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Anisotropic Cosmological Evolution in $f(R,T)$ Gravity: Exact Solutions for Bianchi Type-V Universe

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Abstract − In this study, we investigate the Bianchi type-V cosmological model filled with a perfect fluid characterized by a constant equation of state parameter within the framework of $f(R,T)$ gravity. By adopting the physically justified assumption that the shear scalar is proportional to the expansion scalar, we derive exact analytical solutions to the modified gravitational field equations in $f(R,T)$ gravity. We also derive the models key physical and geometrical parameters, and analyze their dynamical evolution in terms of cosmic time t. We then provide graphical representations to illustrate the temporal behavior of relevant cosmological quantities. The results indicate the model exhibits a phase of accelerated expansion, which is consistent with recent observational data. However, the universe does not evolve toward isotropy; instead, it originates from a Big Bang-type initial singularity.

Keywords − *Bianchi type-V universe, modified gravity, exact solution, anisotropic universe*

1. Introduction

A wide range of contemporary astrophysical observations — including those of high-redshift type Ia supernovae [1–3], the cosmic microwave background (CMB) anisotropies [4], and the distribution of large-scale structures [5] — strongly indicate that the universe is currently undergoing a phase of accelerated expansion. This unexpected late-time dynamical behavior has motivated cosmologists to investigate possible explanatory mechanisms, which are broadly categorized into two main approaches. The first approach involves introducing novel forms of energy-momentum components, often referred to as exotic matter, which are characterized by negative pressure. This category includes theoretical constructs, such as Chaplygin gas, phantom fields, quintessence, tachyonic fields, and K-essence models [7–16]. These exotic fields are generally formulated within the standard framework of General Relativity (GR) and serve as viable candidates for dark energy. The second approach aims to explain cosmic acceleration by modifying the underlying theory of gravity itself. In this context, the standard Einstein-Hilbert action is generalized by replacing the Ricci scalar R with a nonlinear, arbitrary function $f(R)$, thereby giving rise to the so-called $f(R)$ gravity theories [17-19]. Such extensions of GR have proven capable of replicating late-time cosmic acceleration without invoking dark energy [20]. More generally, gravitational action can be further extended to depend not only on curvature invariants but also on the matter content of spacetime, specifically the trace T of the energy-momentum tensor. Among the most prominent theories in this class is $f(R,T)$ gravity, introduced by Harko et al. [21], which incorporates an arbitrary function of both R and T into the gravitational Lagrangian. This formalism allows for

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an explicit coupling between matter and geometry, providing an extended framework to explore gravitational phenomena beyond standard GR.

The prevailing cosmological paradigm, represented by the Friedmann–Lemaître–Robertson–Walker (FLRW) metric, effectively models the universe as spatially homogeneous and isotropic on large scales. However, accumulating observational evidence — particularly from analyses of the cosmic microwave background radiation and large-scale structure — suggests that the early universe may have experienced significant anisotropic phases [22–24]. These findings have led to increased interest in anisotropic cosmological models. Among these, the spatially homogeneous but anisotropic Bianchi-type models are widely studied as generalized frameworks to investigate deviations from isotropy during the early epochs of cosmic evolution.

Motivated by the aforesaid considerations, the present study focuses on exploring the dynamics of the Bianchi type-V cosmological model within the framework of $f(R,T)$ gravity. In recent years, numerous studies on Bianchi type-V models within this modified gravity framework have appeared in the literature. For instance, Ahmed and Pradhan [25] examined a scenario in which the cosmological constant $\Lambda(T)$ is treated as a function of the trace T of the energy-momentum tensor, i.e., $\Lambda(T)$, within the Bianchi type-V background. Rao and Papa Rao [26] analyzed Bianchi type-V spacetime in the presence of cosmic strings within the same theoretical setting. Bishi and Mahanta [27] investigated a Bianchi type-V cosmological model incorporating bulk viscosity and string matter, adopting a specific functional form along with a linearly varying deceleration parameter. Tiwari and Mishra [28] addressed the field equations of $f(R,T)$ gravity by assuming the anisotropy parameter scales inversely with the m^{th} power of the scale factor in a Bianchi type-V geometry. Furthermore, Hasmani and Al-Haysah [29] obtained exact solutions for the Bianchi type-V model by employing a particular formulation of the Hubble parameter within the same gravitational framework.

