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Orbital Period Variation of the Low Mass Ratio Contact Binary HN UMa: O–C Analysis Based on TESS, SuperWASP and Ground Data

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Abstract − The orbital period (OP) variation of the low-mass ratio contact HN Ursae Majoris (UMa) stellar system has been studied in detail for the first time using data from the Transiting Exoplanet Survey Satellite (TESS) and the Super Wide-Angle Search for Planets (SuperWASP) as well as data collected from the literature. We focused on determining the eclipse times (ET) from the light curves (LC) and constructed the O–C diagram from these times. The results obtained indicate the presence of OP variations in the system, where the OP of HN UMa decreases, and this ratio is calculated as $dP/dt = -1.74 \times 10^{-7}$ days/year. Energy and material transfer from the more mass star to the less mass one was proposed as the reason for the OP decrease, and this ratio was obtained as 3.03×10^{-8} $M_{\odot}$/year. Furthermore, a sinusoidal variation was also observed, and it was hypothesized that a possible third object or magnetic activity could cause this variation. The J_s/J_o ratio for HN UMa was calculated as 0.215, and the values of q_{ins} and a_{ins} were determined as 0.036 and 2.21, respectively. In light of these findings, it can be said that the HN UMa system remains stable for the time being. The findings are likely to contribute to the understanding of the evolutionary processes in low-mass ratio binary systems.

Keywords − *W UMa type systems, orbital period change, energy and material transfer, HN UMa*

1. Introduction

Contact binaries (CB) systems (known as W UMa) are short-period eclipsing binaries in which both components share a common convective envelope [1]. These systems are classified into A- and W-subtypes [2]. In A-type systems, the more massive and hotter star is eclipsed at primary minimum, whereas in W-type systems the less massive component appears hotter at the surface due to energy transfer. The transfer of material and energy in these systems makes evolutionary processes highly complex, and systems with low mass ratios ($q = M_2/M_1 < 0.25$) pose essential questions for theoretical models in particular. The origin, stability, and future evolution of low-mass-ratio W UMa systems remain poorly understood [3]. The mass–radius (M–R) and mass–luminosity (M–L) diagrams for W- and A-type systems suggest significant energy transfer from the more mass star to the less mass one, indicating a departure from thermal equilibrium and the presence of thermal relaxation oscillations (TRO) [3]. In such systems, the fact that the secondary component is almost as large as the primary component indicates critical situations within the framework of Roche geometry and increases the probability of instability [4]. The photometric and spectroscopic properties observed in these systems provide crucial clues about the structure and evolution of binary stars in contact [5]. Therefore, detailed photometric and spectroscopic analyses of such systems can provide information on the

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onset, duration, and consequences of the contact phase. Both observational evidence and theoretical analysis regarding thermal and hydrostatic balance indicate a substantial energy exchange between the two components [6]. Utilizing various databases to derive key physical characteristics contributes to a deeper understanding of the evolutionary processes of binary star systems [7].

HN UMa (TIC 450333023, TYC 3010-2322-1, GSC 03010-02322, Gaia DR3 763798502676762752, BD+38 2220, HIP 55030) is a W UMa-type variable binary star with a period of 0.3826 days and was discovered by Hipparcos [8]. Short orbital period HN UMa, A-type W UMa, and spectral type F8V were identified by [9]. Radial velocity (RV) data of HN UMa were obtained by [10]. The Hipparcos light curve analysis was performed by [11], and the mass ratio (q) was calculated as 0.1 and the inclination (i) as $52^0.5$. Light curve (LC) analysis, performed in four different filters (including *UBVR*), and basic astrophysical parameters were determined by [12]. In the same study, [12] reported the temperatures of the first and second components of HN UMa as T_1 = 6100 K and T_2 = 6082 (77) K, respectively. The LC was performed simultaneously with the RV by [13] and reported that HN UMa has a low mass ratio. At the same time, some basic astrophysical parameters of HN UMa were calculated by [14] as M_1=1.78 $M_\odot$, M_2=0.18 $M_\odot$, R_1=1.58 $R_\odot$, R_2=0.64 $R_\odot$, and a=2.78 $R_\odot$ by analyzing the new RV (and also expressed as a mass ratio of about 0.1 in the same study). No study of the orbital period (OP) of HN UMa has been published in the literature.

