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Unbiased ratio estimators of the mean in stratified ranked set sampling

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Abstract

Stratified ranked set sampling (S_tRSS) combines the advantages of stratification and ranked set sampling (RSS). In this paper, we propose several unbiased ratio type estimators using S_tRSS, when population mean of the auxiliary variable is known. The variances of the proposed unbiased ratio-type estimators are obtained to first degree of approximation. In simulation study the proposed estimators are more efficient as compared to other competitor estimators using Percentage Relative Efficiency (PRE), Percentage Relative Bias (PRB) and Percentage Relative Root Mean Square Error ($PRRMSE$).

Keywords: Stratified ranked set sampling, ratio estimators, relative efficiency.

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1. Introduction

The method of ranked set sampling (RSS) was first introduced by Mclntyre [12] as a cost efficient alternative to simple random sampling (SRS) method for those situations where measurement of the units are expensive or difficult to obtain but ranking of units according to the variable of interest is relatively simple and cheap. Section 2 describes RSS design. Takahasi and Wakimoto [21] have provided the necessary mathematical theory of RSS and have showed that the sample mean under RSS is an unbiased estimator of the finite population mean, and is more precise than the sample mean estimator under SRS.

Several population parameters, namely, coefficient of variation (C_x), coefficient of kurtosis (β_{2x}), coefficient of correlation (ρ) etc., play a significant role, when they are

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known, in the estimation of the finite population mean (see Kadilar and Cingi [5, 6, 7], Kadilar et al. [8] and Upadhyaya and Singh [22]).

Stratified ranked set sampling (S_tRSS) was suggested by Samawi and Muttlak [16] to obtain more efficient estimators for population mean. Using S_tRSS, the performances of the combined and separate ratio estimators was obtained by Samawi and Siam [17]. Mandowara and Mehta [11] have used the idea of S_tRSS to obtain efficient ratio-type estimators. Singh et al. [20] have proposed ratio and product type efficient estimators for population mean under S_tRSS.

Hartley and Ross [4] were the first to propose an unbiased ratio-type estimator for finite population mean in SRS. Later on, Pascual [14] proposed an unbiased ratio type estimator in stratified random sampling. Singh et al. [19] and Kadilar and Cekim [9] suggested Hartley-Ross type unbiased estimators of finite population mean using the auxiliary information, such as the population coefficient of variation (C_x), coefficient of kurtosis (β_{2x}) and the coefficient of correlation (ρ) in SRS. Recently Khan and Shabbir [10] suggested a class of Hartely-Ross type unbiased estimators in RSS. In this paper, we investigate the properties of separate ratio type estimator of the finite population mean based on S_tRSS.

2. Sampling scheme

According to Mclntyre [12], we first choose a small number m as a set size such that one can easily rank the m elements of the population with sufficient accuracy. Let Y and X be study and the auxiliary variables. Then randomly select m^2 bivariate sample units from the population and allocate them into m sets, each of size m. Then each sample is ranked with respect to one of the variables Y or X. Here, we assume that the perfect ranking is done on basis of the auxiliary variable X while the ranking of Y is with possible error. An actual measurement from the first sample is then taken on the unit with the smallest rank of X, together with variable Y associated with smallest rank of X. From second sample of size m, the variable Y associated with the second smallest rank of X is measured. The process is continued until the Y value associated with the highest rank of X is measured from the mth sample. This completes one cycle of the sampling. The process is repeated for r cycles to obtain the desired sample of size $n = mr$ units. Thus in a RSS scheme, a total of m^2r units have been drawn from the population and only mr of them are selected for analysis.

In stratified ranked set sampling (see Samawi and Muttlak [16]), first choose m_h independent random samples from the hth stratum of the population, each of size m_h ($h = 1, 2, ..., L$). Rank the observations in each sample and use RSS procedure to get L independent RSS samples, each of size m_h, to get $m_1 + m_2 + ... + m_L = m$ observations. This completes one cycle of S_tRSS. The whole process is repeated r times to get the desired sample size $n = mr$. To estimate population mean ($\bar{Y}$) in S_tRSS using a ratio estimator, the procedure can be summarized as follows:

- **Step 1:** Select m_h^2 bivariate sample units randomly from the h^{th} stratum of the population.
- **Step 2:** Arrange these selected units randomly into m_h sets, each of size m_h.
- **Step 3:** The procedure of ranked set sampling (RSS) is then applied on each of the sets to obtain the m_h sets of ranked set samples, each of size m_h. Here ranking is done with respect to the auxiliary variable X. These ranked set samples are collected together to form m_h sets, each of size m_h units.
- **Step 4:** Repeat the above steps r times for each stratum to get the desired sample of size $n_h = m_h r$.

