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Intuitionistic Fuzzy 2-Absorbing Primary Ideals of Commutative Rings

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Abstract. The aim of this paper is to give a definition of intuitionistic fuzzy 2- absorbing primary ideal and intuitionistic fuzzy weakly completely 2- absorbing primary ideals of commutative rings and to give their properties. Moreover, we give a diagram of transition between definitions of intuitionistic fuzzy 2- absorbing primary ideals of commutative rings.

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1. Introduction

Notion of fuzzy subset is defined by Zadeh [22] and Rosenfeld [17], who is studied to apply fuzzy theory on algebraic structures. After that, different researchers studied concerning it. The concept of fuzzy ideal of a ring is explained by Liu [12]. Fuzzy ideals by introducing the concept of prime fuzzy ideals are explained by Mukherjee and Sen [14].

As for intuitionistic fuzzy set theory, the concept of an intuitionistic fuzzy set as a generalization of Zadeh's fuzzy sets is introduced by Atanassov [2]. Moreover, the notion of intuitionistic fuzzy subring is presented by Hur, Kang and Song [10]. The concept of intuitionistic fuzzy rings based on the concept of fuzzy space is studied by Marasdeh and Salleh [13]. Besides, the translates of intuitionistic fuzzy subrings are explained by Sharma [18].

When it comes to 2-absorbing ideals; the concept of 2- absorbing ideals, which is a generalization of prime ideals are explained by Badawi [3]. Furthermore, Badawi also studied [1] and [5]. Today, study regarding 2-absorbing ideal theory is developing rapidly and many other authors studied extensively about this theory.(e.g. [4], [9], [11]). L-fuzzy 2-absorbing ideals are explained and examined by Darani [7]. Then, the concept of L- fuzzy 2-absorbing ideals in semiring are investigated by Darani and Hashempoor [8].

Prime and primary ideals play an important role in commutative ring theory. Due to the fact that the concept of 2-absorbing ideals, which is generalization of prime ideals [3] and 2- absorbing primary ideals, which is a generilazation of primary ideals [5] were introduced. In the mean time prime fuzzy ideals and primary fuzzy ideals are studied [14] and [15]. Moreover a new study is investigated 2- absorbing primary fuzzy ideals [20].

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The main purpose of this paper is to deal with algebraic structure of 2-absorbing primary ideals by applying intuitionistic fuzzy set theory. The concept of intuitionistic fuzzy 2-absorbing primary ideals are introduced, their characterizations and algebraic properties are investigated by giving some several examples. In addition to this, intuitionistic fuzzy strongly 2-absorbing primary ideals, intuitionistic fuzzy weakly completely 2-absorbing primary ideals, intuitionistic fuzzy K-2-absorbing primary ideals and their properties are introduced. Moreover, image and inverse image of these ideals are studied under ring homomorphism. Finally, a table transition between definitions of intuitionistic fuzzy 2-absorbing primary ideals of a commutative ring is given.

2. Preliminaries

We suppose that all rings are commutative with $1 \neq 0$. Unless stated otherwise $L = [0, 1]$ stands for complete lattice. First of all we will give basic concepts of fuzzy sets and intuitionistic fuzzy sets theory. Furthermore, we will introduce intuitionistic fuzzy ideals, radical of these ideals and basic properties.

Definition 2.1 ([7]). A fuzzy subset μ in a set X is a function $\mu : X \to [0, 1]$.

Definition 2.2 ([6]). The intuitionistic fuzzy sets are defined on a non-empty set X as objects having the form

$$A = \{\langle x, \mu(x), v(x)\rangle \,|x \in X\}$$

where the functions $\mu : X \to [0, 1]$ and $v : X \to [0, 1]$ denote the degrees of membership and of non- membership of each element $x \in X$ to set A, respectively, $0 \leq \mu(x) + v(x) \leq 1$ for all $x \in X$.

Definition 2.3 ([7]). Let $\alpha, \beta \in [0, 1]$ with $\alpha + \beta \leq 1$. An intuitionistic fuzzy point, written as $x_{(\alpha,\beta)}$ is defined to be an intuitionistic fuzzy subset of R, given by

$$x_{(\alpha,\beta)}(y) = \begin{cases} (\alpha, \beta), & \text{if } x = y \\ (0, 1), & \text{if } x \neq y \end{cases}$$

an intuitionistic fuzzy point $x_{(\alpha,\beta)}$ is said to belong in an intuitionistic fuzzy set $\langle \mu, v\rangle$ denoted by $x_{(\alpha,\beta)} \in \langle \mu, v\rangle$ if $\mu(x) \geq \alpha$ and $v(x) \leq \beta$ and we have for $x, y \in R$

i) $x_{(t,s)} + y_{(\alpha,\beta)} = (x + y)_{(t\wedge a, s\vee\beta)}$

ii) $x_{(t,s)} y_{(\alpha,\beta)} = (xy)_{(t\wedge a, s\vee\beta)}$

iii) $\langle x_{(t,s)}\rangle \left\langle y_{(\alpha,\beta)}\right\rangle = \left\langle x_{(t,s)} y_{(\alpha,\beta)}\right\rangle$.

Definition 2.4 ([21]). An intuitionistic fuzzy ideal $I = \langle \mu_I, v_I\rangle$ of R is called an intuitionistic fuzzy prime ideal, if for any intuitionistic fuzzy ideals $A = \langle \mu_A, v_A\rangle$ and $B = \langle \mu_B, v_B\rangle$ of R the condition $AB \subset I$ implies that either $A \subset I$ or $B \subset I$.

Definition 2.5 ([6]). Let R be a ring. An intuitionistic fuzzy set

$$A = \{\langle x, \mu(x), v(x)\rangle \,|x \in R\}$$

is said to be an intuitionistic fuzzy ideal of R if $\forall x, y \in R$

i) $\mu(x - y) \geq \mu(x) \wedge \mu(y)$,

ii) $v(x - y) \leq v(x) \vee v(y)$,

iii) $\mu(xy) \geq \mu(x) \vee \mu(y)$,

iv) $v(xy) \leq v(x) \wedge v(y)$.

Definition 2.6 ([6]). Let $I = \langle \mu_I, v_I\rangle$ be an intuitionistic fuzzy ideal of R. Then, $\sqrt{I}$ is called the radical of I, is defined by $\sqrt{I}(x) = \vee_{n\geq 1} I(x^n)$.

Theorem 2.7 ([19]). *If I and J are intuitionistic fuzzy ideals of R, then*

$$\sqrt{I \cap J} = \sqrt{I} \cap \sqrt{J}.$$

Theorem 2.8 ([19]). *Let $f : R \to S$ be a ring homomorphism and let I be an intuitionistic fuzzy ideal of R such that I is an constant on* ker f *and J be an intuitionistic fuzzy ideal of S. Then,*

i) $\sqrt{f(I)} = f(\sqrt{I})$,

ii) $\sqrt{f^{-1}(J)} = f^{-1}(\sqrt{J})$.

Moreover, we will define intuitionistic fuzzy prime and primary ideals, and intuitionistic fuzzy weakly completely prime ideals. Then we will present 2-absorbing and 2-absorbing primary ideals.

Definition 2.9 ([21]). An intuitionistic fuzzy ideal $I = \langle \mu_I, \nu_I \rangle$ of R is called intuitionistic fuzzy prime ideal, if for any intuitionistic fuzzy ideals $A = \langle \mu_A, \nu_A \rangle$ and $B = \langle \mu_B, \nu_B \rangle$ of R the condition $AB \subset I$ implies that either $A \subset I$ or $B \subset I$.

