Konuralp Journal of Mathematics
Volume 4 No. 1 pp. 203–210 (2016) ©KJM

ANALYTICAL PROBLEM SOLUTION ABOUT INITIAL STEP OF PRESSING POWDER MATERIAL TUBE

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Abstract. The prediction of finite size in the process of hot isostatic pressing (HIP) of powder material tubes is a difficult task which is important for practical purposes. In this paper we propose an analytical problem solution about initial step of the process.

1. Introduction

The difficulty in making a mathematical modeling for the pressing process of powder material tubes consists in predicting the size of a finished product. Thoroughly research [6] investigates the reason for the deviation in this type of modeling and also shows possible ways to determine the movement direction of internal border. It has been noted in research [3] and [5] that most deviations of final form emerge at the initial stage of the process.

The analytical solution has ample areas of application including but not limited to the tubes. The problem of a mathematical modeling of HIP tubes process is a part of many tasks of the mathematical modeling for HIP process which consist of embedded elements with a large radial stiffness.

2. Objectives

The purpose of this research is to find an analytical solution to a stress-strain behavior of powder material in initial stage of hot isostatic tube pressing.

3. Problem Statement

The problem is analyzed in an axisymmetric setting in a cylindrical coordinates system [1]. The domain $R_1 \leq r \leq R_2$, $R_3 \leq r \leq R_4$ - filled by plastically incompressible material (capsule). The domain $R_2 \leq r \leq R_3$- filled by plastically compressible powder material.

2000 *Mathematics Subject Classification.* 74Cxx.

Key words and phrases. Powder Material. Green's Condition. Hot Isostatic Pressing. Analytical Solution. Predictive Analytics.

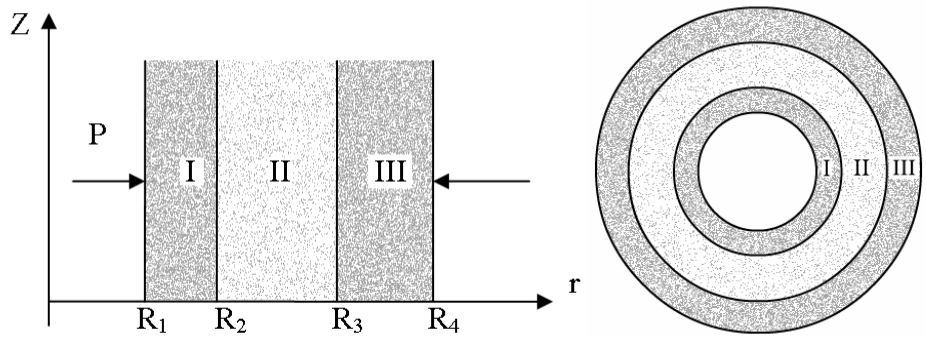

Fig.1.

Deformation rate ε_z is constant throughout the entire volume, $u(r)$ - radial displacement speed, σ_r, σ_φ, σ_z - components of stress tensor dependent only on the coordinate r.

Density is constant throughout the entire volume.

Steady-state equation:

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_\varphi}{r} = 0 \tag{3.1}$$

The equation of the yield surface for the powder material is taken in the form of Green:

$$\frac{(\sigma_r + \sigma_\varphi + \sigma_z)^2}{9f_2^2} + \frac{1}{6f_1^2}\left[(2\sigma_r - \sigma_\varphi - \sigma_z)^2 + (2\sigma_\varphi - \sigma_r - \sigma_z)^2 + (2\sigma_z - \sigma_r - \sigma_\varphi)^2\right] = T^2$$

From the associated flow law it follows:

$$\begin{aligned} \varepsilon_r &= \lambda\left[\frac{2(\sigma_r + \sigma_\varphi + \sigma_z)}{9f_2^2} + \frac{(2\sigma_r - \sigma_\varphi - \sigma_z)}{f_1^2}\right], \\ \varepsilon_\varphi &= \lambda\left[\frac{2(\sigma_r + \sigma_\varphi + \sigma_z)}{9f_2^2} + \frac{(2\sigma_\varphi - \sigma_r - \sigma_z)}{f_1^2}\right], \\ \varepsilon_z &= \lambda\left[\frac{2(\sigma_r + \sigma_\varphi + \sigma_z)}{9f_2^2} + \frac{(2\sigma_z - \sigma_r - \sigma_\varphi)}{f_1^2}\right]. \end{aligned} \tag{3.2}$$