In this study, we investigate the Bianchi type-V cosmological model within the framework of $f(R,T)$ gravity, under the assumption that the shear scalar is proportional to the expansion scalar. Exact solutions to the modified field equations are derived, and the dynamical evolution of the corresponding physical and geometrical parameters is thoroughly examined.

The organization of this paper is as follows. In Section 2, we outline the fundamental field equations governing the $f(R,T)$ gravity framework. Section 3 is devoted to formulating these equations in the context of the Bianchi type-V cosmological model. In Section 4, we derive the exact analytical solutions to the field equations and subsequently determine the associated physical and geometrical parameters of the model. Section 5 presents a detailed analysis of the results, where the temporal evolution of physical and geometrical quantities is visualized through graphical representations as functions of cosmic time, t.

Throughout this study, we adopt the natural units $G = c = 1$. Greek indices $\mu, v, \ldots$ are assumed to run from 1 to 4. A prime symbol “ $'$ ” denotes differentiation concerning the argument of the corresponding function, while an overdot “ $\dot{}$ ” indicates differentiation concerning the cosmic time t.

2. $f(R,T)$ Gravity

The field equations of $f(R,T)$ gravity are derived from the action proposed by Harko et al. [21],

$$S = \frac{1}{16\pi} \int [f(R,T) + L_m] \sqrt{-g} d^4x \tag{2.1}$$

Here, $f(R,T)$ denotes an arbitrary function of the Ricci scalar R, which characterizes the curvature of spacetime, and the trace T of the energy-momentum tensor $T_{\mu v}$, while L_m represents the matter Lagrangian. The variation of the action provided in (2.1) concerning the metric tensor $g^{\mu v}$ yields the field equations of $f(R,T)$ gravity is as follows:

$$f_R(R,T)R_{\mu\nu} - \frac{1}{2}f(R,T)g_{\mu\nu} + \left(g_{\mu\nu}\nabla^\kappa\nabla_\kappa - \nabla_\mu\nabla_\nu\right)f_R(R,T) = 8\pi T_{\mu\nu} - f_T(R,T)T_{\mu\nu} - f_T(R,T)\theta_{\mu\nu} \quad (2.2)$$

Here,

$$\theta_{\mu\nu} = g^{\alpha\beta}\frac{\delta T_{\alpha\beta}}{\delta g^{\mu\nu}} \quad (2.3)$$

and where the energy-momentum tensor $T_{\mu\nu}$ is defined by variation concerning the metric as

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta\sqrt{-g}}{\delta g^{\mu\nu}}L_m \quad (2.4)$$

∇_μ represents the covariant derivative, f_R and f_T are

$$f_R = \frac{\partial f(R,T)}{\partial R} \quad \text{and} \quad f_T = \frac{\partial f(R,T)}{\partial T} \quad (2.5)$$

Harko et.al. proposed three functional forms of $f(R,T)$ in [21]. This paper focuses on the following form

$$f(R,T) = f_1(R) + f_2(T) \quad (2.6)$$

where $f_1(R)$ and $f_2(T)$ are two arbitrary functions of R and T, respectively. This study considers the universe to be dominated by a perfect fluid, for which the energy-momentum tensor is provided by

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu - p g_{\mu\nu} \quad (2.7)$$

where ρ is energy density, p is pressure, and $u^\mu = (1,0,0,0)$ is the four-velocity of the fluid. Since the metric signature used in the study is $(-,+,+,+)$, the norm of the four-velocity must be normalized as $u_\mu u^\mu = -1$. Considering the energy-momentum tensor of a perfect fluid, given in mixed components as $T^\mu{}_\nu = (\rho,-p,-p,-p)$, and adopting the matter Lagrangian as $L_m = -p$, by using the perfect fluid assumption in (2.3), we obtain

$$\theta_{\mu\nu} \equiv -2T_{\mu\nu} - g_{\mu\nu}p \quad (2.8)$$

Substituting (2.6)-(2.8) into the field equations provided in (2.2), we get

$$f_1'(R)R_{\mu\nu} - \frac{1}{2}f_1(R)g_{\mu\nu} = 8\pi T_{\mu\nu} + f_2'(T)T_{\mu\nu} + \left[f_2'(T)p + \frac{1}{2}f_2(T)\right]g_{\mu\nu} \quad (2.9)$$

