

The Eurasia Proceedings of Science, Technology, Engineering and Mathematics (EPSTEM), 2025

Volume 34, Pages 120-138

ICBASSET 2025: International Conference on Basic Sciences, Engineering and Technology

Soft Semi # Generalized α -Connectedness in Soft Topological Spaces

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Abstract: Soft set theory is a newly emerging tool to deal with uncertain problems and has been studied by researchers in theory and practice. The concept of soft topological space is a very recently developed area having many research scopes. Soft sets have been studied in proximity spaces, multi-criteria decision-making problems, medical problems, mobile cloud computing networks, defense learning systems, and approximate reasoning, etc. The objective of this paper is to use $S_{\#}S_{emi} \# \alpha$ -open sets and $S_{\#}S_{emi} \# \alpha$ -closed sets to introduce the concepts of $S_{\#}S_{emi} \# \alpha$ -separated sets, $S_{\#}S_{emi} \# \alpha$ -connected space, $S_{\#}S_{emi} \# \alpha$ -disconnected space, $S_{\#}S_{emi} \# \alpha$ -component of a soft set, locally $S_{\#}S_{emi} \# \alpha$ -connected space, and totally $S_{\#}S_{emi} \# \alpha$ -disconnected space. We investigate and study the properties and characterizations of these spaces in soft topological spaces.

Keywords: $S_{\#}S_{emi} \# \alpha$ -closed set, $S_{\#}S_{emi} \# \alpha$ -separated set, $S_{\#}S_{emi} \# \alpha$ -connected space

I. Introduction

The real world is too complex for our immediate and direct understanding. We create “models” of reality that are simplifications of aspects of the real world. Unfortunately, these mathematical models are too complicated, and we cannot find the exact solutions. The uncertainty of data while modeling problems in engineering, physics, computer sciences, economics, social sciences, medical sciences, and many other diverse fields makes it unsuccessful to use the traditional classical methods. These may be due to the uncertainties of natural environmental phenomena, of human knowledge about the real world, or to the limitations of the means used to measure objects. For example, vagueness or uncertainty in the boundary between states or between urban and rural areas, or the exact growth rate of population in a country’s rural area, or making decisions in a machine-based environment using database information. Thus, classical set theory, which is based on the crisp and exact case, may not be fully suitable for handling such problems of uncertainty. There are several theories, for example, the theory of fuzzy sets, theory of intuitionistic fuzzy sets, theory of vague sets, theory of interval mathematics, and the theory of rough sets. These can be considered as tools for dealing with uncertainties, but all these theories have their own difficulties. The reason for these difficulties is, possibly, the inadequacy of the parametrization tool of the theory as it was mentioned by Molodtsov. He initiated the concept of soft set theory as a new mathematical tool that is free from the problems mentioned above. He presented the fundamental results of the new theory and successfully applied it to several directions such as smoothness of functions, game theory, operations research, Riemann-integration, Perron integration, theory of probability, etc. A soft set is a collection of approximate descriptions of an object. He also showed how soft set theory is free from the parametrization inadequacy syndrome of fuzzy set theory, rough set theory, probability theory, and game theory. Soft systems provide a very general framework with the involvement of parameters. Soft set theory has been applied in several directions. The researchers introduced the concept of soft sets to deal with uncertainty and to solve complicated problems in economics, engineering, sociology, and environment because of unsuccessful use of classical methods. The well-known theories that can be considered as mathematical tools for dealing with uncertainties and imperfect knowledge are theory of fuzzy sets, theory of vague sets, theory of interval

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mathematics, theory of intuitionists fuzzy sets, theory of rough sets and theory of probability. During recent years, soft set theory emerged as a best mathematical tool to deal with uncertainties, imprecision and vagueness. Many engineering, medical science, economics, environment problems have various uncertainties, and the soft set theory came up with the reasonable solutions to these problems. A soft set is a collection of approximate descriptions of objects. Some researchers have presented a systematic survey of the literature and the developments of Topological Spaces in soft set theory. They have also provided some applications of soft set theory in software engineering, innovation, medical diagnosis, data analysis, decision making etc. All these tools require the specification of some parameter to start with. The theory of soft sets gives a vital mathematical tool for handling uncertainties and vague concepts. Recently several researchers introduced the notion of soft topology and established that every soft topology induces a collection of topologies called the parametrized family of topologies induced by the soft topology. They discussed soft set-theoretical operations and gave an application of soft set theory to a decision-making problem. Several mathematicians published papers on applications of soft sets and soft topology. Soft sets and soft topology have applications in data mining, image processing, decision-making problems, spatial modeling, and neural patterns. Research works on soft set theory and its applications in various fields are progressing rapidly. Decision-making and topology have a long joint tradition since the modern statement of the classical Weierstrass extreme value theorem. It combines two topological concepts called continuity of a real-valued function and compactness of the domain (both with respect to a given topology). They represent a necessary and sufficient condition to guarantee the existence of the maximum and minimum values of the function. The success of the Mathematical Problems in Engineering technique was amplified by its adoption in fields like engineering sciences, computer sciences, and mathematical economics. This matter can be adopted in the version of soft setting by replacing the classical notions(compactness, function, and real numbers) by their soft counterparts (soft compactness, soft function, and soft real numbers). Some practical experiments in civil engineering require classification of the materials according to their characteristics (attribute set or parameter set E), which can be expressed using the concept of soft sets. We study the separation of them concerning the group of soft sets, which are constructed from the practical experiments. In this group of soft sets, we add the absolute and null soft sets to initiate a soft weak structure. The researchers in the communication engineering endeavor to select the best protocol to solve the noisy problems in wireless networks. They evaluate the performance of these protocols according to the proposed scenarios. The researchers in Soft Theory may plan with some engineers to propose some protocols using the appropriate soft structure to select the optimal protocol to solve the interference problems in wireless networks. The objective of this paper is to use $S_{\#S_{emi}}^{\#ga}$ -open sets and $S_{\#S_{emi}}^{\#ga}$ -closed sets to introduce the concepts of $S_{\#S_{emi}}^{\#ga}$ -separated sets, $S_{\#S_{emi}}^{\#ga}$ -connected space, $S_{\#S_{emi}}^{\#ga}$ -disconnected space, $S_{\#S_{emi}}^{\#ga}$ -component of a soft set, locally $S_{\#S_{emi}}^{\#ga}$ -connected space, and totally $S_{\#S_{emi}}^{\#ga}$ -disconnected space. We investigate and study the properties and characterizations of these in soft topological spaces.

II. Preliminaries

Definition 2.1. Let X be an initial universe set and E be a collection of all possible parameters with respect to X , where parameters are the characteristics or properties of objects in X . Let $P(X)$ denote the power set of X , and let A be a non-empty subset of E . A pair (F, A) is called a soft set over X , where F is a mapping given by $F : A \rightarrow P(X)$. In other words, a soft set over X is a parameterized family of subsets of the universe X . For $e \in A$, $F(e)$ may be considered as the set e -approximate elements of the soft set (F, A) . Clearly, a soft set is not a set. For two soft sets (F, A) and (G, B) over the common universe X , we say that (F, A) is a soft subset of (G, B) if (i) $A \subseteq B$ and (ii) for all $e \in A$, $F(e)$ and $G(e)$ are identical approximations. We write $(F, A) \subseteq (G, B)$. (G, B) is said to be a soft superset of (F, A) , if (F, A) is a soft subset of (G, B) . Two soft sets (F, A) and (G, B) over a common universe X are said to be soft equal if (F, A) is a soft subset of (G, B) and (G, B) is a soft subset of (F, A) .

Definition 2.2. The union of two soft sets (F, A) and (G, B) over the common universe X is soft set $(H, C) = (F, A) \cup (G, B)$, where $C = A \cup B$, and $H(e) = F(e)$ if $e \in A - B$, $H(e) = G(e)$ if $e \in B - A$ and $H(e) = F(e) \cup G(e)$ if $e \in A \cap B$.

Definition 2.3. The Intersection (H, C) of two soft sets (F, A) and (G, B) over a common universe X denoted $(F, A) \cap (G, B)$ is defined as $C = A \cap B$ and $H(e) = F(e) \cap G(e)$ for all $e \in C$.

Definition 2.4. For a soft set (F, A) over the universe U , the relative complement of (F, A) is denoted by $(F, A)^C$ and is defined by $(F, A)^C = (F^C, A)$, where $F^C: A \rightarrow P(X)$ is a mapping defined by $F^C(e) = X - F(e)$ for all $e \in A$.

Definition 2.5. A soft set (F, E) over X is said to be (i) A null soft set, denoted by $\tilde{\phi}$, if $\forall e \in E, F(e) = \phi$. (ii) An absolute soft set, denoted by $\tilde{X}$, if $\forall e \in E, F(e) = X$.

Definition 2.6. Let $\tilde{\tau}$ be the collection of soft sets over X . Then $\tilde{\tau}$ is said to be a soft Topology on X if

- (i) $\tilde{\phi}, \tilde{X}$ belong to $\tilde{\tau}$
- (ii) The union of any number of soft sets in $\tilde{\tau}$ belongs to $\tilde{\tau}$
- (iii) The intersection of any two soft sets belongs to $\tilde{\tau}$.

The triplet $(\tilde{X}, \tilde{\tau}, E)$ is called a soft topological space over X . The complement of a soft open set is called soft closed set over X .

Definition 2.7. Let $(\tilde{X}, \tilde{\tau}, E)$ be a soft topological space over X and (F, E) be a soft set over X . Then

- (i) Soft interior of a soft set (F, E) denoted by $S_{ft}Int(F, E)$ is defined as the union of all soft open sets over X contained in (F, E) .
- (ii) Soft closure of a soft set (F, E) denoted by $S_{ft}Cl(F, E)$ is defined as the intersection of all soft closed super sets over X containing (F, E) .

Definition 2.8. A S_{ft} -subset (F, A) of a S_{ft} -Top-Space $(\tilde{X}, \tilde{\tau}, E)$ is called a $S_{ft}\alpha$ -closed set if $S_{ft}Cl[S_{ft}Int(S_{ft}Cl(F, A))] \subseteq (F, A)$. The complement of a $S_{ft}\alpha$ -closed set is called $S_{ft}\alpha$ -open set.

Definition 2.9. A S_{ft} -subset (F, A) of a S_{ft} -Top-Space $(\tilde{X}, \tilde{\tau}, E)$ is called a $S_{ft}g$ -closed set if $S_{ft}Cl(A, E) \subseteq (U, E)$ whenever $(F, A) \subseteq (U, E)$ and (U, E) is soft open in $(\tilde{X}, \tilde{\tau}, E)$.

Definition 2.10. A S_{ft} -subset (F, A) of a S_{ft} -Top-Space $(\tilde{X}, \tilde{\tau}, E)$ is called a $S_{ft}S_{emi}$ -closed set if $S_{ft}Int[S_{ft}Cl(F, A)] \subseteq (F, A)$. The complement of a $S_{ft}S_{emi}$ -closed set is called a $S_{ft}S_{emi}$ -open set.

Definition 2.11. A S_{ft} -subset (F, A) of a S_{ft} -Top-Space $(\tilde{X}, \tilde{\tau}, E)$ is called a soft $S_{ft}g^\# \alpha$ -closed set if $S_{ft}\alpha Cl(F, A) \subseteq (U, E)$, whenever $(F, A) \subseteq (U, E)$ and (U, E) is $S_{ft}g$ -open in $(\tilde{X}, \tilde{\tau}, E)$. The complement of a $S_{ft}g^\# \alpha$ -closed set is called a $S_{ft}g^\# \alpha$ -open set.

Definition 2.12. A S_{ft} -subset (F, A) of a S_{ft} -Top-Space $(\tilde{X}, \tilde{\tau}, E)$ is called a soft $S_{ft}^{\#}g\alpha$ -closed set if $S_{ft}\alpha Cl(F, A) \subseteq (U, E)$, whenever $(F, A) \subseteq (U, E)$ and (U, E) is $S_{ft}g^{\#}\alpha$ -open in (X, τ, E) . The complement of a $S_{ft}^{\#}g\alpha$ -closed set is called a $S_{ft}^{\#}g\alpha$ -open set.