For the j^{th} cycle and the h^{th} stratum, the S_tRSS is denoted by $(Y_{h[1:m_h]j}, X_{h(1:m_h)j})$, $(Y_{h[2:m_h]j}, X_{h(2:m_h)j}), \ldots, (Y_{h[mh:m_h]j}, X_{h(mh:m_h)j})$, $(j = 1, 2, \ldots, r)$ and $h = 1, 2, \ldots, L$. Here $Y_{h[i:m_h]j}$ and $X_{h[i:m_h]j}$ are the i^{th} ranked units in the i^{th} sample at the j^{th} cycle of the h^{th} stratum for the study and the auxiliary variables respectively.

Under S_tRSS scheme, the estimator $\bar{y}_{S_tRSS}$, is given by

$$(2.1) \qquad \bar{y}_{S_tRSS} = \sum_{h=1}^{L} P_h \bar{y}_{h[rss]},$$

where $\bar{y}_{h[rss]} = (1/m_h r) \sum_{j=1}^{r} \sum_{i=1}^{m_h} y_{h[i:m_h]j}$ and $P_h = N_h/N$ is the known hth stratum weight. Also, N_h is the hth stratum size and N is the population size. The variance of $\bar{y}_{S_tRSS}$, is given by

$$(2.2) \qquad V(\bar{y}_{S_tRSS}) = \sum_{h=1}^{L} P_h^2 \bar{Y}_h^2 \left(\gamma_h C_{y_h}^2 - W_{y_h}^2\right),$$

where $\gamma_h = (\frac{1}{m_h r})$, C_{y_h} is the coefficient of variation of Y_h, $W_{y_h}^2 = \frac{1}{m_h^2 r \bar{Y}_h^2} \sum_{i=1}^{m_h} \tau_{y_h[i:m_h]}^2$, where $\tau_{y_h[i:m_h]} = (\mu_{y_h[i:m_h]} - \bar{Y}_h)$ and $\mu_{y_h[i:m_h]}$ is an order statistic from some specific distribution.

The aim of this paper is to estimate the population mean $(\bar{Y})$ of a variable of interest under S_tRSS. To estimate the population mean, we proposed the following class of ratio-type estimators.

3. Proposed class of unbiased ratio estimators in S_tRSS

Consider a separate ratio estimator

$$(3.1) \qquad \bar{y}_{LJ(k)} = \sum_{h=1}^{L} P_h \bar{r}_{h[rss]}^{(k)} \bar{X}_h^{(k)}, k = 1, 2, \ldots, 9$$

