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NEW STRUCTURE TO CONSTRUCT NEW SOLITARY WAVE SOLUTIONS FOR PERTURBED NLSE WITH POWER LAW NONLINEARITY

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Abstract. In this paper we applied new structure to constructing new solitary wave solutions for perturbed nonlinear Schrodinger equation with power law nonlinearity, which describes the effects of quantic nonlinearity on the ultrashort optical solitons pulse propagation in non-Kerr media. These solitary wave solutions demonstrate the fact that solutions to the perturbed nonlinear Schrodinger equation with power law nonlinearity model can exhibit a variety of behaviors.

1. Introduction

Exact solutions can serve as a basis for perfecting and testing computer algebra software packages for solving NLEEs. It is significant that many equations of physics, chemistry, and biology contain empirical parameters or empirical functions. Exact solutions allow researchers to design and run experiments, by creating appropriate natural conditions, to determine these parameters or functions. Therefore, investigation of exact traveling wave solutions is becoming successively attractive in nonlinear sciences day by day. However, not all equations posed of these models are solvable. Exact solutions can serve as a basis for perfecting and testing computer algebra software packages for solving NLEEs. It is significant that many equations of physics, chemistry, and biology contain empirical parameters or empirical functions. Exact solutions allow researchers to design and run experiments, by creating appropriate natural conditions, to determine these parameters or functions. Therefore, investigation of exact traveling wave solutions is becoming successively attractive in nonlinear sciences day by day. Hence it becomes increasingly important to be familiar with all traditional and recently developed methods for solving these models and the implementation of new methods. As a result, many new techniques have been successfully developed by diverse groups of mathematicians and physicists, such as, the trigonometric function series method [5], the modified mapping

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method and the extended mapping method [6], homogeneous balance method [7], tanh function method [8], extended tanh function method [9], hyperbolic function method [10], rational expansion method [11], sine-cosine method [12].
In this present paper we applied the direct algebraic method for finding new exact solitary wave solutions of perturbed NLSE with power law nonlinearity in the following form [13],

$$iq_t + aq_{xx} + b|q|^{2m}q = icq_x - i\gamma q_{xxx} + is(|q|^{2m}q)x + ir(|q|_{2m})_x q, \tag{1.1}$$

Where a, b, c, γ, s and r are all real valued constants. Also, the exponent m represents the power law nonlinearity parameter. For the perturbation terms on the right hand side represents the inter-modal dispersion, γ is the coefficient of third order dispersion, s is the coefficient of self-steepening term while r is the coefficient of nonlinear dispersion. The self-steepening and nonlinear dispersion terms are considered with full nonlinearity, namely their intensities are considered with an exponent m, in order to maintain the problem on a generalized setting [14].

2. Our methodology

For a given partial differential equation

$$G(u, u_x, u_t, u_{xx}, u_{tt}, ...) = 0, \tag{2.1}$$

Our method mainly consists of four steps:
Step 1: We seek complex solutions of Eq. (2.1) as the following form:

$$u = u(\xi), \qquad \xi = ik(x - ct), \tag{2.2}$$

Where k and c are real constants. Under the transformation (2.2), Eq. (2.1) becomes an ordinary differential equation

$$N(u, iku', -ikcu', -k^2u'', ...) = 0, \tag{2.3}$$

Where $u' = \frac{du}{d\xi}$.
Step 2: We assume that the solution of Eq. (2.3) is of the form

$$u(\xi) = \sum_{i=0}^{n} a_i F^i(\xi), \tag{2.4}$$

Where $a_i(i = 1, 2, .., n)$ are real constants to be determined later.$F(\xi)$expresses the solutions of the auxiliary ordinary differential equation

$$F'(\xi) = b + F^2(\xi), \tag{2.5}$$

Eq. (2.5) admits the following solutions:

$$\begin{aligned} F(\xi) &= \begin{cases} -\sqrt{-b}\tanh(\sqrt{-b}\xi), & b \prec 0 \quad (a) \\ -\sqrt{-b}\coth(\sqrt{-b}\xi), & b \prec 0 \quad (b) \end{cases} \\ F(\xi) &= \begin{cases} \sqrt{b}\tan(\sqrt{b}\xi), & b \succ 0 \quad (c) \\ -\sqrt{b}\cot(\sqrt{b}\xi), & b \succ 0 \quad (d) \end{cases} \\ F(\xi) &= -\frac{1}{\xi}, \qquad b = 0 \quad (e) \end{aligned} \tag{2.6}$$

