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HERMITE-HADAMARD TYPE INEQUALITIES FOR h-CONVEX FUNCTIONS VIA FRACTIONAL INTEGRALS

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Abstract. By making use of identity of the established by Sarıkaya [4], some new Hermite-Hadamard type inequalities for Riemann-Liouville fractional integral are established. Our results are the generalizations of the results obtain by Sarıkaya [4].

1. Introduction

The following inequlity is well known in the literature as the Hermite-Hadamard integral inequality

$$f\left(\frac{a+b}{2}\right)\leq\frac{1}{b-a}\int_a^b f(x)dx\leq\frac{f(a)+f(b)}{2} \tag{1.1}$$

where $f: I\subseteq\mathbb{R}\to\mathbb{R}$ is a convex function on the interval I of real numbers and $a,b\in I$ with $a<b$.

Both inequalities hold in the reversed direction if f is concave.

The notion of $s-$convex function was introduced in Breckner's paper [1] and a number of properties and connections with $s-$convexity in the first sense discussed in paper [7].

Definition 1.1. A function $f:[0,\infty)\to\mathbb{R}$ is said to be $s-$convex in the second sense if

$$f\left(\lambda x+(1-\lambda)y\right)\leq\lambda^s f(x)+(1-\lambda)^s f(y)$$

for all $x,y\in[0,\infty)$, $\lambda\in[0,1]$ and for some fixed $s\in(0,1]$. This class of $s-$convex function is usually denoted by K_s^2.

It can be easily seen that for $s=1$, $s-$convexity reduces to ordinary convexity of functions defined on $[0,\infty)$.

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Definition 1.2. [6] We say that $f : I \to \mathbb{R}$ is a $P-$function or that f belongs to the class $P(I)$ if f is nonnegative and for all $x, y \in I$ and $\alpha \in [0, 1]$, we have

$$f(\lambda x + (1 - \lambda) y) \leq f(x) + f(y)$$

Definition 1.3. [15] Let $h : J \subseteq \mathbb{R} \to \mathbb{R}$ be a nonnegative function. We say that $f : I \subseteq \mathbb{R} \to \mathbb{R}$ is $h-$convex function, or f belongs to the class $SX(h, I)$, if f is nonnegative and for all $x, y \in I$ and $\alpha \in (0, 1)$, we have

$$f(\lambda x + (1 - \lambda) y) \leq h(\lambda) f(x) + h(1 - \lambda) f(y) \tag{1.2}$$

If inequality (1.2) is reversed, then f is said to be $h-$concave, i.e. $f \in SV(h, I)$.

Obviously, if $h(\lambda) = \lambda$, then all nonnegative convex functions belongs to $SX(h, I)$ and all nonnegative concave functions belongs to $SV(h, I)$; if $h(\lambda) = 1$, then $SX(h, I) \supseteq P(I)$; and if $h(\lambda) = \lambda^s$, where $s \in (0, 1)$, then $SX(h, I) \supseteq K_s^2$.

Sarıkaya ([9]) is generalized Kırmacı's ([8]) results for fractional integral. These results are given below.

Theorem 1.1. *Let $f : [a, b] \to \mathbb{R}$ be a differentiable mapping on (a, b) with $a < b$. If $|f'|$ is convex on $[a, b]$ then the following inequality for fractional integrals holds:*

$$\begin{aligned} &\left| \frac{2^{\alpha-1}\Gamma(\alpha+1)}{(b-a)^{\alpha}} \left[J^{\alpha}_{\left(\frac{a+b}{2}\right)+} f(b) + J^{\alpha}_{\left(\frac{a+b}{2}\right)-} f(a) \right] - f\left(\frac{a+b}{2}\right) \right| \\ \leq\ & \frac{b-a}{4(\alpha+1)} \left[|f'(a)| + |f'(b)| \right] \end{aligned} \tag{1.3}$$