We study herein with $f_1(R) = R$ and $f_2(T) = \lambda T$ functions, where λ is an arbitrary constant. The simplest linear forms $f_1(R) = R$ and $f_2(T) = \lambda T$ are chosen in order to reduce the complexity of the field equations and to enable an exact analytical treatment of the Bianchi type-V cosmological model. Then, the modified field equations of $f(R,T)$ gravity, as given in (2.9), can be rewritten in the form of the original Einstein field equations as follows:

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = (8\pi + \lambda)T_{\mu\nu} + \lambda\left(p + \frac{T}{2}\right)g_{\mu\nu} \quad (2.10)$$

where $G_{\mu\nu}$ is the Einstein tensor.

3. Metric and Field Equations

We consider the spatially homogeneous and anisotropic Bianchi type-V metric of the form

$$ds^2 = -dt^2 + A^2(t)dx^2 + B^2(t)e^{2x}dy^2 + C^2(t)e^{2x}dz^2 \tag{3.1}$$

where the scale factors $A(t), B(t)$, and $C(t)$ are functions of the cosmic time t alone. Substituting the metric given in (3.1) into the field (2.10), we obtain the following components of the modified field equations:

$$-\frac{\dot{A}}{A}\frac{\dot{B}}{B} - \frac{\dot{A}}{A}\frac{\dot{C}}{C} - \frac{\dot{B}}{B}\frac{\dot{C}}{C} + \frac{3}{A^2} = (8\pi + \lambda)\rho + \lambda\left(\frac{\rho - p}{2}\right) \tag{3.2}$$

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} - \frac{1^2}{A^2} = (8\pi + \lambda)p - \lambda\left(\frac{\rho - p}{2}\right) \tag{3.3}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} - \frac{1}{A^2} = (8\pi + \lambda)p - \lambda\left(\frac{\rho - p}{2}\right) \tag{3.4}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} - \frac{1}{A^2} = (8\pi + \lambda)p - \lambda\left(\frac{\rho - p}{2}\right) \tag{3.5}$$

$$\left(2\frac{\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right) = 0 \tag{3.6}$$

(3.6) implies a constraint among the scale factors and can be integrated to yield a relation between them. The unit scale factor a and the volume scalar V are defined in terms of scale factors A, B, and C as

$$a^3 = V = ABC \tag{3.7}$$

and then the Hubble parameter H is defined in terms of the unit scale factor as

$$H = \frac{\dot{a}}{a} \tag{3.8}$$

We consider the following directional Hubble parameters

$$H_1 = \frac{\dot{A}}{A}, \qquad H_2 = \frac{\dot{B}}{B}, \qquad H_3 = \frac{\dot{C}}{C} \tag{3.9}$$

From (3.8) and (3.9), we lead to

$$H = \frac{1}{3}(H_1 + H_2 + H_3) \tag{3.10}$$

Let $\theta_{\mu v}$ denote the expansion tensor, θ the expansion scalar, σ the shear scalar, and q the deceleration parameter. Accordingly, some kinematic quantities of the universe are defined as

$$\theta = 3H = 3\left[\frac{1}{3}(H_1 + H_2 + H_3)\right] = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \tag{3.11}$$

$$3\sigma^2 = (\theta_{11} + \theta_{22} + \theta_{33})^2 - (\theta_{11}\theta_{22} + \theta_{22}\theta_{33} + \theta_{11}\theta_{33}) \tag{3.12}$$

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 \tag{3.13}$$

The deceleration parameter q provides insights into the expansion behavior of the universe: $q > 0$ indicates decelerated expansion, $q = 0$ corresponds to a constant expansion rate, $-1 \leq q < 0$ implies accelerated expansion, and $q < -1$ represents super-exponential expansion.