In this study, we present the first O–C analysis of HN UMa based on ET extracted from TESS and SuperWASP observations, complemented with literature ET. By analyzing the O–C diagram, we investigate both the secular and cyclic period variations of the HN UMa and discuss their possible physical causes.

2. Datasets

Observational data were obtained from the TESS [15] and SuperWASP [16] archives to study the OP variation of the HN UMa system, a low-mass ratio contact binary (LMRCB) star. HN UMa's data collected by TESS has been archived within the Mikulski Archive for Space Telescopes (MAST) [17]. The TESS satellite provides high-precision light curves of short-period variable stars through wide-field photometry [15]. TESS and SuperWASP data were utilized in this study, with details about these datasets provided in Table 1.

Table 1. TESS and SuperWASP observation data information

Provenance Name	Waveband	Project– Mission	Start Time	Exposure Length (s)	Min.–Max. Wavelength (nm)	Sector
TESS-SPOC	Optical	TESS–HLSP	2020-02-19	1800	600-1000	22
TESS-SPOC	Optical	TESS–HLSP	2022-01-28	600	600-1000	48
SuperWASP	Optical	SuperWASP-V+R (Filter)	2004-05-01	~30	400-700	-

The most basic parameter for OP analysis is the ET, and the ET calculated from TESS and SuperWASP data are presented in Table 2, along with their associated errors. A total of ten ET in different years (five first and five second eclipses) were calculated from the TESS data. At the same time, eighteen ET in different years were obtained from SuperWASP data: nine first and nine second ET. The ET were calculated using a code written to perform this calculation, which is based on the least squares method. The template was derived from a symmetric polynomial fit and applied to each ET individually. Uncertainties were estimated using bootstrap resampling with 100 iterations. Moreover, previously published ET were also collected from the O–C Gateway database [18]. In total, 58 eclipse timings spanning about 35 years were used in this study. TESS data are in Barycentric Julian Date (BJD_{TDB}), and SuperWASP and literature data are in Heliocentric Julian Day (HJD), and HJD data were converted to BJD_{TDB}. The HJD2BJD calculator [19] was utilised to effect the conversion to BJD_{TDB}.

Table 2. ET derived from TESS and SuperWASP datasets were incorporated into the O–C analysis

BJD_{TDB} (2400000+)	Error	Type (P/S)	Mission
58900.6248	0.0019	S	TESS
58900.8076	0.0022	P	TESS
58912.1037	0.0062	S	TESS
58925.8701	0.0019	S	TESS
58926.0677	0.0015	P	TESS
59610.3597	0.0001	S	TESS
59610.5511	0.0001	P	TESS
59620.8876	0.0001	P	TESS
59635.6108	0.0001	S	TESS
59635.8027	0.0001	P	TESS
53827.5677	0.0011	S	SuperWASP
53837.5067	0.0011	S	SuperWASP
53853.3877	0.0004	P	SuperWASP
54092.7128	0.0003	S	SuperWASP
54103.6228	0.0008	P	SuperWASP
54111.6558	0.0006	P	SuperWASP
54121.5958	0.0014	P	SuperWASP
54140.5358	0.0008	S	SuperWASP
54150.4858	0.0015	S	SuperWASP
54160.4298	0.0057	S	SuperWASP
54165.6028	0.0008	P	SuperWASP
54192.3848	0.0011	P	SuperWASP
54195.4428	0.0004	P	SuperWASP
54202.5218	0.0003	S	SuperWASP
54220.5098	0.0021	S	SuperWASP
54221.4598	0.0009	P	SuperWASP
54574.4208	0.0008	S	SuperWASP
54575.3808	0.0038	P	SuperWASP