The equation of the yield surface for a plastic incompressible material is:

$$\frac{1}{6}\left[(2\sigma_r - \sigma_\varphi - \sigma_z)^2 + (2\sigma_\varphi - \sigma_r - \sigma_z)^2 + (2\sigma_z - \sigma_r - \sigma_\varphi)^2\right] = T_1^2$$

From the associated flow law it follows:

$$\begin{aligned} \varepsilon_r &= \lambda(2\sigma_r - \sigma_\varphi - \sigma_z), \\ \varepsilon_\varphi &= \lambda(2\sigma_\varphi - \sigma_r - \sigma_z), \\ \varepsilon_z &= \lambda(2\sigma_z - \sigma_r - \sigma_\varphi). \end{aligned} \tag{3.3}$$

External pressure P is given on the outer boundary.

Then boundary conditions are:

$$\sigma_r = -P, r = R_1, r = R_4 \tag{3.4}$$

The equilibrium equation in z-axis is satisfied by the integral form:

$$2\pi\int_{R_1}^{R_4} \sigma_z(r)\, r dr = -P\pi\left(R_4^2 - R_1^2\right).$$

Suppose that tube end is affected by the same pressure as a side wall tube.

At the boundaries "Powder-Capsule" $r = R_2$, $r = R_3$ it is assumed a condition for continuity of displacement and stress equality σ_r.

A connection of rate of deformation tensor with a displacement rate $u\,(r)$ is determined by relations:

$$\varepsilon_r = \frac{du}{dr}, \varepsilon_\varphi = \frac{u}{r}.$$

In the domains $R_1 < r < R_2$, $R_3 < r < R_4$ a condition of incompressibility is:

$$\frac{du}{dr} + \frac{u}{r} + \varepsilon_z = 0.$$

Then displacement rate is

$$u = -\frac{\varepsilon_z}{2} r + \frac{C}{r}, \tag{3.5}$$

where C is a constant.

4. Solution

To describe the behavior of powder material a relation (3.2) can be rewritten as:

$$\begin{aligned} \varepsilon_r &= \lambda \left\{ A\sigma_r + B\sigma_\varphi + B\sigma_z \right\}, \\ \varepsilon_\varphi &= \lambda \left\{ B\sigma_r + A\sigma_\varphi + B\sigma_z \right\}, \\ \varepsilon_z &= \lambda \left\{ B\sigma_r + B\sigma_\varphi + A\sigma_z \right\}, \end{aligned}$$

where

$$A = \frac{18f_2^2 + 2f_1^2}{9f_2^2 f_1^2}, B = \frac{-9f_2^2 + 2f_1^2}{9f_2^2 f_1^2}.$$

Then

$$\begin{aligned} \sigma_r &= \frac{1}{\lambda} \left\{ C\varepsilon_r + D\varepsilon_\varphi + D\varepsilon_z \right\}, \\ \sigma_\varphi &= \frac{1}{\lambda} \left\{ D\varepsilon_r + C\varepsilon_\varphi + D\varepsilon_z \right\}, \\ \sigma_z &= \frac{1}{\lambda} \left\{ D\varepsilon_r + D\varepsilon_\varphi + C\varepsilon_z \right\}, \end{aligned} \tag{4.1}$$

where

$$\begin{aligned} C &= \frac{A+B}{A-B} \cdot \frac{1}{A+2B} = \frac{1}{18} \left(9f_2^2 + 4f_1^2 \right), \\ D &= -\frac{B}{A-B} \cdot \frac{1}{A+2B} = \frac{1}{18} \left(9f_2^2 - 2f_1^2 \right). \end{aligned}$$

Using the equation of the yield surface an expression for λ-parameter is

$$\frac{1}{\lambda} = \frac{6T}{\left[18C \left(\varepsilon_r^2 + \varepsilon_\varphi^2 + \varepsilon_z^2 \right) + 36D \left(\varepsilon_r \varepsilon_\varphi + \varepsilon_r \varepsilon_z + \varepsilon_\varphi \varepsilon_z \right) \right]^{\frac{1}{2}}}. \tag{4.2}$$

Then from Steady-state equation (3.1) and relations (4.1), (4.2) it follows:

$$\frac{d\varepsilon_r}{dr} \left[(C+D)\,\varepsilon_\varphi^2 + (C+D)\,\varepsilon_z^2 + 2D\varepsilon_\varphi \varepsilon_z \right] + \frac{d\varepsilon_\varphi}{dr} \left[D\varepsilon_z^2 - (C+D)\,\varepsilon_z \varepsilon_\varphi - D\varepsilon_r \varepsilon_z - D\varepsilon_\varphi \varepsilon_z \right] +$$
$$+ \frac{\varepsilon_r - \varepsilon_\varphi}{r} \left[C \left(\varepsilon_r^2 + \varepsilon_\varphi^2 + \varepsilon_z^2 \right) + 2D \left(\varepsilon_r \varepsilon_\varphi + \varepsilon_r \varepsilon_z + \varepsilon_\varphi \varepsilon_z \right) \right] = 0$$

Let $u\,(r) = kr + u_1\,(r)$.