Theorem 1.2. *Let $f : [a, b] \to \mathbb{R}$ be a differentiable mapping on (a, b) with $a < b$. If $|f'|^q$ is convex on $[a, b]$ for $q > 1$, then the following inequality for fractional integrals holds:*

$$\begin{aligned} &\left| \frac{2^{\alpha-1}\Gamma(\alpha+1)}{(b-a)^{\alpha}} \left[J^{\alpha}_{\left(\frac{a+b}{2}\right)+} f(b) + J^{\alpha}_{\left(\frac{a+b}{2}\right)-} f(a) \right] - f\left(\frac{a+b}{2}\right) \right| \\ \leq\ & \frac{b-a}{4} \left(\frac{1}{\alpha p + 1} \right)^{\frac{1}{p}} \left[\left(\frac{|f'(a)| + 3|f'(b)|}{4} \right)^{\frac{1}{q}} + \left(\frac{3|f'(a)| + |f'(b)|}{4} \right)^{\frac{1}{q}} \right] \\ \leq\ & \frac{b-a}{4} \left(\frac{4}{\alpha p + 1} \right)^{\frac{1}{p}} \left[|f'(a)| + |f'(b)| \right] \end{aligned} \tag{1.4}$$

where $\frac{1}{p} + \frac{1}{q} = 1$.

Theorem 1.3. *Let $f : [a, b] \to \mathbb{R}$ be a differentiable mapping on (a, b) with $a < b$. If $|f'|^q$ is convex on $[a, b]$ for $q \geq 1$, then the following inequality for fractional integrals holds:*

$$\begin{aligned} &\left| \frac{2^{\alpha-1}\Gamma(\alpha+1)}{(b-a)^{\alpha}} \left[J^{\alpha}_{\left(\frac{a+b}{2}\right)+} f(b) + J^{\alpha}_{\left(\frac{a+b}{2}\right)-} f(a) \right] - f\left(\frac{a+b}{2}\right) \right| \\ \leq\ & \frac{b-a}{4(\alpha+1)} \left(\frac{1}{2(\alpha+2)} \right)^{\frac{1}{q}} \left[\left((\alpha+1)|f'(a)|^q + (\alpha+3)|f'(b)|^q \right)^{\frac{1}{q}} \right. \\ & \left. + \left((\alpha+3)|f'(a)|^q + (\alpha+1)|f'(b)|^q \right)^{\frac{1}{q}} \right] \end{aligned} \tag{1.5}$$

We give some necessary definitions and mathematical preliminaries of fractional calculus theory which are used further this paper.

Definition 1.4. Let $f \in L[a,b]$. The Riemann-Liouville integrals $J_{a^+}^{\alpha} f$ and $J_{b^-}^{\alpha} f$ of order $\alpha > 0$ with a≥ 0 are defined by

$$J_{a^+}^{\alpha} f(x) = \frac{1}{\Gamma(\alpha)} \int_a^x (x-t)^{\alpha-1} f(t)dt, \qquad x > \alpha$$

and

$$J_{b^-}^{\alpha} f(x) = \frac{1}{\Gamma(\alpha)} \int_x^b (t-x)^{\alpha-1} f(t)dt, \qquad x < \alpha$$

respectively. Here, $\Gamma(\alpha)$ is the Gamma function and $J_{a^+}^{0} f(x) = J_{b^-}^{0} f(x) = f(x)$.

For some recent results connected with fractional integral ineqalities, see([2]-[5],[10]-[14]).

2. Main Results

In order to prove our main theorems we need the following lemma see ([9]).

Lemma 2.1. *Let $f : [a,b] \to \mathbb{R}$ be a differentiable mapping on (a,b) with $a < b$. If $f' \in L\,[a,b]$, then the following inequality for fractional integrals holds:*

$$\begin{aligned} (2.1) \qquad & \frac{2^{\alpha-1}\Gamma(\alpha+1)}{(b-a)^{\alpha}} \left[J_{\left(\frac{a+b}{2}\right)^+}^{\alpha} f(b) + J_{\left(\frac{a+b}{2}\right)^-}^{\alpha} f(a) \right] - f\left(\frac{a+b}{2}\right) \\ &= \frac{b-a}{4} \left\{ \int_0^1 t^{\alpha} f'\left(\frac{t}{2}a + \frac{2-t}{2}b\right) dt - \int_0^1 t^{\alpha} f'\left(\frac{2-t}{2}a + \frac{t}{2}b\right) dt \right\} \end{aligned}$$

with $\alpha > 0$.