3. Orbital Period Analysis

OP analysis plays an essential role in understanding the evolutionary processes of CB. The contact ratios of the systems will increase as their OP decrease, and eventually they will merge into a single rapidly rotating star [20]. In this context, the study of orbital period variations provides direct clues to the system's evolutionary state. The first step in the OP analysis is to collect the minimum light times of the system, covering as long a time interval as possible. These times can be obtained from visual, photoelectric, CCD, or modern photometric observations, such as those from TESS and SuperWASP, published in the literature. For each ET, a weighted analysis is performed, taking into account observational uncertainties. Observed-Calculated (O–C) analysis is the main tool to reveal the variations in the OP of the system. The O–C diagram of the system is created using all collected ET. In the O-C analysis, the O–C value is calculated for each observed ET using in (3.1).

$$O - C = T_{obs} - (T_0 + E \times P) \quad (3.1)$$

Here, T_{obs}: Observed ET, T_0: Reference ET, P: Period and E: Epoch number. The O–C values obtained as a result of these calculations were plotted against time to visualize the possible changes in the OP. There are many studies in the literature on OP change using this method (e.g. [21], [22]). If the O–C graph shows a linear curve, the OP is constant, and the system is stable. Parabolic O–C curves indicate the presence of mass transfer or mass loss in the system. In such a case, the O-C curve is represented by applying a second order polynomial to it and is represented by in (3.2).

$$MinI\ (BJD_{TDB}) = T_0 + P \times E \pm Q \times E^2 \tag{3.2}$$

In (3.2) Q: represents the quadratic term and if its value is positive it means that the OP increases and if its value is negative it means that the OP decreases. The O–C analysis for HN UMa was conducted for the first time, and the light elements for HN UMa are presented in (3.3). The parameters obtained as a result of the analysis are listed in Table 2 and the application of in (3.2) is given as in (3.3). The variations in the OP of the HN UMa were examined utilizing the software developed by [23].

$$MinI\ (BJD_{TDB}) = 2456010.5638\,(12) + 0.3826102\,(2) \times E - 0.91\,(3) \times 10^{-10} \times E^2 \tag{3.3}$$

If fluctuations are observed in the diagram, the light-time effect (LITE) should be considered, indicating the presence of a third object. If irregular fluctuations are present, it may be due to magnetic activity. The equations pertaining to these fluctuations are provided in comprehensive detail by [7], [21], and [22]. The OP analysis for HN UMa showed a sinusoidal variation (see Figure 1 and Figure 2), and the possible causes of the sinusoidal variation, i.e., whether it is due to a possible third star or magnetic activity, are discussed in detail in Section 4. To calculate the value of the orbital period increase or decrease, in (3.4), which is widely known in the literature, is used.

$$\frac{\Delta P}{P} = \frac{2 \times Q}{P} \tag{3.4}$$

The OP analysis for HN UMa indicates that the OP of HN UMa is decreasing (see Figure 1, Table 3), with a calculated decrease rate of 0.015 seconds per year. As illustrated in Figure 2, a sinusoidal change was observed, and the amplitude of this change was determined as 0.016 (1) day.