where $\bar{r}_{h[rss]}^{(k)} = \frac{\sum_{j=1}^{r} \sum_{i=1}^{m_h} r_{h[i:m_h]j}^{(k)}}{m_h r}$, $r_{h[i:m_h]j}^{(k)} = \frac{y_{h[i:m_h]j}}{x_{h(i:m_h)j}^{(k)}}$, $\bar{R}_h^{(k)} = E(\bar{r}_{h[rss]}^{(k)})$, $x_{h(i:m_h)j}^{(1)} = x_{h(i:m_h)j}$, $\bar{X}_h^{(1)} = \bar{X}_h$, $x_{h(i:m_h)j}^{(2)} = x_{h(i:m_h)j} + C_{x_h}$, $\bar{X}_h^{(2)} = \bar{X}_h + C_{x_h}$, $x_{h(i:m_h)j}^{(3)} = x_{h(i:m_h)j} + \beta_{2(x_h)}$, $\bar{X}_h^{(3)} = \bar{X}_h + \beta_{2(x_h)}$, $x_{h(i:m_h)j}^{(4)} = x_{h(i:m_h)j}\beta_{2(x_h)} + C_{x_h}$, $\bar{X}_h^{(4)} = \bar{X}_h\beta_{2(x_h)} + C_{x_h}$, $x_{h(i:m_h)j}^{(5)} = x_{h(i:m_h)j}C_{x_h} + \beta_{2(x_h)}$, $\bar{X}_h^{(5)} = \bar{X}_h C_{x_h} + \beta_{2(x_h)}$, $x_{h(i:m_h)j}^{(6)} = x_{h(i:m_h)j}C_{x_h} + \rho_h$, $\bar{X}_h^{(6)} = \bar{X}_h + C_{x_h} + \rho_h$, $x_{h(i:m_h)j}^{(7)} = x_{h(i:m_h)j}\rho_h + C_{x_h}$, $\bar{X}_h^{(7)} = \bar{X}_h\rho_h + C_{x_h}$, $x_{h(i:m_h)j}^{(8)} = x_{h(i:m_h)j}\beta_{2(x_h)} + \rho_h$, $\bar{X}_h^{(8)} = \bar{X}_h\beta_{2(x_h)} + \rho_h$, $x_{h(i:m_h)j}^{(9)} = x_{h(i:m_h)j}\rho_h + \beta_{2(x_h)}$, $\bar{X}_h^{(9)} = \bar{X}_h\rho_h + \beta_{2(x_h)}$, for each estimator. Here, ρ_h is the coefficient of correlation, C_{x_h} is the coefficient of variation and $\beta_{2(x_h)}$ is the coefficient of kurtosis of the auxiliary variable for the hth stratum.

The bias of $\bar{y}_{LJ(k)}$, is given by

$$B(\bar{y}_{LJ(k)}) = -\sum_{h=1}^{L} \left[P_h \frac{(N_h - 1)}{N_h} S_{r_{h[i:m_h]}^{(k)} x_{h(i:m_h)}^{(k)}} \right], \; k = 1, 2, \ldots, 9$$

where $S_{r_{h[i:m_h]}^{(k)} x_{h(i:m_h)}^{(k)}} = \frac{1}{N_h - 1} \sum_{j=1}^{N_h} (r_{h[i:m_h]j}^{(k)} - \bar{R}_h^{(k)})(x_{h(i:m_h)j}^{(k)} - \bar{X}_h^{(k)})$.

3.1. Theorem. *An unbiased estimator of*

$$S_{r^{(k)}_{h[i:m_h]}x^{(k)}_{h(i:m_h)}} = \frac{1}{N_h-1}\sum_{j=1}^{N_h}(r^{(k)}_{h[i:m_h]j} - \bar{R}^{(k)}_h)(x^{(k)}_{h(i:m_h)j} - \bar{X}^{(k)}_h)$$

is given by

$$s_{r^{(k)}_{h[i:m_h]}x^{(k)}_{h(i:m_h)}} = \frac{n_h}{n_h-1}(\bar{y}_{h[i:m_h]} - \bar{r}^{(k)}_{h(i:m_h)}\bar{x}^{(k)}_{h(i:m_h)}).$$

Proof. We have to prove that $E(s_{r^{(k)}_{h[i:m_h]}x^{(k)}_{h(i:m_h)}}) = S_{r^{(k)}_{[i:m_h]}x^{(k)}_{h(i:m_h)}}$. For fixed i, $j = 1, 2, ..., n_h$, $r^{(k)}_{h[i:m_h]j}$ and $x^{(k)}_{h(i:m_h)j}$ are simple random samples of size n_h.