Definition 2.10 ([16]). A fuzzy ideal I of R is called an intuitionistic fuzzy primary ideal if for any $a, b \in R$, either

$$\begin{aligned} \mu(ab) &= \mu(a) \text{ and } \nu_I(ab) = \nu_I(a), \text{or} \\ \mu(ab) &\leq \mu_I(b^m) \text{ and } \nu_I(ab) \geq \nu_I(b^m), \text{ for some } m \in Z^+. \end{aligned}$$

Definition 2.11 ([21]). An intuitionistic fuzzy ideal $I = \langle \mu_I, \nu_I \rangle$ of R is called an intuitionistic fuzzy weakly completely prime ideal, if for any $x, y \in R$ such that

$$I(xy) = (\mu_I(x) \vee \mu_I(y), \nu_I(x) \wedge \nu_I(y)).$$

Definition 2.12 ([5]). Let R be a commutative ring with identity. A proper ideal I of R is said to be a 2- absorbing provided that whenever $a, b, c \in R$ with

$$abc \in I \text{ then either } ab \in I \text{ or } ac \in I \text{ or } bc \in I.$$

R is called a 2- absorbing ring if and only if its zero ideal is 2- absorbing.

Definition 2.13 ([5]). A proper ideal I of R is called a 2- absorbing primary ideal of R if whenever $a, b, c \in R$ with $abc \in I$, then either $ab \in I$ or $ac \in \sqrt{I}$ or $bc \in \sqrt{I}$.

3. Intuitionistic Fuzzy 2-Absorbing Primary Ideals

In this section, firstly we will define intuitionistic fuzzy 2- absorbing primary ideals, then we will give some operations on these ideals.Throughout the section, R is a commutative ring with identity.

Definition 3.1. Let $I = \langle \mu_I, \nu_I \rangle$ be an intuitionistic fuzzy ideal of R. I is called an intuitionistic fuzzy 2- absorbing ideal of R if for any intuitionistic fuzzy points $x_{(a,b)}, y_{(c,d)}, z_{(e,f)}$ such that for all $x_{(a,b)}y_{(c,d)}z_{(e,f)} \in I$ implies that

$$\text{either } x_{(a,b)}y_{(c,d)} \in I \text{ or } y_{(c,d)}z_{(e,f)} \in I \text{ or } x_{(a,b)}z_{(e,f)} \in I$$

$x, y, z \in R$ and $a, b, c, d, e, f \in L$.

Definition 3.2. Let $I = \langle \mu_I, \nu_I \rangle$ be an intuitionistic fuzzy ideal of R. I is called intuitionistic fuzzy 2- absorbing primary ideal of R if for any intuitionistic fuzzy points $x_{(a,b)}, y_{(c,d)}, z_{(e,f)}$ such that for all $x_{(a,b)}y_{(c,d)}z_{(e,f)} \in I$ implies that

$$\text{either } x_{(a,b)}y_{(c,d)} \in I \text{ or } y_{(c,d)}z_{(e,f)} \in \sqrt{I} \text{ or } x_{(a,b)}z_{(e,f)} \in \sqrt{I}$$

$x, y, z \in R$ and $a, b, c, d, e, f \in L$.

Theorem 3.3. *Every intuitionistic fuzzy primary ideal of R is an intuitionistic fuzzy 2- absorbing primary ideal of R.*

Proof. Proof is straightforward. □

Theorem 3.4. *Every intuitionistic fuzzy 2- absorbing ideal of R is an intuitionistic fuzzy 2- absorbing primary ideal.*

Proof. Proof is straightforward by the definition of the intuitionistic fuzzy 2- absorbing ideal. □

The following example shows that the converse of theorem is not true.

Example 3.5. Let $I = \langle \mu_I, \nu_I \rangle$ be an intuitionistic fuzzy ideal defined as

$$\mu_I(x) = \begin{cases} 1, & x \in 50\mathbb{Z} \\ 0, & \text{otherwise} \end{cases} \qquad \nu_I(x) = \begin{cases} 0, & x \in 50\mathbb{Z} \\ 1, & \text{otherwise} \end{cases}$$

We will check easy that I is an intuitionistic fuzzy 2- absorbing primary ideal. Because of $5_1 5_1 2_1 \in \mu_I$ but $5_1 5_1 \notin \mu_I$ and $5_1 2_1 \notin \mu_I$, I is not an intuitionistic fuzzy 2- absorbing ideal.

Lemma 3.6. *Let $I = \langle \mu_I, \nu_I \rangle$ is an intuitionistic fuzzy 2- absorbing primary ideal of R. Then, for every $s, t \in L$ with $I^{(t,s)} \neq R$, $I^{(t,s)}$ is an intuitionistic fuzzy 2- absorbing primary ideal.*

Proof. Suppose that $a, b, c \in R$ such that $abc \in I^{(t,s)}$. ($a_{(t,s)}b_{(t,s)}c_{(t,s)} = (abc)_{(t,s)} \in I$ is known.) Since I is an intuitionistic fuzzy 2- absorbing primary ideal of R, $(ab)_{(t,s)} \in I$ or $(ac)_{(t,s)} \in \sqrt{I}$ or $(bc)_{(t,s)} \in \sqrt{I}$. Hence, $(ab) \in I^{(t,s)}$ or $(ac) \in \sqrt{I^{(t,s)}}$ or $(bc) \in \sqrt{I^{(t,s)}}$. Therefore $I^{(t,s)}$ is an intuitionistic fuzzy 2- absorbing primary ideal. □

Note that if $I^{(t,s)}$ is an intuitionistic fuzzy 2- absorbing primary ideal, then I need not be an intuitionistic fuzzy 2- absorbing primary ideal. Following example shows that this statement.

Example 3.7. Let $I = \langle \mu_I, \nu_I \rangle$ is an intuitionistic fuzzy ideal of R defined by

$$\mu_I(x) = \begin{cases} 1, & x = 0, \\ 1/4 & x \in 9\mathbb{Z} - 0, \\ 0 & x \in \mathbb{Z} - 9\mathbb{Z} \end{cases} \qquad \nu_I(x) = \begin{cases} 0, & x = 0, \\ 3/4 & x \in 9\mathbb{Z} - 0, \\ 1 & x \in \mathbb{Z} - 9\mathbb{Z} \end{cases}$$

Then, μ_I^t is $(0), 9\mathbb{Z}, \mathbb{Z}$ in case $t \geq 1, t \geq 1/4, t \geq 0$ respectively. Similarly we will easy to define ν_I^s. Thus it is seen that $I^{(t,s)}$ is an intuitionistic fuzzy 2- absorbing primary ideal. Since $3_{1/2}.3_{1/2}.1_{1/4}(9) = 1/4 \leq \mu_I(9) = 1/4$ so $3_{1/2}.3_{1/2}.1_{1/4} \in \mu_I$ but $3_{1/2}.3_{1/2}(9) = 1/2 > \mu_I(9) = 1/4$ and $3_{1/2}.1_{1/4}(3) = 1/4 > \sqrt{\mu_I}(3) = 1/4$. Hence, μ_I is not an intuitionistic fuzzy 2- absorbing primary ideal. Hence,
$I = \langle \mu_I, \nu_I \rangle$ is not an intuitionistic fuzzy 2- absorbing primary ideal.