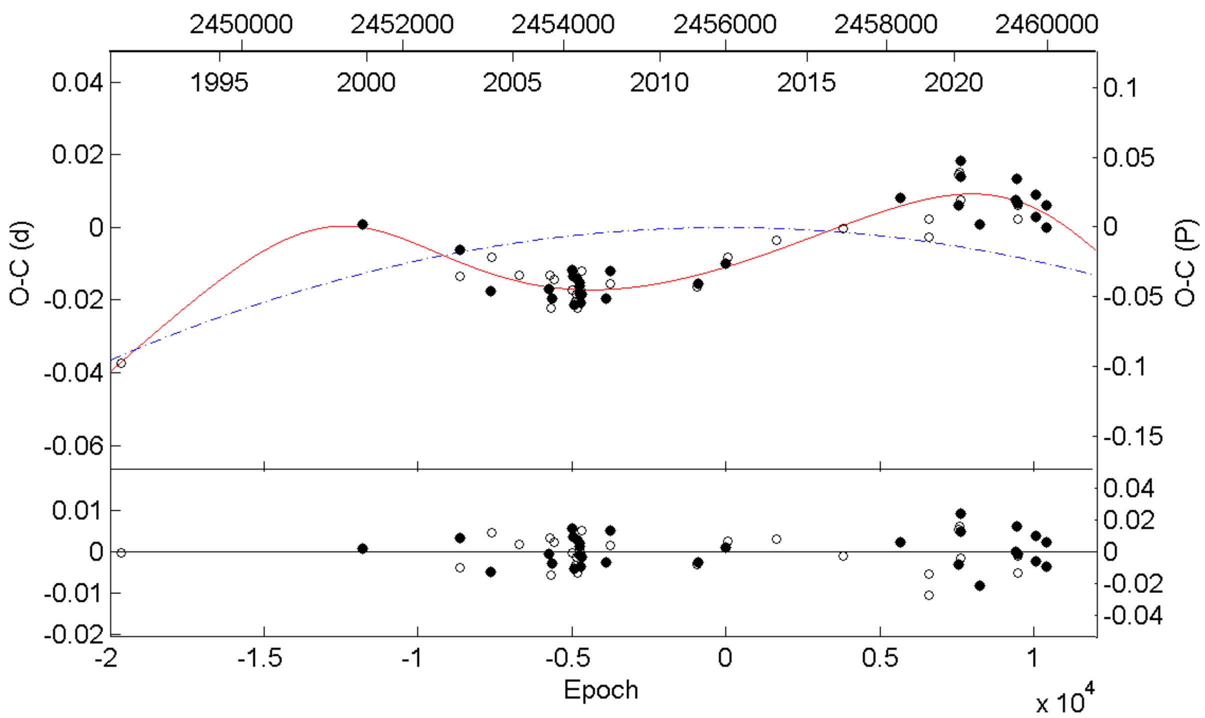

Figure 1. O–C diagram for HN UMa presents a continuous red curve that represents a sinusoidal fluctuation, whereas the dashed blue curve reflects a parabolic variation associated with HN UMa

