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Features of a New Method for Solving Practical Problems of Continuum Mechanics Under Conditions of Complex Loading

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Abstract: This work, carried out within the framework of grant № AP23488953, funded by the Science Committee of the Ministry of Science and Higher Education of the Republic of Kazakhstan, considers an applied problem of continuum mechanics associated with solving a new closed problem of plasticity theory with respect to an asymmetric deformation focus during loading. The analytical solution of this problem is based on the method of argument of functions of a complex variable. The proposed approach is invariant to various areas of continuum mechanics, including not only the theory of plasticity, but also the theory of elasticity, the theory of dynamic processes. In particular, using the example of the rolling process, a closed planar problem of the theory of plasticity in an analytical form was posed and solved. The formulated system of equations includes: differential equations of equilibrium, the Huber-Mises plasticity condition, coupling equations, the equation of continuity of strain rates, the condition of constancy of volume, the equation of thermal conductivity, boundary conditions for stresses and strain rates. The solution uses approaches of limited nonlinearity, fundamental and trigonometric substitution. Under the conditions of new variables through the function argument, it was possible to simplify the intermediate result and determine not the solutions themselves, but the conditions of their existence under given boundary conditions. The solution of the asymmetric problem contributed to the identification of the effects of shape change in the processing center, which make it possible to control the processes of plastic deformation. The fundamental approach considered in this paper makes it possible to determine not only the stress-strain state of the plastic medium, but also the mathematical model of the deformed space and further possibilities for finding generalized solutions to problems of continuum mechanics.

Keywords: Continuum mechanics, Rolling, Closed problem, Asymmetric problem, Plasticity theory

Introduction

At the present stage of production development, technological processes are becoming more complicated, modern computer systems are being introduced, boundary and edge conditions are becoming more complicated, and their number is increasing, which indicates the increasing demands on technical and scientific developments of modern science. Theoretical solutions are overloaded with all kinds of assumptions and limitations, which reduces their effectiveness. There is an urgent need to generalize solutions to various processes and applied theoretical, practical and fundamental problems. One of the ways to increase the effectiveness of scientific approaches in continuum mechanics is to use invariant generalizing variables, which contribute not to obtaining the result itself, but to finding conditions for the existence of solutions. Then it becomes possible to find not just one analytical solution, but an indefinite set. At the same time, the question of how the solution is determined - by analytical, approximate or numerical methods - becomes unimportant. Such defining fundamental methods of finding conditions for the existence of solutions include the method of function argument of a complex variable (Chigirinsky et al., 2021).

Method

This approach is based on a modern method of solving continuum mechanics problems, the method of function argument of a complex variable. The proposed method is invariant to various fields of mechanics, including not only the plasticity theory, but also the elasticity theory and the theory of dynamic processes. In particular, using the example of the rolling process, a closed problem of the theory of plasticity was posed and solved in an analytical form, in which this approach is demonstrated. The formulated system of equations of the theory of plasticity includes: differential equations of equilibrium, the Huber-Mises plasticity condition, the coupling equation, the equation of continuity of deformation rates, the condition of constancy of volume, the equation of thermal conductivity, boundary conditions. For the completeness of the presentation of this problem, the formulation and elements of solving the planar problem of the theory of plasticity in a closed form are considered:

$$\begin{gathered}
\frac{\partial \sigma_x}{\partial x}+\frac{\partial \tau_{xy}}{\partial y}=0;\ \frac{\partial \tau_{yx}}{\partial x}+\frac{\partial \sigma_y}{\partial y}=0; \\
\left(\sigma_x-\sigma_y\right)^2+4\tau_{xy}^2=4k^2; \\
\frac{\sigma_x-\sigma_y}{2\cdot\tau_{xy}}=\frac{\xi_x-\xi_y}{\gamma'_{xy}}=F_1; \\
\xi_x+\xi_y=0; \\
\frac{\partial^2 \xi_x}{\partial y^2}+\frac{\partial^2 \xi_y}{\partial x^2}=\frac{\partial^2 \dot{\gamma}_{xy}}{\partial y \partial x}; \\
\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial x^2}=0,
\end{gathered} \tag{1}$$

where σ_x – normal stress; τ_{xy} – tangential stress; k – plastic shear resistance (variable); ξ_x, $\dot{\gamma}_{xy}$ – linear and shear strain rate; T – temperature of the metal.