$$\begin{aligned}
E(s_{r^{(k)}_{h[i:m_h]}x^{(k)}_{h(i:m_h)}}) &= E\left[\frac{n_h}{n_h-1}(\bar{y}_{h[i:m_h]} - \bar{r}^{(k)}_{h(i:m_h)}\bar{x}^{(k)}_{h(i:m_h)})\right],\\
&= E\left[\frac{1}{n_h-1}\sum_{j=1}^{n_h}(r^{(k)}_{h[i:m_h]j} - \bar{r}^{(k)}_{h[i:m_h]})(x^{(k)}_{h(i:m_h)j} - \bar{x}^{(k)}_{h(i:m_h)})\right],\\
&= \frac{1}{n_h-1}E\left[\sum_{j=1}^{n_h} r^{(k)}_{h[i:m_h]j}x^{(k)}_{h(i:m_h)j} - n_h\bar{r}^{(k)}_{h[i:m_h]}\bar{x}^{(k)}_{h(i:m_h)}\right],\\
&= \frac{1}{n_h-1}\left[\sum_{j=1}^{n_h} E(r^{(k)}_{h[i:m_h]j}x^{(k)}_{h(i:m_h)j}) - n_hE(\bar{r}^{(k)}_{h[i:m_h]}\bar{x}^{(k)}_{h(i:m_h)})\right],\\
&= \frac{1}{n_h-1}\left[\frac{n_h}{N_h}\sum_{j=1}^{N_h} r^{(k)}_{h[i:m_h]j}x^{(k)}_{h(i:m_h)j} - n_h\left(Cov(\bar{r}^{(k)}_{h[i:m_h]}, \bar{x}^{(k)}_{h(i:m_h)}) + \bar{R}^{(k)}_h\bar{X}^{(k)}_h\right)\right],\\
&= \frac{n_h}{n_h-1}\left[\frac{1}{N_h}\sum_{j=1}^{N_h} r^{(k)}_{[i:m_h]j}x^{(k)}_{(i:m_h)j} - \bar{R}^{(k)}\bar{X}^{(k)} - \frac{S_{r^{(k)}_{[i:m_h]}x^{(k)}_{(i:m_h)}}}{n_h}\right],\\
&= \frac{n_h}{n_h-1}\left(S_{r^{(k)}_{h[i:m_h]}x^{(k)}_{h(i:m_h)}} - \frac{S_{r^{(k)}_{h[i:m_h]}x^{(k)}_{h(i:m_h)}}}{n_h}\right),\\
&= S_{r^{(k)}_{h[i:m_h]}x^{(k)}_{h(i:m_h)}}.
\end{aligned}$$

□

So bias of $\bar{y}_{LJ(k)}$ can be estimated by

$$\hat{B}(\bar{y}_{LK(k)}) = -\sum_{h=1}^{L} P_h\left[\frac{n_h(N_h-1)}{N_h(n_h-1)}(\bar{y}_{h[i:m_h]} - \bar{r}^{(k)}_{h(i)}\bar{x}^{(k)}_{h(i:m_h)})\right], \quad k = 1, 2, \ldots, 9. \tag{3.2}$$

Thus, a class of unbiased ratio-type estimators of population mean based on S_tRSS is

$$\bar{y}^{(u)}_{LJ(k)} = \sum_{h=1}^{L} P_h\left[\bar{r}^{(k)}_{h[i:m_h]}\bar{X}^{(k)}_h + \frac{n_h(N_h-1)}{N_h(n_h-1)}\left(\bar{y}_{h[i:m_h]} - \bar{r}^{(k)}_{h[i:m_h]}\bar{x}^{(k)}_{h(i:m_h)}\right)\right], \tag{3.3}$$

where $k = 1, 2, \ldots, 9$.

To find the variances of the proposed unbiased estimators, we define the following notations: Let $\bar{y}_{h[i:m_h]} = \bar{Y}_h(1+\delta_{0h})$, $\bar{x}^{(k)}_{h(i:m_h)} = \bar{X}^{(k)}_h(1+\delta_{1h})$, $\bar{r}^{(k)}_{h(i)} = \bar{R}^{(k)}_h(1+\delta_{2h})$,
such that $E(\delta_{ph}) = 0$, $(p = 0, 1, 2)$, $(h = 1, 2, ..., L)$ and
$E(\delta^2_{0h}) = \gamma_h C^2_{y_h} - W^2_{y_h}$, $E(\delta^2_{1h}) = \gamma_h C^2_{x^{(k)}_h} - W^2_{x^{(k)}_h}$,
$E(\delta_{0h}\delta_{1h}) = \gamma_h C_{y_h x^{(k)}_h} - W_{y_h x^{(k)}_h}$, $E(\delta_{1h}\delta_{2h}) = \gamma_h C_{r^{(k)}_h x^{(k)}_h} - W_{r^{(k)}_h x^{(k)}_h}$,
where

