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The robustness of proximal penalty algorithms in restoration of noisy image

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Abstract

The nondifferentiable convex optimization has an importance crucial in the image restoration for this and in this article we present the performance of the Prox method adapted to the restoration of noisy images. Following of our article ([12]), we illustrate in this work the superior efficacy of this algorithm "*Prox*" ([12]) then we are comparing the obtained numerical results with the algorithms of Wiener filtering ([7], [16]), total variation ([5]) and wavelet soft-thresholding denoising ([1], [12], [13]), in terms of image quality and convergence.
Our first experiments showed that by applying of *Prox* algorithm for restoration of noised image by the white Gaussian noise we obtain a top results of denosed image with high quality (net, not rehearsed and unsmoothed; textures are preserved) in addition to the convergence of the algorithm is ensured whatever the values of SNR.

Keywords: proximal penalty algorithms, image restoration, SNR, convergence.

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1. Problematic and Results

Let us given an original image u, we suppose that it was degraded by an additive noise.

From the observed image Im (which is thus a degraded version of the original image u), we try to reconstruct u. If we suppose that the additive noise is Gaussian, the Maximum likelihood method leads us to look u as solution of the following problem of optimization:

$$(\mathcal{P}) \quad \alpha := \underset{u \in U^{ad}}{Inf} \left\{ \frac{1}{2} \|Im - u\|_2^2 \right\}$$

where

$$U^{ad} = \{u \in BV(\Omega) : \ J(u) \leq 0\}$$

and $J(u)$ here denotes the total variation of u

$$J(u) = Sup \left\{ \int_\Omega u(x) div(\varphi(x)) dx : \varphi \in C_c^1(\Omega, \ \mathbb{R}^2), \ \|\varphi\|_\infty \leq 1 \right\}.$$

Our problem $(\mathcal{P})$ is equivalent to the following unconstrained problem:

$$(\mathcal{P}'_r) \quad \alpha_r := \inf_{u \in BV} \left\{ \frac{1}{2} \|Im - u\|_2^2 \ + r. \max(0, \Pi_K(u)) \right\}.$$

where $r > 0$ and where $\Pi_K(u)$ is the projection of u in K:

$$K = \left\{ div(\varphi(x)) : \varphi \in C_c^1(\Omega, \ \mathbb{R}^2), \ \|\varphi\|_\infty \leq 1 \right\}.$$

so $(r \to \infty)$ that when the solution $u(r)$ obtained is a solution of $(\mathcal{P})$.

By the Proximal method ([1], [2], [10]), we associate to the problem $(\mathcal{P}'_r)$ the following problem :

$$(\mathcal{P}_r) \quad \alpha_r(w) := \min_{(u,w) \in BV^2} \left\{ \frac{1}{2} \|Im - u\|_2^2 \ + r. \max(0, \Pi_K(u)) + \frac{1}{2} \|u - w\|_2^2 \right\}.$$

The algorithm applied to this problem engenders a sequence $\left\{u^k, w^k\right\}_k$ such that u^{k+1} be a solution of the problem

$$(\mathcal{P}_r) \quad \alpha_r = \underset{u \in BV}{Inf} \left\{ \frac{1}{2} \|Im - u\|_2^2 \ + r. \max(0, \Pi_K(u)) + \frac{1}{2} \left\|u - w^k\right\|_2^2 \right\}$$

and w^{k+1} be a solution of the problem

$$(\mathcal{P}_r) \quad \alpha_r = \underset{w \in BV}{Inf} \left\{ \frac{1}{2} \|Im - u\|_2^2 \ + r * \max(0, \Pi_K(u^{k+1})) + \frac{1}{2} \left\|u^{k+1} - w\right\|_2^2 \right\}$$

u^{k+1} be a solution of the problem

$$(\mathcal{P}_r) \qquad \alpha_r = \underset{u \in BV}{Inf} \left\{ \frac{1}{2} \|Im - u\|_2^2 \ + rh(u) + \frac{1}{2} \left\|u - u^k\right\|_2^2 \right\}.$$

We propose the following proximal-penalty algorithm (see [2], [8], [9], [10], [15], [18]):

Proximal Penalty Algorithm
Step 0: $(k = 0)$
Let $u^0 \in \mathbb{R}^n$, $\varepsilon > 0$ be a precision.