The anisotropy parameter is defined in terms of the shear scalar and Hubble parameter as

$$\Delta = \frac{2}{3}\frac{\sigma^2}{H^2} \tag{3.14}$$

In the present study in order to find a tractable solution, we assume that the expansion scalar θ is proportional to the shear scalar σ. This widely adopted assumption in anisotropic cosmology studies simplifies the field equations, making them analytically solvable in the framework of $f(R,T)$ gravity. In addition this approach has been successfully used in other anisotropic models within modified gravity frameworks, such as $f(R)$ or $f(R,T)$, to obtain exact solutions. No additional symmetries or constraints beyond this proportionality are imposed in the model.

4. Resolution of the Gravitational Field Equations

Integration of (3.6) yields the following relation among the scale factors

$$A^2 = kBC \tag{4.1}$$

where k is a constant of integration, without loss of generality, we set $k = 1$. Using (4.1) and adopting a linear barotropic equation of state (EoS)

$$p = w\rho \tag{4.2}$$

where w $(-1 \leq w \leq 1)$ is a constant EoS parameter, from (3.2) – (3.6) become

$$-2\frac{\dot{B}}{B}\frac{\dot{C}}{C} - \frac{\dot{B}^2}{2B^2} - \frac{\dot{C}^2}{2C^2} + \frac{3}{BC} = (8\pi + \lambda)\rho + \frac{\lambda}{2}(1-w)\rho \tag{4.3}$$

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} - \frac{1}{BC} = (8\pi + \lambda)w\rho - \frac{\lambda}{2}(1-w)\rho \tag{4.4}$$

$$\frac{\ddot{B}}{2B} + 3\frac{\ddot{C}}{2C} + \frac{\dot{C}^2}{4C^2} - \frac{\dot{B}^2}{4B^2} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} - \frac{1}{BC} = (8\pi + \lambda)w\rho - \frac{\lambda}{2}(1-w)\rho \tag{4.5}$$

$$3\frac{\ddot{B}}{2B} + \frac{\ddot{C}}{2C} - \frac{\dot{C}^2}{4C^2} + \frac{\dot{B}^2}{4B^2} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} - \frac{1}{BC} = (8\pi + \lambda)w\rho - \frac{\lambda}{2}(1-w)\rho \tag{4.6}$$

In order to solve the system of (4.3) - (4.6), we assume that the expansion scalar and the shear scalar are proportional as:

$$B = C^n \tag{4.7}$$

where n is a constant and $n \neq \pm 1$.

Subtracting (4.5) from (4.6) and integrating the equation obtained, we find the scale factor C, k_1 and k_2 being constants of integration, as

$$C = \left[\frac{3}{2}(n+1)(k_1 t + k_2)\right]^{\frac{2}{3(n+1)}} \qquad (n \neq -1) \tag{4.8}$$

Assuming and using (4.8) in (4.7), we obtain B as

$$B = \left[\frac{3}{2}(n+1)(k_1 t + k_2)\right]^{\frac{2n}{3(n+1)}} \tag{4.9}$$

Then, from (4.1), (4.8), and (4.9), yield

$$A = \left[\frac{3}{2}(n+1)(k_1 t + k_2)\right]^{\frac{1}{3}} \tag{4.10}$$

Substituting (4.8)–(4.10) into the definition of volume, we obtain that the mean scale factor a becomes

$$a = \left[\frac{3}{2}(n+1)T\right] \tag{4.11}$$

where T the trace of the energy-momentum tensor $T_{\mu v}$ is given by $T = k_1 t + k_2$.

The Hubble parameters, expansion scalar, shear scalar, deceleration parameter, and anisotropy parameter of the model are obtained respectively as:

$$H_1 = \frac{k_1}{3T} \tag{4.12}$$

$$\frac{H_2}{n} = H_3 = \frac{2}{n+1} H_1 \tag{4.13}$$

$$\theta = 3H = \frac{k_1}{T} \tag{4.14}$$

$$\sigma = \frac{k_1}{3T}\left(\frac{n-1}{n+1}\right) \tag{4.15}$$

$$q = 0 \tag{4.16}$$

$$\Delta = \frac{2}{3}\frac{(n-1)^2}{(n+1)^2} \tag{4.17}$$

4.1. Physical Properties of the Model

We find the energy density from (4.3) and (4.4). By substituting (4.8) and (4.9) into (4.3), and into (4.4), we find the energy density as, respectively;

$$\rho = \frac{\sqrt[3]{96}}{\left((n+1)T\right)^{\frac{2}{3}}(\lambda w + \lambda + 16\pi)} \tag{4.18}$$

$$\rho = -\frac{2\sqrt[3]{4}}{(3(n+1)T)^{\frac{2}{3}}(3\lambda w + 16\pi w - \lambda)} \tag{4.19}$$