Then

$$\varepsilon_r = k + \varepsilon_{1r};$$
$$\varepsilon_{1r} = \frac{du_1}{dr};$$
$$\varepsilon_\varphi = k + \varepsilon_{1\varphi};\ \varepsilon_{1\varphi} = \tfrac{u_1}{r},$$

$$\frac{d\varepsilon_{1r}}{dr}\left[(C+D)\,\varepsilon_{1\varphi}^2 + 2\,(C+D)\,k\varepsilon_{1\varphi} + (C+D) + (C+D)\,\varepsilon_z^2 + 2D\varepsilon_{1\varphi}\varepsilon_z + 2Dk\varepsilon_z\right] +$$
$$+\frac{\varepsilon_{1r}-\varepsilon_{1\varphi}}{r}2Dk\varepsilon_z = 0.$$

Choose k from condition: $2\,(C+D)\,k\varepsilon_{1\varphi} + 2D\varepsilon_{1\varphi}\varepsilon_z = 0$. Then $k = -\frac{D}{C+D}\varepsilon_z$. Then the following condition will be

$$\frac{d\varepsilon_{1r}}{dr}\left[(C+D)\,\varepsilon_{1\varphi}^2 + \frac{C\,(C+2D)}{(C+D)}\varepsilon_z^2\right] +$$
$$+\frac{\varepsilon_{1r}-\varepsilon_{1\varphi}}{r}\left[C\varepsilon_{1r}^2 + C\varepsilon_{1\varphi}^2 - (C-D)\,\varepsilon_{1r}\varepsilon_{1\varphi} + C\frac{C+2D}{C+D}\varepsilon_z^2\right] = 0.$$

For resolving this equation the change of variables is made.

Let $u_1\,(r) = rv\,(\ln r)$, $\ln r = z$.

Then $\varepsilon_{1r} = v + v'$, $\varepsilon_{1\varphi} = v$, $\frac{d\varepsilon_1}{dr} = (v'' + v')\,\frac{1}{r}$.

The equation for the function $v\,(z)$ definition is:

$$v''\left[(C+D)\,v^2 + \frac{C\,(C+2D)}{C+D}\varepsilon_z^2\right] +$$
$$+v'\left[2\,(C+D)\,v^2 + 2C\frac{C+2D}{C+D}\varepsilon_z^2 + (C+D)\,vv' + Cv'^2\right] = 0.$$

Then parametric dependence can be written as:

$$r = \frac{R_0}{\sqrt{ch\delta\,(\psi-\psi_0)\sin\psi + \delta sh\delta\,(\psi-\psi_0)\cos\psi}},$$
$$u = \varepsilon_z\frac{R_0}{\sqrt{ch\delta\,(\psi-\psi_0)\sin\psi + \delta sh\delta\,(\psi-\psi_0)\cos\psi}}\left[-\gamma sh\delta\,(\psi-\psi_0) - \frac{\cos\alpha}{1+\cos\alpha}\right]\varepsilon_\varphi =$$
$$= \varepsilon_z\left[-\gamma sh\delta\,(\psi-\psi_0) - \frac{\delta^2-1}{2\delta^2}\right],$$
$$\varepsilon_r = \varepsilon_z\left[-\gamma sh\delta\,(\psi-\psi_0) - \frac{\delta^2-1}{2\delta^2} + \frac{2\gamma\delta}{1+\delta^2}\left(ch\delta\,(\psi-\psi_0)\,tg\psi + \delta sh\delta\,(\psi-\psi_0)\right)\right],$$

where ψ is a parameter, R_0, ψ_0 - arbitrary constants,

$$\beta = \frac{C}{C+D} = \frac{1}{2\cos^2\frac{\alpha}{2}},$$
$$\gamma^2 = \frac{1+2\frac{D}{C}}{\left(1+\frac{D}{C}\right)^2} = \frac{1+2\cos\alpha}{(1+\cos\alpha)^2} = \frac{\left(3\delta^2-1\right)\left(\delta^2+1\right)}{4\delta^4},$$
$$\delta = ctg\frac{\alpha}{2} = \sqrt{\frac{9f_2^2+f_1^2}{3f_1^2}}.$$