Definition 2.13. A S_{ft} -subset (F, A) of a S_{ft} -Top-Space $(\tilde{X}, \tilde{\tau}, E)$ is called a soft $S_{ft}S_{emi}^{\#}g\alpha$ -closed set if $S_{ft}S_{emi}Cl(F, A) \subseteq (U, E)$, whenever $(F, A) \subseteq (U, E)$ and (U, E) is $S_{ft}^{\#}g\alpha$ -open in (X, τ, E) . The complement of a $S_{ft}S_{emi}^{\#}g\alpha$ -closed set is called a $S_{ft}S_{emi}^{\#}g\alpha$ -open set in (X, τ, E) .

Definition 2.14. Let $(\tilde{X}, \tilde{\tau}, E)$ be a soft topological space over X and (F, A) be a soft set over X . Then

(i) $S_{ft}S_{emi}^{\#}g\alpha$ -interior of a soft set (F, A) denoted by $S_{ft}S_{emi}^{\#}g\alpha-Int(F, A)$ is defined as the union of all soft $S_{ft}S_{emi}^{\#}g\alpha$ -open sets over X contained in (F, A) .

(ii) $S_{ft}S_{emi}^{\#}g\alpha$ -closure of a soft set (F, A) denoted by $S_{ft}S_{emi}^{\#}g\alpha-Cl(F, A)$ is defined as the intersection of all soft $S_{ft}S_{emi}^{\#}g\alpha$ -closed sets over X containing (F, A) .

Theorem 2.15. (i) Every S_{ft} -closed set in a soft topological space is $S_{ft}S_{emi}^{\#}g\alpha$ -closed set.

(ii) Every S_{ft} -open set in a S_{ft} -Top-Space is $S_{ft}S_{emi}^{\#}g\alpha$ -open set.

Theorem 2.16. Let (F, A) and (G, B) be two S_{ft} -subsets of a S_{ft} -Top-Space $(\tilde{X}, \tilde{\tau}, E)$. Then the following statements are true.

(i) (F, A) is $S_{ft}S_{emi}^{\#}g\alpha$ -open if and only if $S_{ft}S_{emi}^{\#}g\alpha-Int(F, A) = (F, A)$.

(ii) $S_{ft}S_{emi}^{\#}g\alpha-Int(F, A)$ is $S_{ft}S_{emi}^{\#}g\alpha$ -open.

(iii) (F, A) is $S_{ft}S_{emi}^{\#}g\alpha$ -closed if and only if $S_{ft}S_{emi}^{\#}g\alpha-Cl(F, A) = (F, A)$.

(iv) $S_{ft}S_{emi}^{\#}g\alpha-Cl(F, A)$ is $S_{ft}S_{emi}^{\#}g\alpha$ -closed..

(v) $S_{ft}S_{emi}^{\#}g\alpha-Cl[(X, E) - (F, A)] = (X, E) - S_{ft}S_{emi}^{\#}g\alpha-Int(F, A)$.

(vi) $S_{ft}S_{emi}^{\#}g\alpha-Int[(X, E) - (F, A)] = (X, E) - S_{ft}S_{emi}^{\#}g\alpha-Cl(F, A)$.

(vii) If (F, A) is $S_{ft}S_{emi}^{\#}g\alpha$ -open in $(\tilde{X}, \tilde{\tau}, E)$ and (G, B) is S_{ft} -open in $(\tilde{X}, \tilde{\tau}, E)$, then $(F, A) \cap (G, B)$ is $S_{ft}S_{emi}^{\#}g\alpha$ -open in $(\tilde{X}, \tilde{\tau}, E)$.

(viii) A point $x_{\alpha} \in S_{ft}S_{emi}^{\#}g\alpha-Cl(F, A)$ if and only if every $S_{ft}S_{emi}^{\#}g\alpha$ -open set in $(\tilde{X}, \tilde{\tau}, E)$ containing x intersects (F, A) .

(ix) Arbitrary intersection of $S_{ft}S_{emi}^{\#}g\alpha$ -closed sets in $(\tilde{X}, \tilde{\tau}, E)$ is $S_{ft}S_{emi}^{\#}g\alpha$ -closed in $(\tilde{X}, \tilde{\tau}, E)$.

(x) An arbitrary union of $S_{ft}S_{emi}^{\#}g\alpha$ -open sets in $(\tilde{X}, \tilde{\tau}, E)$ is $S_{ft}S_{emi}^{\#}g\alpha$ -open in (X, τ, E) .

Definition 2.17. A S_{ft} -Top-Space $(\tilde{X}, \tilde{\tau}, E)$ is said to be soft connected if there does not exist a pair F_A and G_B of nonempty, disjoint soft open subsets of $(\tilde{X}, \tilde{\tau}, E)$ such that $\tilde{X} = F_A \cup G_B$.

III. Soft $S_{emi}^{\#}g\alpha$ -Separated Sets

In this section, we will introduce and investigate the basic properties of $S_{ft}S_{emi}^{\#}g\alpha$ -separated sets.

Definition 3.1. Two non-null soft disjoint $S_{\#}$ -sets F_E and G_E in a $S_{\#}$ -Top-Space $(\tilde{X}, \tilde{T}, E)$ are called Soft $S_{emi}^{\#}g\alpha$ -separated ($S_{ft}S_{emi}^{\#}g\alpha$ -separated) sets if $G_E \tilde{\cap} [S_{ft}S_{emi}^{\#}g\alpha-Cl(F_E)] = [S_{ft}S_{emi}^{\#}g\alpha-Cl(G_E)] \tilde{\cap} F_E = \tilde{\phi}$

Definition 3.2. A Soft $S_{emi}^{\#}g\alpha$ -separation ($S_{\#}S_{emi}^{\#}g\alpha$ -separation) of a S_{ft} -Top-Space $(\tilde{X}, \tilde{T}, E)$ is a pair of $S_{\#}S_{emi}^{\#}g\alpha$ -separated sets such that $F_E \tilde{\cup} G_E = \tilde{X}$.

Definition 3.3. Let $(\tilde{X}, \tilde{T}, E)$ be a $S_{\#}$ -Top-Space. A Soft $S_{emi}^{\#}g\alpha$ -connected soft connected ($S_{ft}S_{emi}^{\#}g\alpha$ -connected) set over X is a $S_{\#}$ -set $F_E \tilde{\in} S_{\#}S(X)_E$ which does not have a $S_{ft}S_{emi}^{\#}g\alpha$ -separation in the soft relative topology induced on the soft subset F_E .

Proposition 3.4. Every $S_{\#}S_{emi}^{\#}g\alpha$ -connected set in a $S_{\#}$ -Top-Space $(\tilde{X}, \tilde{T}, E)$ is a $S_{\#}$ -connected set.

Proof. Let F_E be a $S_{\#}S_{emi}^{\#}g\alpha$ -connected set in a $S_{\#}$ -Top-Space $(\tilde{X}, \tilde{T}, E)$. Then there does not exist a $S_{ft}S_{emi}^{\#}g\alpha$ -separation of F_E . Since every $S_{\#}$ -open set is a $S_{ft}S_{emi}^{\#}g\alpha$ -open set, there does not exist a $S_{\#}$ -separation of F_E . Hence, F_E is a $S_{\#}$ -connected set in the $S_{\#}$ -Top-Space $(\tilde{X}, \tilde{T}, E)$.

Theorem 3.5. Let $(\tilde{X}, \tilde{T}, E)$ be a $S_{\#}$ -Top-Space and F_E be a $S_{\#}S_{emi}^{\#}g\alpha$ -connected set. Let G_A and H_B be $S_{\#}S_{emi}^{\#}g\alpha$ -separated sets. If $F_E \tilde{\subseteq} G_A \tilde{\cup} H_B$. Then either $F_E \tilde{\subseteq} G_A$ or $F_E \tilde{\subseteq} H_B$.

Proof. Let $(\tilde{X}, \tilde{T}, E)$ be a $S_{\#}$ -Top-Space and $F_E \tilde{\in} S_{\#}S(X)_E$ be a $S_{\#}S_{emi}^{\#}g\alpha$ -connected set. Let G_A and H_B be $S_{\#}S_{emi}^{\#}g\alpha$ -separated sets such that $F_E \tilde{\subseteq} G_A \tilde{\cup} H_B$. We have to prove either $F_E \tilde{\subseteq} G_A$ or $F_E \tilde{\subseteq} H_B$. Suppose not. Then $F_E \not\tilde{\subseteq} G_A$ and $F_E \not\tilde{\subseteq} H_B$. Then, $K_E = G_A \tilde{\cap} F_E \neq \tilde{\phi}$ and $L_E = H_B \tilde{\cap} F_E \neq \tilde{\phi}$ and $F_E = K_E \tilde{\cup} L_E$. Since $K_E \tilde{\subseteq} G_A$ implies that $S_{ft}S_{emi}^{\#}g\alpha-Cl(K_E) \tilde{\subseteq} S_{ft}S_{emi}^{\#}g\alpha-Cl(G_A)$. Since G_A and H_B are $S_{ft}S_{emi}^{\#}g\alpha$ -separated sets, we have $[S_{\#}S_{emi}^{\#}g\alpha-Cl(G_A)] \tilde{\cap} H_B = \tilde{\phi}$. Therefore, $[S_{ft}S_{emi}^{\#}g\alpha-Cl(G_A)] \tilde{\cap} H_B = [S_{ft}S_{emi}^{\#}g\alpha-Cl(K_E)] \tilde{\cap} H_B = [S_{ft}S_{emi}^{\#}g\alpha-Cl(K_E)] \tilde{\cap} L_E = \tilde{\phi}$. Again $L_E \tilde{\subseteq} H_B$ implies $S_{ft}S_{emi}^{\#}g\alpha-Cl(L_E) \tilde{\subseteq} S_{ft}S_{emi}^{\#}g\alpha-Cl(H_B)$. Since G_A and H_B are $S_{ft}S_{emi}^{\#}g\alpha$ -separated sets, we have $G_A \tilde{\cap} [S_{ft}S_{emi}^{\#}g\alpha-S_{\#}(H_B)] = \tilde{\phi}$. Therefore, $G_A \tilde{\cap} [S_{\#}S_{emi}^{\#}g\alpha-Cl(H_B)] = G_A \tilde{\cap} [S_{\#}S_{emi}^{\#}g\alpha-Cl(L_E)] = K_E \tilde{\cap} [S_{\#}S_{emi}^{\#}g\alpha-Cl(L_E)] = \tilde{\phi}$. But $F_E = K_E \tilde{\cup} L_E$. Therefore, there exists a $S_{\#}S_{emi}^{\#}g\alpha$ -separation of F_E . Hence, F_E is not a $S_{\#}S_{emi}^{\#}g\alpha$ -connected set. This is a contradiction. Therefore, either $F_E \tilde{\subseteq} G_A$ or $F_E \tilde{\subseteq} H_B$.

Theorem 3.6. If F_E is $S_{\#}S_{emi}^{\#}g\alpha$ -connected set, then $S_{\#}S_{emi}^{\#}g\alpha-Cl(F_E)$ is $S_{\#}S_{emi}^{\#}g\alpha$ -connected set.

Proof. Let F_E be $S_{\#}S_{emi}^{\#}g\alpha$ -connected set in a $S_{\#}$ -Top-Space $(\tilde{X}, \tilde{T}, E)$. We have to prove that $S_{\#}S_{emi}^{\#}g\alpha-Cl(F_E)$ is a $S_{\#}S_{emi}^{\#}g\alpha$ -connected set. Suppose not. Then there exists a $S_{\#}S_{emi}^{\#}g\alpha$ -separation of $S_{\#}S_{emi}^{\#}g\alpha-Cl(F_E)$. Therefore there exists a pair of $S_{\#}S_{emi}^{\#}g\alpha$ -separated sets G_A and H_B such that $[S_{\#}S_{emi}^{\#}g\alpha-Cl(F_E)] = G_A \tilde{\cup} H_B$. But $F_E \subseteq S_{\#}S_{emi}^{\#}g\alpha-Cl(F_E) = G_A \tilde{\cup} H_B$. Since F_E is $S_{\#}S_{emi}^{\#}g\alpha$ -connected set, then by Theorem 3.5 either $F_E \subseteq G_A$ or $F_E \subseteq H_B$. If $F_E \subseteq G_A$ then $S_{\#}S_{emi}^{\#}g\alpha-Cl(F_E) \subseteq S_{\#}S_{emi}^{\#}g\alpha-Cl(G_A)$. Since $H_B \subseteq S_{\#}S_{emi}^{\#}g\alpha-Cl(F_E)$, then $H_B = \tilde{\emptyset}$. This is a contradiction. Similarly, if $F_E \subseteq H_B$, we can prove $G_A = \tilde{\emptyset}$ which is a contradiction. Thus, there is not any $S_{\#}S_{emi}^{\#}g\alpha$ -separation of $S_{\#}S_{emi}^{\#}g\alpha-Cl(F_E)$. Hence $S_{\#}S_{emi}^{\#}g\alpha-Cl(F_E)$ is $S_{\#}S_{emi}^{\#}g\alpha$ -connected set.