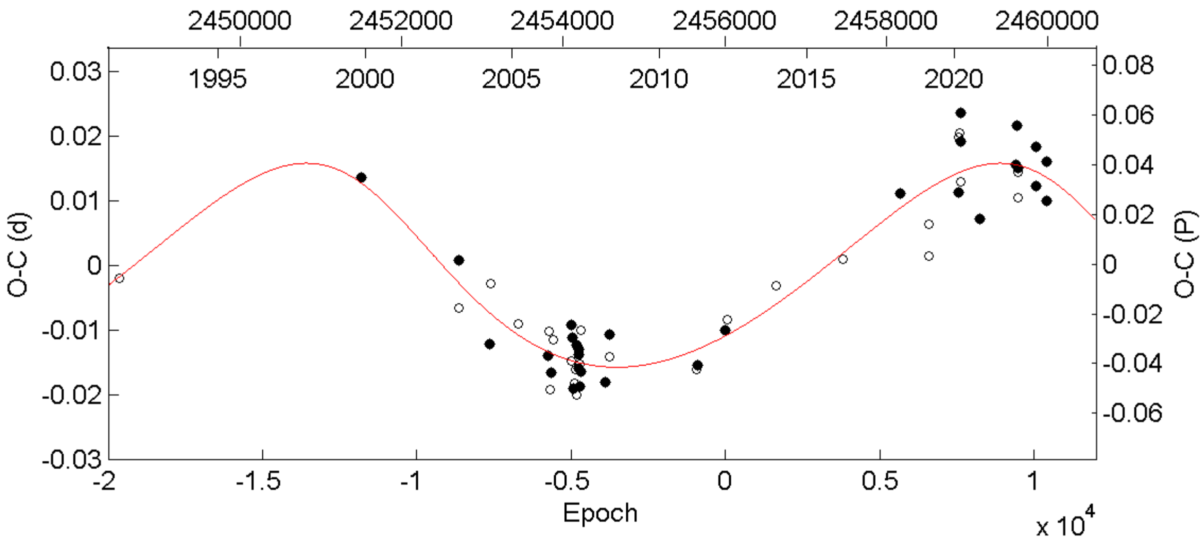

Figure 2. Periodic fluctuation observed in the O-C diagram of HN UMa is modeled through a theoretical curve (solid red line)

4. Results and Discussion

In this research, we performed a detailed investigation of the OP variations of the LMRCB system HN UMa for the first time. A total of 58 ET were analyzed, including 10 from TESS photometry, 18 from SuperWASP archival data, and previously published studies. Utilizing these high-precision ET, which collectively span approximately 35 years, we identified a decrease in the system's OP. In W UMa systems with low mass ratio, if the OP variation is negative, it can be said that the systems are evolving towards dynamical instability. Therefore, OP analyses of such systems are critical to understand not only the evolutionary state of the system but also its termination scenarios.

Table 3. Parameter values and corresponding errors for HN UMa were extracted based on an OP-based analysis

Parameters	HN UMa
T_o (BJD_{TDB} +2400000)	56010.5638 (12)
P_{orb} (day)	0.3826102 (2)
Q (day)	-0.91 (3) × 10^{-10}
dP/dt (days/year)	-1.74 × 10^{-7}
dM/dt ($M_\odot$/year)	3.03 × 10^{-8}
a_{12} sin i (AB)	2.77 (12)
e	0.24 (5)
ω (deg)	134 (9)
A_3 (day)	0.016 (1)
P_{12} (year)	24 (1)
$f(m_3)(M_\odot)$	0.038 (6)
$m_3(M_\odot)$ (for i=90)	0.66

Table 4. Applegate model parameters for HN UMa were subsequently derived

Parameters	HN UMa
P_{mod} (year)	24
$\Delta P/P$	1.15×10^{-5}
ΔJ (erg s^{-1})	7.84×10^{47}
$\Delta\Omega/\Omega$	0.002
ΔE (erg)	4.98×10^{41}
I_s (g cm^2)	2.47×10^{54}
$\Delta L/(L_1)$	0.19
B (kG)	14.2
ΔQ_1 (g cm^2)	1.9×10^{50}
ΔQ_2 (g cm^2)	1.9×10^{49}

The parabolic fit to the O–C data yielded a coefficient indicating an OP decrease at a rate of 1.50 seconds per century. Assuming conservative mass transfer within the binary, this decrease can be attributed to the transfer of material from the more-mass star to the less mass star, with a calculated mass transfer rate of dM/dt=3.03 · 10^{-8} $M_{\odot}$/year. Additionally, a sinusoidal modulation superimposed on the parabolic trend in the O–C diagram was analyzed using the LITE model (Model parameters are listed in Table 3). This analysis supports the presence of a tertiary star orbiting the binary with a period of approximately 24 (1) years. The minimum mass of this third object was estimated to be M_3=0.66 $M_{\odot}$. Interestingly, photometric analyses in the literature have not revealed a third light (L_3) contribution, suggesting that this object may be faint in the optical band despite its mass exceeding that of the system's secondary star. This opens the possibility that the third object could be a compact object such as a white dwarf or a faint main-sequence star. Alternatively, the observed cyclical variations could arise from magnetic activity cycles, as described by the Applegate mechanism [24], which links period modulations to changes in stellar magnetic quadrupole moments. Parameters related to this model were computed (see Table 4) and found to be within the expected theoretical ranges suggested by [24]. However, the calculated change in the quadrupole moment, based on the formula $\Delta P/P = -9\Delta Q/Ma^2$ ([24]), is significantly below the typical range of 10^{51}-10^{52} g cm² suggested in the literature ([25-27]), as can be seen in Table 4. This discrepancy suggests that magnetic activity is unlikely to be the sole cause of the observed periodicity. In addition, no asymmetries were observed in any LC analysis in the literature, indicating that magnetic activity is unlikely. When all these results are evaluated, the cause of the sinusoidal variation can be interpreted as a possible third star and not magnetic activity. Moreover, Figure 1 and Figure 2 show a regular distribution of the sinusoidal variation, which is more likely to be explained by a possible third star. In addition, as can be seen in Table 2, the eccentricity (e) and argument of periastron (w) values of HN UMa are given, which also strengthen the possibility of a possible third star.