Step 1:
We choose a penalty coefficient r^0, a precision $\delta > 0$.
Apply the minimization algorithm to find u^1 solution of the problem

$$\underset{u \in \mathbb{R}^n}{Inf} \left\{ \frac{1}{2} \left\| Im - u \right\|_2^2 \; + r_0 \max(0, \Pi_K(u)) + \frac{1}{2} \left\| u - u^0 \right\|^2 \right\}$$

Step 2:
Let $u^1(r_0) = u^1$ be the obtained solution.
If $\left\| u^1 - u^0 \right\| < \varepsilon$ and $r_0 h(u^1(r_0)) < \delta$, then u^1 is the best and good approximation of the optimum and the calculations end in the iteration $k+1$.

Else, we choose a penalty coefficient $r_1 > r_0$, put $r_0 = r_1$ and $u^0 = u^1$; $k = k+1$ and return to the step 1.

2. Numerical results:

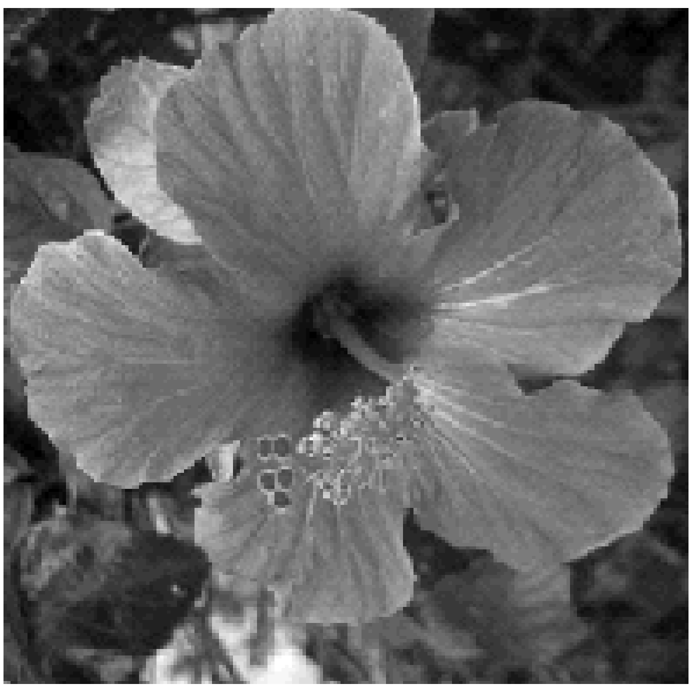

Figure 1. Original image

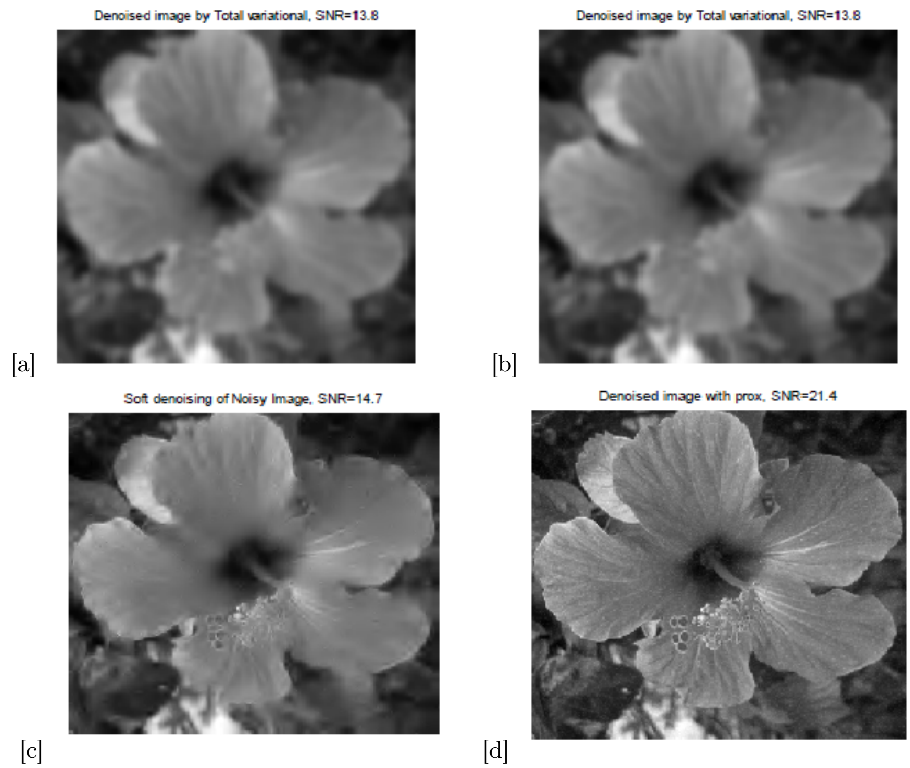

Figure 2. The SNR of denosed images for segma 0.08

[a]:Denoised of noisy Image by generalized Wiener filtering.
[b]:Denoised of noisy Image by Total Variational.
[c]:Soft Denoising of noisy Image.
[d]:Denoised of noisy Image by Prox.