Theorem 3.7. If F_E is $S_{\#}S_{emi}^{\#}g\alpha$ -connected set and $F_E \subseteq G_E \subseteq S_{\#}S_{emi}^{\#}g\alpha-Cl(F_E)$, then G_E is $S_{\#}S_{emi}^{\#}g\alpha$ -connected set.

Proof. Let $F_E \in S_{\#}S(X)_E$ be a $S_{\#}S_{emi}^{\#}g\alpha$ -connected set such that $F_E \subseteq G_E \subseteq S_{\#}S_{emi}^{\#}g\alpha-Cl(F_E)$. We have to prove that G_E is $S_{\#}S_{emi}^{\#}g\alpha$ -connected set. Suppose G_E is not a $S_{\#}S_{emi}^{\#}g\alpha$ -connected set. Then there exists a pair of $S_{\#}S_{emi}^{\#}g\alpha$ -separated sets K_A and L_B such that $G_E = K_A \tilde{\cup} L_B$. Since $F_E \subseteq G_E$ and $F_E \subseteq K_A \tilde{\cup} L_B$. We claim that either $F_E \subseteq K_A$ or $F_E \subseteq L_B$. For, $F_E \tilde{\cap} K_A \neq \tilde{\emptyset}$ and $F_E \tilde{\cap} L_B \neq \tilde{\emptyset}$. Then $F_E = (F_E \tilde{\cap} K_A) \tilde{\cup} (F_E \tilde{\cap} L_B)$. But $F_E \tilde{\cap} K_A$ and $F_E \tilde{\cap} L_B$ are $S_{\#}S_{emi}^{\#}g\alpha$ -separated sets. This is a contradiction to that $S_{\#}S_{emi}^{\#}g\alpha$ -connectivity of F_E . Hence, our claim is proved that G_E is $S_{\#}S_{emi}^{\#}g\alpha$ -separated set. Suppose $F_E \subseteq K_A$, then $S_{\#}S_{emi}^{\#}g\alpha-Cl(F_E) \subseteq S_{\#}S_{emi}^{\#}g\alpha-Cl(K_A)$. Since K_A and L_B are $S_{\#}S_{emi}^{\#}g\alpha$ -separated sets. $[S_{\#}S_{emi}^{\#}g\alpha-Cl(K_A)] \tilde{\cap} L_B = \tilde{\emptyset}$. Therefore, $[S_{\#}S_{emi}^{\#}g\alpha-Cl(F_E)] \tilde{\cap} L_B = \tilde{\emptyset}$. But $L_B \subseteq G_E$. Then by hypothesis $L_B \subseteq G_E \subseteq S_{\#}S_{emi}^{\#}g\alpha-Cl(F_E)$. Therefore, $[S_{\#}S_{emi}^{\#}g\alpha-Cl(F_E)] \tilde{\cap} L_B = L_B$. Thus, we have $[S_{\#}S_{emi}^{\#}g\alpha-Cl(F_E)] \tilde{\cap} L_B = \tilde{\emptyset}$ and $[S_{\#}S_{emi}^{\#}g\alpha-Cl(F_E)] \tilde{\cap} L_B = L_B$. Hence $L_B = \tilde{\emptyset}$, which is a contradiction. Similarly, if $F_E \subseteq L_B$, then we can prove $K_A = \tilde{\emptyset}$. This is a contradiction. Therefore, there does not exist a $S_{\#}S_{emi}^{\#}g\alpha$ -separation of G_E . Hence, G_E is a $S_{\#}S_{emi}^{\#}g\alpha$ -connected set.

Theorem 3.8. The soft union F_E of any family $\{F_{E_j} : j \in J\}$ of $S_{\#}S_{emi}^{\#}g\alpha$ -connected sets having a nonempty soft intersection is $S_{\#}S_{emi}^{\#}g\alpha$ -connected set.

Proof. Suppose that $F_E = G_A \tilde{\cup} H_B$ where G_A and H_B form a $S_{\#}S_{emi}^{\#}g\alpha$ -separation of F_E . By hypothesis, we may choose a soft point $x_e \in \bigcap_{j \in J} F_{E_j}$. Then $x_e \in F_{E_j}$ for all $j \in J$. If $x_e \in F_E$, then either $x_e \in G_A$ or $x_e \in H_B$ but not both. Since G_A and H_B are disjoint soft sets, we must have $F_{E_j} \subseteq G_A$, since F_{E_j} is $S_{\#}S_{emi}^{\#}g\alpha$ -connected and it is true for all $j \in J$, and so $F_E \subseteq G_A$. From this, we obtain that $H_B = \tilde{\emptyset}$, which is a contradiction. Thus, there does not exist a $S_{\#}S_{emi}^{\#}g\alpha$ -separation of F_E . Therefore, F_E is $S_{\#}S_{emi}^{\#}g\alpha$ -connected.

IV. Soft $S_{emi}^{\#}g\alpha$ -Connected Space

In this section, we introduce the $S_{\#}S_{emi}^{\#}g\alpha$ -connected space and discuss its properties in $S_{\#}$ -Top-Space.

Definition 4.1. Let $(\tilde{X}, \tilde{T}, E)$ be a $S_{\#}$ -Top-Space. If there does not exist a $S_{\#}S_{emi}^{\#}g\alpha$ -separation of $\tilde{X}$, then it is said to be $S_{\#}S_{emi}^{\#}g\alpha$ -connected space.

Proposition 4.2. Every $S_{\#}S_{emi}^{\#}g\alpha$ -connected space $(\tilde{X}, \tilde{T}, E)$ is $S_{\#}S_{emi}^{\#}g\alpha$ -connected space.

Proof. Let $(\tilde{X}, \tilde{T}, E)$ be a $S_{\#}S_{emi}^{\#}g\alpha$ -connected space. Then there does not exist a $S_{\#}S_{emi}^{\#}g\alpha$ -separation of $\tilde{X}$. Since every $S_{\#}$ -open set is a $S_{\#}^{\#}g\alpha$ -open set, there does not exist a $S_{\#}$ -separation of $\tilde{X}$. Therefore, $(\tilde{X}, \tilde{T}, E)$ is a $S_{\#}$ -connected space.

Theorem 4.3. Let $(\tilde{X}, \tilde{T}, E)$ be a $S_{\#}$ -Top-Space. Then the following statements are equivalent:

- (i) $\tilde{\phi}$ and $\tilde{X}$ are the only $S_{\#}S_{emi}^{\#}g\alpha$ -clopen sets in $(\tilde{X}, \tilde{T}, E)$.
- (ii) $\tilde{X}$ is not the soft union of two soft disjoint non-empty $S_{\#}S_{emi}^{\#}g\alpha$ -open sets.
- (iii) $\tilde{X}$ is not the soft union of two soft disjoint non-empty $S_{\#}S_{emi}^{\#}g\alpha$ -closed sets.
- (iv) $\tilde{X}$ is not the soft union of two $S_{\#}S_{emi}^{\#}g\alpha$ -separated sets.

Proof. (i) $\Rightarrow$ (ii): Assume that $\tilde{\phi}$ and $\tilde{X}$ are the only $S_{\#}S_{emi}^{\#}g\alpha$ -clopen sets in $(\tilde{X}, \tilde{T}, E)$. Suppose (ii) is false. Then $\tilde{X} = G_A \tilde{\cup} H_B$, where G_A and H_B are two soft disjoint, non-empty $S_{\#}S_{emi}^{\#}g\alpha$ -open sets. Clearly $H_B = G_A^C$ is $S_{\#}S_{emi}^{\#}g\alpha$ -closed and non-empty. Thus H_B is a non-empty proper $S_{\#}S_{emi}^{\#}g\alpha$ -clopen set in $\tilde{X}$, which shows that (i) is false.

(ii) $\Rightarrow$ (iii): Assume that, $\tilde{X}$ is not the soft union of two soft disjoint non-empty $S_{\#}S_{emi}^{\#}g\alpha$ -open sets. Suppose (iii) is false. Then $\tilde{X} = G_A \tilde{\cup} H_B$ where G_A and H_B are two soft disjoint non-empty $S_{\#}S_{emi}^{\#}g\alpha$ -closed sets. Now G_A and H_B being respectively the soft complement of H_B and G_A . Therefore G_A and H_B are $S_{\#}S_{emi}^{\#}g\alpha$ -open sets. This contradicts (ii).

(iii) $\Rightarrow$ (iv): Assume that $\tilde{X}$ is not the soft union of two soft disjoint non-empty $S_{\#}S_{emi}^{\#}g\alpha$ -closed sets. If (iv) is false, then $\tilde{X} = G_A \tilde{\cup} H_B$, where G_A and H_B are $S_{\#}S_{emi}^{\#}g\alpha$ -separated sets. Then $G_A \tilde{\cap} [S_{\#}S_{emi}^{\#}g\alpha\text{-Cl}(H_B)] = \tilde{\phi}$ and this implies that $S_{\#}S_{emi}^{\#}g\alpha\text{-Cl}(H_B) \subseteq H_B$. But $H_B \subseteq S_{\#}S_{emi}^{\#}g\alpha\text{-Cl}(H_B)$. Therefore, $S_{\#}S_{emi}^{\#}g\alpha\text{-Cl}(H_B) = H_B$ and hence H_B is a $S_{\#}S_{emi}^{\#}g\alpha$ -closed set. Similarly, G_A is a $S_{\#}S_{emi}^{\#}g\alpha$ -closed set, which is a contradiction to (iii). Therefore (iv) is true.

(iv) $\Rightarrow$ (i): Assume that $\tilde{X}$ is not the soft union of two $S_{\#}S_{emi}^{\#}g\alpha$ -separated sets. Suppose (i) is false. Then there exists a non-empty proper $S_{\#}S_{emi}^{\#}g\alpha$ -clopen subset H_A of $\tilde{X}$. Then $H_B = H_A^C$ is a non-empty, $S_{\#}S_{emi}^{\#}g\alpha$ -clopen set and $\tilde{X} = G_A \tilde{\cup} H_B$, where G_A and H_B are non-empty disjoint

$S_{\pm}S_{emi} \#g\alpha$ -clopen sets. Since G_A and H_B are $S_{\pm}S_{emi} \#g\alpha$ -clopen sets, $S_{\pm}S_{emi} \#g\alpha -Cl(G_A) = G_A$ and $S_{\pm}S_{emi} \#g\alpha -Cl(H_B) = H_B$. Therefore, $S_{\pm}S_{emi} \#g\alpha -Cl(G_A) \tilde{\cap} H_B = G_A \tilde{\cap} H_B = \tilde{\phi} = G_A \tilde{\cap} [S_{\pm}S_{emi} \#g\alpha -Cl(H_B)]$. Thus $\tilde{X}$ is a soft union of $S_{\pm}S_{emi} \#g\alpha$ -separated sets. This is a contradiction to (iv). Therefore (i) must be true.

Theorem 4.4. Let $(\tilde{X}, \tilde{T}, E)$ be a $S_{\pm}$ -Top-Space such that any two soft points x_α and y_β of $\tilde{X}$ are contained in some $S_{\pm}S_{emi} \#g\alpha$ -connected subspace $\tilde{Y}$ of $\tilde{X}$. Then $\tilde{X}$ is $S_{\pm}S_{emi} \#g\alpha$ -connected space.