The boundary conditions are set in stresses and in terms of the strain rate:

$$\begin{gathered}
\tau_n=-\left[\frac{\sigma_x-\sigma_y}{2}\cdot sin(2\varphi)-\tau_{xy}\,cos(2\varphi)\right], \\
\dot{\gamma}_n=-2\left[\frac{\xi_x-\xi_y}{2}\cdot sin(2\varphi)-\dot{\gamma}_{xy}\,cos(2\varphi)\right].
\end{gathered} \tag{2}$$

where $\dot{\gamma}_n$ – the shear strain rate characterizing the boundary condition; τ_n – tangential stress characterizing the boundary condition.

Based on the system of equations (1) and (2), a simpler problem is solved to determine the stress state of the plastic medium. The statement has the form:
- differential equations of equilibrium:

$$\frac{\partial \sigma_x}{\partial x}+\frac{\partial \tau_{xy}}{\partial y}=0;\ \frac{\partial \tau_{yx}}{\partial x}+\frac{\partial \sigma_y}{\partial y}=0; \tag{3}$$

- Huber-Mises plasticity equation:

$$\left(\sigma_x - \sigma_y\right)^2 + 4\tau_{xy}^2 = 4k^2; \tag{4}$$

- differential equation of continuity of stress deformations:

$$\Delta^2 n\sigma_0 = \frac{\partial^2 n\sigma_0}{\partial x^2} + \frac{\partial^2 n\sigma_0}{\partial y^2} = 0, \tag{5}$$

- boundary conditions:

$$\tau_n = -\left[\frac{\sigma_x - \sigma_y}{2} \cdot sin(2\varphi) - \tau_{xy} cos(2\varphi)\right]. \tag{6}$$

Taking into account (3) and (4), a generalized differential equation of equilibrium for tangential stresses is defined in the form (Arkulis & Dorogobid, 1987):

$$\frac{\partial^2 \tau_{xy}}{\partial x^2} - \frac{\partial^2 \tau_{xy}}{\partial y^2} = \pm 2\frac{\partial^2}{\partial x \partial y} T_i \cdot \sqrt{1 - \left(\frac{\tau_{xy}}{T_i}\right)^2}. \tag{7}$$

Ratio τ_{xy}/T_i using the trigonometric function, the boundary conditions and equation (7) are simplified.

Tangential stresses can then be represented as:

$$\tau_{xy} = H_\sigma \cdot exp\,\theta \cdot sin\,A\Phi, \tag{8}$$

where $T_i -$ intensity of tangential stresses; $A\Phi$ and $\theta -$ unknown function arguments to be defined.

If we use the function of a complex variable and reformat expression (8), after substitution in (7):

$$\begin{aligned}
&\frac{1}{2i} exp(\theta + i \cdot A\Phi) \cdot \Big\{\Big[(H_\sigma)_{xx} - (H_\sigma)_{yy} - 2i(H_\sigma)_{xy}\Big] + 2(H_\sigma)_x\Big[\left(\theta_x + A\Phi_y\right) - \\
&- i\left(\theta_y - A\Phi_x\right)\Big] - 2(H_\sigma)_y\Big[i\left(\theta_x + A\Phi_y\right) - \left(\theta_y - A\Phi_x\right)\Big] + H_\sigma\Big[\left(\theta_{xx} - \theta_{yy} + 2A\Phi_{xy}\right) + \\
&+ i\left(A\Phi_{xx} - A\Phi_{yy} - 2\theta_{xy}\right)\Big] + H_\sigma\Big[\left(\theta_x + A\Phi_y\right) - i\left(\theta_y - A\Phi_x\right)\Big]^2\Big\} - \\
&- \frac{1}{2i} exp(\theta - i \cdot A\Phi) \cdot \Big\{\Big[(H_\sigma)_{xx} - (H_\sigma)_{yy} + 2i(H_\sigma)_{xy}\Big] + 2(H_\sigma)_x\Big[\left(\theta_x + A\Phi_y\right) + \\
&+ i\left(\theta_y - A\Phi_x\right)\Big] - 2(H_\sigma)_y\Big[\left(\theta_y - A\Phi_x\right) - i\left(\theta_x + A\Phi_y\right)\Big] + H_\sigma\Big[\left(\theta_{xx} - \theta_{yy} + 2A\Phi_{xy}\right) - \\
&- i\left(A\Phi_{xx} - A\Phi_{yy} - 2\theta_{xy}\right)\Big] + H_\sigma\Big[\left(\theta_x + A\Phi_y\right) + i\left(\theta_y - A\Phi_x\right)\Big]^2\Big\} = 0.
\end{aligned} \tag{9}$$

