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Effective estimation of population mean, ratio and product in two-phase sampling in presence of random non-response

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Abstract

This paper presents some effective estimation strategies of population mean, ratio and product of two population means in two-phase (double) sampling when random non-response observed in the sample data. Proposed estimators are defined using a random imputation method which is capable in reducing the negative impact of random non-response in two-phase (double) sampling setup. Properties of proposed estimators have been examined and their performances are compared with sample mean, ratio and product estimators under the similar situations while the complete response was observed in the sample data. Empirical studies are carried out to validate the theoretical results, which are subsequently well interpreted followed by suitable recommendations.

Keywords: Two-phase sampling, random non-response, imputation, auxiliary variable, bias, mean square error.

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1. Introduction

In sample surveys sometimes we need to estimate population mean, ratio and product of two characteristics of interest simultaneously in the field of agricultural, socioeconomic, medical sciences etc. For example the ratio of corn acres to wheat acres, the ratio of expenditure on labor to total expenditure, the product of cultivated area and yield rate etc. The use of auxiliary information at estimation stage is a well sought technique to produce the precise estimates of population parameters in sample surveys. Sometimes, information on auxiliary variable is not available for all the units of population, for such

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situation two-phase (double) sampling is a cost effective methodology for generating the valid estimates of unknown population parameters of auxiliary variable in first phase sample. Some of the novel works in two-phase (double) sampling may be referred as Chand [2], Kiregyera [4][5], Mukharjee *et al.* [7], Srivastava *et al.* [26], Singh and Singh [23], Singh *et al.* [24], Singh and Upadhyaya [15], Upadhyaya and Singh [27], Singh [13], Pradhan [8], Bandyopadhyay and Singh [1] and Singh and Sharma [16] [17] among others. In sample surveys due to various reasons often it is not possible to collect the desired information from all the units selected in the sample. For example in socioeconomic surveys the selected families may not be at home at the first attempt and some of them may refuse to cooperate with the interviewer even if contacted. As many respondents do not reply, available sample of returns is incomplete. The resulting incompleteness is called non-response and is sometimes so large that can completely vitiate the survey results. A natural question arises what one needs to assume to justify ignoring the incompleteness in the survey data. For such situations, the problems of estimation of population mean, ratio and product of two population means were addressed by Singh *et al.* [18] and Singh and Kumar [19] under various non-response cases with the utilization of auxiliary information at estimation stage. However, it may be noted that most of the related works on estimation of population mean, ratio and product in survey sampling are either based on complete response situation or traditional non-response situation in survey data. It may also be seen in survey literatures that no enough mechanism has been developed to reduce the negative impact of random non-response in the estimation procedures of population mean, ratio and product of two population means. Rubin [9] addressed three concepts on missing pattern of survey data such as missing at random (MAR), observed at random (OAR) and parameter distribution (PD). He defined 'The data are MAR, if the probability of the observed missingness pattern, given the observed and unobserved data, does not depend on the value of the unobserved data '. Recently imputation methods fascinated survey statisticians to deal with the problems of non-response in survey data. Imputation method describes the filling up of incomplete data for adapting the standard analytic model in statistics. It is typically used when needed to substitute missing item value with certain fabricated values in the sample surveys. To deal with missing values effectively, Sande [10] suggested imputation methods that make incomplete data sets structurally complete. Several authors including Lee *et al.* [6], Singh [14] and Diana and Perri [3] among others have contributed a lot towards formulating efficient estimation procedures of population mean in sample surveys which reduced the negative impact of non-response through imputation methods. So far, no attempt has been made to cease the non-response situation in the estimation of population mean, ratio and product of two population means using imputation methods. Motivated with above arguments and using information on two auxiliary variables, we have suggested an imputation method for a random non-response situation under missing at random (MAR) response mechanism and subsequently proposed the estimators of population mean, ratio and product of two populations means in two-phase (double) sampling setup. The properties of the proposed estimators have been studied and their efficacies are demonstrated empirically which is followed by suitable recommendations to the survey practitioners.

2. Sample structures and notations

Consider a finite population $U = (U_1, U_2, ..., U_N)$ of N units where y and x are the variables under study and z_1 and z_2 are the first and second auxiliary variables respectively. Let y_k, x_k, z_{1k} and z_{2k} be the values of y, x, z_1 and z_2 for the k-th $(k = 1, 2, ..., N)$ unit of the population respectively. The purpose is to estimate the population mean, ratio and product of two population means of study variables y and x when the population mean of the first auxiliary variable z_1 is unknown but information on second auxiliary variable z_2 is available for all the units of the population. A first phase sample S_n of size n is drawn by simple random sampling without replacement (SRSWOR) from population U and observed for the auxiliary variables z_1 and z_2. Again a second-phase sample S_m of size $m\,(m < n)$ is drawn according to following SRSWOR schemes and observed for the characteristics y and x.

Case I: Second phase sample S_m is drawn as a sub-sample of the first phase sample S_n $(S_m \subset S_n)$.

Case II: Second phase sample S_m is drawn independently of the first phase sample S_n $(S_m \not\subset S_n)$.

It is assumed that random non-response situations occurs only for the study variables y and x while the sampled units give complete response for the auxiliary variables z_1 and z_2. We have considered the following non-response probability model for such random non-response situations.

3. Non-response probability model

Since, we have considered random non-response situations found on the study variables y and x and we observe the said characteristics from the second phase sample S_m , therefore, we investigate the random non-response conditions from the second phase sample S_m only. Let $r\,(r = 0, 1, 2, ..., (m-2))$ denote the number of non-responding units on second phase sample S_m. Accordingly, we denote the set of non-responding units by S_r and the set of responding units containing $(m-r)$ units denoted by S_r^c. The observations for the variables on which random non-response occur can be derived from the remaining $(m-r)$ units of the second phase samples such that $0 \le r \le (m-2)$. We also assume that if p denotes the probability of non-response among $(m-2)$ possible cases of non-responses, then r follows the following probability distribution:

$$P\,(r) = \frac{m-r}{mq+2p}\,{}^{m-2}C_r p^r q^{m-2-r};\; r = 0, 1, 2, ..., (m-2)$$

where $q = 1 - p$.

Here ${}^{m-2}C_r$ denotes the total number of ways for obtaining random non-response cases out of $(m-2)$ non-responses, for instance, see Singh and Joarder [21].

Hence, onwards we use the following notations:

$\bar{Y}$, $\bar{X}$: Population means of the study variables y and x respectively.

$\bar{Z}_1$, $\bar{Z}_2$: Population means of the auxiliary variables z_1 and z_2 respectively.