Proof. Let $(\tilde{X}, \tilde{T}, E)$ be a $S_{\pm}$ -Top-Space and $(\tilde{Y}, \tilde{T}_Y, E)$ be a $S_{\pm}S_{emi} \#g\alpha$ -connected subspace of $\tilde{X}$. Let x_α, y_β be two disjoint soft points in $\tilde{X}$. Suppose that x_α and y_β in $\tilde{X}$ are contained in the $S_{\pm}S_{emi} \#g\alpha$ -connected subspace $\tilde{Y}$ of $\tilde{X}$, we have to prove, $(\tilde{X}, \tilde{T}, E)$ is a $S_{\pm}S_{emi} \#g\alpha$ -connected space. Suppose $(\tilde{X}, \tilde{T}, E)$ is not a $S_{\pm}S_{emi} \#g\alpha$ -connected space. Then there exists a $S_{\pm}S_{emi} \#g\alpha$ -separation in $\tilde{X}$. Therefore, there exists a pair of non-empty, soft disjoint $S_{\pm}S_{emi} \#g\alpha$ -separated sets G_A and H_B . Since G_A and H_B are non-empty soft sets, there exist x_α and y_β such that $x_\alpha \tilde{\in} G_A$ and $y_\beta \tilde{\in} H_B$. Since $(\tilde{Y}, \tilde{T}_Y, E)$ is a $S_{\pm}S_{emi} \#g\alpha$ -connected soft subspace of $(\tilde{X}, \tilde{T}, E)$, which contains x_α and y_β , therefore by Theorem 3.5, either $\tilde{Y} \tilde{\subseteq} G_A$ or $\tilde{Y} \tilde{\subseteq} H_B$. But $G_A \tilde{\cap} H_B = \tilde{\phi}$, which is a contradiction. Therefore $(\tilde{X}, \tilde{T}, E)$ is $S_{\pm}S_{emi} \#g\alpha$ -connected space.

Corollary 4.5. If a $S_{\pm}$ -Top-Space $(\tilde{X}, \tilde{T}, E)$ contains a $S_{\pm}S_{emi} \#g\alpha$ -connected subspace $(\tilde{Y}, \tilde{T}_Y, E)$ such that $S_{\pm}S_{emi} \#g\alpha -Cl(\tilde{Y}) = \tilde{X}$, then $\tilde{X}$ is $S_{\pm}S_{emi} \#g\alpha$ -connected space.

Proof. Let $(\tilde{Y}, \tilde{T}_Y, E)$ be a $S_{\pm}S_{emi} \#g\alpha$ -connected subspace of $(\tilde{X}, \tilde{T}, E)$ such that $S_{\pm}S_{emi} \#g\alpha -Cl(\tilde{Y}) = \tilde{X}$. Since $(\tilde{Y}, \tilde{T}_Y, E)$ is a $S_{\pm}S_{emi} \#g\alpha$ -connected subspace, $\tilde{Y}$ is a $S_{\pm}S_{emi} \#g\alpha$ -connected set. Then by Theorem 3.6, $S_{\pm}S_{emi} \#g\alpha -Cl(\tilde{Y})$ is $S_{\pm}S_{emi} \#g\alpha$ -connected set. Therefore, $\tilde{X} = S_{\pm}S_{emi} \#g\alpha -Cl(\tilde{Y})$ is a $S_{\pm}S_{emi} \#g\alpha$ -connected set. Hence $(\tilde{X}, \tilde{T}, E)$ is a $S_{\pm}S_{emi} \#g\alpha$ -connected space.

Theorem 4.6. If $(\tilde{Y}, \tilde{T}_Y, E)$ and $(\tilde{Z}, \tilde{T}_Z, E)$ are $S_{\pm}S_{emi} \#g\alpha$ -connected subspaces of a $S_{\pm}$ -Top-Space $(\tilde{X}, \tilde{T}, E)$ are such that $\tilde{Y} \tilde{\cap} \tilde{Z} \neq \tilde{\phi}$, then $\tilde{Y} \tilde{\cup} \tilde{Z}$ is $S_{\pm}S_{emi} \#g\alpha$ -connected subspace.

Proof. Suppose $\tilde{Y} \tilde{\cup} \tilde{Z}$ is not $S_{\pm}S_{emi} \#g\alpha$ -connected subspace, then there exists a $S_{\pm}S_{emi} \#g\alpha$ -separation of $\tilde{Y} \tilde{\cup} \tilde{Z}$. Thus, there exists a pair of $S_{\pm}S_{emi} \#g\alpha$ -separated sets G_A and H_B such that $G_A \tilde{\cup} H_B = \tilde{Y} \tilde{\cup} \tilde{Z}$. Since $\tilde{Y} \tilde{\subseteq} \tilde{Y} \tilde{\cup} \tilde{Z} = G_A \tilde{\cup} H_B$ and $\tilde{Y}$ is $S_{\pm}S_{emi} \#g\alpha$ -connected, then by Theorem 3.5, either $\tilde{Y} \tilde{\subseteq} G_A$ or $\tilde{Y} \tilde{\subseteq} H_B$. Since $\tilde{Z} \tilde{\subseteq} \tilde{Y} \tilde{\cup} \tilde{Z} = G_A \tilde{\cup} H_B$ and $\tilde{Z}$ is $S_{\pm}S_{emi} \#g\alpha$ -connected, then either $\tilde{Z} \tilde{\subseteq} G_A$ or $\tilde{Z} \tilde{\subseteq} H_B$.

(i) If $\tilde{Y} \tilde{\subseteq} G_A$ and $\tilde{Z} \tilde{\subseteq} G_A$, then $\tilde{Y} \tilde{\cup} \tilde{Z} \tilde{\subseteq} G_A$. Hence $H_B = \tilde{\phi}$. This is a contradiction.

(ii) If $\tilde{Y} \subseteq G_A$ and $\tilde{Z} \subseteq H_B$, then $\tilde{Y} \cap \tilde{Z} \subseteq G_A \cap H_B = \tilde{\phi}$, which is a contradiction to our assumption. In the same way, we can get a contradiction if $\tilde{Y} \subseteq H_B$ and $\tilde{Z} \subseteq G_A$ or if $\tilde{Y} \subseteq H_B$ and $\tilde{Z} \subseteq H_B$. Therefore $\tilde{Y} \cup \tilde{Z}$ is $S_{\#}S_{emi} \# \mathcal{G}\alpha$ -connected subspace of $(\tilde{X}, \tilde{T}, E)$.

Corollary 4.7. Let $(\tilde{X}, \tilde{T}_1, E)$ and $(\tilde{X}, \tilde{T}_2, E)$ be $S_{\#}$ -Top-Spaces such that $\tilde{T}_2 \subseteq \tilde{T}_1$ and $(\tilde{X}, \tilde{T}_1, E)$ is $S_{\#}S_{emi} \# \mathcal{G}\alpha$ -connected space. Then $(\tilde{X}, \tilde{T}_2, E)$ is a $S_{\#}S_{emi} \# \mathcal{G}\alpha$ -connected space.

Proof. Suppose $(\tilde{X}, \tilde{T}_2, E)$ is not $S_{\#}S_{emi} \# \mathcal{G}\alpha$ -connected space. Then there exists a $S_{\#}S_{emi} \# \mathcal{G}\alpha$ -separation G_A and H_B of $S_{\#}S_{emi} \# \mathcal{G}\alpha$ -separated sets in $\tilde{X}$ with soft topology $\tilde{T}_2$. Since $\tilde{T}_2 \subseteq \tilde{T}_1$, G_A and H_B are $S_{\#}S_{emi} \# \mathcal{G}\alpha$ -separated set in $\tilde{X}$ with soft topology $\tilde{T}_1$. This is a contradiction. Hence, the result follows.

Proposition 4.8. Let $(\tilde{X}, \tilde{T}, E)$ be a $S_{\#}$ -Top-Space over X . Then the following statements are equivalent:

- (1) $(\tilde{X}, \tilde{T}, E)$ is $S_{\#}S_{emi} \# \mathcal{G}\alpha$ -connected.
- (2) There exists no $F_E, G_E \in S_{\#}S_{emi} \# \mathcal{G}\alpha - C(\tilde{X}, \tilde{T}, E) - \{\tilde{\phi}\}$ such that $F_E \cap G_E = \tilde{\phi}$ and $F_E \cup G_E = \tilde{X}$, where $S_{\#}S_{emi} \# \mathcal{G}\alpha - C(\tilde{X}, \tilde{T}, E) = \{F_E^C \in S_{\#}S_{emi} \# \mathcal{G}\alpha - O(\tilde{X}, \tilde{T}, E)\}$.
- (3) There exists no $F_E, G_E \in S_{\#}S(X)_E - \{\tilde{\phi}\}$ such that $[F_E \cap S_{\#}S_{emi} \# \mathcal{G}\alpha - Cl(G_E)] \cup [S_{\#}S_{emi} \# \mathcal{G}\alpha - Cl(F_E) \cap G_E] = \tilde{\phi}$ and $F_E \cup G_E = \tilde{X}$.
- (4) There exists no $F_E \in S_{\#}S(X)_E - \{\tilde{\phi}, \tilde{X}\}$ such that $F_E \in [S_{\#}S_{emi} \# \mathcal{G}\alpha - O(\tilde{X}, \tilde{T}, E)] \cap [S_{\#}S_{emi} \# \mathcal{G}\alpha - C(\tilde{X}, \tilde{T}, E)]$.

Proof. (1) $\Rightarrow$ (2). Assume there exist $F_E, G_E \in S_{\#}S_{emi} \# \mathcal{G}\alpha - C(\tilde{X}, \tilde{T}, E) - \{\tilde{\phi}\}$ such that $F_E \cap G_E = \tilde{\phi}$ and $F_E \cup G_E = \tilde{X}$. Then $\forall e \in E$, $F_E(e) \cap G_E(e) = \tilde{\phi}$ and $F_E(e) \cup G_E(e) = \tilde{X}$. Thus $\forall e \in E$, $F_E^C(e) = X - F(e) = G(e)$ and $G_E^C(e) = X - G(e) = F(e)$, which implies that $F_E^C = G_E \in S_{\#}S_{emi} \# \mathcal{G}\alpha - C(\tilde{X}, \tilde{T}, E) - \{\tilde{\phi}\}$ and $G_E^C = F_E \in S_{\#}S_{emi} \# \mathcal{G}\alpha - C(\tilde{X}, \tilde{T}, E) - \{\tilde{\phi}\}$. Then there exist $F_E, G_E \in S_{\#}S_{emi} \# \mathcal{G}\alpha - O(\tilde{X}, \tilde{T}, E) - \{\tilde{\phi}\}$ such that $F_E \cap G_E = \tilde{\phi}$ and $F_E \cup G_E = \tilde{X}$. However, $(\tilde{X}, \tilde{T}, E)$ is $S_{\#}S_{emi} \# \mathcal{G}\alpha$ -connected. This is a contradiction.

(2) $\Rightarrow$ (3). Assume there exist $F_E, G_E \in S_{\#}S(X)_E - \{\tilde{\phi}\}$ such that $[F_E \cap S_{\#}S_{emi} \# \mathcal{G}\alpha - Cl(G_E)] \cup [S_{\#}S_{emi} \# \mathcal{G}\alpha - Cl(F_E) \cap G_E] = \tilde{\phi}$ and $F_E \cup G_E = \tilde{X}$. Obviously, $F_E \cap G_E = \tilde{\phi}$. Now $S_{\#}S_{emi} \# \mathcal{G}\alpha - Cl(G_E) = S_{\#}S_{emi} \# \mathcal{G}\alpha - Cl(G_E) \cap \tilde{X} = S_{\#}S_{emi} \# \mathcal{G}\alpha - Cl(G_E) \cap (F_E \cup G_E) = [S_{\#}S_{emi} \# \mathcal{G}\alpha - Cl(G_E) \cap F_E] \cup [S_{\#}S_{emi} \# \mathcal{G}\alpha - Cl(G_E) \cap G_E] = G_E$, which implies that G_E is a $S_{\#}S_{emi} \# \mathcal{G}\alpha$ -closed set. By using the same methods, we can show that F_E is also a $S_{\#}S_{emi} \# \mathcal{G}\alpha$ -closed set. Hence there exist $F_E, G_E \in S_{\#}S_{emi} \# \mathcal{G}\alpha - C(\tilde{X}, \tilde{T}, E) - \{\tilde{\phi}\}$ such that $F_E \cap G_E = \tilde{\phi}$ and $F_E \cup G_E = \tilde{X}$. This is a contradiction. So (3) holds.

(3) $\Rightarrow$ (4). Assume there exists $F_E \in [S_{\#}S_{emi} \# \mathcal{G}\alpha - O(\tilde{X}, \tilde{T}, E)] \tilde{\cap} [S_{\#}S_{emi} \# \mathcal{G}\alpha - C(\tilde{X}, \tilde{T}, E)] - \{\tilde{\phi}, \tilde{X}\}$.