$W^2_{y_h} = \frac{1}{m_h^2 r \bar{Y}_h^2} \sum_{i=1}^{m_h} \tau^2_{y_h[i:m_h]}, W_{y_h x_h^{(k)}} = \frac{1}{m_h^2 r \bar{X}_h^{(k)} \bar{Y}_h} \sum_{i=1}^{m_h} \tau_{y_h x_h^{(k)}(i:m_h)},$
$W^2_{x_h^{(k)}} = \frac{1}{m_h^2 r \bar{X}_h^{(k)2}} \sum_{i=1}^{m_h} \tau^2_{x_h^{(k)}(i)}, \quad W_{r_h^{(k)} x_h^{(k)}} = \frac{1}{m_h^2 r \bar{X}_h^{(k)} \bar{R}_h^{(k)}} \sum_{i=1}^{m_h} \tau_{r_h^{(k)} x_h^{(k)}(i:m_h)},$
$\tau_{y_h[i:m_h]} = (\mu_{y_h[i:m_h]} - \bar{Y}_h), \quad \tau_{x_h^{(k)}(i:m_h)} = (\mu_{x_h^{(k)}(i:m_h)} - \bar{X}_h^{(k)}),$
$\tau_{y_h x_h^{(k)}(i:m_h)} = (\mu_{y_h[i:m_h]} - \bar{Y}_h)(\mu_{x_h^{(k)}(i:m_h)} - \bar{X}_h^{(k)})$ and
$\tau_{r_h^{(k)} x_h^{(k)}(i:m_h)} = (\mu_{r_h^{(k)}(i:m_h)} - \bar{R}_h^{(k)})(\mu_{x_h^{(k)}(i:m_h)} - \bar{X}_h^{(k)}).$

Here, $C_{y_h x_h^{(k)}} = \rho C_{y_h} C_{x_h^{(k)}}$, where C_{y_h} and $C_{x_h^{(k)}}$ are the coefficients of variation of Y_h and $X_h^{(k)}$ respectively. Also $\bar{Y}_h$ and $\bar{X}_h^{(k)}$ are the population means of Y_h and $X_h^{(k)}$ respectively. The values of $\mu_{y_h[i:m_h]}$ and $\mu_{x_h^{(k)}(i:m_h)}$ depend on order statistic from some specific distributions (see Arnold et al.[1]).

In terms of $\delta's$, we get
$\bar{y}^{(u)}_{LJ(k)} = \sum_{h=1}^{L} P_h \left[\bar{X}_h^{(k)} \bar{R}_h^{(k)} (1+\delta_{2h}) + \frac{n_h(N_h-1)}{N_h(n_h-1)} \{\bar{Y}_h(1+\delta_{0h}) - \bar{X}_h^{(k)} \bar{R}_h^{(k)} (1+\delta_{1h})(1+\delta_{2h})\}\right]$. Assuming $\frac{n_h(N_h-1)}{N_h(n_h-1)} \cong 1$ and considering first order approximation, we get

$$(\bar{y}^{(u)}_{LJ(k)} - \bar{Y}) \cong \sum_{h=1}^{L} P_h \left[(\bar{Y}_h \delta_{0h} - \bar{X}_h^{(k)} \bar{R}_h^{(k)} \delta_{1h})\right].$$

Taking square and then expectation, the variance of $\bar{y}^{(u)}_{LJ(k)}$, is given by

$$V(\bar{y}^{(u)}_{LJ(k)}) \cong \sum_{h=1}^{L} P_h^2 \left[\bar{Y}_h^2(\gamma_h C^2_{yh} - W^2_{yh}) + \bar{X}_h^{(k)2} \bar{R}_h^{(k)2} (\gamma_h C^2_{xh^{(k)}} - W^2_{xh^{(k)}})\right]$$
$$- \sum_{h=1}^{L} P_h^2 \left[2\bar{R}_h^{(k)} \bar{X}_h^{(k)} \bar{Y}_h (\gamma_h C_{yhxh^{(k)}} - W_{yhxh^{(k)}})\right], k = 1, 2, \ldots, 9. \quad (3.4)$$

4. Simulation study

To compare the performances of the proposed estimators, a simulation study is conducted. Ranking is performed on basis of the auxiliary variable X. Bivariate random observations (X_h, Y_h), $h = 1, 2, \ldots, L$ were generated from a bivariate gamma population with correlation parameter ρ_{yxh}. Using $20,000$ simulations, estimates of variances for unbiased ratio-type estimators are computed under stratified ranked sampling scheme as described in Section 2. Estimators are compared in terms of percentage relative efficiencies ($PREs$).