Integer n in (2.4) can be determined by considering direct algebraic [3] between the nonlinear terms and the highest derivatives of $u(\xi)$in Eq. (2.3).
Step 3: Substituting (2.4) into (2.3) with (2.5), then the left hand side of Eq. (2.3) is converted into a polynomial in$F(\xi)$, equating each coefficient of the polynomial to zero yields a set of algebraic equations for a_i, k, c.

Step 4: Solving the algebraic equations obtained in step 3, and substituting the results into (2.4), then we obtain the exact traveling wave solutions for Eq. (2.1).

3. Application to the perturbed NLSE with power law nonlinearity

We assume Eq. (2.5) has the traveling wave solution of the form

$$q(x,\ t)=U(\xi)e^{i(\alpha x+\beta t)},\ \xi=i(kx-\omega t), \tag{3.1}$$

where α,β,k and $\omega$are constants, all of them are to be determined. Thus, from the wave transformation (3.1), we have

$$\begin{aligned}
&q_t=i\ \left(\beta U-\omega U'\right)e^{i(\alpha x+\beta t)},\\
&q_x=i\ \left(\alpha U+kU'\right)e^{i(\alpha x+\beta t)},\\
&q_{xx}=-\ \left(\alpha^2U+2\alpha kU'+k^2U''\right)e^{i(\alpha x+\beta t)},\\
&q_{xxx}=-i\ \left(\alpha^3U+3\alpha^2kU'+3\alpha k^2U''+k^3U'''\right)e^{i(\alpha x+\beta t)},\\
&\left(|q|^{2m}q\right)_x=i\ \left(\alpha U^{2m+1}+k\left(U^{2m+1}\right)'\right)e^{i(\alpha x+\beta t)},\\
&\left(|q|^{2m}\right)_x q=ik\ \left(U^{2m}\right)'Ue^{i(\alpha x+\beta t)},
\end{aligned} \tag{3.2}$$

Inserting the expressions (3.2) into Eq. (1.1), we obtain nonlinear ODE in the form

$$\begin{aligned}
&(c\alpha+\gamma\alpha^3-\beta-a\alpha^2)U+(\omega-2a\alpha k+ck+3\alpha^2k\gamma)U'+(3\alpha k^2\gamma-ak^2)U''\\
&+(b+s\alpha)U^{2m+1}+k^3\gamma U'''+sk(U^{2m+1})'+rk(U^{2m})'U=0.
\end{aligned} \tag{3.3}$$

Balancing U''' with $U'U^{2m}$in Eq. (3.3) give

$$N+3=N+1+2mN\Leftrightarrow 3=2mN+1\Leftrightarrow N=\frac{1}{m}.$$

We then assume that Eq. (3.3) has the following formal solutions:

$$U(\xi)=AF^{\frac{1}{m}},\ A\neq 0 \tag{3.4}$$

Substituting Eq (3.4) into Eq. (3.3) and collecting all terms with the same order of F^j together, we convert the left-hand side of Eq. (3.3) into a polynomial in F^j. Setting each coefficient of each polynomial to zero, we derive a set of algebraic equations forα,β,k,ω and A. By solving these algebraic equations we have