Proposition 3.8. *If I is an intuitionistic fuzzy 2- absorbing primary ideal of R, then $\sqrt{I}$ is an intuitionistic fuzzy 2- absorbing ideal of R.*

Proof. Let $x_{(a,b)}, y_{(c,d)}, z_{(e,f)}$ are any intuitionistic fuzzy points of R such that $x_{(a,b)}y_{(c,d)}z_{(e,f)} \in \sqrt{I}$ and $x_{(a,b)}y_{(c,d)} \notin \sqrt{I}$. Since $x_{(a,b)}y_{(c,d)}z_{(e,f)} \in \sqrt{I}$, we get

$$\begin{Bmatrix} a \wedge c \wedge e = x_a y_c z_e(xyz) \leq \sqrt{\mu}(xyz) \\ and \\ b \vee d \vee f = x_b y_d z_f\ (xyz) \leq \sqrt{\nu}(xyz) \end{Bmatrix}$$

By the definiton of $\sqrt{\mu}$,

$$\begin{aligned} \mu(xyz) &= \vee_{n\geq 1}\mu(x^n y^n z^n) \geq a \wedge c \wedge e \text{ and} \\ \nu(xyz) &= \vee_{n\geq 1}\nu(x^n y^n z^n) \geq b \vee d \vee f. \end{aligned}$$

Then, there exist a $k \in \mathbb{Z}^+$ such that

$$\begin{aligned} a \wedge c \wedge e &\leq \mu(x^k y^k z^k) = \mu((xyz)^k) \text{ and} \\ b \vee d \vee f &\leq \nu(x^k y^k z^k) = \nu((xyz)^k). \end{aligned}$$

It implies that $(x_a y_c z_e)^k \in \mu$ and $(x_b y_d z_f)^k \in \nu$. If $x_a y_c \notin \sqrt{\mu}$ and $x_b y_d \notin \sqrt{\nu}$, then for all $k \in \mathbb{Z}^+$, $(x_a y_c)^k = x_a^k y_c^k \notin \mu$ and $(x_b y_d)^k = x_b^k y_d^k \notin \nu$. Since I is an intuitionistic fuzzy 2- absorbing primary ideal, we conclude that

$$\begin{Bmatrix} (x_a z_e) \in \sqrt{\mu} \\ and \\ \left(x_b z_f\right) \in \sqrt{\nu} \end{Bmatrix} \ or \ \begin{Bmatrix} (y_c z_e) \in \sqrt{\mu} \\ and \\ (y_d z_e) \in \sqrt{\nu} \end{Bmatrix}.$$

Thus, $x_{(a,b)}z_{(e,f)} \in \sqrt{I}$ or $y_{(c,d)}z_{(e,f)} \in \sqrt{I}$ and $\sqrt{I}$ is an intuitionistic fuzzy 2- absorbing ideal. □

Definition 3.9. Let I be an intuitionistic fuzzy 2- absorbing primary ideal of R. Then, $\gamma = \sqrt{I}$ is an intuitionistic fuzzy 2- absorbing ideal by previous proposition. We say that I is an intuitionistic fuzzy γ-2- absorbing primary ideal of R.

Theorem 3.10. *Let $I_1, I_2, ..., I_n$ be intuitionistic fuzzy γ-2- absorbing primary ideals of R for some intuitionistic fuzzy 2- absorbing ideal γ of R. Then,*
$I = \cap_{i=1}^n I_i$ is an intuitionistic fuzzy γ-2- absorbing primary ideal of R.

Proof. Suppose that $x_{(a,b)}y_{(c,d)}z_{(e,f)} \in I$ and $x_{(a,b)}y_{(c,d)} \notin I$. Then, $x_{(a,b)}y_{(c,d)} \notin I_j$ for some $n \geq j \geq 1$ and $x_{(a,b)}y_{(c,d)}z_{(e,f)} \in I_j$ for all $n \geq j \geq 1$. Since I_j is an intuitionistic fuzzy γ-2- absorbing primary ideal, we have

$$\begin{aligned} y_{(c,d)}z_{(e,f)} &\in \sqrt{I_j} = \gamma = \cap_{i=1}^{n} \sqrt{I_i} = \sqrt{\cap_{i=1}^{n} I_i} = \sqrt{I} \text{ or} \\ x_{(a,b)}z_{(e,f)} &\in \sqrt{I_j} = \gamma = \cap_{i=1}^{n} \sqrt{I_i} = \sqrt{\cap_{i=1}^{n} I_i} = \sqrt{I}. \end{aligned}$$

Thus, I is an intuitionistic fuzzy γ-2- absorbing primary ideal of R. □

In the following example, we show that if I_1, I_2 are intuitionistic fuzzy 2- absorbing primary ideals of R, then $I_1 \cap I_2$ need not to be an intuitionistic fuzzy 2- absorbing primary ideal of R.

Example 3.11. Let $R = \mathbb{Z}$, the ring of integers. Define the intuitionistic fuzzy ideals $I_1 = \langle \mu_{I_1}, \nu_{I_1} \rangle$ and $I_2 = \langle \mu_{I_2}, \nu_{I_2} \rangle$ of $\mathbb{Z}$ by

$$\mu_{I_1}(x) = \begin{cases} 1, & x \in 8\mathbb{Z} \\ 0, & \text{otherwise} \end{cases} \qquad \nu_{I_1}(x) = \begin{cases} 0, & x \in 8\mathbb{Z} \\ 1, & \text{otherwise} \end{cases}$$

and

$$\mu_{I_2}(x) = \begin{cases} 1, & x \in 75\mathbb{Z} \\ 0, & \text{otherwise} \end{cases} \qquad \nu_{I_2}(x) = \begin{cases} 0, & x \in 75\mathbb{Z} \\ 1, & \text{otherwise} \end{cases}$$

Here I_1, I_2 are intuitionistic fuzzy 2- absorbing primary ideals but it is not difficult to show that $I_1 \cap I_2$ is not an intuitionistic fuzzy 2- absorbing primary ideal. Since,

$$(\mu_{I_1} \cap \mu_{I_2})(x) = \begin{cases} 1, & x \in 600\mathbb{Z} \\ 0, & \text{otherwise} \end{cases} \qquad (\nu_{I_1} \cap \nu_{I_2})(x) = \begin{cases} 0, & x \in 600\mathbb{Z} \\ 1, & \text{otherwise} \end{cases}$$

then, $25_1 3_1 8_1 \in \mu_{I_1} \cap \mu_{I_2}$ but $25_1 3_1 \notin \mu_{I_1} \cap \mu_{I_2}$, $3_1 8_1 \notin \mu_{I_1} \cap \mu_{I_2}$, $25_1 8_1 \notin \mu_{I_1} \cap \mu_{I_2}$. Moreover by the definiton of

$$\sqrt{\mu_{I_1} \cap \mu_{I_2}} = \begin{cases} 1, & x \in 30\mathbb{Z} \\ 0, & \text{otherwise} \end{cases} \qquad \sqrt{\nu_{I_1} \cap \nu_{I_2}} = \begin{cases} 0, & x \in 30\mathbb{Z} \\ 1, & \text{otherwise} \end{cases}$$

we conclude that $25_1 3_1 8_1 \in \mu_{I_1} \cap \mu_{I_2}$ but $25_1 3_1 \notin \sqrt{\mu_{I_1} \cap \mu_{I_2}}$, $3_1 8_1 \notin \sqrt{\mu_{I_1} \cap \mu_{I_2}}$ and $25_1 8_1 \notin \sqrt{\mu_{I_1} \cap \mu_{I_2}}$. Hence, $I_1 \cap I_2$ is not intuitionistic fuzzy 2- absorbing primary ideal of $\mathbb{Z}$.