Then for the stress the following conditions will be

$$\sigma_r = -\sqrt{2C}T sin\psi,$$
$$\sigma_\varphi = -\sqrt{2C}T sin\psi \left[\frac{\delta^2-1}{\delta^2+1} - 2\frac{\delta}{\delta^2+1} th\delta\left(\psi-\psi_0\right) ctg\psi\right],$$
$$\sigma_z = -\sqrt{2C}T sin\psi \left[\frac{\delta^2-1}{\delta^2+1} - \frac{\delta^2-1}{\delta^2+1}\frac{1}{\delta} th\delta\left(\psi-\psi_0\right) ctg\psi + \frac{2\delta\gamma}{\left(\delta^2+1\right)}\frac{ctg\psi}{ch\delta\left(\psi-\psi_0\right)}\right].$$

The solution for a research of a capsule behavior ($R_1 < r < R_2$,$R_3 < r < R_4$): Using (3.5) we get [2]:

$$\varepsilon_r = -\frac{\varepsilon_z}{2} - \frac{C}{r^2},$$
$$\varepsilon_\varphi = -\frac{\varepsilon_z}{2} + \frac{C}{r^2}.$$

From the equation (3.3) it follows:

$$S^2 = \frac{1}{3\lambda^2}\left(\varepsilon_r^2 + \varepsilon_\varphi^2 + \varepsilon_r\varepsilon_\varphi\right).$$

Therefore

$$\lambda = \frac{\sqrt{\frac{3}{4}\varepsilon_z^2 + \frac{C^2}{r^4}}}{\sqrt{3}T_1}.$$

As far as

$$\sigma_r - \sigma_\varphi = \frac{1}{3\lambda}\left(\varepsilon_r - \varepsilon_\varphi\right) = \frac{\sqrt{3}T_1}{3\sqrt{\frac{3}{4}\varepsilon_z^2 + \frac{C^2}{r^4}}}\left(-\frac{2C}{r^2}\right),$$

then from the equilibrium equation (3.1) it follows

$$\frac{d\sigma_r}{dr} = \frac{\sqrt{3}}{3}\frac{T_1}{\sqrt{\frac{3}{4}\varepsilon_z^2 + \frac{C^2}{r^4}}}\frac{2C}{r^3}. \tag{4.3}$$

Using (3.5), $R_1 < r < R_2$, an expression for the radial displacement speed will be

$$u = -\frac{\varepsilon_z}{2}r + \frac{C_1}{r}.$$

As regards to (3.4) $\sigma_r = -P$ at which $r = R_1$, then from (4.3) it follows, that at which $r \in [R_1; R_2]$

$$\sigma_r = -P - \frac{\sqrt{3}}{3}T_1 \ln\frac{\frac{C_1}{r^2} + \sqrt{\frac{3}{4}\varepsilon_z^2 + \frac{C_1^2}{r^4}}}{\frac{C_1}{R_1^2} + \sqrt{\frac{3}{4}\varepsilon_z^2 + \frac{C_1^2}{R_1^4}}}.$$

Accordingly for $r \in [R_1; R_2]$:

$$\sigma_\varphi = \sigma_r + \frac{\sqrt{3}}{3}T_1\frac{2C_1}{r^2}\frac{1}{\sqrt{\frac{3}{4}\varepsilon_z^2 + \frac{C_1^2}{r^4}}}.$$

As $\sigma_z = \frac{1}{2}\left[\sigma_r + \sigma_\varphi - \frac{1}{\lambda}\left(\varepsilon_r + \varepsilon_\varphi\right)\right] = \frac{1}{2}\left[\sigma_r + \sigma_\varphi + \frac{\varepsilon_z}{\lambda}\right]$, then

$$\sigma_z = -P - \frac{\sqrt{3}}{3}T_1 \ln\frac{\frac{C_1}{r^2} + \sqrt{\frac{3}{4}\varepsilon_z^2 + \frac{C_1^2}{r^4}}}{\frac{C_1}{R_1^2} + \sqrt{\frac{3}{4}\varepsilon_z^2 + \frac{C_1^2}{R_1^4}}} + \frac{\sqrt{3}}{3}T_1\frac{C_1}{r^2}\frac{1}{\sqrt{\frac{3}{4}\varepsilon_z^2 + \frac{C_1^2}{r^4}}} + \sqrt{3}\,T_1\frac{1}{\sqrt{\frac{3}{4}\varepsilon_z^2 + \frac{C_1^2}{r^4}}}\frac{\varepsilon_z}{2}.$$

Similarly for $r \in [R_3; R_4]$ the relations are the same with replacement $C_1$to C_2, $R_1$to R_3, $R_2$to R_4.