$$\begin{aligned}
&A=\Big[-\tfrac{sn(s+5sn+8rn+8sn^2+12rn^2+4n^3s+8n^3r)(4sn+4sn^2+2rn+s+4rn^2)\gamma}{(s+5sn+8rn+8sn^2+12rn^2+4n^3s+8n^3r)^2}\times\\
&\left(\tfrac{1}{n^2}+\tfrac{3}{n}+2\right)-\tfrac{s}{2r}\left(\tfrac{1}{n}+2\right)\Big]^{\frac{1}{2n}},\\
&\alpha=\tfrac{a}{3\gamma},\beta=\tfrac{9ca\gamma-2a^3}{27\gamma^2},\\
&k=\pm\tfrac{\sqrt{2\gamma rs(s+5sn+8rn+8sn^2+12rn^2+4n^3s+8n^3r)(4sn+4sn^2+2rn+s+4rn^2)n}}{\gamma(s+5sn+8rn+8sn^2+12rn^2+4n^3s+8n^3r)}.
\end{aligned} \tag{3.5}$$

$$\begin{aligned}
&\omega=\tfrac{a^2}{3\gamma^2}\tfrac{\sqrt{2\gamma rs(s+5sn+8rn+8sn^2+12rn^2+4n^3s+8n^3r)(4sn+4sn^2+2rn+s+4rn^2)n}}{(s+5sn+8rn+8sn^2+12rn^2+4n^3s+8n^3r)}-\\
&\tfrac{c\sqrt{2\gamma rs(s+5sn+8rn+8sn^2+12rn^2+4n^3s+8n^3r)(4sn+4sn^2+2rn+s+4rn^2)n}}{\gamma(s+5sn+8rn+8sn^2+12rn^2+4n^3s+8n^3r)}-\\
&\tfrac{b\left(2\gamma rs(s+5sn+8rn+8sn^2+12rn^2+4n^3s+8n^3r)(4sn+4sn^2+2rn+s+4rn^2)\right)^{\frac{3}{2}}n^3}{\gamma^2(s+5sn+8rn+8sn^2+12rn^2+4n^3s+8n^3r)^3}\times\\
&\left(\tfrac{1}{n}-1\right)\left(\tfrac{1}{n}-2\right)-\\
&\tfrac{2bn\left(2\gamma rs(s+5sn+8rn+8sn^2+12rn^2+4n^3s+8n^3r)(4sn+4sn^2+2rn+s+4rn^2)\right)^{\frac{3}{2}}}{\gamma^2(s+5sn+8rn+8sn^2+12rn^2+4n^3s+8n^3r)^3},
\end{aligned} \tag{3.6}$$

From Eq. (2.6)(a) and relations (3.5) , (3.6) along with (3.4) we have

$$U(\xi)=\Big[-\frac{sn(s+5sn+8rn+8sn^2+12rn^2+4n^3s+8n^3r)(4sn+4sn^2+2rn+s+4rn^2)\gamma}{(s+5sn+8rn+8sn^2+12rn^2+4n^3s+8n^3r)^2}\times \left(\frac{1}{n^2}+\frac{3}{n}+2\right)-\frac{s}{2r}\left(\frac{1}{n}+2\right)\Big]^{\frac{1}{2n}}\left(-\sqrt{-b}\tanh(\sqrt{-b}\xi)\right)^{\frac{1}{n}},$$