$\bar{y}_m$, $\bar{x}_m$, $\bar{z}_{1m}$, $\bar{z}_{2m}$: Sample means of the respective variables based on second phase sample S_m of size m.

$\bar{z}_{1n}$, $\bar{z}_{2n}$: Sample means of the respective variables based on first phase sample S_n of size n.

ρ_{yx}, ρ_{yz_1}, ρ_{yz_2}, ρ_{xz_1}, ρ_{xz_2} : Correlation coefficients between the variables shown in subscripts.

C_y, C_x, C_{z_1}, C_{z_2} : Coefficients of variation of respective variables shown in subscripts.

4. Formulation of estimators

To estimate the parametric function $\bar{R} = \bar{Y}/\bar{X}^{\alpha}$, where $\bar{X} \neq 0$, the natural estimator is considered as

(4.1) $\hat{\bar{R}}_{(\alpha)} = \bar{y}/\bar{x}^{\alpha}\ (\bar{x} \neq 0)$

where α is a scalar, which assumes value 0, 1 and -1 as per the requirement such that
(i) $\hat{\bar{R}}_{(\alpha=0)} = \bar{y}$ is an estimator of $\bar{R}_{(\alpha=0)} = \bar{Y}$,
(ii) $\hat{\bar{R}}_{(\alpha=1)} = \bar{y}/\bar{x}$ is an estimator of $\bar{R}_{(\alpha=1)} = \bar{Y}/\bar{X}$
and
(iii) $\hat{\bar{R}}_{(\alpha=-1)} = \bar{y}.\bar{x}$ is an estimator of $\bar{R}_{(\alpha=-1)} = \bar{Y}.\bar{X}$.

To estimate the population mean, ratio and product of two population means from the second phase sample, a set of estimators $T_{(\alpha)i}\,(i = 1, 2, ..., 4)$ are suggested. The proposed estimators are structured to cope with the problems of random non-response situations under missing at random (MAR) response mechanism which are occurred for the study variables y and x in the second phase sample S_m.

Since, the information on auxiliary variables z_1 and z_2 are available on all the units of the samples, therefore, motivated by some imputation methods for estimating the population mean as considered by Singh [14] and Diana and Perri [3] among others, we suggest the following imputation method based on responding and non-responding units in the sample S_m to estimate mean, ratio and product of two population means, which is described as

$$\hat{\bar{R}}_{(\alpha)i} = \begin{cases} \hat{\bar{R}}^*_{(\alpha)m} \frac{\bar{z}_1^*}{\bar{z}_{1m}} \exp\left(\frac{\bar{Z}_2 - \bar{z}_{2m}}{\bar{Z}_2 + \bar{z}_{2m}}\right), & i \epsilon S_r \\ \frac{\hat{\bar{R}}^*_{(\alpha)m}}{\left(\bar{z}_{1n} - \bar{z}_{1(m-r)}\right)} \frac{\bar{z}_1^*}{\bar{z}_{1m}} \exp\left(\frac{\bar{Z}_2 - \bar{z}_{2m}}{\bar{Z}_2 + \bar{z}_{2m}}\right) \left(\bar{z}_{1n} - \bar{z}_{1i}\right), & i \epsilon S_r^c \end{cases}$$

where $\hat{\bar{R}}^*_{(\alpha)m} = \bar{y}^*_m / \left(\bar{x}^*_m\right)^\alpha$ such that, $\bar{y}^*_m = \frac{1}{m-r}\sum_{i=1}^{m-r} y_i$, $\bar{x}^*_m = \frac{1}{m-r}\sum_{i=1}^{m-r} x_i$, $\bar{z}_{1(m-r)} = \frac{1}{m-r}\sum_{i=1}^{m-r} z_{1i}$, $\bar{z}_1^* = \bar{z}_{1n} + b_{z_1 z_2}\left(\bar{Z}_2 - \bar{z}_{2n}\right)$ and $b_{z_1 z_2}$ is a sample regression coefficient between variables z_1 and z_2.

Under the above method of imputation, the estimator $\hat{\bar{R}}_{(\alpha)m}$ of $\bar{R}_{(\alpha)}$ derived as
$\hat{\bar{R}}_{(\alpha)m} = \frac{1}{m}\sum_{i\epsilon S_m} \hat{\bar{R}}_{(\alpha).i} = \frac{1}{m}\left[\sum_{i\epsilon S_m} \hat{\bar{R}}_{(\alpha).i} + \sum_{i\epsilon S_m^c} \hat{\bar{R}}_{(\alpha).i}\right]$

After simplification of the above expression, the final structure of $\hat{\bar{R}}_{(\alpha)m}$ is obtained as

$$\hat{\bar{R}}_{(\alpha)m} = \hat{\bar{R}}^*_{(\alpha)m} \frac{\bar{z}_1^*}{\bar{z}_{1m}} \exp\left(\frac{\bar{Z}_2 - \bar{z}_{2m}}{\bar{Z}_2 + \bar{z}_{2m}}\right) \tag{4.2}$$

In follow up of above discussion and motivated by Singh and Vishwakarma [25] and Sanaullah *et al.* [11], we suggest the following estimators of $\bar{R}_{(\alpha)}$ as

$$T_{(\alpha)1} = \hat{\bar{R}}_{(\alpha)m} \tag{4.3}$$

$$T_{(\alpha)2} = \hat{\bar{R}}_{(\alpha)m} \exp\left(\frac{\hat{\bar{R}}_{(z)n} - \hat{\bar{R}}_{(z)m}}{\hat{\bar{R}}_{(z)n} + \hat{\bar{R}}_{(z)m}}\right) \tag{4.4}$$

$$(4.5)\qquad T_{(\alpha)3} = \hat{\hat{R}}_{(\alpha)m} \exp\left(\frac{\bar{Z}_1 - \bar{z}_{1m}}{\bar{Z}_1 + \bar{z}_{1m}}\right)$$

$$(4.6)\qquad T_{(\alpha)3} = \hat{\hat{R}}_{(\alpha)m} \exp\left(\frac{\frac{\bar{z}_{1n}}{\bar{z}_{2n}}\bar{Z}_2 - \bar{z}_{1m}}{\frac{\bar{z}_{1n}}{\bar{z}_{2n}}\bar{Z}_2 + \bar{z}_{1m}}\right)$$

where $\hat{\hat{R}}_{(z)n} = \frac{\bar{z}_{2n}}{\bar{z}_{1n}}$ and $\hat{\hat{R}}_{(z)m} = \frac{\bar{z}_{2m}}{\bar{z}_{1m}}$.