Theorem 3.12. *Let I be an intuitionistic fuzzy ideal of R. If $\sqrt{I}$ is an intuitionistic fuzzy prime ideal of R, then I is an intuitionistic fuzzy 2- absorbing primary ideal of R.*

Proof. Assume that $x_{(a,b)}y_{(c,d)}z_{(e,f)} \in I$ and $x_{(a,b)}y_{(c,d)} \notin I$ for any $x, y, z \in R$ and $a, b, c, d, e, f \in L$. Since $x_{(a,b)}y_{(c,d)}z_{(e,f)} \in I$ and R is a commutative ring, we have $x_{(a,b)}y_{(c,d)}z_{(e,f)}z_{(e,f)} = (x_{(a,b)}z_{(e,f)})(y_{(c,d)}z_{(e,f)}) \in I \subseteq \sqrt{I}$. Since $\sqrt{I}$ is an intuitionistic fuzzy prime ideal, then $(x_{(a,b)}z_{(e,f)}) \in \sqrt{I}$ or $(y_{(c,d)}z_{(e,f)}) \in \sqrt{I}$. Hence, I is an intuitionistic fuzzy 2- absorbing primary ideal of R. □

Corollary 3.13. *If I is an intuitionistic fuzzy prime ideal of R, then I^n is an intuitionistic fuzzy 2- absorbing primary ideal of R for any $n \in \mathbb{Z}^+$.*

Proof. Let I is an intuitionistic fuzzy prime ideal and $x_{(a,b)}y_{(c,d)}z_{(e,f)} \in I^n$ but $x_{(a,b)}y_{(c,d)} \notin I^n$ for all $x, y, z \in R$ and $a, b, c, d, e, f \in L$ any $n \in \mathbb{Z}^+$. Since $x_{(a,b)}y_{(c,d)}z_{(e,f)} \in I^n$ and R is commutative ring, we conclude that $x_{(a,b)}y_{(c,d)}z_{(e,f)}z_{(e,f)} = (x_{(a,b)}z_{(e,f)})(y_{(c,d)}z_{(e,f)}) \in I^n \subseteq I$. Since I is an intuitionistic fuzzy prime ideal of R, $(x_{(a,b)}z_{(e,f)}) \in I = \sqrt{I^n}$ or $(y_{(c,d)}z_{(e,f)}) \in I = \sqrt{I^n}$. Hence, I^n is an intuitionistic fuzzy 2- absorbing primary ideal of R for any $n \in \mathbb{Z}^+$. □

Theorem 3.14. *Let $\{I_i \mid i \in I\}$ be a directed collection of intuitionistic fuzzy 2- absorbing primary ideals of R. Then, the intuitionistic fuzzy ideal $I = \cup_{i \in I} I_i$ is an intuitionistic fuzzy 2- absorbing primary ideal of R.*

Proof. Suppose that $x_{(a,b)}y_{(c,d)}z_{(e,f)} \in I$ and $x_{(a,b)}y_{(c,d)} \notin I$ for some $x_{(a,b)}, y_{(c,d)}, z_{(e,f)}$ are intuitionistic fuzzy points for R. Then, there are some $j \in I$ such that $x_{(a,b)}y_{(c,d)}z_{(e,f)} \in I_j$ and $x_{(a,b)}y_{(c,d)} \notin I_j$ for all $j \in I$. Since I_j is an intuitionistic fuzzy 2- absorbing primary ideal, we have $(y_{(c,d)}z_{(e,f)}) \in \sqrt{I_j}$ or $(x_{(a,b)}z_{(e,f)}) \in \sqrt{I_j}$. Thus,

$$\begin{aligned} (y_{(c,d)}z_{(e,f)}) &\in \sqrt{I_j} \subseteq \cup_{i \in I} \sqrt{I_i} = \sqrt{\cup_{i \in I} I_i} = I \text{ or} \\ (x_{(a,b)}z_{(e,f)}) &\in \sqrt{I_j} \subseteq \cup_{i \in I} \sqrt{I_i} = \sqrt{\cup_{i \in I} I_i} = I. \end{aligned}$$

Hence, $I = \cup_{i\in I} I_i$ is an intuitionistic fuzzy 2- absorbing primary ideal of R. □

Theorem 3.15. *Let $f : R \to S$ be a surjective ring homomorphism. If I is an intuitionistic fuzzy 2- absorbing primary ideal of R which is constant on* $\ker f$, *then $f(I)$ is an intuitionistic fuzzy 2- absorbing primary ideal of S.*

Proof. Suppose that $a_{(r,k)}b_{(s,d)}c_{(t,n)} \in f(I)$ where $a_{(r,k)}, b_{(s,d)}, c_{(t,n)}$ are any intuitionistic fuzzy points of S. Since f is surjective ring homomorphism, we have there exist $x, y, z \in R$ such that $f(x) = a$, $f(y) = b$, $f(z) = c$. Thus,

$$\begin{aligned} a_{(r,k)}b_{(s,d)}c_{(t,n)}(abc) &= r \wedge s \wedge t \leq f(\mu)(abc) \\ &= f(\mu)(f(x)f(y)f(z)) = f(\mu)(f(xyz)) = \mu(xyz) \end{aligned}$$

because μ is constant of $\ker f$. Then, we get $x_r y_s z_t \in \mu$. Since μ is an intuitionistic fuzzy 2- absorbing primary ideal we conclude that

$$x_r y_s \in \mu \text{ or } x_r z_t \in \sqrt{\mu} \text{ or } y_s z_t \in \sqrt{\mu}.$$

Thus,

$$r \wedge s \leq \mu(xy) = f(\mu)(f(xy)) = f(\mu)(f(x)f(y)) = f(\mu)(ab),\ a_r b_s \in f(\mu).$$

or

$$\begin{aligned} r \wedge t &\leq \sqrt{\mu}(xz) = \vee_{n\geq 1}\mu(x^n z^n) = \vee_{n\geq 1} f(\mu)(f(x^n)f(z^n)) \\ &= \vee_{n\geq 1} f(\mu)(a^n c^n) = \sqrt{f(\mu)}(ac),\ a_r c_t \in \sqrt{f(\mu)}. \end{aligned}$$

By a similar way it is easy to see that $b_s c_t \in \sqrt{f(\mu)}$ if $y_s z_t \in \sqrt{\mu}$. Similarly,

$$\begin{aligned} a_{(r,k)}b_{(s,d)}c_{(t,n)}(abc) &= k \vee d \vee n \leq f(\nu)(abc) \\ &= f(\nu)(f(x)f(y)f(z)) = f(\nu)(f(xyz)) = \nu(xyz) \end{aligned}$$

because ν is constant of $\ker f$. Then, we get $x_r y_s z_t \in \nu$. Since ν is an intuitionistic fuzzy 2- absorbing primary ideal we conclude that

$$x_k y_d \in \nu \text{ or } x_k z_n \in \sqrt{\nu} \text{ or } y_d z_n \in \sqrt{\nu}.$$

Thus,

$$k \vee d \leq \nu(xy) = f(\nu)(f(xy)) = f(\nu)(f(x)f(y)) = f(\nu)(ab),\ a_k b_d \in f(\nu).$$

or

$$\begin{aligned} k \vee n &\leq \sqrt{\nu}(xz) = \vee_{n\geq 1}\nu(x^n z^n) = \vee_{n\geq 1} f(\nu)(f(x^n)f(z^n)) \\ &= \vee_{n\geq 1} f(\nu)(a^n c^n) = \sqrt{f(\nu)}(ac),\ a_k c_n \in \sqrt{f(\nu)}. \end{aligned}$$

By a similar way it is easy to see that $b_d c_n \in \sqrt{f(\nu)}$ if $y_d z_n \in \sqrt{\nu}$. Thus, $f(I)$ is an intuitionistic fuzzy 2- absorbing primary ideal of S. □

Theorem 3.16. *Let $f : R \to S$ be a ring homomorphism. If I is an intuitionistic fuzzy 2- absorbing primary ideal of S, then $f^{-1}(I)$ is an intuitionistic fuzzy 2- absorbing primary ideal of R.*