The differential equation (9) is a function of three variables $A\Phi$, θ and H, and it turns into an identity if the conditions are met:

$$\begin{aligned}
&\theta_x = -A\Phi_x,\ \theta_y = A\Phi_x. \\
&\theta_{xx} + \theta_{yy} = 0, A\Phi_{xx} + A\Phi_{yy} = 0.
\end{aligned} \tag{10}$$

Using the relations (10), the problem of identifying the argument of functions is eliminated. It should be added that the differential relations (10) are generalized invariant characteristics that make it possible to close equations (9). Thus, taking into account formulas (10), the solution of equation (9) is the expression:

$$\tau_{xy} = H_\sigma \cdot \exp\theta \cdot \sin A\Phi. \tag{11}$$

Substituting solution (11) into the equilibrium equations (3), taking into account the complex variable, boundary conditions, and the equation of continuity of strain rates, we have expressions for normal stresses:

$$\sigma_x = -H_\sigma \cdot exp\theta \cdot cos\,A\Phi + C,$$

$$\sigma_y = -3H_\sigma \cdot exp\,\theta \cdot cos\,A\Phi + C, \tag{12}$$

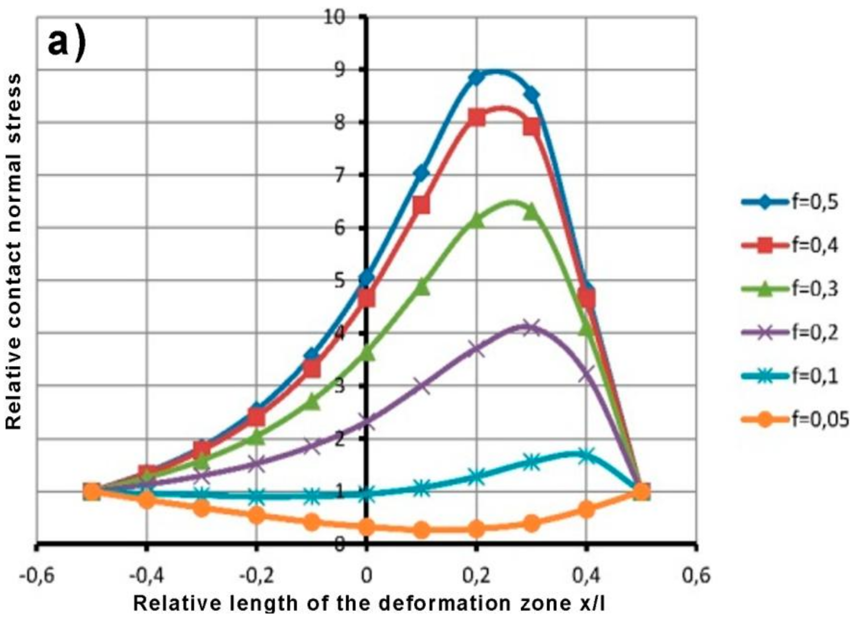
$$\theta_x = -A\Phi_y,\, \theta_y = A\Phi_x, \quad \theta_{xx} + \theta_{yy} = 0, \;\; A\Phi_{xx} + A\Phi_{yy} = 0\,.$$

A special feature of the new method for solving continuum mechanics problems is the relations (10). They have a number of useful qualities, simplify the process of solving problems. They can be interpreted as fundamental variables of continuum mechanics. One of the notable properties of these ratios is the ability to switch from one variable to another. This is important when solving multicomponent problems, and the integration of differential equations is simplified.

The next feature is analytical solutions of different types of differential equations. For example, equation (7) is an equation of the hyperbolic type, and the equation of continuity of deformations (5) of the elliptical type is quite acceptable for solving equations of the parabolic type. This combination is explained by the use of fundamental variables in different tasks. In relation to these studies, variables close the solution of the problem, i.e. they limit the functions θ and $A\Phi$ thus, the differential equations of equilibrium, the generalized equation of equilibrium, and the continuity of deformation are identically satisfied. Such relations are known as the Cauchy-Riemann relations and the Laplace equations. Therefore, the function argument is represented as harmonic functions.

Results and Discussion

According to expressions (12), the stress states were calculated during a simple rolling process, with different technological parameters. Figures 1 and 2 show the distribution of contact normal and tangential stresses along the length of the deformation zone at different coefficients of friction and shape factor (Chigirinsky et al., 2024).