Using lemma 2.1, we obtain the following fractional integral inequality for $h-$convex functions.

Theorem 2.1. *Let $h : J \subseteq \mathbb{R} \to \mathbb{R}$ be a nonnegative function. $f : I \subseteq \mathbb{R} \to \mathbb{R}$ be a differentiable mapping on (a,b) with $a < b$. If $|f'|$ is $h-$convex on $[a,b]$ then the following inequality for fractional integrals holds:*

$$\begin{aligned} (2.2) \qquad & \left| \frac{2^{\alpha-1}\Gamma(\alpha+1)}{(b-a)^{\alpha}} \left[J_{\left(\frac{a+b}{2}\right)^+}^{\alpha} f(b) + J_{\left(\frac{a+b}{2}\right)^-}^{\alpha} f(a) \right] - f\left(\frac{a+b}{2}\right) \right| \\ &\leq \frac{b-a}{4} \left\{ \int_0^1 t^{\alpha} \left[h\left(\frac{t}{2}\right) + h\left(1-\frac{t}{2}\right) \right] dt \right\} \left[|f'(a)| + |f'(b)| \right] \end{aligned}$$

Proof. From Lemma 2.1 since $|f'|$ is $h-$convex, we have

$$\begin{aligned} & \left| \frac{2^{\alpha-1}\Gamma(\alpha+1)}{(b-a)^{\alpha}} \left[J_{\left(\frac{a+b}{2}\right)^+}^{\alpha} f(b) + J_{\left(\frac{a+b}{2}\right)^-}^{\alpha} f(a) \right] - f\left(\frac{a+b}{2}\right) \right| \\ \leq\ & \frac{b-a}{4} \left\{ \int_0^1 t^{\alpha} \left| f'\left(\frac{t}{2}a + \frac{2-t}{2}b\right) \right| dt + \int_0^1 t^{\alpha} \left| f'\left(\frac{2-t}{2}a + \frac{t}{2}b\right) \right| dt \right\} \\ \leq\ & \frac{b-a}{4} \left\{ \int_0^1 t^{\alpha} \left[h\left(\frac{t}{2}\right) |f'(a)| + h\left(1-\frac{t}{2}\right) |f'(b)| \right] dt \right. \\ & \left. + \int_0^1 t^{\alpha} \left[h\left(1-\frac{t}{2}\right) |f'(a)| + h\left(\frac{t}{2}\right) |f'(b)| \right] \right\} \\ =\ & \frac{b-a}{4} \left\{ \int_0^1 t^{\alpha} \left[h\left(\frac{t}{2}\right) + h\left(1-\frac{t}{2}\right) \right] dt \right\} \left[|f'(a)| + |f'(b)| \right] \end{aligned}$$

□

Corollary 2.1. *If we choose $h(t) = t^s$, then the inequality (2.2) of Theorem 2.1 becomes the following inequality:*

$$\begin{aligned} &\left| \frac{2^{\alpha-1}\Gamma(\alpha+1)}{(b-a)^{\alpha}} \left[J^{\alpha}_{\left(\frac{a+b}{2}\right)+} f(b) + J^{\alpha}_{\left(\frac{a+b}{2}\right)-} f(a) \right] - f\left(\frac{a+b}{2}\right) \right| \\ \leq\ & \frac{b-a}{4} \left\{ \frac{1}{2^s(\alpha+s+1)} + 2^{\alpha+1} B\left(\frac{1}{2}; \alpha+1, s+1\right) \right\} \left[|f'(a)| + |f'(b)| \right] \end{aligned}$$

The incomplete beta function, a generalization of the beta function is defined as

$$B(x; a, b) = \int_0^x t^{a-1} (1-t)^{b-1} \, dt, \quad a > 0, b > 0 \text{ and } 0 \leq x \leq 1$$

and we used the fact that

$$\int_0^1 t^{\alpha} \left(1 - \frac{t}{2}\right)^s dt = 2^{\alpha+1} B\left(\frac{1}{2}; \alpha+1, s+1\right)$$