For the energy densities (4.18) and (4.19) to be equal to each other, we obtain the following condition on λ,

$$\lambda = -\frac{8\pi(3w+1)}{5w-1} \qquad (5w-1 \neq 0) \tag{4.20}$$

If the condition holds, we see that (4.19) and (4.20) give us the energy density of our model as

$$\rho = -\frac{(5w-1)}{(12(n+1)T)^{\frac{2}{3}}\,\pi(w^2-2w+1)} \qquad (w \neq 1) \tag{4.21}$$

For ρ to be physically acceptable (i.e., positive), the equation of state parameter w must satisfy the inequality $-1 \leq w < \frac{1}{5}$.

Using (4.8)-(4.10), the Bianchi type-V cosmological model in (3.1) takes the form

$$ds^2 = -dt^2 + \left[\frac{3}{2}(n+1)T\right]^{\frac{2}{3}} dx^2 + \left[\frac{3}{2}(n+1)T\right]^{\frac{4n}{3(n+1)}} (e^{2x})dy^2 + \left[\frac{3}{2}(n+1)T\right]^{\frac{4}{3(n+1)}} e^{2x}dz^2) \tag{4.22}$$

4. Results and Discussion

In the present study, we have investigated a Bianchi type-V cosmological model filled with a perfect fluid in the context of $f(R,T)$ gravity theory. We have considered the $f(R,T)$ functional form as $f(R,T) = R + \lambda T$, where λ is a coupling constant. An exact analytical solution to the modified field equations was obtained by assuming the relation $B = C^n$ where $n \neq \pm 1$ is a real constant. The corresponding physical and geometrical parameters were explicitly derived. Additionally, to examine the dynamical behavior of the model and the evolution of the universe, the time evolution of the average scale factor $a(t)$, energy density $\rho(t)$, Hubble parameter $H(t)$, shear scalar $\sigma(t)$, and deceleration parameter $q(t)$, were graphically analyzed and interpreted.

Furthermore, the coupling parameter λ, derived as a function of the equation of state parameter w, introduces additional physical constraints into the model. Specifically, the analytical form λ, which is given in (4.20), exhibits a singularity at $w = -1$, indicating a breakdown of the theory in the presence of a cosmological constant-like fluid. In addition, requiring the positivity of the energy density imposes the bound $-1 \leq w < \frac{1}{5}$. These theoretical limitations delineate the range of physically viable models within the considered $f(R,T)$ framework and are essential for consistent interpretation of dynamical behavior in the context of observational cosmology.

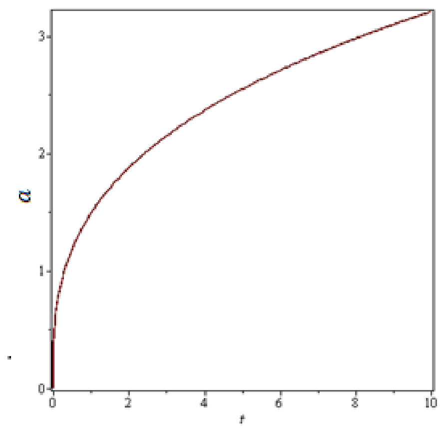

Figure 1. Plot of average scale factor a versus cosmic time t with $n = 1.2, k_1 = 1$, and $k_2 = 0$

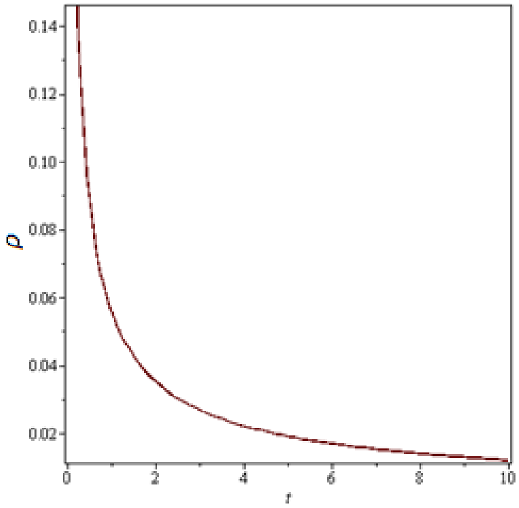

Figure 2. Plot of energy density ρ versus cosmic time t for $w = -\frac{2}{3}$, with $n = 1.2, k_1 = 1$, and $k_2 = 0$