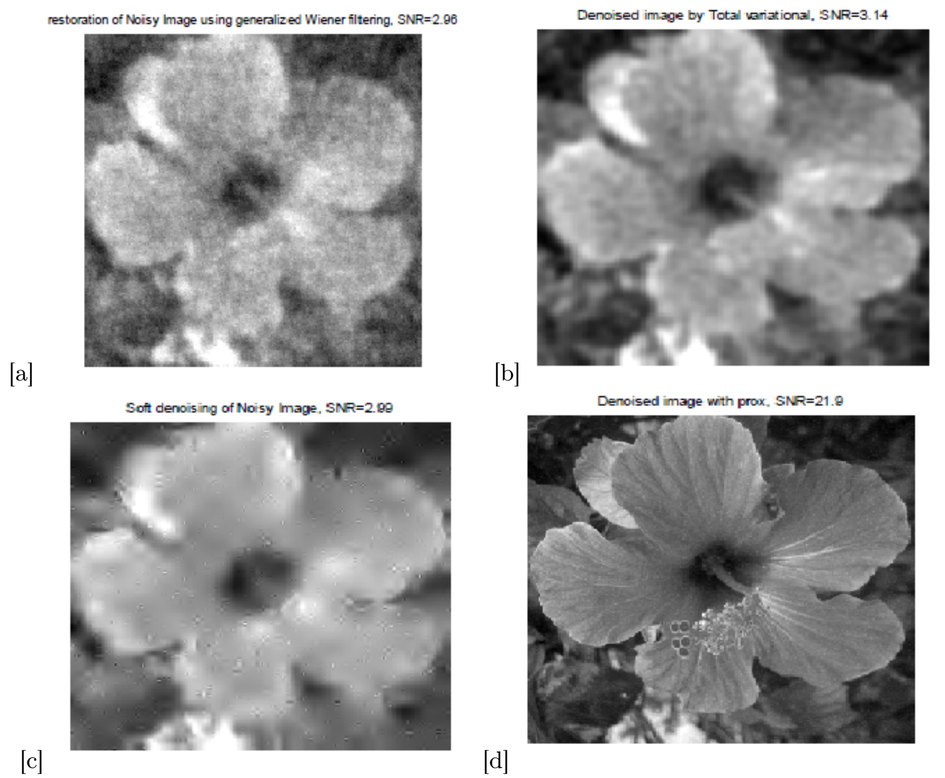

Figure 3. The SNR of denosed images for segma 0.35

[a]:Denoised of noisy Image by generalized Wiener filtering.
[b]:Denoised of noisy Image by Total Variational.
[c]:Soft Denoising of noisy Image.
[d]:Denoised of noisy Image by Prox.