5. Bias and mean square errors of the proposed estimators $T_{(\alpha)i}\ (i = 1, 2, 3, 4)$ under case I

The bias and mean square errors of proposed estimators $T_{(\alpha)i}\ (i = 1, 2, 3, 4)$ are derived for the Case I in two-phase sampling setup up to the first order of approximations under large sample assumptions and presented as

$$(5.1)\qquad B\left(T_{(\alpha)1}\right) = R_{(\alpha)}\left[\begin{array}{c} f_m\left\{\begin{array}{c}\left(C_{z_1} - \rho_{yz_1}C_y - \alpha\rho_{xz_1}C_x + \frac{1}{2}\rho_{z_1z_2}C_{z_2}\right)C_{z_1} \\ +\left(\frac{3}{8}C_{z_2} - \frac{1}{2}\rho_{yz_2}C_y + \frac{1}{2}\alpha\rho_{xz_2}C_x\right)C_{z_2}\end{array}\right\} \\ +f_n\left\{\begin{array}{c}\left(\rho_{yz_1}C_y - \alpha\rho_{xz_1}C_x - \frac{1}{2}\rho_{z_1z_2}C_{z_2}\right)C_{z_1} - B \\ -\rho_{z_1z_2}C_{z_1}\left(\rho_{yz_2}C_y - \alpha\rho_{xz_2}C_x - \frac{1}{2}C_{z_2}\right)\end{array}\right\}\end{array}\right]$$

$$(5.2)\qquad B\left(T_{(\alpha)2}\right) = R_{(\alpha)}\left[\begin{array}{c} \frac{1}{2}f_{mn}\left\{\begin{array}{c}\frac{3}{4}C_{z_1}^2 + \frac{5}{4}C_{z_2}^2 - \left(\rho_{yz_1}C_y - \alpha\rho_{xz_1}C_x\right)C_{z_1} \\ -\left(\rho_{yz_2}C_y + \alpha\rho_{xz_2}C_x - \rho_{z_1z_2}C_{z_1}\right)C_{z_2}\end{array}\right\} \\ +f_n\left\{\left(\rho_{xz_2}C_x - \rho_{yz_2}C_y + \frac{1}{2}\rho_{z_1z_2}C_{z_1}\right)C_{z_2}\right\}\end{array}\right]$$

$$(5.3)\qquad B\left(T_{(\alpha)3}\right) = R_{(\alpha)}\left[\begin{array}{c} \frac{1}{2}f_m\left\{\begin{array}{c}\left(\frac{11}{4}C_{z_1} - \rho_{yz_1}C_y - 3\alpha\rho_{xz_1}C_x\right)C_{z_1} \\ +\left(\frac{3}{4}C_{z_2} - \rho_{yz_2}C_y + \alpha\rho_{xz_2}C_x\right)C_{z_2}\end{array}\right\} \\ +f_n\left\{\begin{array}{c}\frac{1}{2}\left(\rho_{yz_1}C_y - \alpha\rho_{xz_1}C_x - \frac{1}{2}\rho_{z_1z_2}C_{z_2}\right)C_{z_1} \\ -\rho_{z_1z_2}C_{z_1}\left(\rho_{yz_2}C_y - \alpha\rho_{xz_2}C_x + \frac{1}{2}C_{z_2}\right)\end{array}\right\} \\ -f_n\left(\frac{19}{8}C_{z_2}^2 + B\right)\end{array}\right]$$

$$(5.4)\qquad B\left(T_{(\alpha)4}\right) = R_{(\alpha)}\left[\begin{array}{c} f_n\left(\rho_{yz_2}C_yC_{z_2} + \frac{1}{2}\alpha\rho_{xz_1}C_xC_{z_1}\right) \\ +f_n\left\{-\rho_{z_1z_2}C_{z_1}\left(\rho_{yz_2}C_y - \alpha\rho_{xz_2}C_x + \frac{1}{2}C_{z_2}\right) - B\right\} \\ +f_m\left\{\left(\frac{3}{8}C_{z_2} + \frac{1}{2}\alpha\rho_{xz_2}C_x\right)C_{z_2} - \alpha\rho_{xz_1}C_xC_{z_1}\right\} \\ +\frac{1}{2}f_{mn}\left(\frac{13}{4}C_{z_1} - 3\rho_{yz_1}C_y + \alpha\rho_{xz_1}C_x\right)C_{z_1} \\ +\frac{3}{4}f_{mn}\rho_{z_1z_2}C_{z_2}C_{z_1}\end{array}\right]$$

$$(5.5)\qquad M\left(T_{(\alpha)1}\right) = R_{(\alpha)}^2\left[\begin{array}{c} f_{mn}\left(C_{z_1} - 2\rho_{yz_1}C_y + 2\alpha\rho_{xz_1}C_x + \rho_{z_1z_2}C_{z_2}\right)C_{z_1} \\ +f_m^*A + f_m\left(\frac{1}{4}C_{z_2} - 2\rho_{yz_2}C_y + \alpha\rho_{xz_2}C_x\right)C_{z_2} \\ +f_n\left(\rho_{z_1z_2}C_{z_1} - 2\rho_{yz_2}C_y + \alpha\rho_{xz_2}C_x + C_{z_2}\right)\rho_{z_1z_2}C_{z_1}\end{array}\right]$$

$$(5.6)\qquad M\left(T_{(\alpha)2}\right) = R_{(\alpha)}^2\left[\begin{array}{c} f_{mn}\left(\frac{1}{4}C_{z_1} - \rho_{yz_1}C_y + \alpha\rho_{xz_1}C_x + \frac{1}{2}\rho_{z_1z_2}C_{z_2}\right)C_{z_1} \\ +f_m^*A + f_m\left(C_{z_2} - 2\rho_{yz_2}C_y + 2\alpha\rho_{xz_2}C_x\right)C_{z_2} \\ +f_n\left((g-2)C_{z_2} + 2\rho_{yz_2}C_y + 2\alpha\rho_{xz_2}C_x\right)C_{z_2}\end{array}\right]$$

$$(5.7)\qquad M\left(T_{(\alpha)3}\right) = R_{(\alpha)}^2\left[\begin{array}{c} f_{mn}\left(\frac{1}{4}C_{z_1} - \rho_{yz_1}C_y + \alpha\rho_{xz_1}C_x + \frac{1}{2}\rho_{z_1z_2}C_{z_2}\right)C_{z_1} \\ +f_m\left(\frac{1}{4}C_{z_2} - \rho_{yz_2C_y} + \alpha\rho_{xz_2}C_x - 2\rho_{yz_2}C_y\right)C_{z_2} \\ +f_n\left(\rho_{z_1z_2}C_{z_1} + 2\rho_{yz_2}C_y + \alpha\rho_{xz_2}C_x\right)\rho_{z_1z_2}C_{z_1} \\ +f_m^*A\end{array}\right]$$