Complete system of equations for arbitrary constants definition in case of a continuous field of velocities is

1) Stress equality at with $R = R_2$ is

$$-\sqrt{2C}T\sin\psi_2 = -P - \frac{\sqrt{3}}{3}T_1 \ln \frac{\frac{C_1}{R_2^2} + \sqrt{\frac{3}{4}\varepsilon_z^2 + \frac{C_1^2}{R_2^4}}}{\frac{C_1}{R_2^2} + \sqrt{\frac{3}{4}\varepsilon_z^2 + \frac{C_1^2}{R_1^4}}},$$

$$R_2^2 = \frac{R_0^2}{ch\delta\left(\psi_2 - \psi_0\right)\sin\psi_2 + \delta sh\delta\left(\psi_2 - \psi_0\right)\cos\psi_2}.$$

2) Stress equality at with $R = R_3$ is

$$-\sqrt{2C}T\sin\psi_3 = -P - \frac{\sqrt{3}}{3}T \ln \frac{\frac{C_2}{R_3^2} + \sqrt{\frac{3}{4}\varepsilon_z^2 + \frac{C_2^2}{R_3^4}}}{\frac{C_2}{R_4^2} + \sqrt{\frac{3}{4}\varepsilon_z^2 + \frac{C_2^2}{R_4^4}}},$$

$$R_3^2 = \frac{R_0^2}{ch\delta\left(\psi_3 - \psi_0\right)\sin\psi_3 + \delta sh\delta\left(\psi_3 - \psi_0\right)\cos\psi_0}.$$

3) Displacement rate equality at with $R = R_2$ is

$$-\frac{\varepsilon_z}{2}R_2 + \frac{C_1}{R_2} = \varepsilon_z \frac{R_0\left[-\gamma sh\delta\left(\psi_2 - \psi_0\right) - \frac{\delta^2 - 1}{2\delta^2}\right]}{\sqrt{ch\delta\left(\psi_2 - \psi_0\right)\sin\psi_2 + \delta sh\delta\left(\psi_2 - \psi_0\right)\cos\psi_2}}.$$

4) Displacement rate equality at with $R = R_3$ is

$$-\frac{\varepsilon_z}{2}R_3 + \frac{C_2}{R_3} = \varepsilon_z \frac{R_0\left[-\gamma sh\delta\left(\psi_3 - \psi_0\right) - \frac{\delta^2 - 1}{2\delta^2}\right]}{\sqrt{ch\delta\left(\psi_3 - \psi_0\right)\sin\psi_3 + \delta sh\delta\left(\psi_3 - \psi_0\right)\cos\psi_3}}.$$

5) Equilibrium condition about the z-axis

$$\int_{R_1}^{R_4} \sigma_z 2\pi r dr = -P\pi\left(R_4^2 - R_1^2\right).$$

In practice, we define the relation $\frac{C_1}{\varepsilon_z}$, $\frac{C_2}{\varepsilon_z}$.

The research shows, that in geometric parameter domain there are 4 areas for different deformation modes (Fig. 2).

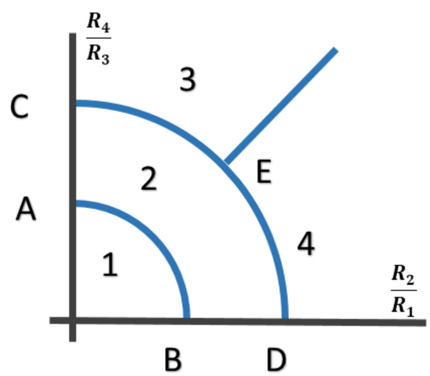

Fig. 2

Area 1 - continuous field of velocities in entire system.

Area 2 - displacement rate with a displacement rate with a discontinuity in internal boundary $r = R_2$.

Area 3 - plane deformation with a fixed outer boundary and a localization of deformation in internal border.

Area 4 – plane deformation with a fixed internal boundary.

5. Conclusions

The result shows that we get a plane deformation from a certain capsule wall thickness which has an important practical application. It demonstrates the possibility to create a radially directional effect capsule. As per the rough scheme axis direction effect capsules are constructed as follows. Side walls become quite thin while top and bottom ones become thick. In this case it is mostly an axial shrinkage. The possibility of transition into a plane deformation demonstrates a possibility to get only a radial shrinkage during the HIP process. It is important when a shape of the surface of powder product is rather complex.

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