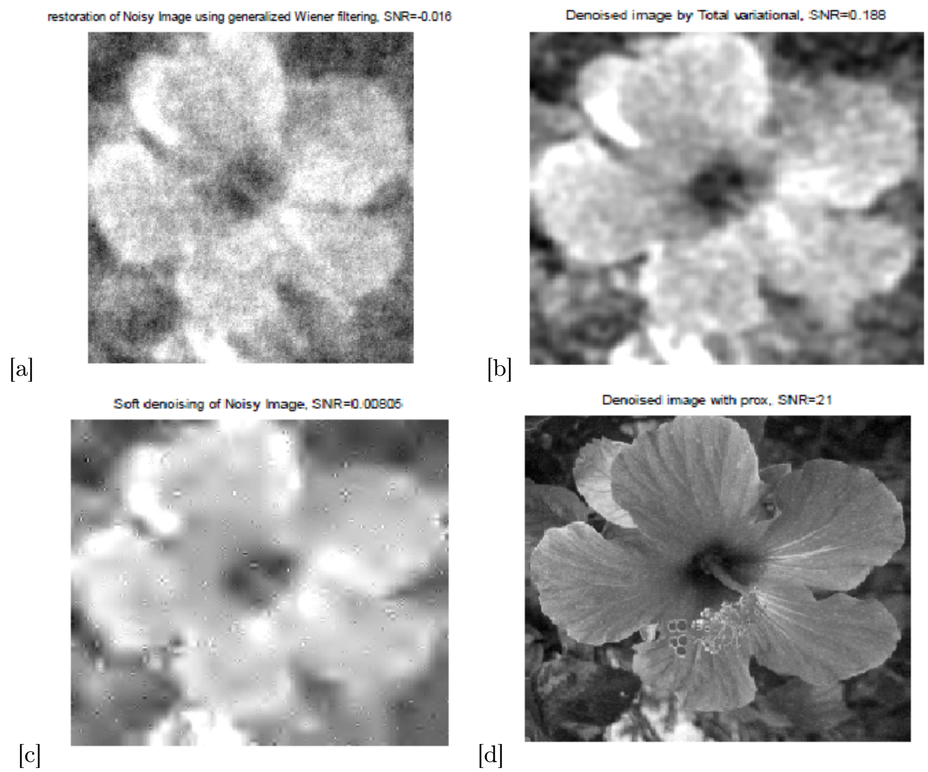

Figure 4. The SNR of denosed images for segma 0.501

[a]:Denoised of noisy Image by generalized Wiener filtering.
[*b*]:Denoised of noisy Image by Total Variational.
[*c*]:Soft Denoising of noisy Image.
[*d*]:Denoised of noisy Image by Prox.

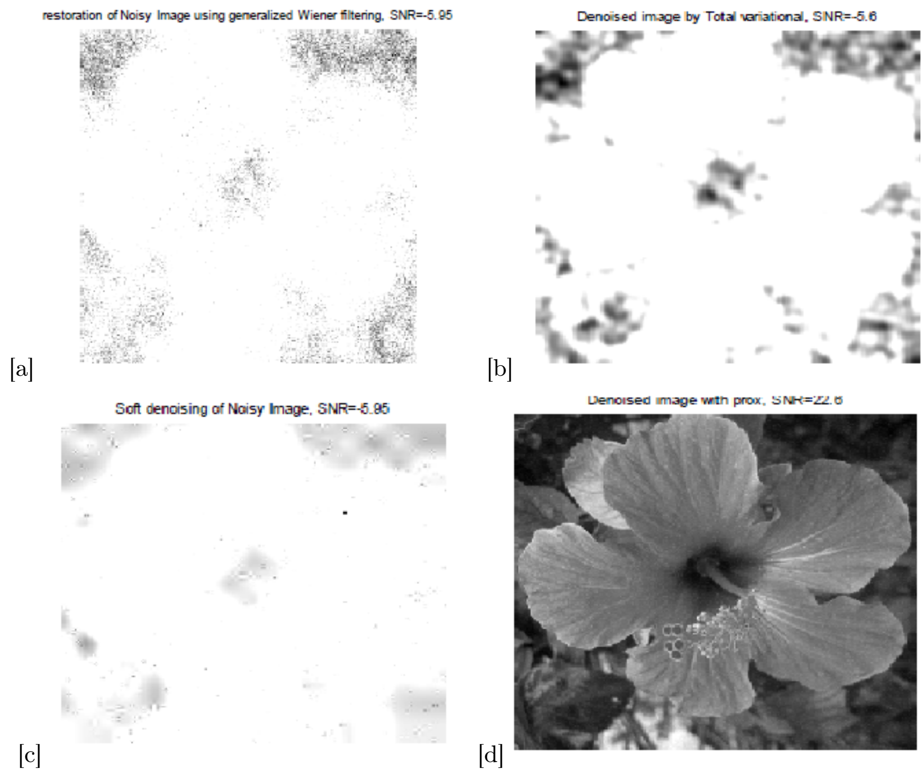

Figure 5. The SNR of denosed images for segma 1

[a]:Denoised of noisy Image by generalized Wiener filtering.
[b]:Denoised of noisy Image by Total Variational.
[c]:Soft Denoising of noisy Image.
[d]:Denoised of noisy Image by Prox.