$$(5.8)\qquad M\left(T_{(\alpha)4}\right)=R_{(\alpha)}^{2}\left[\begin{array}{c} f_{mn}\left(\frac{9}{4}C_{z_1}-3\rho_{yz_1}C_y+3\alpha\rho_{xz_1}C_x+\frac{3}{2}\rho_{z_1z_2}C_{z_2}\right)C_{z_1}\\ +f_m^{*}A+f_m\left(\frac{1}{4}C_{z_2}-\rho_{yz_2}C_y+\alpha\rho_{xz_2}C_x\right)C_{z_2}\\ +f_n\left(\frac{3}{4}C_{z_2}-\rho_{yz_2}C_y+\alpha\rho_{xz_2}C_x-2\rho_{z_1z_2}C_{z_1}\right)C_{z_2}\\ +f_n\left(\rho_{z_1z_2}C_{z_1}+2\rho_{yz_2}C_y-2\alpha\rho_{xz_2}C_x\right)\rho_{z_1z_2}C_{z_1}\end{array}\right]$$

where $A=C_y^2+\alpha^2C_x^2-2\alpha\rho_{yx}C_yC_x$, $B=\rho_{z_1z_2}\frac{C_{z_1}}{C_{z_2}}\left\{\frac{1}{\bar{Z}_2}\left(\frac{\mu_{0012}}{\mu_{0011}}-\frac{\mu_{0003}}{\mu_{0002}}\right)-\rho_{z_1z_2}C_{z_1}C_{z_2}\right\}$, $g=\frac{1}{2}-\rho_{z_1z_2}\frac{C_{z_1}}{C_{z_2}}$, $f_n^{*}=\left(\frac{1}{nq+2p}-\frac{1}{N}\right)$, $f_m^{*}=\left(\frac{1}{mq+2p}-\frac{1}{N}\right)$, $f_m=\left(\frac{1}{m}-\frac{1}{N}\right)$, $f_n=\left(\frac{1}{n}-\frac{1}{N}\right)$ and $\mu_{abcd}=E\left[\left(y_i-\bar{Y}\right)^a\left(x_i-\bar{X}\right)^b\left(z_{1i}-\bar{Z}_1\right)^c\left(z_{2i}-\bar{Z}_2\right)^d\right]$; $(a,b,c,d)\geq 0$.

6. Bias and mean square errors of the proposed estimators $T_{(\alpha)i}(i=1,2,3,4)$ under case II

If sample of size m is directly drawn independently from the population (Case II of two phase sampling setup as defined in section 2) then the bias and mean square errors of estimators $T_{(\alpha)i}(i=1,2,3,4)$ are derived as

$$(6.1)\qquad B\left(T_{(\alpha)1}\right)=R_{(\alpha)}\left[\begin{array}{c} f_m\left\{\begin{array}{c}\left(C_{z_1}-\rho_{yz_1}C_y-\alpha\rho_{xz_1}C_x\right)C_{z_1}+\\ \frac{1}{2}\left(\frac{3}{4}C_{z_2}-\rho_{yz_2}C_y+\alpha\rho_{xz_2}C_x\right)C_{z_2}\\ +\frac{1}{2}\rho_{z_1z_2}C_{z_1}C_{z_2}\end{array}\right\}\\ -f_nB\end{array}\right]$$

$$(6.2)\qquad B\left(T_{(\alpha)2}\right)=R_{(\alpha)}\left[\begin{array}{c}\frac{1}{2}f_m\left\{\begin{array}{c}\left(\frac{3}{4}C_{z_1}^2+\frac{5}{4}C_{z_2}^2-\left(\rho_{yz_1}C_y-\alpha\rho_{xz_1}C_x\right)C_{z_1}\right.\\ \left.+\left(\rho_{yz_2}C_y+\rho_{z_1z_2}C_{z_1}\right)C_{z_2}\right.\\ \left.-f_n\left(C_{z_1}^2+C_{z_2}^2+\frac{1}{4}\rho_{z_1z_2}C_{z_1}C_{z_2}\right)\right.\end{array}\right\}\end{array}\right]$$

$$(6.3)\qquad B\left(T_{(\alpha)3}\right)=R_{(\alpha)}\left[\begin{array}{c}\frac{1}{2}f_m\left\{\begin{array}{c}\left(\frac{11}{4}C_{z_1}-\rho_{yz_1}C_y-3\alpha\rho_{xz_1}C_x\right)C_{z_1}\\ +\left(\frac{3}{4}C_{z_2}-\rho_{yz_2}C_y+\alpha\rho_{xz_2}C_x\right)C_{z_2}\end{array}\right\}\\ -f_n\left(\frac{1}{8}C_{z_1}^2-B\right)\end{array}\right]$$

$$(6.4)\qquad B\left(T_{(\alpha)4}\right)=R_{(\alpha)}\left[\begin{array}{c}\frac{1}{2}f_m\left\{\begin{array}{c}\left(\frac{3}{4}C_{z_2}^2+C_{z_1}\left(\frac{13}{4}C_{z_1}-3\rho_{yz_1}C_y-\alpha\rho_{xz_1}C_x\right)\right)\\ +\left(\alpha\rho_{xz_2}C_x-\rho_{yz_2}C_y\right)C_{z_2}+\frac{3}{2}\rho_{z_1z_2}C_{z_1}c_{z_2}\end{array}\right\}\\ +\frac{1}{2}f_n\left\{\begin{array}{c}\frac{5}{4}C_{z_1}^2+\frac{1}{4}C_{z_2}^2-\frac{3}{2}\rho_{z_1z_2}C_{z_1}C_{z_2}\\ -\rho_{z_1z_2}\left(\rho_{z_1z_2}C_{z_1}-C_{z_2}\right)-B\end{array}\right\}\end{array}\right]$$

$$(6.5)\qquad M\left(T_{(\alpha)1}\right)=R_{(\alpha)}^2\left[\begin{array}{c}f_m\left\{\begin{array}{c}\left(C_{z_1}-2\rho_{yz_1}C_y+2\alpha\rho_{xz_1}C_x+\rho_{z_1z_2}C_{z_2}\right)C_{z_1}\\ +\left(\frac{1}{4}C_{z_2}-\rho_{yz_2}C_y+\alpha\rho_{xz_2}C_x\right)C_{z_2}\\ +f_m^{*}A+f_n\left(1-\rho_{z_1z_2}^2\right)C_{z_1}^2\end{array}\right\}\end{array}\right]$$