So from (3.1) we have solitary wave solutions of Eq. (1.1) as follows

$$q_1(x,\ t)=\Big[-\frac{sn(s+5sn+8rn+8sn^2+12rn^2+4n^3s+8n^3r)(4sn+4sn^2+2rn+s+4rn^2)\gamma}{(s+5sn+8rn+8sn^2+12rn^2+4n^3s+8n^3r)^2}\times \left(\frac{1}{n^2}+\frac{3}{n}+2\right)-\frac{s}{2r}\left(\frac{1}{n}+2\right)\Big]^{\frac{1}{2n}}\times\Big[-\sqrt{-b}\tanh(\sqrt{-b}i(\frac{\sqrt{2\gamma rs(s+5sn+8rn+8sn^2+12rn^2+4n^3s+8n^3r)(4sn+4sn^2+2rn+s+4rn^2)n}}{\gamma(s+5sn+8rn+8sn^2+12rn^2+4n^3s+8n^3r)}x- (\frac{a^2}{3\gamma^2}\frac{\sqrt{2\gamma rs(s+5sn+8rn+8sn^2+12rn^2+4n^3s+8n^3r)(4sn+4sn^2+2rn+s+4rn^2)n}}{(s+5sn+8rn+8sn^2+12rn^2+4n^3s+8n^3r)}- \frac{c\sqrt{2\gamma rs(s+5sn+8rn+8sn^2+12rn^2+4n^3s+8n^3r)(4sn+4sn^2+2rn+s+4rn^2)n}}{\gamma(s+5sn+8rn+8sn^2+12rn^2+4n^3s+8n^3r)}- \frac{b\left(2\gamma rs(s+5sn+8rn+8sn^2+12rn^2+4n^3s+8n^3r)(4sn+4sn^2+2rn+s+4rn^2)\right)^{\frac{3}{2}}n^3}{\gamma^2(s+5sn+8rn+8sn^2+12rn^2+4n^3s+8n^3r)^3}\times \left(\frac{1}{n}-1\right)\left(\frac{1}{n}-2\right)- \frac{2bn\left(2\gamma rs(s+5sn+8rn+8sn^2+12rn^2+4n^3s+8n^3r)(4sn+4sn^2+2rn+s+4rn^2)\right)^{\frac{3}{2}}}{\gamma^2(s+5sn+8rn+8sn^2+12rn^2+4n^3s+8n^3r)^3})t\Big]^{\frac{1}{n}}\times exp\left(i(\frac{a}{3\gamma}x+\frac{9ca\gamma-2a^3}{27\gamma^2}t)\right),$$

From (2.6)(b) and relations (3.5) and (3.6) along with (3.1) and (3.4) we obtaion solitary wave solutions of Eq. (1.1) in following form

$$q_2(x,\ t)=\Big[-\frac{sn(s+5sn+8rn+8sn^2+12rn^2+4n^3s+8n^3r)(4sn+4sn^2+2rn+s+4rn^2)\gamma}{(s+5sn+8rn+8sn^2+12rn^2+4n^3s+8n^3r)^2}\times \left(\frac{1}{n^2}+\frac{3}{n}+2\right)-\frac{s}{2r}\left(\frac{1}{n}+2\right)\Big]^{\frac{1}{2n}}\times\Big[-\sqrt{-b}\coth(\sqrt{-b}i(\frac{\sqrt{2\gamma rs(s+5sn+8rn+8sn^2+12rn^2+4n^3s+8n^3r)(4sn+4sn^2+2rn+s+4rn^2)n}}{\gamma(s+5sn+8rn+8sn^2+12rn^2+4n^3s+8n^3r)}x- (\frac{a^2}{3\gamma^2}\frac{\sqrt{2\gamma rs(s+5sn+8rn+8sn^2+12rn^2+4n^3s+8n^3r)(4sn+4sn^2+2rn+s+4rn^2)n}}{(s+5sn+8rn+8sn^2+12rn^2+4n^3s+8n^3r)}- \frac{c\sqrt{2\gamma rs(s+5sn+8rn+8sn^2+12rn^2+4n^3s+8n^3r)(4sn+4sn^2+2rn+s+4rn^2)n}}{\gamma(s+5sn+8rn+8sn^2+12rn^2+4n^3s+8n^3r)}- \frac{b\left(2\gamma rs(s+5sn+8rn+8sn^2+12rn^2+4n^3s+8n^3r)(4sn+4sn^2+2rn+s+4rn^2)\right)^{\frac{3}{2}}n^3}{\gamma^2(s+5sn+8rn+8sn^2+12rn^2+4n^3s+8n^3r)^3}\times \left(\frac{1}{n}-1\right)\left(\frac{1}{n}-2\right)- \frac{2bn\left(2\gamma rs(s+5sn+8rn+8sn^2+12rn^2+4n^3s+8n^3r)(4sn+4sn^2+2rn+s+4rn^2)\right)^{\frac{3}{2}}}{\gamma^2(s+5sn+8rn+8sn^2+12rn^2+4n^3s+8n^3r)^3})t\Big]^{\frac{1}{n}}\times exp\left(i(\frac{a}{3\gamma}x+\frac{9ca\gamma-2a^3}{27\gamma^2}t)\right),$$