Corollary 2.2. *If we choose $h(t) = 1$, then the inequality (2.2) of Theorem 2.1 becomes the following inequality:*

$$\begin{aligned} &\left| \frac{2^{\alpha-1}\Gamma(\alpha+1)}{(b-a)^{\alpha}} \left[J^{\alpha}_{\left(\frac{a+b}{2}\right)+} f(b) + J^{\alpha}_{\left(\frac{a+b}{2}\right)-} f(a) \right] - f\left(\frac{a+b}{2}\right) \right| \\ \leq\ & \frac{b-a}{2(\alpha+1)} \left[|f'(a)| + |f'(b)| \right] \end{aligned}$$

Remark 2.1. If we choose $h(t) = t$, then the inequality (2.2) of Theorem 2.1 reduces the inequality (1.3) of Theorem 1.1.

Theorem 2.2. *Let $h : J \subseteq \mathbb{R} \to \mathbb{R}$ be a nonnegative function. $f : I \subseteq \mathbb{R} \to \mathbb{R}$ be a differentiable mapping on (a, b) with $a < b$. If $|f'|^q$ is $h-$convex on $[a, b]$ for $q > 1$, then the following inequality for fractional integrals holds:*

$$\begin{aligned} &\left| \frac{2^{\alpha-1}\Gamma(\alpha+1)}{(b-a)^{\alpha}} \left[J^{\alpha}_{\left(\frac{a+b}{2}\right)+} f(b) + J^{\alpha}_{\left(\frac{a+b}{2}\right)-} f(a) \right] - f\left(\frac{a+b}{2}\right) \right| \\ \leq\ & \frac{b-a}{4} \left(\frac{1}{\alpha p+1}\right)^{\frac{1}{p}} \left\{ \left(|f'(a)|^q \int_0^1 h\left(\frac{t}{2}\right) dt + |f'(b)|^q \int_0^1 h\left(1-\frac{t}{2}\right) dt \right)^{\frac{1}{q}} \right. \\ & \left. + \left(|f'(a)|^q \int_0^1 h\left(1-\frac{t}{2}\right) dt + |f'(b)|^q \int_0^1 h\left(\frac{t}{2}\right) dt \right)^{\frac{1}{q}} \right\} \end{aligned} \tag{2.3}$$

Proof. From Lemma 2.1, using the Hölder inequality and $|f'|^q$ is $h-$convex we have

$$\begin{aligned}
&\left|\frac{2^{\alpha-1}\Gamma(\alpha+1)}{(b-a)^{\alpha}}\left[J^{\alpha}_{\left(\frac{a+b}{2}\right)+}f(b)+J^{\alpha}_{\left(\frac{a+b}{2}\right)-}f(a)\right]-f\left(\frac{a+b}{2}\right)\right| \\
\leq\ & \frac{b-a}{4}\left\{\int_0^1 t^{\alpha}\left|f'\left(\frac{t}{2}a+\frac{2-t}{2}b\right)\right|dt+\int_0^1 t^{\alpha}\left|f'\left(\frac{2-t}{2}a+\frac{t}{2}b\right)\right|dt\right\} \\
\leq\ & \frac{b-a}{4}\left(\int_0^1 t^{\alpha p}dt\right)^{\frac{1}{p}}\left\{\left(\int_0^1\left|f'\left(\frac{t}{2}a+\frac{2-t}{2}b\right)\right|^q dt\right)^{\frac{1}{q}}\right. \\
&\left.+\left(\int_0^1\left|f'\left(\frac{2-t}{2}a+\frac{t}{2}b\right)\right|^q dt\right)^{\frac{1}{q}}\right\} \\
\leq\ & \frac{b-a}{4}\left(\frac{1}{\alpha p+1}\right)^{\frac{1}{p}}\left\{\left(|f'(a)|^q\int_0^1 h\left(\frac{t}{2}\right)dt+|f'(b)|^q\int_0^1 h\left(1-\frac{t}{2}\right)dt\right)^{\frac{1}{q}}\right. \\
&\left.+\left(|f'(a)|^q\int_0^1 h\left(1-\frac{t}{2}\right)dt+|f'(b)|^q\int_0^1 h\left(\frac{t}{2}\right)dt\right)^{\frac{1}{q}}\right\}
\end{aligned}$$