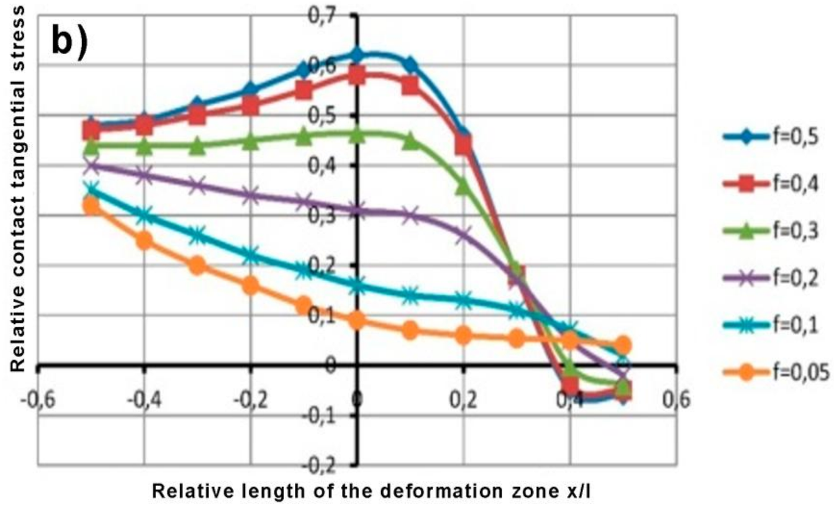


Figure 1. Distribution of contact stresses along the length of the deformation zone, depending on contact friction, at l/h=11.04, α=0.129, f=0.05-0.5: a – distribution of normal stresses; b – distribution of tangential stresses

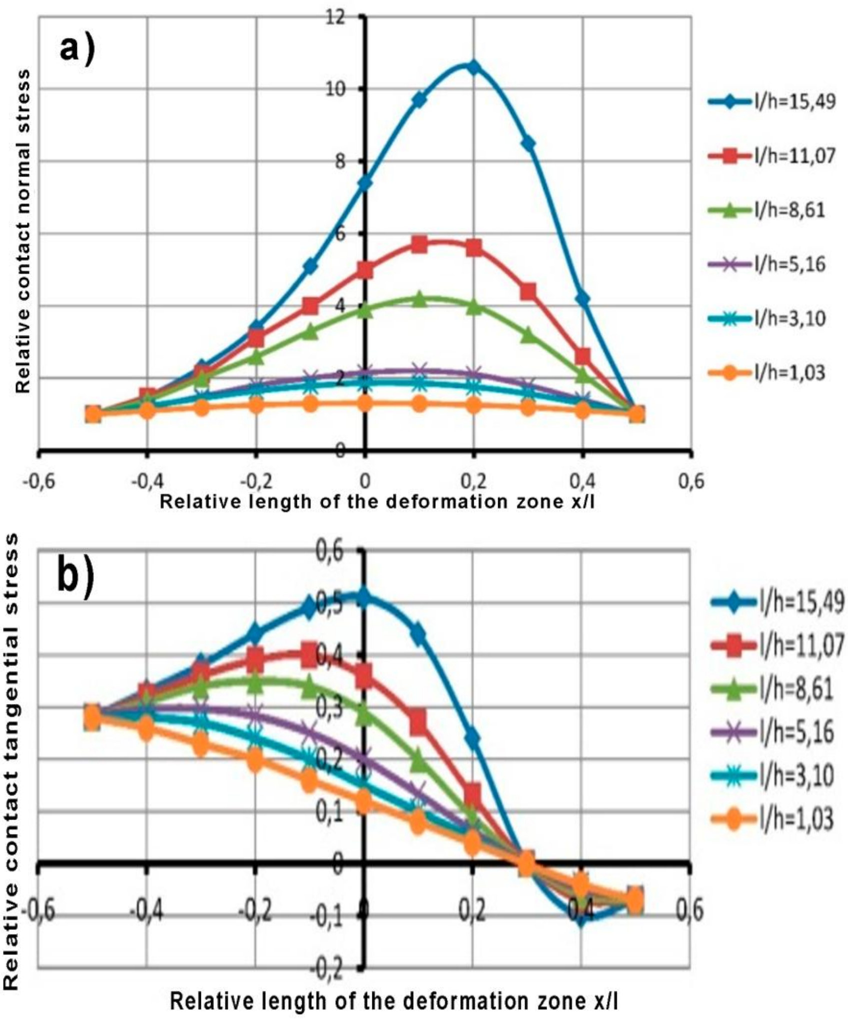


Figure 2. Distribution of contact stresses along the length of the deformation zone depending on the shape factor at l/h=1.03-15.49; α=0.077; f=0.3: a – distribution of normal stresses; b – distribution of tangential stresses

Figure 3 shows diagrams of contact stresses under different process conditions and different combinations of deformation and force loading, as well as the process of loss of stability.