The simulation results show that with decrease in correlation coefficients ρ_{yxh}, $PREs$ decrease, as expected. The numerical values given in the first five rows are obtained by assuming equal correlations across the strata, whereas the last three rows assume unequal correlations across the strata. The simulation results are presented in Table 1. We use the following expression to obtain the PRE:

$$PRE_{(k)} = \frac{V(\bar{y}_{(S_t RSS)})}{V(\bar{y}^{(u)}_{LJ(k)})} \times 100, \quad k = 1, 2, \ldots, 9.$$

The expression of variance is as follows:

$$V(\bar{y}_P) = \frac{1}{20000} \sum_{i=1}^{20000} (\bar{y}_P - \bar{Y})^2, \; P = \bar{y}_{(S_t RSS)}, \bar{y}^{(u)}_{LJ(k)}.$$

The simulation study is completed with some alternative empirical measures, such as the relative bias (RB) and the relative root mean square error (RRMSE), which are used to compare the precision of different estimators. The values of RB allow us to analyze the empirical bias of the different estimators, whereas the values of RRMSE reveal the most efficient estimator from an empirical point of view. Chambers and Dunstan [2], Rao et al. [15], Silva and Skinner [18], Harms and Duchesne [3] and Munoz and Rueda [13] used RB and RRMSE. We define percentage relative bias (PRB) and percentage relative root mean square error ($PRRMSE$) as

$$PRB_{(k)} = \frac{1}{\bar{Y}}\left[\frac{1}{20000}\sum_{i=1}^{20000}\left(\bar{y}_{LJ(k)i} - \bar{Y}\right)\right] \times 100, \quad k = 1, 2, \ldots, 9.$$

and

$$PRRMSE_{(k)} = \frac{1}{\bar{Y}}\left[\frac{1}{20000}\sum_{i=1}^{20000}\left(\bar{y}_{LJ(k)i} - \bar{Y}\right)^2\right]^{\frac{1}{2}} \times 100, \quad k = 1, 2, \ldots, 9.$$

Here $\bar{Y}$ is the parameter of interest and $\bar{y}_{LJ(k)i}$ is the value of $\hat{\bar{Y}}$ for the ith simulated sample. The values of PRB and $PRRMSE$ are presented in Tables 2 and 3 respectively. The simulation results indicate that for a given sample size $n = mr$, PRB decreases with increase in correlation coefficient. Among all estimators, $\bar{y}_{LJ(3)}$ has minimum absolute PRB and $PRRMSE$.

Table 1. Percentage Relative Efficiencies $PREs$ for separate ratio-type estimators for the simulated data, obtained through bivariate gamma distribution.

$L = 3, \quad P_h = (.30, .30, .40), \quad m_h = (3, 4, 5), \quad r = 5 \quad n_h = (15, 20, 25).$

ρ_{yxh}	$PRE_{(1)}$	$PRE_{(2)}$	$PRE_{(3)}$	$PRE_{(4)}$	$PRE_{(5)}$	$PRE_{(6)}$	$PRE_{(7)}$	$PRE_{(8)}$	$PRE_{(9)}$
0.90, 0.90, 0.90	120.7	137.5	242.9	127.2	123.0	144.5	140.2	130.1	130.4
0.80, 0.80, 0.80	115.7	130.9	231.8	121.6	120.8	138.8	133.9	124.3	127.5
0.70, 0.70, 0.70	113.4	125.2	221.8	116.8	118.8	133.9	128.4	119.2	124.7
0.60, 0.60, 0.60	106.6	119.3	209.8	114.5	116.8	128.9	122.8	113.8	121.9
0.50, 0.50, 0.50	102.2	113.7	198.5	106.6	114.7	123.9	117.5	108.7	119.1
0.90, 0.80, 0.70	113.8	128.6	226.9	119.6	119.5	136.8	131.7	122.1	125.4
0.80, 0.70, 0.60	110.1	123.5	217.2	115.3	117.5	132.1	126.9	117.6	122.8
0.70, 0.60, 0.50	105.4	117.5	206.9	110.1	115.3	126.5	121.2	112.2	119.7