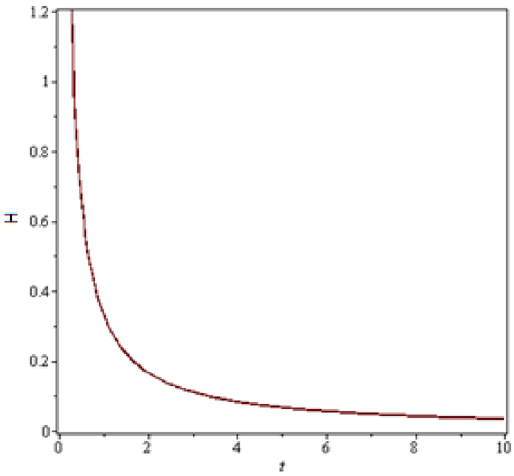

Figure 3. Plot of Hubble parameter H versus cosmic time t for $w = -\frac{2}{3}$, with $n = 1.2, k_1 = 1$, and $k_2 = 0$

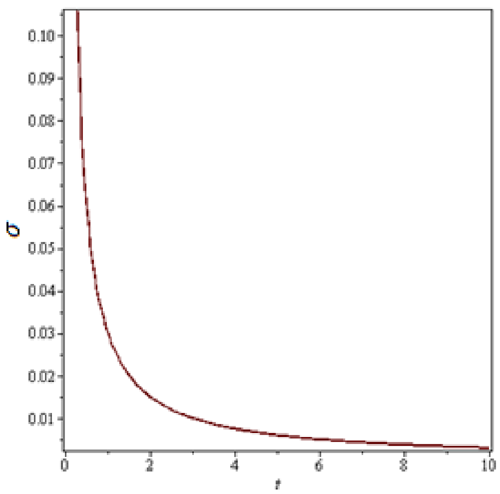

Figure 4. Plot of shear scalar σ versus cosmic time t for $w = -\frac{2}{3}$, with $n = 1.2, k_1 = 1$, and $k_2 = 0$

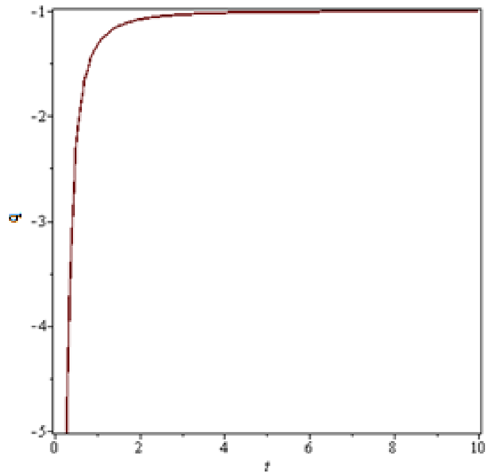

Figure 5. Plot of deceleration parameter q versus cosmic time t for $w = -\frac{2}{3}$, with $n = 1.2, k_1 = 1$, and $k_2 = 0$

Figure 1 depicts the evolution of the average scale factor $a(t)$ concerning cosmic time. It is evident that $a(t)$ is a monotonically increasing function of t, vanishing at $t = 0$. This behavior signifies an expanding universe that originates from a Big Bang-type singularity. Figure 2 illustrates the evolution of the energy density $\rho(t)$. The graph shows that $\rho(t)$ is initially extremely large at $t = 0$, and asymptotically approaches zero as approaches infinity; $t \to \infty$. This implies a rapidly decreasing energy content in the universe as it evolves. Furthermore, for the chosen values of the equation of state parameter, $w = -\frac{2}{3}$. The isotropic pressure is found to be negative, which aligns with dark energy-like behavior. As shown in Figure 3, the Hubble parameter $H(t)$ remains positive, signifying expansion, but decreases with time and tends to zero as t approaches infinity. This indicates that the expansion rate of the universe is high during early times and gradually slows down as the universe ages. Figure 4 presents the variation of the shear scalar $\sigma(t)$ with time. The plot reveals that $\sigma(t)$ is positive and initially large at $t = 0$, but diminishes with cosmic time and asymptotically approaches zero $t \to \infty$, signifying that the anisotropic effects weaken as the universe expands, although they never completely vanish.