Assuming the conservation of both angular momentum and mass, the thermal timescale (τ_{th}) of HN UMa was determined using the formula $\tau_{th} = GM^2/RL$. Based on this expression, the thermal timescale for HN UMa was estimated to be approximately 1.8×10^7 years. According to the CB age catalog compiled by [28] and the statistical distribution analysis carried out by [29], the majority of CB systems have ages falling within the 4–5 Gyr range. In addition, [30] found that A-type and W-type CB are 4.4 Gyr and 4.6 Gyr, respectively. In light of these comparative age estimates, it can be inferred that HN UMa is likely in a state of thermal equilibrium. Furthermore, the small temperature disparity between the components of HN UMa lends additional support to this conclusion. The mass-to-thermal-timescale ratio (M/τ_{th}) was calculated to be approximately 9.8×10^{-8} $M_{\odot}$/year, which aligns with the expected mass transfer rates associated with orbital period variations.

According to Darwin's instability criterion, a CB system is expected to undergo a merger if the spin angular momentum (J_s) exceeds one-third of the orbital angular momentum (J_o), as described by [31] and further discussed by [32]. For HN UMa, the ratio J_s/J_o was computed using (4.1) from [32], providing insight into the system's dynamic stability.

$$\frac{J_s}{J_o} = \frac{q}{1+q}[(k_1 r_1)^2 + (k_2 r_2)^2 q] \tag{4.1}$$

In the referenced formulation, $r_{1,2}$ and $k_{1,2}$ represent the relative radii and the gyration radii of the two stellar components, respectively. From (4.1), the squared gyration radii were taken to be k_1^2 = k_2^2 = 0.06, based on the values reported by [33, 34]. This value is widely used in the literature for late spectral main-sequence stars that form components of W UMa systems [29, 33, 34]. Stellar structure models show that the gyration radii of F-K spectral main sequence stars are typically in the range 0.05-0.07, and therefore adopting 0.06 provides a physically realistic and consistent approximation for CB. Using these parameters, the spin-to-orbital angular momentum ratio (J_s/J_o) for HN UMa was found to be 0.215, indicating that the system lies within the stable regime. Furthermore, the critical separation parameter (a_{ins}) for HN UMa was calculated to be 2.21. The critical mass ratio (q_{ins}), derived using the empirical relations established by [35], was found to range between 0.03 and 0.04. Since both a_{ins} and q_{ins} values calculated for HN UMa are smaller than the a and q values given in the first part of this study, it can be concluded that HN UMa remains dynamically stable.

In summary, the observed long-term decrease in the OP of HN UMa is most plausibly attributed to a conservative mass transfer process occurring from the more mass star to its less mass one. The cyclical variations detected in the O–C diagram are most consistently interpreted as the influence of a tertiary object within the system; however, the possibility of magnetic activity as a contributing factor cannot be excluded entirely without additional supporting observations. Future studies incorporating high-resolution spectroscopy and multi-band photometry will be essential for better determining the properties of the potential third object and assessing the extent to which magnetic phenomena may be affecting the system's orbital evolution.

Author Contributions

The author read and approved the final version of the paper.

Conflict of Interest

The author declares no conflict of interest.

Ethical Review and Approval

No approval from the Board of Ethics is required.

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