$$(6.6)\qquad M\left(T_{(\alpha)2}\right)=R_{(\alpha)}^2\left[\begin{array}{c}f_m\left\{\begin{array}{c}\left(C_{z_2}-2\rho_{yz_2}C_y+2\alpha\rho_{xz_2}C_x+\rho_{z_1z_2}C_{z_1}\right)C_{z_2}\\ +\left(\frac{1}{4}C_{z_2}-\rho_{yz_1}C_y+\alpha\rho_{xz_1}C_x\right)C_{z_1}\\ +f_m^{*}A+f_n\left(\frac{1}{4}C_{z_1}^2-g^2C_{z_2}^2\right)\end{array}\right\}\end{array}\right]$$

$$(6.7)\qquad M\left(T_{(\alpha)3}\right)=R_{(\alpha)}^2\left[\begin{array}{c}f_m\left\{\begin{array}{c}\left(\alpha\rho_{xz_1}C_x-\rho_{yz_1}C_y+\frac{1}{2}\rho_{z_1z_2}C_{z_2}\right)C_{z_1}\\ \frac{1}{4}C_{z_1}^2+\left(\frac{1}{4}C_{z_2}-\rho_{yz_2}C_y+\alpha\rho_{xz_2}C_x\right)C_{z_2}\\ +f_m^{*}A+f_n\left(2\rho_{z_1z_2}^2+\frac{1}{4}\right)C_{z_1}^2\end{array}\right\}\end{array}\right]$$

and

$$(6.8) \qquad M\left(T_{(\alpha)4}\right) = R_{(\alpha)}^2 \left[\begin{array}{c} f_m \left\{ \begin{array}{c} \left(3\alpha\rho_{xz_1}C_x - 3\rho_{yz_1}C_y + \frac{3}{2}\rho_{z_1z_2}C_{z_2}\right)C_{z_1} \\ \frac{9}{4}C_{z_1}^2 + \left(\frac{1}{4}C_{z_2} - \rho_{yz_2}C_y + \alpha\rho_{xz_2}C_x\right)C_{z_2} \end{array} \right\} \\ +f_n\left(4\rho_{z_1z_2}C_{z_1} - \frac{3}{2}C_{z_2}\right)\rho_{z_1z_2}C_{z_1} \\ +f_n\left(\frac{3}{2}C_{z_1}^2 - \frac{1}{2}C_{z_2}^2\right) + f_m^* A \end{array} \right]$$

7. Efficiency comparisons

It is to be noted that random non-response situations may be misleading because the estimate based on them may be biased. Thus, to examine the effect of non-response on the performances of proposed estimators $T_{(\alpha)i}\,(i = 1, 2, 3, 4)$ their absolute percent relative bias (PRB) and percent relative losses in efficiencies (PRLE) with respect to the sample mean, ratio and product of two sample means estimator $\tau_{(\alpha)} = \bar{y}_m / \left(\bar{x}_m\right)^{\alpha}; (\alpha = 0, 1, -1)$ for the similar circumstances but under the complete response case (with no missing data) have been obtained. Since, bias of the estimator $\tau_{(0)} = \bar{y}_m$ is theoretically zero; where as its simulated values for large set of independent samples are usually different from zero. Therefore, following Senapati and Sahoo (2006), we calculated absolute percent relative bias (PRB) of proposed estimators $T_{(\alpha)i}\,(i = 1, 2, 3, 4)$ using the following expressions

$$(7.1) \qquad PRB\left(T_{(\alpha)i}\right) = \frac{\left|B\left(T_{(\alpha)i}\right)\right|}{\bar{R}_{(\alpha)}} \times 100; (i = 1, 2, 3, 4)$$

The percent relative losses in precision of the estimators $T_{(\alpha)i}\,(i = 1, 2, 3, 4)$ with respect to the estimator $\tau_{(\alpha)}$ is defined as

$$(7.2) \qquad PRLE\left(T_{(\alpha)i}\right) = \left[\frac{M\left(T_{(\alpha)i}\right) - M\left(\tau_{(\alpha)}\right)}{M\left(T_{(\alpha)i}\right)}\right] \times 100; (i = 1, 2, 3, 4)$$

where

$$(7.3) \qquad M\left(\tau_{(\alpha)}\right) = R_{(\alpha)}^2 f_m\left(C_y^2 + \alpha^2 C_x^2 - 2\alpha\rho_{yx}C_yC_x\right)$$

The empirical studies are carried out through a natural population data set and artificially generated population data. The merits of the proposed works are also shown for different choices of the non-response probability p in Tables 3, 4, 7 and 8.

Population I- Source: Ministry of Statistics & Programme Implementation, Chapter-2(2.1)

For empirical studies, we have chosen available free access data of 35 states of India on the website of the Ministry of Statistics & Programme Implementation of Government of India, Chapter-2 (2.1), 2013.

To validate the performance of proposed estimators, we considered the following variables as study and auxiliary characters:
Y: Male population of a state in India as per 2011 census.
X: Area of a state in India as per 2011 census.
Z1: Rural population of a state in India as per 2011 census.
Z2: Urban population of a state in India as per 2011 census.

Table 1. Absolute bias and absolute percent relative bias of $T_{(\alpha)i}$ $(i = 1, 2, 3, 4)$ estimators with respect to the estimator τ_α for Case I .

N=35			Case-I			
			$T_{(\alpha)1}$	$T_{(\alpha)2}$	$T_{(\alpha)3}$	$T_{(\alpha)4}$
n	m		$\alpha = -1$			
30	25	$\|B\|$	0.0165	0.0031	0.0041	0.0097
		PRB	9.8E-13	1.85E-13	2.44E-13	5.79E-13
26	19	$\|B\|$	0.0347	0.0011	0.0093	0.0208
		PRB	2.07E-12	6.57E-14	5.55E-12	1.24E-12
20	12	$\|B\|$	0.0785	0.0029	0.0273	0.0506
		PRB	4.69E-12	1.73E-13	1.63E-12	3.02E-12
			$\alpha = 0$			
30	25	$\|B\|$	0.0096	0.0126	0.0087	0.0135
		PRB	4.01E-12	5.26E-21	2.20E-12	9.21E-31
26	19	$\|B\|$	0.0202	0.0132	0.0175	0.0287
		PRB	8.44E-12	5.52E-21	2.30E-12	9.65E-31
20	12	$\|B\|$	0.0456	0.0642	0.0337	0.0677
		PRB	1.90E-11	2.68E-21	1.12E-12	4.69E-30
			$\alpha = 1$			
30	25	$\|B\|$	0.0270	0.0210	0.0215	0.0273
		PRB	0.0142	0.0110	0.0113	0.0143
26	19	$\|B\|$	0.0260	0.0224	0.0244	0.0266
		PRB	0.0137	0.0118	0.0128	0.0140
20	12	$\|B\|$	0.0127	0.0142	0.0147	0.0148
		PRB	0.0116	0.0127	0.0137	0.0138

On the basis of above description, the values of the different required parameters for the population are calculated as follows:

$N = 35, \bar{Y} = 17820692.8, \bar{X} = 93921.14, \rho_{yx} = 0.7111, \rho_{yz_1} = 0.9826, \rho_{xz_1} = 0.6696, \rho_{yz_2} = 0.8809, \rho_{xz_2} = 0.7078, \rho_{z_1z_2} = 0.7787, C_y = 1.2949, C_x = 1.1047, C_{z_1} = 1.3853, C_{z_2} = 1.2654.$

Table 2. Absolute bias and absolute percent relative bias of $T_{(\alpha)i}\,(i=1,2,3,4)$ estimators with respect to the estimator τ_α for Case II .