In (4.16), the deceleration parameter is found to be constant as $q = 0$, which corresponds to a uniform expansion rate. This behavior represents a critical boundary separating decelerating ($q > 0$) and accelerating ($q < 0$) cosmological regimes and can be interpreted as a transitional state. However, Figure 5 shows the deceleration parameter $q(t)$ to be negative and evolving with time, indicating an accelerating expansion. This apparent discrepancy may be attributed to the limitations of the analytical approximation or specific assumptions made in deriving (4.16). As time progresses, $q(t)$ asymptotically approaches the value -1 as $t \to \infty$, which is characteristic of a super-exponential (de Sitter-like) expansion. This behavior is in qualitative agreement with recent observational data that suggests a late-time acceleration of the universe.

On the other hand, the anisotropy parameter Δ, given by (4.17), remains constant. The invariance of the anisotropy parameter Δ over cosmic time implies that the model preserves a non-vanishing level of anisotropy throughout its evolution. This sustained anisotropic feature, despite the monotonic decay of the shear scalar, suggests that directional dependence in the expansion dynamics remains embedded in the spacetime geometry. This constant anisotropic behavior is consistent with certain Bianchi-type models in modified gravity frameworks, including Bianchi types V and VI_0 in $f(R,T)$, $f(R)$, and scalar–tensor theories, where anisotropy persists throughout cosmic evolution [25–29]. This indicates that although the shear scalar decreases over time, the model retains a persistent anisotropic character. In other words, the universe does not evolve toward a state of isotropy. This behavior implies a sustained directional dependence in the expansion rate, likely arising from the underlying geometry of the Bianchi type-V spacetime, which is inherently anisotropic and does not fully isotropize under the conditions assumed in this study.

5. Conclusion

This study presents exact analytical solutions to the modified field equations within the framework of $f(R,T) = R + \lambda T$ gravity for a Bianchi type-V cosmological model filled with a perfect fluid. By adopting the physically motivated assumption that the shear scalar is proportional to the expansion scalar, the model describes a universe that originates from a Big Bang-type initial singularity and undergoes a super-exponential (de Sitter-like) expansion. Throughout its evolution, the model retains a non-vanishing level of anisotropy, as indicated by the constant anisotropy parameter, despite the monotonic decay of the shear scalar. The energy density decreases with cosmic time, and both the Hubble and shear scalars tend asymptotically toward zero. The deceleration parameter evolves toward $q = -1$ signifying late-time acceleration. These results are in qualitative agreement with current observational data and reinforce the viability of anisotropic models within modified gravity frameworks.

A key novelty of this work lies in the use of a proportionality condition between the expansion scalar θ and the shear scalar σ, which, without invoking additional symmetries, leads to tractable exact solutions within the modified field equations. This assumption has been employed in earlier literature [30,31] but is here applied in the specific framework of $f(R,T)$ gravity for the Bianchi type-V spacetime, resulting in a sustained anisotropy behavior.

Future studies may focus on extending the model by considering more general forms of the $f(R,T)$ function, including nonlinear or multiplicative couplings, or by incorporating additional physical components such as bulk viscosity, scalar fields, or magnetized fluids. Moreover, comparative analyses of different Bianchi-type geometries under varying initial conditions and kinematical constraints could offer deeper insights into anisotropy-driven dynamics in the early universe. Additionally, confronting such extended models with observational datasets—such as cosmic microwave background (CMB) anisotropies and Type Ia supernovae—would be instrumental in constraining free parameters and assessing the empirical consistency of $f(R,T)$ cosmologies.

Author Contributions

The author read and approved the final version of the paper.

Conflict of Interest

The author declares no conflict of interest.

Ethical Review and Approval

No approval from the Board of Ethics is required.

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