Proof. Assume that $x_{(r,k)}y_{(s,d)}z_{(t,l)} \in f^{-1}(I)$ where $x_{(r,k)}, y_{(s,d)}, z_{(t,l)}$ are any intuitionistic fuzzy points of R. Then,

$$r \wedge s \wedge t \leq f^{-1}(\mu)(xyz) = \mu(f(xyz)) = \mu(f(x)f(y)f(z)).$$

Let $f(x) = a$, $f(y) = b$, $f(z) = c \in S$. Thus, we get that $r \wedge s \wedge t \leq \mu(abc)$ and $a_r b_s c_t \in \mu$. Since μ is an intuitionistic fuzzy 2- absorbing primary ideal we conclude that

$$a_r b_s \in \mu \text{ or } a_r c_t \in \sqrt{\mu} \text{ or } b_s c_t \in \sqrt{\mu}.$$

If $a_r b_s \in \mu$, then

$$r \wedge s \leq \mu(ab) = \mu(f(x)f(y)) = \mu(f(xy)) = f^{-1}(\mu)(xy).$$

Hence, $x_r y_s \in f^{-1}(\mu)$. If $a_r c_t \in \sqrt{\mu}$, then for some $n \in \mathbb{Z}^+$

$$\begin{aligned} r \wedge t &\leq \mu(a^n c^n) = \mu(f(x)^n f(z)^n) \leq \vee_{n\geq 1}\mu(f(x)^n f(z)^n) = \sqrt{\mu}(f(x)f(z)) \\ &= \sqrt{\mu}(f(xz)) = f^{-1}(\sqrt{\mu})(xz) = \sqrt{f^{-1}(\mu)}(xz). \end{aligned}$$

Thus, $x_r z_t \in \sqrt{f^{-1}(\mu)}$. By a similar way it can be see that $y_s z_t \in \sqrt{f^{-1}(\mu)}$. Similarly,

$$k \vee d \vee l \leq f^{-1}(\nu)(xyz) = \nu(f(xyz)) = \nu(f(x)f(y)f(z)).$$

Let $f(x) = a$, $f(y) = b$, $f(z) = c \in S$. Thus, we get that $k \vee d \vee l \leq \nu(abc)$ and $a_r b_s c_t \in \nu$. Since ν is an intuitionistic fuzzy 2- absorbing primary ideal we conclude that

$$a_k b_d \in \nu \text{ or } a_k c_l \in \sqrt{\nu} \text{ or } b_d c_l \in \sqrt{\nu}.$$

If $a_k b_d \in \nu$, then

$$k \vee d \leq \nu(ab) = \nu(f(x)f(y)) = \nu(f(xy)) = f^{-1}(\nu)(xy).$$

Hence, $x_k y_d \in f^{-1}(\nu)$. If $a_k c_l \in \sqrt{\nu}$, then for some $n \in \mathbb{Z}^+$

$$\begin{aligned} k \vee l &\leq \nu(a^n c^n) = \nu(f(x)^n f(z)^n) \leq \vee_{n\geq 1}\nu(f(x)^n f(z)^n) \\ &= \sqrt{\nu}(f(x)f(z)) = \sqrt{\nu}(f(xz)) = f^{-1}(\sqrt{\nu})(xz) = \sqrt{f^{-1}(\nu)}(xz). \end{aligned}$$

Thus, $x_k z_l \in \sqrt{f^{-1}(\nu)}$. By a similar way it can be see that $y_d z_l \in \sqrt{f^{-1}(\nu)}$. Hence, $f^{-1}(I)$ is an intuitionistic fuzzy 2- absorbing primary ideal of R. □

Definition 3.17. Let I be an intuitionistic fuzzy ideal of R. I is called an intuitionistic fuzzy strongly 2- absorbing primary ideal of R if it is constant and for any intuitionistic fuzzy ideals J, K, L of R with $JKL \subseteq I$ implies that $JK \subseteq I$ or $JL \subseteq \sqrt{I}$ or $KL \subseteq \sqrt{I}$.

Theorem 3.18. *Every intuitionistic fuzzy prime ideal of R is an intuitionistic fuzzy strongly 2- absorbing primary ideal of R.*

Proof. Proof is straightforward. □

Theorem 3.19. *Every intuitionistic fuzzy strongly 2- absorbing primary ideal is intuitionistic fuzzy 2- absorbing primary ideal.*

Proof. Suppose that I is an intuitionistic fuzzy strongly 2- absorbing primary ideal of R. Assume that $x_{(a,b)}y_{(c,d)}z_{(e,f)} \in I$ for some intuitionistic fuzzy points. Then, we have

$$\langle x_{(a,b)}\rangle \langle y_{(c,d)}\rangle \langle z_{(e,f)}\rangle = \langle x_{(a,b)}y_{(c,d)}z_{(e,f)}\rangle \subseteq I.$$

Since I is an intuitionistic fuzzy strongly 2- absorbing primary ideal, we have

$$\begin{aligned} \langle x_{(a,b)}y_{(c,d)}\rangle &= \langle x_{(a,b)}\rangle \langle y_{(c,d)}\rangle \subseteq I \text{ or} \\ \langle x_{(a,b)}z_{(e,f)}\rangle &= \langle x_{(a,b)}\rangle \langle z_{(e,f)}\rangle \subseteq \sqrt{I} \text{ or} \\ \langle y_{(c,d)}z_{(e,f)}\rangle &= \langle y_{(c,d)}\rangle \langle z_{(e,f)}\rangle \subseteq \sqrt{I}. \end{aligned}$$

Therefore,

$$\begin{aligned} (x_{(a,b)}y_{(c,d)}) &\in I \text{ or} \\ (x_{(a,b)}z_{(e,f)}) &\in \sqrt{I} \text{ or} \\ (y_{(c,d)}z_{(e,f)}) &\in \sqrt{I}. \end{aligned}$$

Hence, I is an intuitionistic fuzzy 2- absorbing primary ideal of R. □

Theorem 3.20. *Let $f : R \to S$ be a surjective ring homomorphism. If I is an intuitionistic fuzzy strongly 2- absorbing primary ideal of R which is constant on* $\ker f$, *then $f(I)$ is an intuitionistic fuzzy strongly 2- absorbing primary ideal of S.*

Proof. Proof is straightforward. □

Theorem 3.21. *Let $f : R \to S$ be a ring homomorphism. If I is an intuitionistic fuzzy strongly 2- absorbing primary ideal of S, then $f^{-1}(I)$ is an intuitionistic fuzzy strongly 2- absorbing primary ideal of R.*

Proof. Proof is straightforward. □

4. Intuitionistic Fuzzy Weakly Completely 2-Absorbing Primary Ideals

In this section, we will define intuitionistic fuzzy weakly completely 2- absorbing primary ideals and intuitionistic fuzzy K-2- absorbing primary ideals of R and then prove some fundamental properties between these classes of ideals.

Definition 4.1. Let I be an intuitionistic fuzzy ideal of R. I is called an intuitionistic fuzzy weakly completely 2-absorbing ideal of R provided that for all $a, b, c \in R$,

$$\left(\left\{\begin{matrix}\mu(abc) \leq \mu(ab) \\ \text{and} \\ \nu(abc) \geq \nu(ab)\end{matrix}\right\}\right) \text{ or } \left(\left\{\begin{matrix}\mu(abc) \leq \mu(ac) \\ \text{and} \\ \nu(abc) \geq \nu(ac)\end{matrix}\right\}\right) \text{ or } \left(\left\{\begin{matrix}\mu(abc) \leq \mu(bc) \\ \text{and} \\ \nu(abc) \geq \nu(bc)\end{matrix}\right\}\right).$$

Definition 4.2. Let I be an intuitionistic fuzzy ideal of R. I is called an intuitionistic fuzzy weakly completely 2-absorbing primary ideal of R provided that for all $a, b, c \in R$,

$$I(abc) \leq I(ab) \text{ or } I(abc) \leq \sqrt{I}(ac) \text{ or } I(abc) \leq \sqrt{I}(bc) \text{ i.e.}$$

$$\left\{\begin{matrix}\mu(abc) \leq \mu(ab) \\ \text{and} \\ \nu(abc) \geq \nu(ab)\end{matrix}\right\} \text{ or } \left\{\begin{matrix}\mu(abc) \leq \sqrt{\mu}(ac) \\ \text{and} \\ \nu(abc) \geq \sqrt{\nu}(ac)\end{matrix}\right\} \text{ or } \left\{\begin{matrix}\mu(abc) \leq \sqrt{\mu}(bc) \\ \text{and} \\ \nu(abc) \geq \sqrt{\nu}(bc)\end{matrix}\right\}.$$

Theorem 4.3. *Every intuitionistic fuzzy weakly completely 2-absorbing ideal is an intuitionistic fuzzy weakly completely 2-absorbing primary ideal.*

Proof. The proof is straightforward. □

The following example shows that the converse of this theorem is not necessarily true.