If we take $G_E = F_E^C$ then

$F_E, G_E \in [S_{\#}S_{emi} \# \mathcal{G}\alpha - O(\tilde{X}, \tilde{T}, E)] \tilde{\cap} [S_{\#}S_{emi} \# \mathcal{G}\alpha - C(\tilde{X}, \tilde{T}, E)] - \{\tilde{\phi}\} (\subseteq S_{\#}S_{emi} (X)_E - \{\tilde{\phi}\})$. Besides,

we have $[F_E \tilde{\cap} S_{\#}S_{emi} \# \mathcal{G}\alpha - Cl(G_E)] \tilde{\cup} [G_E \tilde{\cap} S_{\#}S_{emi} \# \mathcal{G}\alpha - Cl(F_E)] = F_E \tilde{\cap} G_E = \tilde{\phi}$ and $F_E \tilde{\cup} G_E = \tilde{X}$.

This is a contradiction, so (4) holds.

(4) $\Rightarrow$ (1). Assume $(\tilde{X}, \tilde{T}_2, E)$ is not $S_{\#}S_{emi} \# \mathcal{G}\alpha$ -connected. Then there exist

$F_E, G_E \in S_{\#}S_{emi} \# \mathcal{G}\alpha - O(\tilde{X}, \tilde{T}, E) - \{\tilde{\phi}\}$ such that $F_E \tilde{\cap} G_E = \tilde{\phi}$ and $F_E \tilde{\cup} G_E = \tilde{X}$. It is easy to see that

$F_E^C = G_E$ and $G_E^C = F_E$. Thus

$F_E, G_E \in [S_{\#}S_{emi} \# \mathcal{G}\alpha - O(\tilde{X}, \tilde{T}, E)] \tilde{\cap} [S_{\#}S_{emi} \# \mathcal{G}\alpha - C(\tilde{X}, \tilde{T}, E)] - \{\tilde{\phi}, \tilde{X}\}$. This is a contradiction.

Theorem 4.9. A non-null proper subset F_E of a $S_{\#}S_{emi} \# \mathcal{G}\alpha$ -connected space $(\tilde{X}, \tilde{T}, E)$ has a non-null $S_{\#}S_{emi} \# \mathcal{G}\alpha$ -boundary points.

Proof. Suppose, by contrary, that $S_{\#}S_{emi} \# \mathcal{G}\alpha - Bd(F_E) = \tilde{\phi}$. Then $S_{\#}S_{emi} \# \mathcal{G}\alpha - Cl(F_E) = S_{\#}S_{emi} \# \mathcal{G}\alpha - Int(F_E)$, which means that F_E is both $S_{\#}S_{emi} \# \mathcal{G}\alpha$ -open and $S_{\#}S_{emi} \# \mathcal{G}\alpha$ -closed. But this contradicts the $S_{\#}S_{emi} \# \mathcal{G}\alpha$ -connectedness of $(\tilde{X}, \tilde{T}, E)$. Hence, $S_{\#}S_{emi} \# \mathcal{G}\alpha - Bd(F_E) \neq \tilde{\phi}$, as required.

Definition 4.10. A soft mapping $f: (\tilde{X}, \tilde{\tau}, E) \rightarrow (\tilde{Y}, \tilde{\sigma}, K)$ is said to be $S_{\#}S_{emi} \# \mathcal{G}\alpha$ -continuous (resp. $S_{\#}S_{emi} \# \mathcal{G}\alpha$ -irresolute) if $f^{-1}(H_K)$ is a $S_{\#}S_{emi} \# \mathcal{G}\alpha$ -open set where H_K is a soft open ($S_{\#}S_{emi} \# \mathcal{G}\alpha$ -open) set.

Proposition 4.11. Let f be a $S_{\#}S_{emi} \# \mathcal{G}\alpha$ -continuous mapping of a $S_{\#}S_{emi} \# \mathcal{G}\alpha$ -connected space $(\tilde{X}, \tilde{T}, E)$ onto a $S_{\#}$ -Top-Space $(\tilde{Y}, \tilde{\sigma}, K)$. Then $f(\tilde{X})$ is connected.

Proof. Suppose that $f(\tilde{X}) = \tilde{Y}$ is disconnected. Then we get $\tilde{\sigma}$ contains two non-null disjoint soft open sets F_K and G_K . By hypothesis, $f^{-1}(F_K)$ and $f^{-1}(G_K)$ are disjoint $S_{\#}S_{emi} \# \mathcal{G}\alpha$ -open sets in $(\tilde{X}, \tilde{T}, E)$. Since f is surjective, these $S_{\#}S_{emi} \# \mathcal{G}\alpha$ -open sets are non-null. Thus $(\tilde{X}, \tilde{T}, E)$ is $S_{\#}S_{emi} \# \mathcal{G}\alpha$ -disconnected. But this is a contradiction. Hence, $(\tilde{Y}, \tilde{\sigma}, K)$ is $S_{\#}S_{emi} \# \mathcal{G}\alpha$ -connected.

Theorem 4.12. Let f be a $S_{\#}S_{emi} \# \mathcal{G}\alpha$ -function from a $S_{\#}$ -Top-Space $(\tilde{X}, \tilde{T}, E)$ onto a soft space $(\tilde{Y}, \tilde{\sigma}, K)$. Suppose $(\tilde{X}, \tilde{T}, E)$ is $S_{\#}S_{emi} \# \mathcal{G}\alpha$ -connected. Then $(\tilde{Y}, \tilde{\sigma}, K)$ is $S_{\#}S_{emi} \# \mathcal{G}\alpha$ -connected.

Proof. Our assumption is that $(\tilde{Y}, \tilde{\sigma}, K)$ is not $S_{\#}S_{emi} \# \mathcal{G}\alpha$ -connected. Therefore, there exists a non-empty proper subset H_K of $(\tilde{Y}, \tilde{\sigma}, K)$ which is both $S_{\#}S_{emi} \# \mathcal{G}\alpha$ -open and $S_{\#}S_{emi} \# \mathcal{G}\alpha$ -closed. Then the

inverse image of H_K under f is both $S_{\#}S_{emi}^{\#}g\alpha$ -open and $S_{\#}S_{emi}^{\#}g\alpha$ -closed in $(\tilde{X}, \tilde{T}, E)$, which contradicts our hypothesis.

Corollary 4.13. A $S_{\#}S_{emi}^{\#}g\alpha$ -irresolute function maps $S_{\#}S_{emi}^{\#}g\alpha$ -connected set onto $S_{\#}S_{emi}^{\#}g\alpha$ -connected set.

Theorem 4.14. Consider a $S_{\#}S_{emi}^{\#}g\alpha$ -continuous function f from a $S_{\#}$ -Top-Space $(\tilde{X}, \tilde{T}, E)$ onto a $S_{\#}$ -Top-Space $(\tilde{Y}, \tilde{\sigma}, K)$. If $(\tilde{X}, \tilde{T}, E)$ is $S_{\#}S_{emi}^{\#}g\alpha$ -connected, then $(\tilde{Y}, \tilde{\sigma}, K)$ is soft connected.

Proof. Consider $(\tilde{Y}, \tilde{\sigma}, K)$ is not soft connected. Then there is a nonempty proper subset of $(\tilde{Y}, \tilde{\sigma}, K)$ which is both soft open and soft closed. Then the inverse image of H_K under f is both $S_{\#}S_{emi}^{\#}g\alpha$ -open and $S_{\#}S_{emi}^{\#}g\alpha$ -closed in $(\tilde{X}, \tilde{T}, E)$, a contradiction. Though the concept of soft continuity, $S_{\#}g^{**}$ -continuity and $S_{\#}S_{emi}^{\#}g\alpha$ -irresolute are independent of each other, but they behave similarly in the case of $S_{\#}S_{emi}^{\#}g\alpha$ -connectedness, that is, these functions map a $S_{\#}S_{emi}^{\#}g\alpha$ -connected set onto a soft connected set.

Definition 4.15. A soft function $f : (\tilde{X}, \tilde{\tau}, E) \rightarrow (\tilde{Y}, \tilde{\sigma}, K)$ is called $S_{\#}S_{emi}^{\#}g\alpha$ -homeomorphism if f is both $S_{\#}S_{emi}^{\#}g\alpha$ -continuous and $S_{\#}S_{emi}^{\#}g\alpha$ -open.

Remark 4.16. A soft-homeomorphism preserves $S_{\#}S_{emi}^{\#}g\alpha$ -connectedness.

Theorem 4.17. If $(\tilde{X}, \tilde{T}, E)$ is $S_{\#}S_{emi}^{\#}g\alpha$ -connected space, then $(\tilde{X}, \tilde{T}, E) \times \{a\}$ is also $S_{\#}S_{emi}^{\#}g\alpha$ -connected.

Proof. Obviously, $(\tilde{X}, \tilde{T}, E)$ is soft homeomorphic to $(\tilde{X}, \tilde{T}, E) \times \{a\}$. Then by the previous remark, $(\tilde{X}, \tilde{T}, E) \times \{a\}$ is $S_{\#}S_{emi}^{\#}g\alpha$ -connected.

Theorem 4.18. If $(\tilde{X}, \tilde{T}, E)$ and $(\tilde{Y}, \tilde{\sigma}, K)$ are two $S_{\#}S_{emi}^{\#}g\alpha$ -connected spaces, then $(\tilde{X}, \tilde{T}, E) \times (\tilde{Y}, \tilde{\sigma}, K)$ is also $S_{\#}S_{emi}^{\#}g\alpha$ -connected.

Proof. For any soft point (x_{α}, y_{β}) in the product $(\tilde{X}, \tilde{T}, E) \times (\tilde{Y}, \tilde{\sigma}, K)$, each of the subspaces $(\tilde{X}, \tilde{T}, E) \times \{y_{\beta}\} \cup (\tilde{Y}, \tilde{\sigma}, K) \times \{x_{\alpha}\}$ is $S_{\#}S_{emi}^{\#}g\alpha$ -connected since it is the union of two $S_{\#}S_{emi}^{\#}g\alpha$ -connected subspaces with a point in common. Then by Theorem 4.6, $(\tilde{X}, \tilde{T}, E) \times (\tilde{Y}, \tilde{\sigma}, K)$ is also $S_{\#}S_{emi}^{\#}g\alpha$ -connected.

V. Soft $S_{emi}^{\#}g\alpha$ -D isconnected Space

In this section, we present a new concept of $S_{\#}S_{emi}^{\#}g\alpha$ -disconnected space and discuss some of its properties.

Definition 5.1. A $S_{\pm}$ -Top-Space $(\tilde{X}, \tilde{T}, E)$ is said to be a $S_{\pm}S_{emi}^{\#}g\alpha$ -disconnected ($S_{\pm}S_{emi}^{\#}g\alpha$ -disconnected) space if there exists a $S_{\pm}S_{emi}^{\#}g\alpha$ -separation of $\tilde{X}$.

Theorem 5.2. A $S_{\pm}$ -Top-Space $(\tilde{X}, \tilde{T}, E)$ is $S_{\pm}S_{emi}^{\#}g\alpha$ -disconnected if and only if there exists a non-empty proper soft subset of $\tilde{X}$ which is both $S_{\pm}S_{emi}^{\#}g\alpha$ -open and $S_{\pm}S_{emi}^{\#}g\alpha$ -closed.

Proof. Let $(\tilde{X}, \tilde{T}, E)$ be a $S_{\pm}$ -Top-Space and F_E be a non-empty proper soft subset which is both $S_{\pm}S_{emi}^{\#}g\alpha$ -open and $S_{\pm}S_{emi}^{\#}g\alpha$ -closed set. Since F_E is $S_{\pm}S_{emi}^{\#}g\alpha$ -closed, $S_{\pm}S_{emi}^{\#}g\alpha-Cl(F_E) = F_E \dots (*)$. Also, F_E is $S_{\pm}S_{emi}^{\#}g\alpha$ -open implies $F_E^C = G_E$ (say) is $S_{\pm}S_{emi}^{\#}g\alpha$ -closed. Therefore, $S_{\pm}S_{emi}^{\#}g\alpha-Cl(G_E) = G_E \dots (**)$. Since G_E is the soft complement of F_E in $\tilde{X}$, therefore $F_E \tilde{\cup} G_E = \tilde{X}$ and $F_E \tilde{\cap} G_E = \tilde{\phi}$. Now $F_E \tilde{\cap} G_E = \tilde{\phi}$ implies that $[S_{\pm}S_{emi}^{\#}g\alpha-Cl(F_E)] \tilde{\cap} G_E = \tilde{\phi}$ by $(*)$ and $F_E \tilde{\cap} [S_{\pm}S_{emi}^{\#}g\alpha-Cl(G_E)] = \tilde{\phi}$ by $(**)$. Since $\tilde{X} = G_E \tilde{\cup} F_E$ and $[S_{\pm}S_{emi}^{\#}g\alpha-Cl(F_E)] \tilde{\cap} G_E = \tilde{\phi}$, $F_E \tilde{\cap} [S_{\pm}S_{emi}^{\#}g\alpha-Cl(G_E)] = \tilde{\phi}$, therefore $\tilde{X}$ is $S_{\pm}S_{emi}^{\#}g\alpha$ -disconnected. Therefore, we conclude that $(\tilde{X}, \tilde{T}, E)$ is $S_{\pm}S_{emi}^{\#}g\alpha$ -disconnected.