From (2.6)(c) and relations (3.1),(3.4),(3.5) and (3.6) we obtain solitary wave solutions for nonlinear Schrodinger equation with power law nonlinearity

$$\begin{aligned}
&q_3(x,\ t)=\Big[-\tfrac{sn(s+5sn+8rn+8sn^2+12rn^2+4n^3s+8n^3r)(4sn+4sn^2+2rn+s+4rn^2)\gamma}{(s+5sn+8rn+8sn^2+12rn^2+4n^3s+8n^3r)^2}\times\\
&\left(\tfrac{1}{n^2}+\tfrac{3}{n}+2\right)-\tfrac{s}{2r}\left(\tfrac{1}{n}+2\right)\Big]^{\frac{1}{2n}}\times\Big[\sqrt{b}\tan(\sqrt{b}i(\\
&\tfrac{\sqrt{2\gamma rs(s+5sn+8rn+8sn^2+12rn^2+4n^3s+8n^3r)(4sn+4sn^2+2rn+s+4rn^2)n}}{\gamma(s+5sn+8rn+8sn^2+12rn^2+4n^3s+8n^3r)}x-\\
&(\tfrac{a^2}{3\gamma^2}\tfrac{\sqrt{2\gamma rs(s+5sn+8rn+8sn^2+12rn^2+4n^3s+8n^3r)(4sn+4sn^2+2rn+s+4rn^2)n}}{(s+5sn+8rn+8sn^2+12rn^2+4n^3s+8n^3r)}-\\
&\tfrac{c\sqrt{2\gamma rs(s+5sn+8rn+8sn^2+12rn^2+4n^3s+8n^3r)(4sn+4sn^2+2rn+s+4rn^2)n}}{\gamma(s+5sn+8rn+8sn^2+12rn^2+4n^3s+8n^3r)}-\\
&\tfrac{b\left(2\gamma rs(s+5sn+8rn+8sn^2+12rn^2+4n^3s+8n^3r)(4sn+4sn^2+2rn+s+4rn^2)\right)^{\frac{3}{2}}n^3}{\gamma^2(s+5sn+8rn+8sn^2+12rn^2+4n^3s+8n^3r)^3}\times\\
&\left(\tfrac{1}{n}-1\right)\left(\tfrac{1}{n}-2\right)-\\
&\tfrac{2bn\left(2\gamma rs(s+5sn+8rn+8sn^2+12rn^2+4n^3s+8n^3r)(4sn+4sn^2+2rn+s+4rn^2)\right)^{\frac{3}{2}}}{\gamma^2(s+5sn+8rn+8sn^2+12rn^2+4n^3s+8n^3r)^3})t\Big]^{\frac{1}{n}}\times\\
&exp\left(i(\tfrac{a}{3\gamma}x+\tfrac{9ca\gamma-2a^3}{27\gamma^2}t)\right),
\end{aligned}$$

In this case we obtain solitary wave solution for (1.1) from (2.6)(d) and relations (3.1)-(3.6) as follow