Segma	*SNR Prox*	*SNR TV*	*SNR Soft*	*SNR Wienerfiltre*
0.08	21.4	12.5	14.7	13
0.15	25.1	9.28	9.89	9.47
0.25	20.4	5.68	5.76	5.68
0.35	21.9	3.01	2.98	2.98
0.5	24.1	0.17	0.0173	0.0075
0.501	21	0.126	0.008	/
0.5125	25.1	0.0956	/	/
1	22.6	/	/	/
1.5	23.3	/	/	/
2	20.4	/	/	/
2.5	21.9	/	/	/
3	21.9	/	/	/

Table 1. The different values of SNR of denosed image by: Prox, Wiener filtering, total variation and wavelet soft-thresholding denosing methods

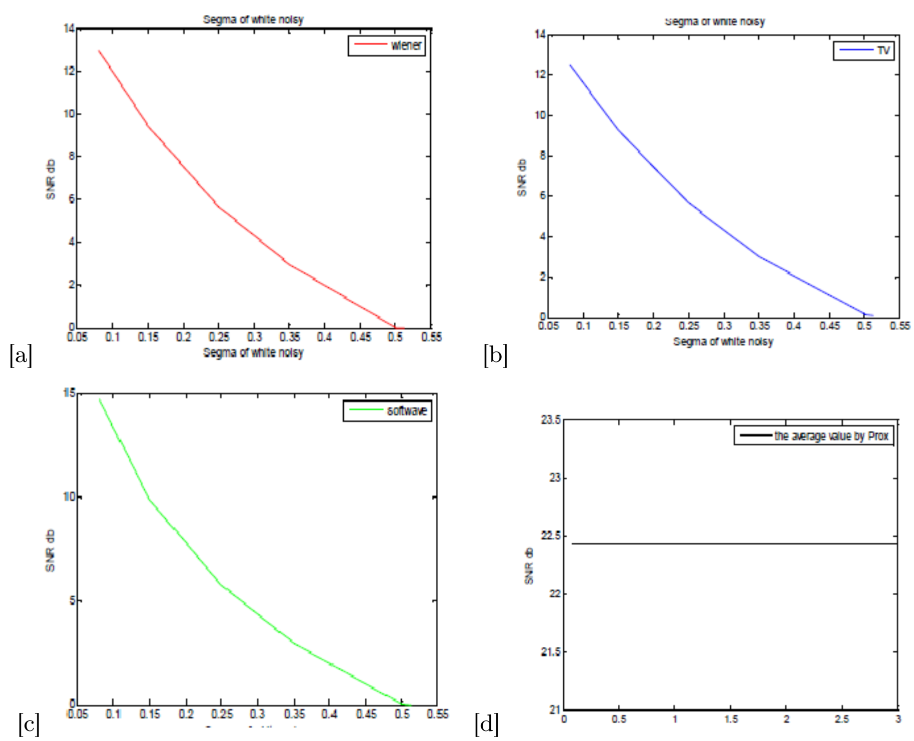

Figure 6. The curves of SNR of denosed images with segma of white noise

[a]:The curve of SNR by generalized Wiener filtering
[b]:The curve of SNR by Total Variational
[c]:The curve of SNR by Soft
[d]:The curve of SNR by Prox

Comment:

We see from the results that the restoration with the total variation of denosed image shows that the regularization term has more influence on energy and therefore on the position of its minimizer. The reverse occurs when the regularization term is higher so the restored image is smoother ([19]).

On the other hand the quality of images denosed with $Prox$ stay fixed with the increasing of the Lagrangian value of $Prox$ wich keeps the performance of texture after denoising.

Also the figures $(2; 3; 4; 5; 6)$ illustrate that the different algorithms applied to restoration image such the total variation, wavelet soft-thresholding, Wiener filtering are not robust they find difficulties during the restoration. In other words, they diverge with increasing of invariace of white noise (sigma), but this is not the case if we apply the $Prox$ algorithm. It seems that this last is very effective and strong. It gives whatever the vriance Sigma high quality of denoised image from Table.1, the SNR of restored images remains almost constant, that its average value is:

$$SNR(prox)db = 22.4250.$$

On the other hand the curve of SNR with $SEGMA$ for other methods seems that it varied under the form of exponential:

$$SNR = \beta EXP(-\alpha.segma).$$

Conclusion

About the above results we can conclud that the $Prox$ algorithm suitable for image restoration is more effective in terms of the convergence to the solution (denoised image) if we compared with an other methods. Therefore the results obtained by this method confirms the validity and performance of our algorithm of $Prox$ for restoration image.

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