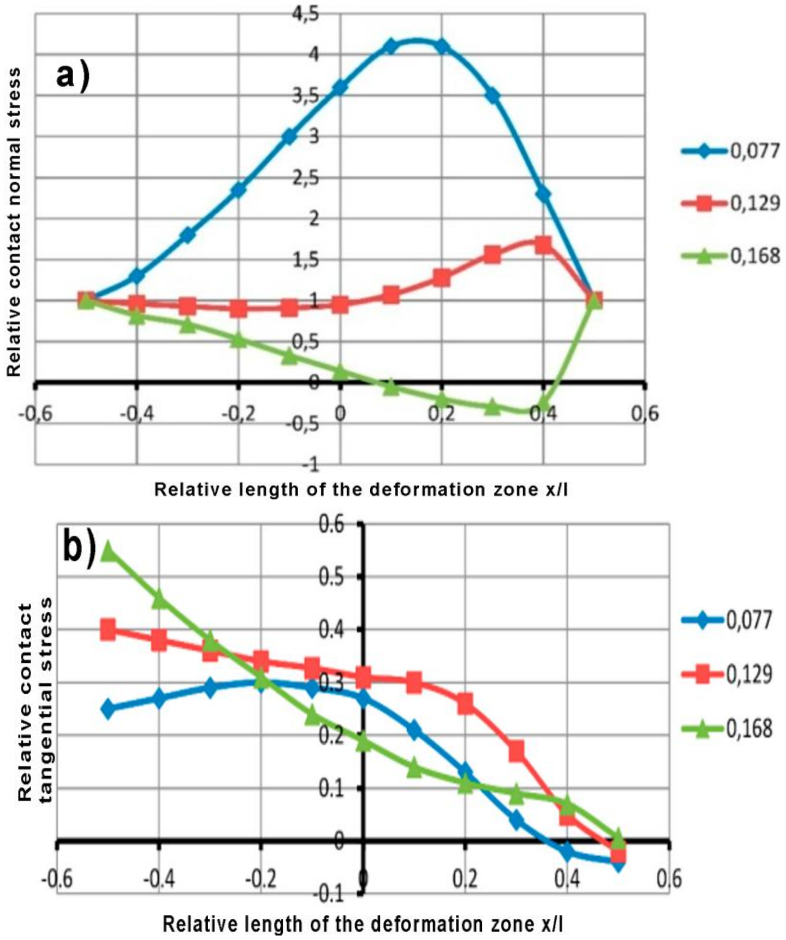


Figure 3. Distribution of contact stresses along the length of the deformation zone depending on the capture angle at: $l/h=11.04$; $\alpha=0.077;0.129;0.168$; $f=0.2$: a – distribution of normal stresses; b – distribution of tangential stresses

Figure 3 shows an interesting process of loss of stability of the process with the designation of options for reducing power characteristics with an increase in deformation. The features of the function argument method make it possible to eliminate some incorrect limitations of problem solving using the analytical approach. It becomes possible to obtain results under complex and diverse boundary conditions due to the multiplicity of solutions. This is postulated by the fact that the function argument method does not show the solutions themselves, but the conditions of their existence.

Conclusion

The method of argument of functions of a complex variable was used as the basis for solving the applied problem of continuum mechanics related to solving the closed problem of plasticity theory in relation to an asymmetric deformation zone during complex loading. The formulated system of equations includes: differential equations of equilibrium, the Huber-Mises plasticity condition, coupling equations, the equation of continuity of strain rates, the condition of volume constancy, the equation of thermal conductivity, boundary conditions for stresses and strain rates. In addition, the approaches of limited nonlinearity, fundamental and trigonometric substitution were used to solve the problem. Under the conditions of new variables through the function argument, it was possible to simplify the intermediate result and determine not the solutions themselves, but the conditions of their existence under the given boundary conditions. The invariant variables in this solution are the Cauchy-Riemann relations and the Laplace differential equations. The solution of the closed problem of the theory of plasticity given in this paper, applied to an asymmetric deformation focus, showed the presence of the effect of plastic shaping. This solution makes it possible to successfully implement the necessary control of plastic deformation processes under difficult loading conditions. At the same time, the fundamental approach considered in this paper makes it possible to determine not only the stress-strain state of the plastic medium, but also the mathematical model of the deformed space and further possibilities for finding generalized solutions to problems of continuum mechanics.

Scientific Ethics Declaration

* The authors declare that the scientific ethical and legal responsibility of this article published in EPSTEM Journal belongs to the authors.

Conflict of Interest

* The authors declare that they have no conflicts of interest

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