N=35			Case-II			
			$T_{(\alpha)1}$	$T_{(\alpha)2}$	$T_{(\alpha)3}$	$T_{(\alpha)4}$
n	m		$\alpha=-1$			
30	25	$\|B\|$	0.0131	0.0142	0.0306	0.0175
		PRB	7.82E-13	8.48E-13	1.82E-12	1.04E-12
26	19	$\|B\|$	0.0277	0.0294	0.0644	0.0368
		PRB	1.65E-12	1.75E-12	3.84E-12	2.19E-12
20	12	$\|B\|$	0.0633	0.0627	0.1466	0.0835
		PRB	3.78E-12	3.74E-12	8.74E-12	4.98E-12
			$\alpha=0$			
30	25	$\|B\|$	0.0071	0.0084	0.0187	0.0173
		PRB	2.96E-12	2.96E-12	7.82E-12	7.23E-12
26	19	$\|B\|$	0.0149	0.0171	0.0393	0.0364
		PRB	6.23E-12	6.23E-12	1.64E-11	1.52E-11
20	12	$\|B\|$	0.0342	0.0346	0.0895	0.0825
		PRB	1.43E-11	1.43E-12	3.74E-11	3.45E-11
			$\alpha=1$			
30	25	$\|B\|$	0.0010	0.0025	0.0068	0.0171
		PRB	0.0005	0.0001	0.0003	0.0012
26	19	$\|B\|$	0.0022	0.0047	0.0143	0.0360
		PRB	0.0004	0.0005	0.0008	0.0190
20	12	$\|B\|$	0.0052	0.0066	0.0324	0.0815
		PRB	0.0002	0.0003	0.0170	0.0429

Population II – Source: Artificially Generated Data Set

An important aspect of simulation is that one builds a simulation model to replicate the actual system. Simulation allows comparison of analytical techniques and helps in concluding whether a newly developed technique is better than the existing ones. Motivated by Singh and Deo [20] and Singh *et al.* [21] those have adopted the artificial population generation techniques. In the present study, to find the percentage relative losses in bias and precision of estimators $T_i\,(i=1,2,3,4)$ with respect to bias and efficiency of estimator respectively as shown in Tables 5 - 8, we have simulated a data set consisting of four normal random variables of size N (N = 1000) namely y_k, x_k, z_{1k} and z_{2k} with the help of Matlab software. We have generated the random variables of the population U with the values of $\bar{Y}=1.4859, \bar{X}=2.0176, \rho_{yx}=0.8272, \rho_{yz_1}=0.8245, \rho_{yz_2}=0.8109, \rho_{xz_1}=0.6165, \rho_{xz_2}=0.5325, \rho_{z_1z_2}=0.4132, C_y=4.1252, C_x=3.0583, C_{z_1}=2.8605$ and $C_{z_2}=5.4265$.

In order to have above correlation coefficient and coefficient of variation in the generated data following steps have been followed.

1. For the desired correlation matrix 'R', find an upper triangular matrix 'U' by Cholesky decomposition.

2. Generated four normal random variables of size 1000 such that $Y\longrightarrow N(1.5,6), X\longrightarrow N(1.8022,6.04), Z_1\longrightarrow N(1.98,5.67), Z_2\longrightarrow N(5.60,3.6)$

Table 3. Percent relative losses in efficiencies of estimators $T_{(\alpha)i}\,(i = 1, 2, 3, 4)$ with respect to the estimator τ_α for Case I with different values of non-response probability p .

N=35			Case-I			
			$T_{(\alpha)1}$	$T_{(\alpha)2}$	$T_{(\alpha)3}$	$T_{(\alpha)4}$
n	m	p	$\alpha = -1$			
30	25	0.02	-176.6934	-165.0334	-488.2893	-488.2893
		0.04	-99.8835	-93.7266	-223.7661	-223.7661
		0.10	-53.7446	-50.0760	-117.8636	-117.8636
26	19	0.02	-199.9935	-186.2512	-591.3294	-591.3294
		0.04	-139.4757	-130.6370	-336.8967	-336.8967
		0.10	-96.6803	-90.6788	-212.7468	-212.7468
20	2	0.02	-221.4395	-205.3752	-677.4488	-677.4488
		0.04	-173.6728	-161.9410	-446.6716	-446.6716
		0.10	-136.0449	-127.2656	-314.6391	-314.6391
			$\alpha = 0$			
30	25	0.02	-97.1816	-147.1736	-25.6024	-47.8585
		0.04	-54.7923	-84.0080	-6.9469	-22.6689
		0.10	-25.6021	-44.1769	7.8495	-3.5904
26	19	0.02	-108.9406	-164.5069	-31.4325	-52.2270
		0.04	-77.6694	-116.3096	-18.3313	-34.9251
		0.10	-52.9745	-80.7793	-6.8438	-20.1905
20	12	0.02	-119.9230	-176.7707	-40.2349	-51.1262
		0.04	-96.4622	-140.6106	-30.3121	-39.6652
		0.10	-76.2884	-111.0338	-21.1186	-29.1579
			$\alpha = 1$			
30	25	0.02	65.7557	58.8576	64.1455	78.5963
		0.04	67.3104	61.0814	65.8462	79.2142
		0.10	68.8397	63.2299	67.5121	79.8432
26	19	0.02	65.4799	58.5844	63.5819	78.5804
		0.04	66.4553	59.9806	64.6659	78.9600
		0.10	67.4475	61.3847	65.7650	79.3547
20	12	0.02	65.4134	58.9271	62.5537	78.8986
		0.04	66.0510	59.8232	63.2999	79.1376
		0.10	66.7093	60.7419	64.0680	79.3881

3. Consider A= [Y, X, Z1, Z2].
4. Consider a transformation $A_C = A * U$

This transformation will eventually determine the mean and standard deviation of the normally distributed random variables.