Conversely, suppose that $S_{\pm}$ -Top-Space $(\tilde{X}, \tilde{T}, E)$ is $S_{\pm}S_{emi}^{\#}g\alpha$ -disconnected space. Therefore, there exists a pair of non-empty proper soft subsets of $\tilde{X}$ such that $\tilde{X} = G_A \tilde{\cup} H_B$, $S_{\pm}S_{emi}^{\#}g\alpha-Cl(G_A) \tilde{\cap} H_B = \tilde{\phi}$, $G_A \tilde{\cap} S_{\pm}S_{emi}^{\#}g\alpha-Cl(H_B) = \tilde{\phi}$. Hence, we conclude that the $S_{\pm}$ -Top-Space $(\tilde{X}, \tilde{T}, E)$ is $S_{\pm}S_{emi}^{\#}g\alpha$ -disconnected space. Now $G_A \subseteq S_{\pm}S_{emi}^{\#}g\alpha-Cl(G_A)$. Therefore, $[S_{\pm}S_{emi}^{\#}g\alpha-Cl(G_A)] \tilde{\cap} H_B = \tilde{\phi}$ implies that $G_A \tilde{\cap} H_B = \tilde{\phi}$. Also $\tilde{X} = G_A \tilde{\cup} H_B$. Therefore, $G_A = H_B^C$. Thus, G_A is a proper soft subset of $\tilde{X}$ which is non-empty. Again, therefore $G_A \tilde{\cup} H_B = \tilde{X}$ implies $G_A \tilde{\cup} [S_{\pm}S_{emi}^{\#}g\alpha-Cl(H_B)] = \tilde{X}$ and $H_B \subseteq [S_{\pm}S_{emi}^{\#}g\alpha-Cl(H_B)]$. We have $G_A \tilde{\cap} H_B = \tilde{\phi}$ implies $G_A \tilde{\cap} [S_{\pm}S_{emi}^{\#}g\alpha-Cl(H_B)] = \tilde{\phi}$. We conclude that $G_A = \tilde{X} - [S_{\pm}S_{emi}^{\#}g\alpha-Cl(H_B)]$. But as H_B is $S_{\pm}S_{emi}^{\#}g\alpha$ -closed set. Therefore $G_A = \tilde{X} - [S_{\pm}S_{emi}^{\#}g\alpha-Cl(H_B)]$ is $S_{\pm}S_{emi}^{\#}g\alpha$ -open set... $(***)$. Similarly, we can prove $H_B = \tilde{X} - [S_{\pm}S_{emi}^{\#}g\alpha-Cl(G_A)]$ is $S_{\pm}S_{emi}^{\#}g\alpha$ -open set. Now $G_A = H_B^C$, where H_B is $S_{\pm}S_{emi}^{\#}g\alpha$ -open set. Therefore, G_A is $S_{\pm}S_{emi}^{\#}g\alpha$ -closed and we have already shown in $(***)$ that G_A is $S_{\pm}S_{emi}^{\#}g\alpha$ -open. Hence, G_A is a non-empty proper soft subset which is both $S_{\pm}S_{emi}^{\#}g\alpha$ -open and $S_{\pm}S_{emi}^{\#}g\alpha$ -closed set.

Theorem 5.3. A $S_{\pm}$ -Top-Space $(\tilde{X}, \tilde{T}, E)$ is $S_{\pm}S_{emi}^{\#}g\alpha$ -disconnected space if and only if any one of the following statements holds good.

- (i) $\tilde{X}$ is the soft union of two non-empty disjoint $S_{\pm}S_{emi}^{\#}g\alpha$ -open sets.
- (ii) $\tilde{X}$ is the soft union of two non-empty disjoint $S_{\pm}S_{emi}^{\#}g\alpha$ -closed sets.

Proof. Let $(\tilde{X}, \tilde{T}, E)$ be a $S_{\pm}S_{emi}^{\#}g\alpha$ -disconnected space. We have to prove $\tilde{X}$ is the soft union of two non-empty disjoint $S_{\pm}S_{emi}^{\#}g\alpha$ -open ($S_{\pm}S_{emi}^{\#}g\alpha$ -closed) subsets of $\tilde{X}$. Since $(\tilde{X}, \tilde{T}, E)$ is a $S_{\pm}S_{emi}^{\#}g\alpha$ -disconnected space, then by Theorem 5.2, there exists a non-empty proper soft subset F_E of

$\tilde{X}$ which is both $S_{\pm}S_{emi}^{\#}g\alpha$ -open and $S_{\pm}S_{emi}^{\#}g\alpha$ -closed. Consequently, F_E^C is non-empty, which is both $S_{\pm}S_{emi}^{\#}g\alpha$ -closed and $S_{\pm}S_{emi}^{\#}g\alpha$ -open. Also, $F_E \cup F_E^C = \tilde{X}$ and $F_E \cap F_E^C = \emptyset$. From above, we conclude that $\tilde{X}$ is the soft union of two disjoint non-empty $S_{\pm}S_{emi}^{\#}g\alpha$ -open ($S_{\pm}S_{emi}^{\#}g\alpha$ -closed) sets. As F_E and F_E^C are both $S_{\pm}S_{emi}^{\#}g\alpha$ -open as well as $S_{\pm}S_{emi}^{\#}g\alpha$ -closed. Conversely suppose, $\tilde{X}$ is the soft union of two non-empty disjoint $S_{\pm}S_{emi}^{\#}g\alpha$ -open sets. That is $\tilde{X} = G_A \cup H_B$ and $G_A \cap H_B = \emptyset$. We need to prove $(\tilde{X}, \tilde{T}, E)$ is a $S_{\pm}S_{emi}^{\#}g\alpha$ -disconnected space. Since $G_A = H_B^C$ is $S_{\pm}S_{emi}^{\#}g\alpha$ -open so that G_A is $S_{\pm}S_{emi}^{\#}g\alpha$ -closed set. Also, H_B is non-empty. Therefore $G_A = H_B^C$ is a proper soft subset of $\tilde{X}$ which is non-empty and is both $S_{\pm}S_{emi}^{\#}g\alpha$ -open and $S_{\pm}S_{emi}^{\#}g\alpha$ -closed set. Hence, by Theorem 5.2, the $S_{\pm}$ -Top-Space $\tilde{X}$ is $S_{\pm}S_{emi}^{\#}g\alpha$ -disconnected space. In the second case, if both G_A and H_B are both non-empty $S_{\pm}S_{emi}^{\#}g\alpha$ -closed sets, then $G_A = H_B^C$ and H_B is $S_{\pm}S_{emi}^{\#}g\alpha$ -closed set. Therefore, G_A is $S_{\pm}S_{emi}^{\#}g\alpha$ -open set. As above G_A will be a proper non-empty soft subset that is both $S_{\pm}S_{emi}^{\#}g\alpha$ -open and $S_{\pm}S_{emi}^{\#}g\alpha$ -closed set. Hence, by Theorem 5.2, the $S_{\pm}$ -Top-Space $(\tilde{X}, \tilde{T}, E)$ is $S_{\pm}S_{emi}^{\#}g\alpha$ -disconnected space.

VI. Soft $S_{emi}^{\#}g\alpha$ -Component of a Soft Set and Soft Topological Space

In this section, we introduce the concept of $S_{\pm}S_{emi}^{\#}g\alpha$ -component of a soft subset as well as $S_{\pm}S_{emi}^{\#}g\alpha$ -component of a $S_{\pm}$ -Top-Space and discuss their properties and characterizations.

Definition 6.1. Let F_E be a soft subset of a $S_{\pm}$ -Top-Space $(\tilde{X}, \tilde{T}, E)$ and x_e be any soft point of F_E , then the soft union of all $S_{\pm}S_{emi}^{\#}g\alpha$ -connected subsets of F_E containing x_e is defined as the $S_{\pm}S_{emi}^{\#}g\alpha$ -component of F_E with respect to x_e and is denoted by $S_{\pm}S_{emi}^{\#}g\alpha-C(F_E, x_e)$.

$$S_{\pm}S_{emi}^{\#}g\alpha-C(F_E, x_e) = \tilde{\cup} \{P_E \subseteq F_E : x_e \in P_E, P_E \text{ is } S_{\pm}S_{emi}^{\#}g\alpha\text{-connected set}\}.$$

Proposition 6.2. $S_{\pm}S_{emi}^{\#}g\alpha-C(F_E, x_e)$ is a $S_{\pm}S_{emi}^{\#}g\alpha$ -connected set.

Proof. The $S_{\pm}S_{emi}^{\#}g\alpha$ -component of F_E with respect to the soft point x_e is the soft union of all $S_{\pm}S_{emi}^{\#}g\alpha$ -connected subsets of F_E which have a nonempty soft intersection (as each contains the soft point x_e). Then by Theorem 3.8, $S_{\pm}S_{emi}^{\#}g\alpha-C(F_E, x_e)$ is a $S_{\pm}S_{emi}^{\#}g\alpha$ -connected set.

Remark 6.3. $S_{\pm}S_{emi}^{\#}g\alpha-C(F_E, x_e)$ is the largest $S_{\pm}S_{emi}^{\#}g\alpha$ -connected subset of F_E containing x_e .

Theorem 6.4. Let $(\tilde{X}, \tilde{T}, E)$ be a $S_{\pm}$ -Top-Space. Then the following statements are true.

- (i) Each soft point in $\tilde{X}$ is contained in exactly one $S_{\pm}S_{emi}^{\#}g\alpha$ -component of $\tilde{X}$.
- (ii) The $S_{\pm}S_{emi}^{\#}g\alpha$ -components with respect to two different soft points of $\tilde{X}$ are either identical or soft disjoint.

Proof. (i) Let x_e be an arbitrary soft point of $\tilde{X}$ and $\{P_{E_\lambda} : \lambda \in \Lambda\}$ be the collection of all $S_{\#}S_{emi} \#g\alpha$ -connected subsets of $\tilde{X}$ which contain x_e . Since the soft singleton $\{x_e\}$ is $S_{\#}S_{emi} \#g\alpha$ -connected, the collection $\{P_{E_\lambda} : \lambda \in \Lambda\}$ of all soft $S_{\#}S_{emi} \#g\alpha$ -connected subsets containing the soft point x_e is non-empty. Then $\bigcap \{P_{E_\lambda} : \lambda \in \Lambda\} \neq \emptyset$, as x_e is contained in each P_{E_λ} . Now, $S_{\#}S_{emi} \#g\alpha-C(\tilde{X}, x_e)$ is the soft union of $\{P_{E_\lambda} : \lambda \in \Lambda\}$ having a non-empty soft intersection. Then by **Theorem 3.8**, $S_{\#}S_{emi} \#g\alpha-C(\tilde{X}, x_e)$ is also $S_{\#}S_{emi} \#g\alpha$ -connected. Again $S_{\#}S_{emi} \#g\alpha-C(\tilde{X}, x_e)$ is the maximal $S_{\#}S_{emi} \#g\alpha$ -connected subset of $\tilde{X}$ containing x_e . Therefore, $S_{\#}S_{emi} \#g\alpha-C(\tilde{X}, x_e)$ is $S_{\#}S_{emi} \#g\alpha$ -component of $\tilde{X}$ with respect to x_e . Now we have to prove that there is no other $S_{\#}S_{emi} \#g\alpha$ -component of $\tilde{X}$ containing x_e . Now, suppose $S_{\#}S_{emi} \#g\alpha-C^*(\tilde{X}, x_e)$ be another $S_{\#}S_{emi} \#g\alpha$ -component of $\tilde{X}$ containing x_e . Then $S_{\#}S_{emi} \#g\alpha-C^*(\tilde{X}, x_e)$ is a $S_{\#}S_{emi} \#g\alpha$ -connected subset of $\tilde{X}$ containing x_e . But $S_{\#}S_{emi} \#g\alpha-C(\tilde{X}, x_e)$ being a $S_{\#}S_{emi} \#g\alpha$ -component is the maximal $S_{\#}S_{emi} \#g\alpha$ -connected subset of $\tilde{X}$ containing x_e . Consequently $S_{\#}S_{emi} \#g\alpha-C^*(\tilde{X}, x_e) \subseteq S_{\#}S_{emi} \#g\alpha-C(\tilde{X}, x_e)$. Similarly $S_{\#}S_{emi} \#g\alpha-C(\tilde{X}, x_e) \subseteq S_{\#}S_{emi} \#g\alpha-C^*(\tilde{X}, x_e)$ and hence $S_{\#}S_{emi} \#g\alpha-C(\tilde{X}, x_e) = S_{\#}S_{emi} \#g\alpha-C^*(\tilde{X}, x_e)$. This implies x_e is contained in exactly one $S_{\#}S_{emi} \#g\alpha$ -component of $\tilde{X}$.