□

Corollary 2.3. *If we choose $h(t)=t^s$, then the inequality (2.3) of Theorem 2.2 becomes the following inequality:*

$$\begin{aligned}
&\left|\frac{2^{\alpha-1}\Gamma(\alpha+1)}{(b-a)^{\alpha}}\left[J^{\alpha}_{\left(\frac{a+b}{2}\right)+}f(b)+J^{\alpha}_{\left(\frac{a+b}{2}\right)-}f(a)\right]-f\left(\frac{a+b}{2}\right)\right| \\
\leq\ & \frac{b-a}{4}\left(\frac{1}{\alpha p+1}\right)^{\frac{1}{p}}\left\{\left(|f'(a)|^q\frac{1}{2^s(s+1)}+|f'(b)|^q\frac{2^{s+1}}{2^s(s+1)}\right)^{\frac{1}{q}}\right. \\
&\left.+\left(|f'(a)|^q\frac{2^{s+1}}{2^s(s+1)}+|f'(b)|^q\frac{1}{2^s(s+1)}\right)^{\frac{1}{q}}\right\}
\end{aligned}$$

Corollary 2.4. *If we choose $h(t)=1$, then the inequality (2.3) of Theorem 2.2 becomes the following inequality:*

$$\begin{aligned}
&\left|\frac{2^{\alpha-1}\Gamma(\alpha+1)}{(b-a)^{\alpha}}\left[J^{\alpha}_{\left(\frac{a+b}{2}\right)+}f(b)+J^{\alpha}_{\left(\frac{a+b}{2}\right)-}f(a)\right]-f\left(\frac{a+b}{2}\right)\right| \\
\leq\ & \frac{b-a}{2}\left(\frac{1}{\alpha p+1}\right)^{\frac{1}{p}}\left(|f'(a)|^q+|f'(b)|^q\right)^{\frac{1}{q}}
\end{aligned}$$

Remark 2.2. If we choose $h(t)=t$, then the inequality (2.3) of Theorem 2.2 reduces the inequality (1.4) of Theorem 1.2.

Theorem 2.3. *Let $h:J\subseteq\mathbb{R}\to\mathbb{R}$ be a nonnegative function. $f:I\subseteq\mathbb{R}\to\mathbb{R}$ be a differentiable mapping on (a,b) with $a<b$. If $|f'|^q$ is $h-$convex on $[a,b]$ for $q\geq 1$,*

then the following inequality for fractional integrals holds:

$$\left|\frac{2^{\alpha-1}\Gamma(\alpha+1)}{(b-a)^{\alpha}}\left[J^{\alpha}_{\left(\frac{a+b}{2}\right)+}f(b)+J^{\alpha}_{\left(\frac{a+b}{2}\right)-}f(a)\right]-f\left(\frac{a+b}{2}\right)\right| \tag{2.4}$$
$$\leq \frac{b-a}{4}\left(\frac{1}{\alpha+1}\right)^{\frac{1}{p}}\left[\left(|f'(a)|^{q}\int_{0}^{1}t^{\alpha}h\left(\frac{t}{2}\right)dt+|f'(b)|^{q}\int_{0}^{1}t^{\alpha}h\left(1-\frac{t}{2}\right)dt\right)^{\frac{1}{q}}\right.$$
$$\left.+\left(|f'(a)|^{q}\int_{0}^{1}t^{\alpha}h\left(1-\frac{t}{2}\right)dt+|f'(b)|^{q}\int_{0}^{1}t^{\alpha}h\left(\frac{t}{2}\right)dt\right)^{\frac{1}{q}}\right]$$