Table 4. Percent relative losses in efficiencies of estimators $T_{(\alpha)i}\,(i=1,2,3,4)$ with respect to the estimator τ_α for Case II with different values of non-response probability p .

N=35			Case-II			
			$T_{(\alpha)1}$	$T_{(\alpha)2}$	$T_{(\alpha)3}$	$T_{(\alpha)4}$
n	m	p	$\alpha=-1$			
30	25	0.02	-295.5552	-270.5377	-55.1720	-330.0892
		0.04	-155.3041	-144.6432	-27.6607	-149.5159
		0.10	-84.5606	-78.9240	-7.1278	-71.6091
26	19	0.02	-342.0850	-310.9190	-63.0342	-399.9794
		0.04	-222.1244	-205.2548	-43.3473	-233.1541
		0.10	-149.1904	-138.9740	-26.8284	-144.9591
20	2	0.02	-378.2187	-341.3597	-72.2545	-451.8645
		0.04	-279.6382	-256.0342	-57.5211	-309.2508
		0.10	-210.8901	-194.8806	-44.2826	-220.3661
			$\alpha=0$			
30	25	0.02	-105.1651	-93.5860	55.5632	-133.6303
		0.04	-59.6697	-52.5677	58.1461	-67.7029
		0.10	-28.7945	-24.1334	60.6207	-28.4957
26	19	0.02	-117.8003	-104.6887	54.7299	-153.5604
		0.04	-84.0351	-74.5856	56.3929	-102.2095
		0.10	-57.6703	-50.6829	58.0548	-65.9457
20	12	0.02	-129.2676	-114.3690	53.2646	-169.1364
		0.04	-103.8857	-92.0179	54.4212	-130.0417
		0.10	-82.2426	-72.7017	55.6000	-99.0059
			$\alpha=1$			
30	25	0.02	71.1341	71.2648	87.6217	72.8514
		0.04	72.2467	72.3675	87.8309	74.0374
		0.10	73.3569	73.4682	88.0492	75.2085
26	19	0.02	70.8763	71.0136	87.5349	72.5550
		0.04	71.5737	71.7045	87.6645	73.2892
		0.10	72.2894	72.4137	87.8012	74.0386
20	12	0.02	65.4134	70.7626	62.5537	78.8986
		0.04	66.0510	71.2196	63.2999	79.1376
		0.10	66.7093	71.6941	64.0680	79.3881

8. Interpretations of empirical results

The following interpretations may be read out from Tables 1- 8:

(1) From Tables 1 - 2, it is cleared that the values of absolute bias and absolute percent relative bias of estimators $T_{(\alpha)i}\,(i=1,2,3,4)$ are almost increasing with respect to the estimator $\tau_{(\alpha)}$ with the decreasing values of sample sizes n and m. These trends show that the proposed estimators produce effective estimation technique with reduced bias. This phenomenon establishes the fact that our imputation method is efficient to cope with the negative impact of random non-response situations.

(2) From Tables 3 - 4, it is stressed that

(a) For fixed values of non-response probability p, percent relative losses of estimators $T_{(\alpha)i}\,(i=1,2,3,4)$ are decreasing with the decreasing values of sample sizes n and m

Table 5. Absolute bias and absolute percent relative bias of $T_{(\alpha)i}$ $(i = 1, 2, 3, 4)$ estimators with respect to the estimator τ_α for Case I .

N=1000			Case-I			
			$T_{(\alpha)1}$	$T_{(\alpha)2}$	$T_{(\alpha)3}$	$T_{(\alpha)4}$
n	m		$\alpha = -1$			
700	500	$\|B\|$	0.0049	0.0030	0.0113	0.0060
		PRB	0.1634	0.1000	0.3768	0.2001
575	365	$\|B\|$	0.0085	0.0054	0.0197	0.0104
		PRB	0.2835	0.0018	0.6570	0.3468
500	300	$\|B\|$	0.0113	0.0069	0.0264	0.0139
		PRB	0.3769	0.2301	0.8805	0.4636
			$\alpha = 0$			
700	500	$\|B\|$	0.0039	0.0057	0.0077	0.0078
		PRB	0.2624	0.3836	0.5182	0.5182
575	365	$\|B\|$	0.0068	0.0101	0.0133	0.0134
		PRB	0.4576	0.6797	0.8950	0.9018
500	300	$\|B\|$	0.0091	0.0132	0.0179	0.0180
		PRB	0.6124	0.8883	1.2046	1.2113
			$\alpha = 1$			
700	500	$\|B\|$	0.0029	0.0084	0.0040	0.0094
		PRB	0.3938	1.1406	0.5431	1.2764
575	365	$\|B\|$	0.0051	0.0148	0.0069	0.0163
		PRB	0.6925	2.0097	0.9363	2.2134
500	300	$\|B\|$	0.0068	0.0195	0.0093	0.0220
		PRB	0.0092	2.6480	1.2629	2.9875

which leads in reduction of reducing the cost of the survey. Since, values of percent relative losses for population mean and product of two population means are negative. Therefore, this phenomenon is highly desirable in terms of losses are decreasing with reduced cost of survey even in random non-response with respect to sample estimator $\tau_{(\alpha)}$ in the absence of non-response. This encouraging behaviour may be seen from both the cases of two-phase sampling.

(b) For fixed values of sample sizes n and m, percent relative losses in precision of estimators $T_{(\alpha)i}$ $(i = 1, 2, 3, 4)$ are found to be negative when applied for estimation of population mean and product of two population means the increasing values of probability of non-response p which lead us to have effective estimation strategy than the sample estimator $\tau_{(\alpha)}$ in the absence of non-response.

(3) From Tables 5 - 8, it is observed similar trend of performance of the proposed estimators as it is for the Cases I and II as shown the Tables 1 - 4. Therefore, percent relative losses in precision of estimators $T_{(\alpha)i}$ $(i = 1, 2, 3, 4)$ are highly acceptable in both the cases of two phase sampling setup in terms of dominating result.

Table 6. Absolute bias and absolute percent relative bias of $T_{(\alpha)i}$ $(i = 1, 2, 3, 4)$ estimators with respect to the estimator τ_α for Case II .