(ii) $S_{\#}S_{emi} \#g\alpha-C(\tilde{X}, x_\alpha)$ and $S_{\#}S_{emi} \#g\alpha-C(\tilde{X}, y_\beta)$ be respectively the $S_{\#}S_{emi} \#g\alpha$ -components of $\tilde{X}$ with respect to two different soft points x_e and y_β of $\tilde{X}$ with $x_e \neq y_\beta$. If $S_{\#}S_{emi} \#g\alpha-C(\tilde{X}, x_\alpha) \cap S_{\#}S_{emi} \#g\alpha-C(\tilde{X}, y_\beta) = \emptyset$, then the proof is over. Suppose $S_{\#}S_{emi} \#g\alpha-C(\tilde{X}, x_\alpha) \cap S_{\#}S_{emi} \#g\alpha-C(\tilde{X}, y_\beta) \neq \emptyset$. We may choose a soft point $z_\gamma \in S_{\#}S_{emi} \#g\alpha-C(\tilde{X}, x_\alpha) \cap S_{\#}S_{emi} \#g\alpha-C(\tilde{X}, y_\beta)$. Then $z_\gamma \in S_{\#}S_{emi} \#g\alpha-C(\tilde{X}, x_\alpha)$ and $z_\gamma \in S_{\#}S_{emi} \#g\alpha-C(\tilde{X}, y_\beta)$. Since $S_{\#}S_{emi} \#g\alpha-C(\tilde{X}, z_\gamma)$ is the maximal $S_{\#}S_{emi} \#g\alpha$ -component of $\tilde{X}$ with respect to z_γ . Therefore $S_{\#}S_{emi} \#g\alpha-C(\tilde{X}, x_\alpha) \subseteq S_{\#}S_{emi} \#g\alpha-C(\tilde{X}, z_\gamma) \dots (1)$. Since $x_\alpha \in S_{\#}S_{emi} \#g\alpha-C(\tilde{X}, x_\alpha) \subseteq S_{\#}S_{emi} \#g\alpha-C(\tilde{X}, z_\gamma)$, we have $x_\alpha \in S_{\#}S_{emi} \#g\alpha-C(\tilde{X}, z_\gamma)$. Also $S_{\#}S_{emi} \#g\alpha-C(\tilde{X}, z_\gamma)$ is a $S_{\#}S_{emi} \#g\alpha$ -connected set containing x_α . But, $S_{\#}S_{emi} \#g\alpha-C(\tilde{X}, x_\alpha)$ is the maximal $S_{\#}S_{emi} \#g\alpha$ -component set containing x_α . Therefore $S_{\#}S_{emi} \#g\alpha-C(\tilde{X}, z_\gamma) \subseteq S_{\#}S_{emi} \#g\alpha-C(\tilde{X}, x_\alpha) \dots (2)$. From (1) and (2), we have $S_{\#}S_{emi} \#g\alpha-C(\tilde{X}, x_\alpha) = S_{\#}S_{emi} \#g\alpha-C(\tilde{X}, z_\gamma)$. Similarly $z_\gamma \in S_{\#}S_{emi} \#g\alpha-C(\tilde{X}, y_\beta)$ implies that $S_{\#}S_{emi} \#g\alpha-C(\tilde{X}, y_\beta) = S_{\#}S_{emi} \#g\alpha-C(\tilde{X}, z_\gamma)$. Thus we have $S_{\#}S_{emi} \#g\alpha-C(\tilde{X}, x_\alpha) = S_{\#}S_{emi} \#g\alpha-C(\tilde{X}, y_\beta)$.

Definition 6.5. The $S_{\#}S_{emi} \#g\alpha$ -component of a space $\tilde{X}$ of a $S_{\#}$ -Top-Space $(\tilde{X}, \tilde{T}, E)$ is the maximal $S_{\#}S_{emi} \#g\alpha$ -connected subspace of $(\tilde{X}, \tilde{T}, E)$. The $S_{\#}S_{emi} \#g\alpha$ -component of a $S_{\#}$ -Top-Space $(\tilde{X}, \tilde{T}, E)$ is denoted as $S_{\#}S_{emi} \#g\alpha-C(\tilde{X})$.

Theorem 6.6. Each $S_{\#}S_{emi}^{\#}g\alpha$ -component of a $S_{\#}$ -Top-Space $(\tilde{X}, \tilde{T}, E)$ is a $S_{\#}S_{emi}^{\#}g\alpha$ -closed set.

Proof. Let $S_{\#}S_{emi}^{\#}g\alpha-C(\tilde{X})$ be a $S_{\#}S_{emi}^{\#}g\alpha$ -component of $\tilde{X}$. Then, $S_{\#}S_{emi}^{\#}g\alpha-C(\tilde{X})$ is the maximal $S_{\#}S_{emi}^{\#}g\alpha$ -connected subset of $\tilde{X}$. By Theorem 3.6, $S_{\#}S_{emi}^{\#}g\alpha-Cl[S_{\#}S_{emi}^{\#}g\alpha-C(\tilde{X})]$ is also $S_{\#}S_{emi}^{\#}g\alpha$ -connected and $S_{\#}S_{emi}^{\#}g\alpha-Cl[S_{\#}S_{emi}^{\#}g\alpha-C(\tilde{X})] \subseteq S_{\#}S_{emi}^{\#}g\alpha-C(\tilde{X})$. But $S_{\#}S_{emi}^{\#}g\alpha-C(\tilde{X}) \subseteq S_{\#}S_{emi}^{\#}g\alpha-Cl[S_{\#}S_{emi}^{\#}g\alpha-C(\tilde{X})]$. Hence, $S_{\#}S_{emi}^{\#}g\alpha-C(\tilde{X}) = S_{\#}S_{emi}^{\#}g\alpha-Cl[S_{\#}S_{emi}^{\#}g\alpha-C(\tilde{X})]$. Therefore, $S_{\#}S_{emi}^{\#}g\alpha-C(\tilde{X})$ is a $S_{\#}S_{emi}^{\#}g\alpha$ -closed set.

Theorem 6.7. In a $S_{\#}$ -Top-Space $(\tilde{X}, \tilde{T}, E)$, the set of all disjoint $S_{\#}S_{emi}^{\#}g\alpha$ -components of soft points of $\tilde{X}$ form a partition of $\tilde{X}$.

Proof. Let $S_{\#}S_{emi}^{\#}g\alpha-C(\tilde{X}, x_{\alpha})$ and $S_{\#}S_{emi}^{\#}g\alpha-C(\tilde{X}, y_{\beta})$ be two $S_{\#}S_{emi}^{\#}g\alpha$ -components of distinct soft points x_{α} and y_{β} in $\tilde{X}$. Now for any soft point $x_{\alpha} \in \tilde{X}$, $x_{\alpha} \in S_{\#}S_{emi}^{\#}g\alpha-C(\tilde{X}, x_{\alpha})$ and hence $\tilde{X} = \bigcup \{x_{\alpha} : x_{\alpha} \in \tilde{X}\} \subseteq \bigcup \{S_{\#}S_{emi}^{\#}g\alpha-C(\tilde{X}, x_{\alpha}) : x_{\alpha} \in \tilde{X}\}$, by considering only the disjoint $S_{\#}S_{emi}^{\#}g\alpha$ -components with respect to the soft point $x_{\alpha} \in \tilde{X}$. This implies that $\tilde{X} \subseteq \bigcup \{S_{\#}S_{emi}^{\#}g\alpha-C(\tilde{X}, x_{\alpha}) : x_{\alpha} \in \tilde{X}\} \subseteq \tilde{X}$. Therefore $\bigcup \{S_{\#}S_{emi}^{\#}g\alpha-C(\tilde{X}, x_{\alpha}) : x_{\alpha} \in \tilde{X}\} = \tilde{X}$.

VII. Locally Soft $S_{emi}^{\#}g\alpha$ -Connected Space

In this section, we introduce the concept locally $S_{\#}S_{emi}^{\#}g\alpha$ -connected space and discuss its properties. Also, we analyse locally $S_{\#}S_{emi}^{\#}g\alpha$ -connected space with other soft connectedness like $S_{\#}S_{emi}^{\#}g\alpha$ -connected space, soft connected space and locally soft connected space.

Definition 7.1. A $S_{\#}$ -Top-Space $(\tilde{X}, \tilde{T}, E)$ is called locally $S_{\#}S_{emi}^{\#}g\alpha$ -connected at $x_e \in \tilde{X}$ if and only if for every $S_{\#}S_{emi}^{\#}g\alpha$ -open set P_E containing x_e , there exist a $S_{\#}S_{emi}^{\#}g\alpha$ -connected soft open set F_E such that $x_e \in F_E \subseteq P_E$.

Definition 7.2. A $S_{\#}$ -Top-Space $(\tilde{X}, \tilde{T}, E)$ is said to be locally $S_{\#}S_{emi}^{\#}g\alpha$ -connected if and only if it is locally $S_{\#}S_{emi}^{\#}g\alpha$ -connected at each of its soft points.

Theorem 7.3. Every $S_{\#}S_{emi}^{\#}g\alpha$ -component of a locally $S_{\#}S_{emi}^{\#}g\alpha$ -connected space is $S_{\#}S_{emi}^{\#}g\alpha$ -open set.

Proof. Let $S_{\#}S_{emi}^{\#}g\alpha-C(\tilde{X})$ be a $S_{\#}S_{emi}^{\#}g\alpha$ -component of a locally $S_{\#}S_{emi}^{\#}g\alpha$ -connected space $(\tilde{X}, \tilde{T}, E)$ and let $x_e \in S_{\#}S_{emi}^{\#}g\alpha-C(\tilde{X}, x_e)$. Now $\tilde{X}$ is locally $S_{\#}S_{emi}^{\#}g\alpha$ -connected. Therefore, it is locally $S_{\#}S_{emi}^{\#}g\alpha$ -connected at each soft point $x_e \in \tilde{X}$. This implies that each $S_{\#}S_{emi}^{\#}g\alpha$ -open set

containing x_e contains a $S_{\#}S_{emi}^{\#}g\alpha$ -connected open set P_E containing the soft point x_e . Since the $S_{\#}S_{emi}^{\#}g\alpha$ -component $S_{\#}S_{emi}^{\#}g\alpha-C(\tilde{X}, x_e)$ is the maximal $S_{\#}S_{emi}^{\#}g\alpha$ -connected set containing x_e , $x_e \in P_E \subseteq S_{\#}S_{emi}^{\#}g\alpha-C(\tilde{X}, x_e)$, where P_E is $S_{\#}S_{emi}^{\#}g\alpha$ -open set. Therefore, $S_{\#}S_{emi}^{\#}g\alpha-C(\tilde{X}, x_e) = \bigcup \{P_E : x_e \in P_E \subseteq S_{\#}S_{emi}^{\#}g\alpha-C(\tilde{X}, x_e)\}$. Thus, $S_{\#}S_{emi}^{\#}g\alpha-C(\tilde{X}, x_e)$ is the soft union of all $S_{\#}S_{emi}^{\#}g\alpha$ -open sets containing x_e . Therefore, $S_{\#}S_{emi}^{\#}g\alpha-C(\tilde{X}, x_e)$ is $S_{\#}S_{emi}^{\#}g\alpha$ -open set.