Example 4.4. Let $R = \mathbb{Z}$, the ring of integers. Define the intuitionistic fuzzy ideals $I = \langle \mu_I, \nu_I \rangle$ of $\mathbb{Z}$ by

$$\mu_I(x) = \begin{cases} 1, & x \in 12\mathbb{Z} \\ 0, & \text{otherwise} \end{cases} \qquad \nu_I(x) = \begin{cases} 0, & x \in 12\mathbb{Z} \\ 1, & \text{otherwise} \end{cases}$$

Assume that $\mu_I(xyz) > \mu_I(xy)$ and $\nu_I(xyz) < \nu_I(xy)$ for any $x, y, z \in R$. Thus, $\mu_I(xyz) = 1$ and $\mu_I(xy) = 0$. Similarly, $\nu_I(xyz) = 0$ and $\nu_I(xy) = 1$. Because of this, we get $xyz \in 12\mathbb{Z}$ and $xy \notin 12\mathbb{Z}$. Since $12\mathbb{Z}$ is a primary ideal of $\mathbb{Z}$, we get $z = 6\mathbb{Z}$. By the definiton of

$$\sqrt{\mu_I}(x) = \begin{cases} 1, & x \in 6\mathbb{Z} \\ 0, & \text{otherwise} \end{cases} \qquad \sqrt{\nu_I}(x) = \begin{cases} 0, & x \in 6\mathbb{Z} \\ 1, & \text{otherwise} \end{cases}$$

$\sqrt{\mu_I}(xz) = 1$ and $\sqrt{\nu_I}(xz) = 0$, and $\sqrt{\mu_I}(yz) = 1$ and $\sqrt{\nu_I}(yz) = 0$. Hence, $\sqrt{\mu_I}(xz) \geq \mu_I(xyz)$ and $\sqrt{\nu_I}(xz) \leq \nu_I(xyz)$. Moreover $\sqrt{\mu_I}(yz) \geq \mu_I(xyz)$ and $\sqrt{\nu_I}(yz) \leq \nu_I(xyz)$. Therefore, I is an intuitionistic fuzzy weakly completely 2-absorbing primary ideal. But since $\mu_I(2.2.3) = 1 > 0 = \mu_I(2.2)$ and
$\mu_I(2.2.3) = 1 > 0 = \mu_I(2.3)$, we conclude that I is not an intuitionistic fuzzy weakly completely 2- absorbing ideal.

Proposition 4.5. *Every intuitionistic fuzzy primary ideal of R is an intuitionistic fuzzy weakly completely 2- absorbing primary ideal.*

Proof. Let I be an intuitionistic fuzzy primary ideal of R. Assume that
$\mu(xyz) > \mu(xy)$ and $\nu(xyz) < \nu(xy)$ for any $x, y, z \in R$. Since μ and ν are intuitionistic fuzzy primary ideals, we have $\sqrt{\mu}(z) \geq \mu(xyz)$ and $\sqrt{\nu}(z) \leq \nu(xyz)$. Since $\sqrt{\mu}$ and $\sqrt{\nu}$ are intuitionistic fuzzy ideals , we have

$$\left\{\begin{matrix}\sqrt{\mu}(xz) \geq \sqrt{\mu}(z) \geq \mu(xyz) \\ \text{and} \\ \sqrt{\nu}(xz) \leq \sqrt{\nu}(z) \leq \nu(xyz)\end{matrix}\right\} \text{ or } \left\{\begin{matrix}\sqrt{\mu}(yz) \geq \sqrt{\mu}(z) \geq \mu(xyz) \\ \text{and} \\ \sqrt{\nu}(yz) \leq \sqrt{\nu}(z) \leq \nu(xyz)\end{matrix}\right\}.$$

Hence, I is an intuitionistic fuzzy weakly completely 2- absorbing primary ideal. □

Lemma 4.6. *Let I be an intuitionistic fuzzy ideal of R. Then, I is an intuitionistic fuzzy weakly completely 2- absorbing primary ideal if and only if $I^{(t,s)}$ is an intuitionistic fuzzy 2- absorbing primary ideal of R.*

Proof. Assume that $xyz \in I^{(t,s)}$ and $xy \notin I^{(t,s)}$ for any $x, y, z \in R$. We show that $yz \in \sqrt{I^{(t,s)}}$ or $xz \in \sqrt{I^{(t,s)}}$. If $xyz \in \sqrt{I^{(t,s)}}$, then $\mu_I(xyz) \geq t$ and
$\nu_I(xyz) \leq s$. Note that $\mu_I(xyz) \geq t > \mu_I(xy)$ and $\nu_I(xyz) \leq s < \nu_I(xy)$. Since μ, ν are intuitionistic fuzzy weakly completely 2- absorbing primary ideals, we have

$$\left\{\begin{array}{c}\sqrt{\mu}(xz) \geq \mu(xyz) \geq t\\ \text{and}\\ \sqrt{\nu}(xz) \leq \nu(xyz) \leq s\end{array}\right\} \text{ or } \left\{\begin{array}{c}\sqrt{\mu}(yz) \geq \mu(xyz) \geq t\\ \text{and}\\ \sqrt{\nu}(yz) \leq \nu(xyz) \leq s\end{array}\right\}.$$

Thus, $xz \in \sqrt{I^{(t,s)}}$ or $yz \in \sqrt{I^{(t,s)}}$. Hence, $I^{(t,s)}$ is an intuitionistic fuzzy 2- absorbing primary ideal. Conversely, assume that $I^{(t,s)}$ is an intuitionistic fuzzy 2- absorbing primary ideal of R. If $\mu_I(xyz) > \mu_I(xy)$ and $\nu_I(xyz) < \nu_I(xy)$, then there are $k \in [0, \mu(0)]$, $t \in [1, \nu(1)]$ such that $\mu_I(xyz) = k$ and $\nu_I(xyz) = t$. Since $k = \mu_I(xyz) > \mu_I(xy)$, then $xyz \in \mu_k$ and $xy \notin \mu_k$. Similarly, $t = \nu_I(xyz) < \nu_I(xy)$, then $xyz \in \nu_t$ and $xy \notin \nu_t$. Since μ_k, ν_t are intuitionistic fuzzy 2- absorbing primary ideals, we get

$$\left\{\begin{array}{c}xz \in \sqrt{\mu_k}\\ \text{and}\\ xz \in \sqrt{\nu_t}\end{array}\right\} \text{ or } \left\{\begin{array}{c}yz \in \sqrt{\mu_k}\\ \text{and}\\ yz \in \sqrt{\nu_t}\end{array}\right\}.$$

Hence,

$$\left\{\begin{array}{c}\sqrt{\mu(xz)} \geq k = \mu(xyz)\\ \text{and}\\ \sqrt{\nu(xz)} \leq t = \nu(xyz)\end{array}\right\} \text{ or } \left\{\begin{array}{c}\sqrt{\mu(yz)} \geq k = \mu(xyz)\\ \text{and}\\ \sqrt{\nu(yz)} \leq t = \nu(xyz)\end{array}\right\}.$$