5. Conclusion

It is observed from the simulation results (see Table 1) that the proposed unbiased ratio type estimators $\bar{y}_{LJ(k)}, (k = 1, 2, \ldots, 9)$, have high PRE in comparison to $\bar{y}_{(S_tRSS)}$. Also, the values of PRE decreases with decrease in correlation. The simulation result of Table 2 indicates that the proposed ratio-type estimators have reasonable biases, since the values of PRB are all less than 2.2% in absolute terms. Also, for a given sample size, PRB decreases with increase in correlation coefficient. Among all estimators, $\bar{y}_{LJ(3)}$ is more efficient as compared to the other considered estimators. So, we conclude that the proposed unbiased ratio-type estimators are preferable over $\bar{y}_{(S_tRSS)}$ under S_tRSS

Table 2. Percentage Relative Bias (PRB) of the different estimators.

$L = 3, \quad P_h = (.30, .30, .40), \; m_h = (3, 4, 5), \quad r = 5 \quad n_h = (15, 20, 25).$

ρ_{yxh}	$\bar{y}_{StRSS}$	$\bar{y}_{LJ(1)}$	$\bar{y}_{LJ(2)}$	$\bar{y}_{LJ(3)}$	$\bar{y}_{LJ(4)}$	$\bar{y}_{LJ(5)}$	$\bar{y}_{LJ(6)}$	$\bar{y}_{LJ(7)}$	$\bar{y}_{LJ(8)}$	$\bar{y}_{LJ(9)}$
0.90, 0.90, 0.90	2.02	1.01	1.99	-0.35	2.01	1.38	1.79	1.97	2.00	1.53
0.80, 0.80, 0.80	2.05	1.02	2.02	-0.33	2.04	1.39	1.79	1.98	2.02	1.53
0.70, 0.70, 0.70	2.08	1.04	2.04	-0.32	2.07	1.40	1.82	2.01	2.06	1.54
0.60, 0.60, 0.60	2.13	1.07	2.08	-0.31	2.12	1.41	1.84	2.03	2.10	1.55
0.50, 0.50, 0.50	2.14	1.08	2.08	-0.30	2.12	1.39	1.85	2.03	2.11	1.57
0.90, 0.80, 0.70	2.05	1.03	2.03	-0.33	2.04	1.36	1.81	1.99	2.03	1.51
0.80, 0.70, 0.60	2.09	1.04	2.06	-0.31	2.08	1.38	1.82	2.00	2.07	1.54
0.70, 0.60, 0.50	2.18	1.14	2.14	-0.31	2.17	1.47	1.92	2.09	2.16	1.61

Table 3. Percentage Relative Root Mean Squarer Error (PRRMSE) of the different estimators.

$L = 3, \quad P_h = (.30, .30, .40), \; m_h = (3, 4, 5), \quad r = 5 \quad n_h = (15, 20, 25).$

ρ_{yxh}	$\bar{y}_{StRSS}$	$\bar{y}_{LJ(1)}$	$\bar{y}_{LJ(2)}$	$\bar{y}_{LJ(3)}$	$\bar{y}_{LJ(4)}$	$\bar{y}_{LJ(5)}$	$\bar{y}_{LJ(6)}$	$\bar{y}_{LJ(7)}$	$\bar{y}_{LJ(8)}$	$\bar{y}_{LJ(9)}$
0.90, 0.90, 0.90	4.91	4.79	4.66	3.53	4.81	4.63	4.47	4.61	4.76	4.56
0.80, 0.80, 0.80	4.90	4.76	4.66	3.37	4.81	4.52	4.44	4.60	4.76	4.48
0.70, 0.70, 0.70	4.89	4.66	4.66	3.27	4.80	4.44	4.42	4.59	4.75	4.41
0.60, 0.60, 0.60	4.88	4.53	4.66	3.16	4.79	4.35	4.39	4.57	4.75	4.34
0.50, 0.50, 0.50	4.84	4.38	4.63	3.05	4.76	4.26	4.35	4.54	4.72	4.25
0.90, 0.80, 0.70	4.87	4.77	4.66	3.36	4.79	4.51	4.42	4.58	4.75	4.46
0.80, 0.70, 0.60	4.86	4.56	4.64	3.17	4.78	4.39	4.41	4.58	4.73	4.37
0.70, 0.60, 0.50	4.85	4.52	4.63	3.15	4.76	4.35	4.38	4.55	4.72	4.33

scheme.

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