Theorem 7.4. A $S_{\#}$ -Top-Space $(\tilde{X}, \tilde{T}, E)$ is locally $S_{\#}S_{emi}^{\#}g\alpha$ -connected if and only if the $S_{\#}S_{emi}^{\#}g\alpha$ -components of $S_{\#}S_{emi}^{\#}g\alpha$ -open sets are soft open.

Proof. Suppose that $(\tilde{X}, \tilde{T}, E)$ is locally $S_{\#}S_{emi}^{\#}g\alpha$ -connected. Let P_E be a $S_{\#}S_{emi}^{\#}g\alpha$ -open set and F_E be a $S_{\#}S_{emi}^{\#}g\alpha$ -component of P_E . If $x_e \in F_E$, then $x_e \in P_E$. Since $(\tilde{X}, \tilde{T}, E)$ is locally $S_{\#}S_{emi}^{\#}g\alpha$ -connected space, there is a $S_{\#}S_{emi}^{\#}g\alpha$ -connected open set O_E such that $x_e \in O_E \subseteq P_E$. Since F_E is the $S_{\#}S_{emi}^{\#}g\alpha$ -component of x_e and O_E is $S_{\#}S_{emi}^{\#}g\alpha$ -connected, we have $x_e \in O_E \subseteq F_E$. This shows that F_E is soft open.

Conversely, let $x_e \in \tilde{X}$ be arbitrary and let P_E be a $S_{\#}S_{emi}^{\#}g\alpha$ -open set containing x_e . Let F_E be the $S_{\#}S_{emi}^{\#}g\alpha$ -component of P_E such that $x_e \in F_E$. Now F_E is a $S_{\#}S_{emi}^{\#}g\alpha$ -connected open set such that $x_e \in F_E \subseteq P_E$.

Theorem 7.5. A $S_{\#}$ -Top-Space $(\tilde{X}, \tilde{T}, E)$ is locally $S_{\#}S_{emi}^{\#}g\alpha$ -connected if and only if, given any soft point $x_e \in \tilde{X}$ and a $S_{\#}S_{emi}^{\#}g\alpha$ -open set P_E containing x_e , there is a soft open set O_E containing x_e such that O_E is contained in a single $S_{\#}S_{emi}^{\#}g\alpha$ -component of P_E .

Proof. Let $\tilde{X}$ be locally $S_{\#}S_{emi}^{\#}g\alpha$ -connected, $x_e \in \tilde{X}$ and P_E be a $S_{\#}S_{emi}^{\#}g\alpha$ -open set containing x_e . Let F_E be a $S_{\#}S_{emi}^{\#}g\alpha$ -component of P_E that contains x_e . Since $\tilde{X}$ is locally $S_{\#}S_{emi}^{\#}g\alpha$ -connected and P_E is $S_{\#}S_{emi}^{\#}g\alpha$ -open, there is a $S_{\#}S_{emi}^{\#}g\alpha$ -connected open set O_E such that $x_e \in O_E \subseteq P_E$. By Remark 6.3, F_E is the maximal $S_{\#}S_{emi}^{\#}g\alpha$ -connected set containing x_e and so $x_e \in O_E \subseteq F_E \subseteq P_E$. Since $S_{\#}S_{emi}^{\#}g\alpha$ -components are disjoint soft sets, it follows that O_E is not contained in any other soft $S_{\#}S_{emi}^{\#}g\alpha$ -component of P_E . Conversely, we suppose that given any soft point $x_e \in \tilde{X}$ and any $S_{\#}S_{emi}^{\#}g\alpha$ -open set P_E containing x_e , there is a soft open set O_E containing x_e which is contained in a soft singleton $S_{\#}S_{emi}^{\#}g\alpha$ -component F_E of P_E . Then, $x_e \in O_E \subseteq F_E \subseteq P_E$. Let $y_\beta \in F_E$, then $y_\beta \in P_E$. Thus, there is a soft open set O_A such that $y_\beta \in O_A$ and O_A is contained in a single $S_{\#}S_{emi}^{\#}g\alpha$ -component of P_E . Since the $S_{\#}S_{emi}^{\#}g\alpha$ -components are disjoint soft sets and $y_\beta \in F_E$, $y_\beta \in G_E \subseteq F_E$. Thus F_E is soft open. Thus, for every $x_e \in \tilde{X}$ and for every $S_{\#}S_{emi}^{\#}g\alpha$ -open set P_E containing x_e , there is a $S_{\#}S_{emi}^{\#}g\alpha$ -connected open set F_E such that $x_e \in F_E \subseteq P_E$. Thus $(\tilde{X}, \tilde{T}, E)$

is locally $S_{\#}S_{emi}^{\#}g\alpha$ -connected at x_e . Since $x_e \in \tilde{X}$ is arbitrary, $(\tilde{X}, \tilde{T}, E)$ is locally $S_{\#}S_{emi}^{\#}g\alpha$ -connected space.

VIII. Totally Soft $S_{emi}^{\#}g\alpha$ -D iscon n e c t e d Space

In this section, we introduce the concept of totally $S_{\#}S_{emi}^{\#}g\alpha$ -disconnected space in a $S_{\#}$ -Top -Space and discuss its properties and characterizations.

Definition 8.1. $S_{\#}S_{emi}^{\#}g\alpha$ -disconnected means $\tilde{X}$ is the soft union of two nonempty disjoint $S_{\#}S_{emi}^{\#}g\alpha$ -open sets or $S_{\#}S_{emi}^{\#}g\alpha$ -closed sets in $\tilde{X}$.

Definition 8.2. A $S_{\#}$ -Top -Space $(\tilde{X}, \tilde{T}, E)$ is said to be totally $S_{\#}S_{emi}^{\#}g\alpha$ -disconnected space if and only if for each pair of distinct soft points $x_{\alpha}, y_{\beta} \in \tilde{X}$, there exists a $S_{\#}S_{emi}^{\#}g\alpha$ -disconnection $F_E \cup G_E$ of $\tilde{X}$ such that $x_{\alpha} \in F_E$ and $y_{\beta} \in G_E$, where F_E, G_E are both either $S_{\#}S_{emi}^{\#}g\alpha$ -connected open sets or $S_{\#}S_{emi}^{\#}g\alpha$ -connected closed sets.

Example 8.3. Every discrete $S_{\#}$ -Top -Space $\tilde{X}$ is totally $S_{\#}S_{emi}^{\#}g\alpha$ -disconnected space. Indeed, let $x_{\alpha}, y_{\beta} \in \tilde{X}$ and $x_{\alpha} \neq y_{\beta}$. Then $\{x_{\alpha}\} \cup (\tilde{X} - \{x_{\alpha}\})$ is $S_{\#}S_{emi}^{\#}g\alpha$ -disconnection of $\tilde{X}$.

Proposition 8.4. A soft subspace of a totally $S_{\#}S_{emi}^{\#}g\alpha$ -disconnected space is totally $S_{\#}S_{emi}^{\#}g\alpha$ -disconnected space. Each totally soft disconnected space is a totally $S_{\#}S_{emi}^{\#}g\alpha$ -disconnected space. The $S_{\#}S_{emi}^{\#}g\alpha$ -components of a totally $S_{\#}S_{emi}^{\#}g\alpha$ -disconnected space $(\tilde{X}, \tilde{T}, E)$ are soft singleton subsets of $\tilde{X}$.

Proof. Let $(\tilde{X}, \tilde{T}, E)$ be a totally $S_{\#}S_{emi}^{\#}g\alpha$ -disconnected space and $\tilde{Y}$ be a soft subset of $\tilde{X}$ consisting of two or more than two soft points. Let x_{α} and y_{β} be two distinct soft points of $\tilde{Y}$. Then x_{α} and y_{β} are also distinct soft points of $\tilde{X}$. Now $(\tilde{X}, \tilde{T}, E)$ is totally $S_{\#}S_{emi}^{\#}g\alpha$ -disconnected. Therefore, there exists a $S_{\#}S_{emi}^{\#}g\alpha$ -disconnection $F_A \cup G_B$ of $\tilde{X}$ such that $x_{\alpha} \in F_A$ and $y_{\beta} \in G_B$. Let $P_C = F_A \cap \tilde{Y}$ and $Q_D = G_B \cap \tilde{Y}$. We claim that $P_C \cup Q_D$ is a $S_{\#}S_{emi}^{\#}g\alpha$ -disconnection of $\tilde{Y}$. For, since $x_{\alpha} \in \tilde{Y}$, $x_{\alpha} \in F_A$, $y_{\beta} \in \tilde{Y}$, $y_{\beta} \in G_B$. Both P_C and Q_D are non-empty. Also, $P_C \cap Q_D = (F_A \cap \tilde{Y}) \cap (G_B \cap \tilde{Y}) = (F_A \cap G_B) \cap \tilde{Y} = \emptyset \cap \tilde{Y} = \emptyset$. That is P_C and Q_D are soft disjoint. Thus, $\tilde{Y}$ is $S_{\#}S_{emi}^{\#}g\alpha$ -disconnected and hence $\tilde{Y}$ a soft subset containing two or more than two soft points cannot be a $S_{\#}S_{emi}^{\#}g\alpha$ -component of $\tilde{X}$. Therefore, the $S_{\#}S_{emi}^{\#}g\alpha$ -components of $\tilde{X}$ are soft singleton subsets of $\tilde{X}$ which are the only $S_{\#}S_{emi}^{\#}g\alpha$ -connected subsets of $\tilde{X}$.

Conclusions

Connectedness is an important and major area of topology, and it can give many relationships between other scientific areas and mathematical models. The notion of connectedness captures the idea of the hanging-togetherness of image elements in an object by giving a firmness of connectedness to every feasible path

between every possible pair of image elements. It is an essential tool for designing algorithms for image segmentation. We have used the notions of $S_{\#}S_{emi}^{\#}\alpha$ -open sets and $S_{\#}S_{emi}^{\#}\alpha$ -closed sets to introduce $S_{\#}S_{emi}^{\#}\alpha$ -separated sets, $S_{\#}S_{emi}^{\#}\alpha$ -connected space, $S_{\#}S_{emi}^{\#}\alpha$ -disconnected space, $S_{\#}S_{emi}^{\#}\alpha$ -component of a soft set and soft topological space, locally $S_{\#}S_{emi}^{\#}\alpha$ -connected space and totally $S_{\#}S_{emi}^{\#}\alpha$ -disconnected space in soft topological spaces. We have investigated properties and characterizations of these new spaces in soft topological spaces. We hope that the concepts initiated herein will be beneficial for researchers and scholars to promote and progress the study of soft topology and decision-making problems with applications in many fields soon.

Recommendations

In the future, it is recommended to utilize soft semi $\#$ generalized α -open sets to initiate new kinds of soft semi $\#$ generalized α -compactness, namely soft semi $\#$ generalized α -Lindelofness, almost (approximately, mildly) soft semi $\#$ generalized α -compactness, and almost (approximately, mildly) soft semi $\#$ generalized α -Lindelofness. The equivalent conditions of each one of them can be investigated. Also, the behavior of these spaces under soft semi $\#$ generalized α -irresolute maps can be investigated. Furthermore, the enough conditions for the equivalence among the four sorts of soft semi $\#$ generalized α -compact spaces and for the equivalence among the four sorts of soft semi $\#$ generalized α -Lindelof spaces can be explored. The relationships between enriched soft topological spaces and the initiated spaces can be investigated in different cases. Some properties which connect some of these spaces with some soft topological notions, such as soft semi $\#$ generalized α - T_2 -spaces and soft subspaces, can be obtained.

Scientific Ethics Declaration

* The author declares that the scientific ethical, and legal responsibility of this article published in EPSTEM journal belongs to the author.

Conflict of Interest

* The authors declare that they have no conflicts of interest

Funding

* The author is highly and greatly indebted to Prince Mohammad bin Fahd University Al Khobar, Saudi Arabia, for providing all necessary research facilities during the preparation of this research paper.

Acknowledgements or Notes

*This article was presented as an oral presentation at the International Conference on Basic Sciences, Engineering and Technology (www.icbaset.net) held in Trabzon/Türkiye on May 01-04, 2025.

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To cite this article:

Latif, R. M. (2025), Soft semi $\#$ generalized α -connectedness in soft topological spaces. *The Eurasia Proceedings of Science, Technology, Engineering and Mathematics (EPSTEM)*, 34, 120-138.