Therefore, I is an intuitionistic fuzzy weakly completely 2- absorbing primary ideal. □

Theorem 4.7. *If I is an intuitionistic fuzzy weakly completely 2-absorbing primary ideal of R, then $\sqrt{I}$ is an intuitionistic fuzzy weakly completely 2-absorbing ideal of R.*

Proof. If I is an intuitionistic fuzzy weakly completely 2-absorbing primary ideal of R, then by lemma, $I^{(t,s)}$ is an intuitionistic fuzzy 2- absorbing primary ideal of R. We know that $\sqrt{I^{(t,s)}} = \sqrt{I}^{(t,s)}$ is an intuitionistic fuzzy 2- absorbing ideal of R. Then, using by definiton $\sqrt{I}^{(t,s)}$ is an intuitionistic fuzzy 2- absorbing ideal if and only if $\sqrt{I}$ is an intuitionistic fuzzy weakly completely 2-absorbing ideal of R. □

Definition 4.8. Let I be an intuitionistic fuzzy ideal of R. I is called intuitionistic fuzzy K-2- absorbing primary ideal of R provided that for all $a, b, c \in R$,

$$I(abc) = (0,1) \text{ implies that } I(ab) = (0,1) \text{ or } \sqrt{I}(ac) = (0,1) \text{ or } \sqrt{I}(bc) = (0,1) \text{ i.e.}$$

$$I(abc) \quad = \quad (0,1) = \left\{\begin{array}{c}\mu(abc) = \mu(0)\\ \text{and}\\ \nu(abc) = \nu(1)\end{array}\right\} \text{ implies that } \left\{\begin{array}{c}\mu(ab) = \mu(0)\\ \text{and}\\ \nu(ab) = \nu(1)\end{array}\right\}$$
$$\text{or } \left\{\begin{array}{c}\sqrt{\mu}(ac) = \mu(0)\\ \text{and}\\ \sqrt{\nu}(ac) = \nu(1)\end{array}\right\} \text{ or } \left\{\begin{array}{c}\sqrt{\mu}(bc) = \mu(0)\\ \text{and}\\ \sqrt{\nu}(bc) = \nu(1)\end{array}\right\}.$$

Theorem 4.9. *Every intuitionistic fuzzy weakly completely 2- absorbing primary ideal is intuitionistic fuzzy K-2-absorbing primary ideal.*

Proof. Let $I = \langle \mu_I, \nu_I \rangle$ is an intuitionistic fuzzy weakly completely 2- absorbing primary ideal. For any $x, y, z \in R$, if

$$\begin{aligned}
\mu(xyz) &= \mu(0), \text{ then } \mu(0) \geq \mu(xy) \geq \mu(xyz) = \mu(0) \text{ and}\\
\nu(xyz) &= \nu(1), \text{ then } \nu(1) \leq \nu(xy) \leq \nu(xyz) = \nu(1) \text{ or}\\
\mu(0) &= \sqrt{\mu}(0) \geq \sqrt{\mu}(xz) \geq \mu(xyz) = \mu(0) \text{ and}\\
\nu(1) &= \sqrt{\nu}(1) \leq \sqrt{\nu}(xz) \leq \nu(xyz) = \nu(1) \text{ or}\\
\mu(0) &= \sqrt{\mu}(0) \geq \sqrt{\mu}(yz) \geq \mu(xyz) = \mu(0) \text{ and}\\
\nu(1) &= \sqrt{\nu}(1) \leq \sqrt{\nu}(yz) \leq \nu(xyz) = \nu(1).
\end{aligned}$$

Due to the fact that μ, ν are fuzzy weakly completely 2- absorbing primary ideals. Hence,

$$\begin{Bmatrix} \mu(xy) = \mu(0) \\ \text{and} \\ \nu(xy) = \nu(1) \end{Bmatrix} \text{ or } \begin{Bmatrix} \sqrt{\mu}(xz) = \mu(0) \\ \text{and} \\ \sqrt{\nu}(xz) = \nu(1) \end{Bmatrix} \text{ or } \begin{Bmatrix} \sqrt{\mu}(yz) = \mu(0) \\ \text{and} \\ \sqrt{\nu}(yz) = \nu(1) \end{Bmatrix}.$$

We conclude that I is an intuitionistic fuzzy K-2- absorbing primary ideal. □

But the converse of theorem is not true.

Example 4.10. Let $R = \mathbb{Z}$, the ring of integers. Define the intuitionistic fuzzy ideal I of $\mathbb{Z}$ by

$$\mu_I(x) = \begin{cases} 1, & x = 0, \\ 1/2 & x \in 150\mathbb{Z} - 0, \\ 1/3 & x \in \mathbb{Z} - 150\mathbb{Z} \end{cases} \qquad \nu_I(x) = \begin{cases} 0, & x = 0, \\ 1/2 & x \in 150\mathbb{Z} - 0, \\ 2/3 & x \in \mathbb{Z} - 150\mathbb{Z} \end{cases}$$

Then, I is an intuitionistic fuzzy K-2-absorbing primary ideal. But since

$$\begin{aligned} \mu_I(25.3.2) &= 1/2 > \vee\{\mu_I(25.3), \mu_I(25.2), \mu_I(3.2)\} = 1/3 \text{ or} \\ \mu_I(25.3.2) &= 1/2 > \vee\{\sqrt{\mu_I}(25.3), \sqrt{\mu_I}(25.2), \sqrt{\mu_I}(3.2)\} = 1/3 \end{aligned}$$

then I is not an intuitionistic fuzzy weakly completely 2- absorbing primary ideal.

Corollary 4.11. *Every intuitionistic fuzzy weakly completely prime ideal is an intuitionistic fuzzy weakly completely 2- absorbing primary ideal.*

Proof. Since every intuitionistic fuzzy weakly completely prime ideal is an intuitionistic fuzzy primary ideal and by proposition every intuitionistic fuzzy weakly completely prime ideal is an intuitionistic fuzzy weakly completely 2- absorbing primary ideal. □

Theorem 4.12. *Every intuitionistic fuzzy K-2- absorbing ideal is an intuitionistic fuzzy K-2- absorbing primary ideal of R.*

Proof. Proof is straightforward but the converse of this theorem is not true. □

Example 4.13. Define the intuitionistic fuzzy ideal I of $\mathbb{Z}$ by

$$\mu_I(x) = \begin{cases} 1, & x \in 50\mathbb{Z} \\ 0, & \text{otherwise} \end{cases} \qquad \nu_I(x) = \begin{cases} 0, & x \in 50\mathbb{Z} \\ 1, & \text{otherwise} \end{cases}$$

then I is an intuitionistic fuzzy K-2- absorbing primary fuzzy ideal but since

$$\begin{aligned} \mu_I(5.5.2) &= 1 = \mu_I(0) \neq \mu_I(5.5) = 0 \text{ and} \\ \mu_I(5.5.2) &= 1 = \mu_I(0) \neq \mu_I(5.2) = 0 \end{aligned}$$

we have I is not an intuitionistic fuzzy K-2- absorbing ideal.