$$\begin{aligned}
&q_4(x,\ t)=\Big[-\tfrac{sn(s+5sn+8rn+8sn^2+12rn^2+4n^3s+8n^3r)(4sn+4sn^2+2rn+s+4rn^2)\gamma}{(s+5sn+8rn+8sn^2+12rn^2+4n^3s+8n^3r)^2}\times\\
&\left(\tfrac{1}{n^2}+\tfrac{3}{n}+2\right)-\tfrac{s}{2r}\left(\tfrac{1}{n}+2\right)\Big]^{\frac{1}{2n}}\times\Big[-\sqrt{b}\cot(\sqrt{b}i(\\
&\tfrac{\sqrt{2\gamma rs(s+5sn+8rn+8sn^2+12rn^2+4n^3s+8n^3r)(4sn+4sn^2+2rn+s+4rn^2)n}}{\gamma(s+5sn+8rn+8sn^2+12rn^2+4n^3s+8n^3r)}x-\\
&(\tfrac{a^2}{3\gamma^2}\tfrac{\sqrt{2\gamma rs(s+5sn+8rn+8sn^2+12rn^2+4n^3s+8n^3r)(4sn+4sn^2+2rn+s+4rn^2)n}}{(s+5sn+8rn+8sn^2+12rn^2+4n^3s+8n^3r)}-\\
&\tfrac{c\sqrt{2\gamma rs(s+5sn+8rn+8sn^2+12rn^2+4n^3s+8n^3r)(4sn+4sn^2+2rn+s+4rn^2)n}}{\gamma(s+5sn+8rn+8sn^2+12rn^2+4n^3s+8n^3r)}-\\
&\tfrac{b\left(2\gamma rs(s+5sn+8rn+8sn^2+12rn^2+4n^3s+8n^3r)(4sn+4sn^2+2rn+s+4rn^2)\right)^{\frac{3}{2}}n^3}{\gamma^2(s+5sn+8rn+8sn^2+12rn^2+4n^3s+8n^3r)^3}\times\\
&\left(\tfrac{1}{n}-1\right)\left(\tfrac{1}{n}-2\right)-\\
&\tfrac{2bn\left(2\gamma rs(s+5sn+8rn+8sn^2+12rn^2+4n^3s+8n^3r)(4sn+4sn^2+2rn+s+4rn^2)\right)^{\frac{3}{2}}}{\gamma^2(s+5sn+8rn+8sn^2+12rn^2+4n^3s+8n^3r)^3})t\Big]^{\frac{1}{n}}\times\\
&exp\left(i(\tfrac{a}{3\gamma}x+\tfrac{9ca\gamma-2a^3}{27\gamma^2}t)\right),
\end{aligned}$$

Finally from (2.6)(e) we obtain solitary wave solutions for perturbed NLSE with power law nonlinearity in following form

$$\begin{aligned}
&q_5(x,\ t)=\Big[-\tfrac{sn(s+5sn+8rn+8sn^2+12rn^2+4n^3s+8n^3r)(4sn+4sn^2+2rn+s+4rn^2)\gamma}{(s+5sn+8rn+8sn^2+12rn^2+4n^3s+8n^3r)^2}\times\\
&\left(\tfrac{1}{n^2}+\tfrac{3}{n}+2\right)-\tfrac{s}{2r}\left(\tfrac{1}{n}+2\right)\Big]^{\frac{1}{2n}}\times\\
&\sqrt[n]{i}\,\Big[\tfrac{\sqrt{2\gamma rs(s+5sn+8rn+8sn^2+12rn^2+4n^3s+8n^3r)(4sn+4sn^2+2rn+s+4rn^2)n}}{\gamma(s+5sn+8rn+8sn^2+12rn^2+4n^3s+8n^3r)}\ x-\\
&(\tfrac{a^2}{3\gamma^2}\tfrac{\sqrt{2\gamma rs(s+5sn+8rn+8sn^2+12rn^2+4n^3s+8n^3r)(4sn+4sn^2+2rn+s+4rn^2)n}}{(s+5sn+8rn+8sn^2+12rn^2+4n^3s+8n^3r)}-\\
&\tfrac{c\sqrt{2\gamma rs(s+5sn+8rn+8sn^2+12rn^2+4n^3s+8n^3r)(4sn+4sn^2+2rn+s+4rn^2)n}}{\gamma(s+5sn+8rn+8sn^2+12rn^2+4n^3s+8n^3r)})t\Big]^{-\frac{1}{n}}\times\\
&exp\left(i(\tfrac{a}{3\gamma}x+\tfrac{9ca\gamma-2a^3}{27\gamma^2}t)\right),
\end{aligned}$$

4. Conclusion

In summary we derive many types of optical solitary wave solutions of perturbed nonlinear Schrodinger equation with power law nonlinearity which include the bright and dark optical solitary wave solutions. The results show that the method is reliable and effective and gives more solutions. We hope that the obtained results will be useful for further studies in mathematical physics and engineering.

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