Proof. From Lemma 2.1, using the power mean inequality and $|f'|^{q}$ is $h-$convex we have

$$\left|\frac{2^{\alpha-1}\Gamma(\alpha+1)}{(b-a)^{\alpha}}\left[J^{\alpha}_{\left(\frac{a+b}{2}\right)+}f(b)+J^{\alpha}_{\left(\frac{a+b}{2}\right)-}f(a)\right]-f\left(\frac{a+b}{2}\right)\right|$$
$$\leq \frac{b-a}{4}\left\{\left(\int_{0}^{1}t^{\alpha}dt\right)^{\frac{1}{p}}\left(\int_{0}^{1}t^{\alpha}\left|f'\left(\frac{t}{2}a+\frac{2-t}{2}b\right)\right|^{q}dt\right)^{\frac{1}{q}}\right.$$
$$\left.+\left(\int_{0}^{1}t^{\alpha}dt\right)^{\frac{1}{p}}\left(\int_{0}^{1}t^{\alpha}\left|f'\left(\frac{t}{2}a+\frac{2-t}{2}b\right)\right|^{q}dt\right)\right\}$$
$$= \frac{b-a}{4}\left(\frac{1}{\alpha+1}\right)^{\frac{1}{p}}\left[\left(|f'(a)|^{q}\int_{0}^{1}t^{\alpha}h\left(\frac{t}{2}\right)dt+|f'(b)|^{q}\int_{0}^{1}t^{\alpha}h\left(1-\frac{t}{2}\right)dt\right)^{\frac{1}{q}}\right.$$
$$\left.+\left(|f'(a)|^{q}\int_{0}^{1}t^{\alpha}h\left(1-\frac{t}{2}\right)dt+|f'(b)|^{q}\int_{0}^{1}t^{\alpha}h\left(\frac{t}{2}\right)dt\right)^{\frac{1}{q}}\right]$$

□

Corollary 2.5. *If we choose $h(t)=t^{s}$, then the inequality (2.4) of Theorem 2.3 becomes the following inequality:*

$$\left|\frac{2^{\alpha-1}\Gamma(\alpha+1)}{(b-a)^{\alpha}}\left[J^{\alpha}_{\left(\frac{a+b}{2}\right)+}f(b)+J^{\alpha}_{\left(\frac{a+b}{2}\right)-}f(a)\right]-f\left(\frac{a+b}{2}\right)\right|$$
$$\leq \frac{b-a}{4}\left(\frac{1}{\alpha+1}\right)^{\frac{1}{p}}\left\{\left(\frac{|f'(a)|^{q}}{2^{s}(\alpha+s+1)}+2^{\alpha+1}|f'(b)|^{q}B\left(\frac{1}{2};\alpha+1,s+1\right)\right)\right.$$
$$\left.+\left(2^{\alpha+1}|f'(a)|^{q}B\left(\frac{1}{2};\alpha+1,s+1\right)+\frac{|f'(b)|^{q}}{2^{s}(\alpha+s+1)}\right)^{\frac{1}{q}}\right\}$$

and we used the fact that

$$\int_{0}^{1}t^{\alpha}\left(1-\frac{t}{2}\right)^{s}dt=2^{\alpha+1}B\left(\frac{1}{2};\alpha+1,s+1\right)$$

Corollary 2.6. *If we choose $h(t)=1$, then the inequality (2.4) of Theorem 2.3 becomes the following inequality:*

$$\left|\frac{2^{\alpha-1}\Gamma(\alpha+1)}{(b-a)^{\alpha}}\left[J^{\alpha}_{\left(\frac{a+b}{2}\right)+}f(b)+J^{\alpha}_{\left(\frac{a+b}{2}\right)-}f(a)\right]-f\left(\frac{a+b}{2}\right)\right|$$
$$\leq \frac{b-a}{2(\alpha+1)}\left(|f'(a)|^{q}+|f'(b)|^{q}\right)^{\frac{1}{q}}$$

Remark 2.3. If we choose $h(t) = t$, then the inequality (2.4) of Theorem 2.3 reduces the inequality (1.5) of Theorem 1.3.

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