Theorem 4.14. *Let $f : R \to S$ be a surjective ring homomorphism. If I is an intuitionistic fuzzy weakly completely 2- absorbing primary ideal of R which is constant on* $\ker f$, *then $f(I)$ is an intuitionistic fuzzy weakly completely 2- absorbing primary ideal of S.*

Proof. Assume that $f(\mu)(abc) > f(\mu)(ab)$ for any $a, b, c \in S$. Since f is a surjective ring homomorphism, $f(x) = a$, $f(y) = b$, $f(z) = c$ for all $x, y, z \in R$. Thus,

$$\begin{aligned} f(\mu)(abc) &= f(\mu)(f(x)f(y)f(z)) = f(\mu)(f(xyz)) > f(\mu)(ab) \\ &= f(\mu)(f(x)f(y)) = f(\mu)(f(xy)). \end{aligned}$$

So, as μ is constant on $\ker f$, $f(\mu)(f(xyz)) = \mu(xyz)$ and $f(\mu)(f(xy)) = \mu(xy)$. This means that

$$f(\mu)(abc) = \mu(xyz) > \mu(xy) = f(\mu)(ab).$$

Since μ is an intuitionistic fuzzy weakly completely 2- absorbing primary ideal of R, then

$$\begin{aligned}\mu(xyz) &= f(\mu)(f(x)f(y)f(z)) = f(\mu)(abc) \leq \sqrt{\mu}(xz) = f(\sqrt{\mu})(f(xz)) \\ &= f(\sqrt{\mu})(ac) = \sqrt{f(\mu)}(ac) \text{ or} \\ \mu(xyz) &= f(\mu)(f(x)f(y)f(z)) = f(\mu)(abc) \leq \sqrt{\mu}(yz) = f(\sqrt{\mu})(f(yz)) \\ &= f(\sqrt{\mu})(bc) = \sqrt{f(\mu)}(bc).\end{aligned}$$

Similarly, assume that $f(v)(abc) < f(v)(ab)$ for any $a, b, c \in S$. Since f is a surjective ring homomorphism, $f(x) = a$, $f(y) = b$, $f(z) = c$ for all $x, y, z \in R$. Thus,

$$\begin{aligned}f(v)(abc) &= f(v)(f(x)f(y)f(z)) = f(v)(f(xyz)) < f(v)(ab) \\ &= f(v)(f(x)f(y)) = f(v)(f(xy)).\end{aligned}$$

So, as v is constant on $\ker f$, $f(v)(f(xyz)) = v(xyz)$ and $f(v)(f(xy)) = v(xy)$. This means that

$$f(v)(abc) = v(xyz) < v(xy) = f(v)(ab).$$

Since v is an intuitionistic fuzzy weakly completely 2- absorbing primary ideal of R, then

$$\begin{aligned}v(xyz) &= f(v)(f(x)f(y)f(z)) = f(v)(abc) \geq \sqrt{v}(xz) = f(\sqrt{v})(f(xz)) \\ &= f(\sqrt{v})(ac) = \sqrt{f(v)}(ac) \text{ or} \\ v(xyz) &= f(v)(f(x)f(y)f(z)) = f(v)(abc) \geq \sqrt{v}(yz) = f(\sqrt{v})(f(yz)) \\ &= f(\sqrt{v})(bc) = \sqrt{f(v)}(bc).\end{aligned}$$

Hence, $f(I)$ is an intuitionistic fuzzy weakly completely 2- absorbing primary ideal of S. □

Theorem 4.15. *Let $f : R \to S$ be a ring homomorphism. If I is an intuitionistic fuzzy weakly completely 2- absorbing primary ideal of S, then $f^{-1}(I)$ is an intuitionistic fuzzy weakly completely 2- absorbing primary ideal of R.*

Proof. Assume that $f^{-1}(\mu)(xyz) > f^{-1}(\mu)(xy)$ for any $x, y, z \in R$. Then,

$$\begin{aligned}f^{-1}(\mu)(xyz) &= \mu(f(xyz)) = \mu(f(x)f(y)f(z)) > f^{-1}(\mu)(xy) \\ &= \mu(f(xy)) = \mu(f(x)f(y)).\end{aligned}$$

Since μ is an intuitionistic fuzzy weakly completely 2- absorbing primary ideal of S, we conclude that

$$\begin{aligned}\mu(f(x)f(y)f(z)) &= f^{-1}(\mu)(xyz) \leq \sqrt{\mu}(f(x)f(z)) = \sqrt{\mu}(f(xz)) \\ &= f^{-1}(\sqrt{\mu}(xz)) = \sqrt{f^{-1}(\mu)}(xz) \text{ or} \\ \mu(f(x)f(y)f(z)) &= f^{-1}(\mu)(xyz) \leq \sqrt{\mu}(f(y)f(z)) = \sqrt{\mu}(f(yz)) \\ &= f^{-1}(\sqrt{\mu}(yz)) = \sqrt{f^{-1}(\mu)}(yz).\end{aligned}$$

Similarly, assume that $f^{-1}(v)(xyz) < f^{-1}(v)(xy)$ for any $x, y, z \in R$. Then,

$$\begin{aligned}f^{-1}(v)(xyz) &= v(f(xyz)) = v(f(x)f(y)f(z)) < f^{-1}(v)(xy) \\ &= v(f(xy)) = v(f(x)f(y)).\end{aligned}$$

Since v is an intuitionistic fuzzy weakly completely 2- absorbing primary ideal of S, we conclude that

$$\begin{aligned}v(f(x)f(y)f(z)) &= f^{-1}(v)(xyz) \geq \sqrt{v}(f(x)f(z)) = \sqrt{v}(f(xz)) \\ &= f^{-1}(\sqrt{v}(xz)) = \sqrt{f^{-1}(v)}(xz) \text{ or} \\ \mu(f(x)f(y)f(z)) &= f^{-1}(\mu)(xyz) \geq \sqrt{v}(f(y)f(z)) = \sqrt{v}(f(yz)) \\ &= f^{-1}(\sqrt{v}(yz)) = \sqrt{f^{-1}(v)}(yz).\end{aligned}$$

Hence, $f^{-1}(I)$ is an intuitionistic fuzzy weakly completely 2- absorbing primary ideal of R. □

Theorem 4.16. *Let $f : R \to S$ be a surjective ring homomorphism. If I is an intuitionistic fuzzy K- 2- absorbing primary ideal of R which is constant on* $\ker f$*, then $f(I)$ is an intuitionistic fuzzy K- 2- absorbing primary ideal of S.*

Proof. It is straightforward. □

Theorem 4.17. *Let $f : R \to S$ be a ring homomorphism. If I is an intuitionistic fuzzy K- 2- absorbing primary ideal of S, then $f^{-1}(I)$ is an intuitionistic fuzzy K- 2- absorbing primary ideal of R.*

Proof. It is straightforward. □

Corollary 4.18. *We obtained following table for intuitionistic fuzzy 2- absorbing primary ideals of commutative rings.*

$$\begin{array}{ccccc}
 & & \text{i.f.K-2-abs. id.} & \to & \text{i.f.K-2-abs.primary id.}\\
 & & \uparrow & & \uparrow \\
\text{i.f.w.c.prime id.} & \to & \text{i.f.w.c.2-abs.id.} & \to & \text{i.f.w.c.2-abs.primary id.}\\
\uparrow & & \uparrow & & \uparrow\\
\text{i.f. prime id.} & \longrightarrow & \text{i.f.2-abs.id.} & \longrightarrow & \text{i.f.2-abs.primary id.}\\
 & & \searrow & & \nearrow \\
 & \searrow & \text{i.f. primary ideal} & \nearrow & \\
 & & \text{i.f.strongly 2-abs.primary.id.} & &
\end{array}$$

5. Conclusion

In this study the theoretical point of view of intuitionistic fuzzy 2-absorbing primary ideals are discussed. The work also is focused on intuitionistic fuzzy strongly 2-absorbing primary ideals, intuitionistic fuzzy weakly completely 2-absorbing primary ideals, intuitionistic fuzzy K-2-absorbing primary ideals and characterized their algebraic properties. Furthermore, under a ring homomorphism, these ideals are investigated. In order to extend this study, one could study other algebraic structures and do some further study on properties them.

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