N=1000			Case-II			
			$T_{(\alpha)1}$	$T_{(\alpha)2}$	$T_{(\alpha)3}$	$T_{(\alpha)4}$
n	m		$\alpha = -1$			
700	500	$\lvert B\rvert$	0.0048	0.0013	0.0113	0.0060
		PRB	0.1601	0.0433	0.3769	0.2001
575	365	$\lvert B\rvert$	0.0083	0.0024	0.0197	0.0104
		PRB	0.2768	0.0800	0.6570	0.3468
500	300	$\lvert B\rvert$	0.0111	0.0031	0.0264	0.0139
		PRB	0.3702	0.1033	0.8805	0.4636
			$\alpha = 0$			
700	500	$\lvert B\rvert$	0.0001	0.0054	0.0077	0.0078
		PRB	0.0006	0.3634	0.5182	0.5182
575	365	$\lvert B\rvert$	0.0003	0.0095	0.0133	0.0134
		PRB	0.0002	0.6393	0.8950	0.9018
500	300	$\lvert B\rvert$	0.0004	0.0126	0.0179	0.0180
		PRB	0.0002	0.8479	1.2046	1.2113
			$\alpha = 1$			
700	500	$\lvert B\rvert$	0.0543	0.0454	0.0401	0.0492
		PRB	7.3737	6.1651	5.4454	6.6811
575	365	$\lvert B\rvert$	0.0763	0.0662	0.0651	0.0773
		PRB	10.3612	8.9896	8.8403	10.4970
500	300	$\lvert B\rvert$	0.1025	0.1014	0.0932	0.1260
		PRB	13.9190	13.7696	12.6561	17.1102

9. Conclusions

From above analyses, it is clear that the proposed estimators $T_{(\alpha)i}$ $(i = 1, 2, 3, 4)$ contribute significantly to handle the different two realistic situations of random non-response while estimating population mean, ratio and product of two population means in two-phase sampling scheme. It is visible that the proposed estimators are more efficient than the sample mean, ratio and product estimator under the similar realistic situations but in the absence of random non-response. It may be noted that the use of imputation methods in the structures of the proposed estimators are highly rewarding in terms increased precession of estimates as well as in the reduction the cost of survey and negative impact of non-responses. Hence, the propositions of the suggested estimators in the present study are justified as they unify several highly desirable results. Therefore, proposed estimators may be recommended for their practical applications to the survey statisticians.

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Table 7. Percent relative losses in efficiencies of estimators $T_{(\alpha)i}\,(i=1,2,3,4)$ with respect to the estimator τ_α for Case I with different values of non-response probability p .

N=1000			Case-I			
			$T_{(\alpha)1}$	$T_{(\alpha)2}$	$T_{(\alpha)3}$	$T_{(\alpha)4}$
n	m	p	$\alpha=-1$			
700	500	0.02	-90.1817	-178.5738	-94.3417	-220.8460
		0.04	-63.3241	-124.4981	-66.3826	-151.1662
		0.10	-41.5632	-85.3374	-43.8553	-103.1441
575	365	0.02	-94.1328	-185.3345	-98.0141	-231.5390
		0.04	-71.5013	-138.9827	-74.5234	-170.5642
		0.10	52.1918	-103.0786	-54.5669	-125.4395
500	300	0.02	-94.5073	-187.9539	-98.8608	-233.3484
		0.04	-73.7009	-144.5822	-77.1646	-176.5722
		0.10	-55.5891	-110.1384	-58.3623	-133.3257
			$\alpha=0$			
700	500	0.02	-84.7125	-102.6155	-203.9218	-67.0151
		0.04	-59.2741	-72.4101	-140.6746	-45.9396
		0.10	-38.5105	-48.3391	-96.2257	-28.3148
575	365	0.02	-89.0202	-105.9911	-213.7430	-69.5660
		0.04	-67.4990	-80.6905	-158.5939	-52.0415
		0.10	-49.0317	-59.3848	-117.0673	-36.6690
500	300	0.02	-88.7903	-107.5325	-215.1210	-70.3762
		0.04	-69.1272	-84.0147	-163.9071	-54.1975
		0.10	-51.9093	-63.8132	-124.2467	-39.7555
			$\alpha=1$			
700	500	0.02	57.8723	71.5193	13.4961	78.1120
		0.04	59.3530	72.2038	19.5161	78.5185
		0.10	60.8507	72.9125	25.1835	78.9442
575	365	0.02	57.6226	71.4661	12.6402	78.1170
		0.04	58.8091	72.0090	17.5370	78.4377
		0.10	60.0272	72.5768	22.2785	78.7763
500	300	0.02	57.6638	71.4241	12.6122	78.0521
		0.04	58.7395	71.9183	17.0748	78.3448
		0.10	59.8497	72.4370	21.4407	78.6546

Table 8. Percent relative losses in efficiencies of estimators $T_{(\alpha)i}\,(i=1,2,3,4)$ with respect to the estimator τ_α for Case II with different values of non-response probability p .

N=1000			Case-II			
			$T_{(\alpha)1}$	$T_{(\alpha)2}$	$T_{(\alpha)3}$	$T_{(\alpha)4}$
n	m	p	$\alpha=-1$			
700	500	0.02	-181.2991	-138.0069	-117.3146	-401.4777
		0.04	-126.2647	-97.3855	-82.9394	-249.8003
		0.10	-86.5397	-66.4610	-56.0677	-163.1607
575	365	0.02	-188.7927	-143.2380	-121.6847	-422.9302
		0.04	-141.4039	-108.7271	-92.6539	-285.7957
		0.10	-104.8243	-80.8075	-68.6212	-200.1341
500	300	0.02	-190.8668	-144.8205	-122.9809	-432.7164
		0.04	-146.6804	-112.7455	-96.0586	-301.1233
		0.10	-111.6854	-86.1984	-73.2898	-216.1394
			$\alpha=0$			
700	500	0.02	-242.7855	-48.4511	-276.8280	-470.2910
		0.04	-164.4143	-31.5635	-184.2203	-281.9479
		0.10	-111.7241	-17.0675	-124.2362	-180.9507
575	365	0.02	-255.1704	-50.6086	-291.1975	-494.9745
		0.04	-186.0990	-36.6218	-209.0237	-323.6411
		0.10	-136.1223	-24.0807	-151.5219	-222.5508
500	300	0.02	-257.0994	-51.0736	-294.1982	-511.0394
		0.04	-192.7256	-38.2148	-217.1962	-343.9742
		0.10	-144.7183	-26.4977	-161.5894	-242.1674
			$\alpha=1$			
700	500	0.02	56.3661	75.6104	18.6557	65.6652
		0.04	57.9525	76.1141	24.0011	66.6551
		0.10	59.5532	76.6393	29.0744	67.6698
575	365	0.02	56.1118	75.5408	17.8564	65.6143
		0.04	57.3832	75.9408	22.2005	66.3997
		0.10	58.6857	76.3616	26.4346	67.2146
500	300	0.02	56.1423	75.5406	17.8745	65.5268
		0.04	57.2957	75.9036	21.8281	66.2434
		0.10	58.4838	76.2865	25.7195	66.9902

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