \cap X$ with $st_{\mathfrak{B}} - \lim x = x_0$. We will show that $Ax \in Y$. We can assume $x_0 = 0$ because $Ae \in Y$. If $x \in c$ then $Ax \in Y$ follows directly from $A \in (c \cap X, Y)$. But if $x \in st_{\mathfrak{B}} \setminus c$ then there exists an infinite index set K with $\delta_{\mathfrak{B}}(K) = 0$ such that $\lim z_k = 0$ where $z = (z_k)$ is the K-section of x. Since $z \in X$ then $Az \in Y$, so by $A^{[K]}x \in Y$ and $Ax = Az + A^{[K]}x$ we get $Ax \in Y$. This completes the proof. □

Theorem 3.7. *Let X be a section-closed sequence space and Y be an arbitrary sequence space. Then $A \in (st^0_{\mathfrak{B}} \cap X, Y)$ if and only if $A \in (c_0 \cap X, Y)$ and $A^{[K]} \in (X, Y)$ whenever $\delta_{\mathfrak{B}}(K) = 0$.*

In some special cases for X and Y one can obtain the followings

- In the case of $X = l_\infty$, $A \in (st_{\mathfrak{B}} \cap l_\infty, Y)$ if and only if $A \in (c, Y)$ and $A^{[K]} \in (l_\infty, Y)$ whenever $\delta_{\mathfrak{B}}(K) = 0$.
- In the case of $X = l_\infty$, $A \in (st^0_{\mathfrak{B}} \cap l_\infty, Y)$ if and only if $A \in (c_0, Y)$ and $A^{[K]} \in (l_\infty, Y)$ whenever $\delta_{\mathfrak{B}}(K) = 0$.
- In the case of $X = l_\infty$ and $Y = c$, $A \in (st_{\mathfrak{B}} \cap l_\infty, c)$ if and only if $A \in (c, c)$ and $A^{[K]} \in (l_\infty, c)$ $\lim\limits_n \sum\limits_{k \in K} |a_{nk} - a_k| = 0$ whenever $\delta_{\mathfrak{B}}(K) = 0$, where $a_k = \lim\limits_n a_{nk}$.
- In the case of $X = l_\infty$ and $Y = c_0$, $A \in (st_{\mathfrak{B}} \cap l_\infty, c_0)$ if and only if $A \in (c_0, c_0)$ and $A^{[K]} \in (l_\infty, c_0)$ $\lim\limits_n \sum\limits_{k \in K} |a_{nk}| = 0$ whenever $\delta_{\mathfrak{B}}(K) = 0$, where $a_k = \lim\limits_n a_{nk}$.

Notice that if $\mathfrak{B} = (A)$ for all i, our results reduce to those given in [9].

Connor, Demirci and Orhan [2], Khan and Orhan [8] and also Orhan and Dirik [14] have studied multipliers for bounded statistically convergent sequences. Özgüç and Yurdakadim [15] have answered this question for quasi-statistical convergence. Now we get similar results for $\mathfrak{B}$-statistical convergence. Suppose that two sequence spaces X and Y are given. A multiplier from X into Y is a sequence u such that $ux = (u_n x_n) \in Y$ whenever $x \in X$. The linear space of such multipliers is denoted by $m(X, Y)$.

Theorem 3.8. *$x \in m(st_{\mathfrak{B}}, st_{\mathfrak{B}})$ if and only if $x \in st_{\mathfrak{B}}$.*

Proof. Let $x \in m(st_{\mathfrak{B}}, st_{\mathfrak{B}})$. Then we have $xy \in st_{\mathfrak{B}}$ for every $y \in st_{\mathfrak{B}}$. Also since $y = \chi_{\mathbb{N}} \in st_{\mathfrak{B}}$, we have $xy = x \in st_{\mathfrak{B}}$.
Conversely, let $x \in st_{\mathfrak{B}}$ and $y \in st_{\mathfrak{B}}$. Without loss of generality we can let x and y are $\mathfrak{B}$-statistically convergent to 0. From the inequality

$$|\{k \in \mathbb{N} : |x_k y_k| \geq \varepsilon\}| \leq |\{k \in \mathbb{N} : |x_k| \geq \sqrt{\varepsilon}\}| + |\{k \in \mathbb{N} : |y_k| \geq \sqrt{\varepsilon}\}|,$$

we obtain $xy \in st_{\mathfrak{B}}$. That is, $x \in m(st_{\mathfrak{B}}, st_{\mathfrak{B}})$. □

Theorem 3.9. *$x \in m(N_{\mathfrak{B}}, st_{\mathfrak{B}})$ if and only if $x \in st_{\mathfrak{B}}$,
where $N_{\mathfrak{B}} = \{x \in \omega :$ there exists L such that $\lim_n \sum_k b_{nk}^{(i)} |x_k - L| = 0$ uniformly in $n\}$.*

Proof. Let $x \in m(N_{\mathfrak{B}}, st_{\mathfrak{B}})$. So we have $xy \in st_{\mathfrak{B}}$ for every $y \in N_{\mathfrak{B}}$. Also since $y = \chi_{\mathbb{N}} \in N_{\mathfrak{B}}$, we have $xy = x \in st_{\mathfrak{B}}$. Conversely, let $x \in st_{\mathfrak{B}}$ and $y \in N_{\mathfrak{B}}$. If $y \in N_{\mathfrak{B}}$, y is also in $st_{\mathfrak{B}}$. From the above theorem, the proof is completed. □

Now we give a decomposition theorem for $\mathfrak{B}$-statistical convergence.

Theorem 3.10. *$x \in st_{\mathfrak{B}}$ if and only if there exist $y \in N_{\mathfrak{B}}$ and $z \in st_{\mathfrak{B}}$ such that $x = yz$.*

Proof. Let $x \in st_{\mathfrak{B}}$. Because of $\chi_{\mathbb{N}} \in N_{\mathfrak{B}}$, we get $x = \chi_{\mathbb{N}} x$. Now let $y \in N_{\mathfrak{B}}$ and $z \in st_{\mathfrak{B}}$ exist such that $x = yz$. From the above theorem we easily obtain the proof. □

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